MA101, Real Analysis
Series and Power Series

1. Discuss the convergence and divergence of the following series:

   (a) $\sum\left[\sqrt{n+1}-\sqrt{n}\right]$.

   (b) $\sum(1/2)^n\left(50+\frac{2}{n}\right)$.

   (c) $\sum\frac{100^n}{n!}$.

   (d) $\sum\frac{1}{\sqrt{n!}}$.

   (e) $\sum\frac{n^3}{3^n}$.

   (f) $\sum\frac{(-1)^n n!}{2^n}$.

   (g) $\sum_2^\infty\frac{1}{\sqrt{n}\log n}$.

   (h) $\sum_2^\infty\frac{\log n}{n}$.

   (i) $\sum_4^\infty\frac{1}{n\log n\log\log n}$.

   (j) $\sum_2^\infty\frac{\log n}{n^2}$.

   (k) $\sum_2^\infty\frac{1}{n(\log n)^p}$.

   (l) $\sum_2^\infty(-1)^n\left(\frac{\ln n}{(\log n)^2}\right)^n$.

   (m) $\sum_1^\infty(-1)^n\text{sech } n$.

   (n) $\sum_2^\infty(-1)^n\left(\frac{\tan^{-1}n}{1+n^2}\right)$.

2. Let $\sum_{n=1}^\infty a_n$ be a convergent series of positive terms. What can be said about the convergence of
$$\sum_{n=1}^\infty\frac{a_1+a_2+\cdots+a_n}{n}.$$

3. Let p_n be a sequence of all consecutive prime numbers. Study convergence of $\sum_{n=1}^\infty\frac{1}{p_n}$.

4. Decide whether the series
$$\sum_{n=1}^\infty\frac{(-1)^{[\ln n]}}{n}$$
is absolutely convergent, conditional convergent or divergent.

5. For a sequence $\{a_n\}$ tends to zero and for a,b,c such that $a+b+c\neq 0$, prove that the series $\sum_{n=1}^\infty a_n$ and $\sum_{n=1}^\infty(aa_n+ba_{n+1}+ca_{n+2})$ either both converge or both diverge.

6. Apply Dirichlet's test and study the convergence of the series where $a\in\mathbb{R}$

   (a)
$$\sum_{n=1}^\infty\frac{\sin(na)\sin(n^2a)}{n},$$

   (b)
$$\sum_{n=1}^\infty\frac{\sin(na)\cos(n^2a)}{n}.$$

7. In the following exercises (a) find the series' radius and interval of convergence. For what values of x does the series converge (b) absolutely, (c) conditionally?

   (a) $\sum_0^\infty(x+5)^n$.

   (b) $\sum_0^\infty(-2)^n(n+1)(x-1)^n$.

   (c) $\sum_2^\infty\frac{x^n}{n\ln n}$.

   (d) $\sum_1^\infty\frac{(4x-5)^{2n+1}}{n^{3/2}}$.

   (e) $\sum_1^\infty\frac{\left(x-\sqrt{2}\right)^{2n+1}}{2^n}$.

8. If $\sum a_n$ with $a_n>0$ is convergent, then is $\sum(a_n)^2$ always convergent. Either prove it or give a counter example.

9. If $\sum a_n$ with $a_n>0$ is convergent, then is $\sqrt{\sum a_n}$ always convergent. Either prove it or give a counter example.

10. If $\sum a_n$ with $a_n>0$ is convergent, then is $\sum\sqrt{a_na_{n+1}}$ always convergent. Either prove it or give a counter example.

11. If $\sum a_n$ with $a_n>0$ is convergent, and if
$$b_n:=\frac{(a_1+a_2+\cdots+a_n)}{n}$$
for $n\in\mathbb{N}$, then show that $\sum b_n$ is always divergent.

12. Use Series and Plot command in Mathematica and Plot function and corresponding series and study the convergence and how accuracy increases with increase in the number of terms in series.