Benchmark

Implementations for option prices can be evaluated using a benchmark. The benchmark itself was published in [3] and is available online[1]. It consists of three parts. One contains the parameters for the Heston model, the second the option parameters and the third the (approximated) results.

To clarify the meaning of the parameters the Heston SDE is restated here.

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^S \tag{1}$$

$$dV_t = \kappa\left(\theta - V_t\right) dt + \sigma\sqrt{V_t} dW_t^V \tag{2}$$

We first propose a set of different calibration parameters for the Heston SDE (see Equation 1 and 2) with starting condition $S(0) = 100$ and $V(0) = V_0$. Note that setting $S(0) = 100$ does not reduce the generality of the proposed problems. The asset simulation can be adjusted to fit $S(0) = 100$. A similar adjustment is not possible for the volatility process. The details of the parameters can be found in Table 1.

	κ	θ	σ	r	V_0	ρ	T
I	2.75	0.035	0.425	0	0.0384	−0.4644	1
II	2	0.09	1	0.05	0.09	−0.3	5
III	0.5	0.04	1	0	0.04	0	1
IV	1	0.09	1	0	0.09	−0.3	5
V	0.5	0.04	1	0.08	0.04	−0.9	10
VI	2.75	0.35	0.425	0	0.384	−0.4644	1

Table 1: The Test Parameters

The parameters characterize the behavior of the asset price and the volatility process. The parameter T already belongs to the option, but is also important for the discretization of the two processes and therefore is provided here.

The test cases can be found in the available literature. Parameter set I is from [7] and is a calibration to the S&P500 index. Sets II and III are taken from [6] and IV and V are due to [1] with an additional value for r. The set VI is a modified version of I and has a higher long term average and starting volatility to account for a more extreme case. These cases span a wide range of parameters observable on the markets.

The Feller condition – $\sigma^2 < 2\kappa\theta$ – guarantees that the volatility process does not hit zero. For the parameters of set I the condition does hold. In all other cases the Feller condition is not satisfied and therefore the volatility can reach zero. Nevertheless, the process is always non-negative. This reflects the observation that the Feller condition is seldom satisfied in real-world applications [2].

In case III the riskless interest rate r and the correlation of the two driving Brownian motions ρ is zero. For this special case, there is a closed form solution for the price

[1] www.uni-kl.de/benchmarking

of double barrier options available due to Lipton [5], however, this method cannot be generalized [4].

Observations of the financial mathematics group indicate the mean reversion level θ is between 0.05 and 0.4 with a similar initial variance, the mean reversion speed κ being between 0.5 and 3. The volatility of volatility σ is between 0.2 and 0.7. The correlation is strongly negative and mostly between -0.9 and -0.5. However, this also depends on the market and the observed time. For this benchmark, most of the parameters are taken from the literature to have more continuity, acceptance of the parameters and including other opinions of the relevant parameters.

The parameters for the Heston model are only one part of the benchmark. Next we need options on these parameter sets. Table 2 illustrates the used options. The focus is on no-touch or knock-out double barrier options. A few abbreviations are used. ATM stands for *at-the-money* and means that the strike price is the same as the current asset price. In our case we fixed the current asset price to be 100. ITM stands for *in-the-money* and indicates that the option would provide a positive payoff if it could be exercised now. Thus, for a call option ITM means the current asset value is higher than the strike price. For a put option it has to be lower. OTM stands for *out-of-the-money* and indicates that the current asset value is below the strike price in case of a call and above in case of a put option.

All considered options are European options. Thus, they can only be exercised at maturity time. Option 1 is a plain vanilla call and thus can be calculated using a (semi-)closed form solution. Options 2 and 3 are single barrier options and options $4-10$ are double barrier knock-out options. These are the main focus of the benchmark and this type of derivative is presented with different strikes, barriers and Heston model parameters. Option 11 is a kick-in or knock-in single barrier option. Finally option 12 is a double barrier knock-out digital call. Thus, it pays a fixed amount of money (here 1) if the barrier is not hit and the asset price at maturity is above the strike price. The payoff of this option is highly discontinuous and can therefore be difficult to price.

In order to be able to use the benchmark set, it is important to have exact (or very good approximations thereof) results for the different prices. In some cases (semi-)closed form solutions could be used, in others a Monte Carlo simulation with a fine discretization and a high number of simulations to provide a good precision. In Table 3 these numbers are provided. The word precision is to be understood in the sense of root MSE in the Monte Carlo setting.

References

[1] Leif Andersen. Efficient Simulation of the Heston Stochastic Volatility Model. *SSRN eLibrary*, 2007.

[2] Iain J. Clark. *Foreign Exchange Option Pricing: A Practitioners Guide.* The Wiley Finance Series, 2011.

[3] Christian de Schryver, Matthias Jung, Norbert Wehn, Henning Marxen, Anton

Number	Parameters from Table 1	Description of the Details
1	II	ATM European Call
2	II	ATM single barrier call with barrier 120
3	IV	ATM single barrier call with barrier 120
4	III	ATM double barrier call with barrier 90 and 110
5	I	ITM double barrier call with barrier 80 and 120, strike 90
6	IV	ATM double barrier call with barrier 66 and 150
7	V	ITM double barrier call with barrier 66 and 150, strike 90
8	VI	ATM double barrier call with barrier 66 and 150
9	I	ATM double barrier put with barrier 80 and 120
10	VI	OTM double barrier call with barrier 66 and 150, strike 120
11	I	ATM single barrier kick-in call option with barrier 120
12	IV	ATM double barrier digital call with barrier 66 and 150

Table 2: Table of Options

number	1	2	3	4	5	6
price	34.9998	0.10280	0.31606	0.74870	5.7531	3.0421
obtained by	closed form	MC	MC	closed form	MC	MC
precision		0.0001	0.0003		0.001	0.005
number	7	8	9	10	11	12
price	0.017117	0.82226	1.5294	0.17167	4.9783	0.16805
obtained by	MC	MC	MC	MC	MC	MC
precision	0.0002	0.0005	0.0005	0.0005	0.0005	0.0001

Table 3: Price Results for the Benchmark Set

Kostiuk, and Ralf Korn. Energy Efficient Acceleration and Evaluation of Financial Computations Towards Real-Time Pricing. In Andreas Knig, Andreas Dengel, Knut Hinkelmann, Koichi Kise, Robert J. Howlett, and Lakhmi C. Jain, editors, *Knowledge-Based and Intelligent Information and Engineering Systems*, volume 6884 of *Lecture Notes in Computer Science*, pages 177–186. Springer, September 2011. Proceedings of 15th International Conference on Knowledge-Based and Intelligent Information & Engineering Systems (KES).

[4] Oliver Faulhaber. Analytic Methods for Pricing Double Barrier Options in the Presence of Stochastic Volatility. Diploma thesis, University of Kaiserslautern, 2002.

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[7] J. E. Zhang and Jinghong Shu. Pricing s&p 500 index options with heston's model. In *Proc. IEEE Int Computational Intelligence for Financial Engineering Conf*, pages 85–92.