EE101 Tutorial 12

Topics: **RC, RL and RLC Circuits**

1.

Chapter 7, Problem 7.

Assuming that the switch in Fig. 7.87 has been in position *A* for a long time and is moved to position *B* at $t = 0$, find $v_0(t)$ for $t \geq 0$.

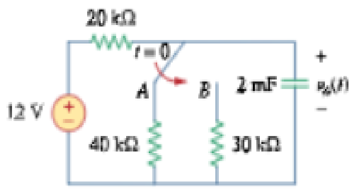


Figure 7.87
For Prob. 7.7.

Chapter 7, Solution 7.

When the switch is at position A, the circuit reaches steady state. By voltage division,

$$v_o(0) = \frac{40}{40+20}(12V) = 8V$$

When the switch is at position B, the circuit reaches steady state. By voltage division,

$$v_o(\infty) = \frac{30}{30+20}(12V) = 7.2V$$

$$R_{Th} = 20k // 30k = \frac{20x30}{50} = 12k\Omega$$

$$\tau = R_{Th}C = 12x10^3 x2x10^{-3} = 24s$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} = 7.2 + (8-7.2)e^{-t/24} = \underline{7.2 + 0.8e^{-t/24} \ \text{V}}$$

2.

Chapter 7, Problem 10.

For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

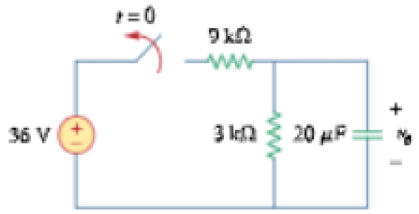


Figure 7.90
For Prob. 7.10.

Chapter 7, Solution 10.

For t<0, $$v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ V}$$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$$v_o(t) = \underline{\mathbf{9e^{-16.667t}\ V}}$$

Let the time be t_o.

$3 = 9e^{-16.667to}$ or $e^{16.667to} = 9/3 = 3$

$$t_o = \ln(3)/16.667 = \underline{\mathbf{65.92\ ms}}.$$

3.

Chapter 7, Problem 11.

For the circuit in Fig. 7.91, find i_o for $t > 0$.

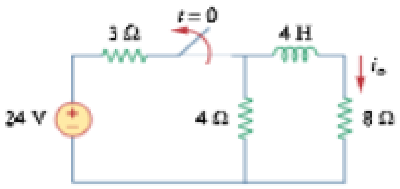
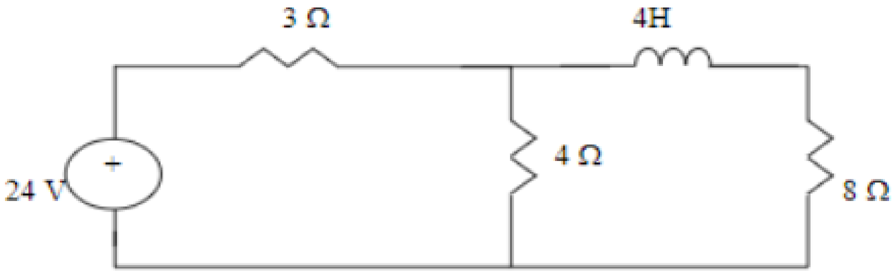


Figure 7.91
For Prob. 7.11.

Chapter 7, Solution 11.

For t<0, we have the circuit shown below.



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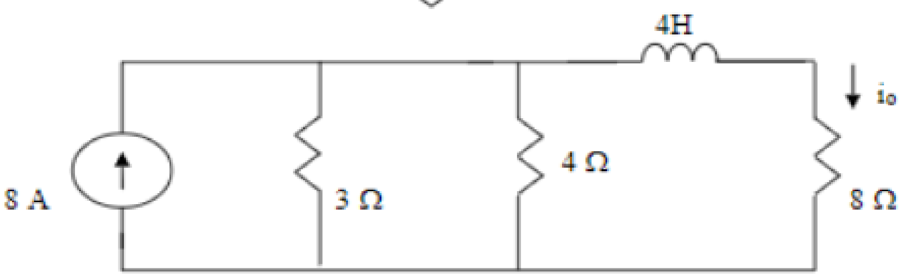


3//4= 4x3/7=1.7143

$$i_o(0^-) = \frac{1.7143}{1.7143+8}(8) = 1.4118 \text{ A}$$

For t >0, we have a source-free RL circuit.

$$\tau = \frac{L}{R} = \frac{4}{4+8} = 1/3$$

$$i_o(t) = i_o(0)e^{-t/\tau} = \underline{1.4118e^{-3t} \text{ A}}$$

4.

Chapter 7, Problem 13.

In the circuit of Fig. 7.93,

$v(t) = 20e^{-10^3 t}$ V, $\quad t > 0$

$i(t) = 4e^{-10^3 t}$ mA, $\quad t > 0$

(a) Find R, L, and τ.
(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

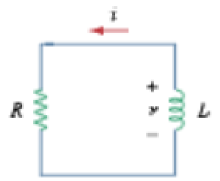


Figure 7.93
For Prob. 7.13.

Chapter 7, Solution 13.

(a) $\tau = \dfrac{1}{10^3} = \underline{1ms}$

$v = iR \longrightarrow 20e^{-1000t} = Rx4e^{-1000t}x10^{-3}$
From this, R = 20/4 kΩ= $\underline{5\ k\Omega}$

But $\tau = \dfrac{L}{R} = 1/10^3 \qquad \longrightarrow \qquad L = \dfrac{5x1000}{1000} = \underline{5H}$

(b) The energy dissipated in the resistor is

$$w = \int_0^t p\,dt = \int_0^t 80x10^{-3}e^{-2x10^3}\,dt = -\frac{80x10^{-3}}{2x10^3}e^{-2x10^3 t}\Bigg|_0^{0.5x10^{-3}}$$

$$= 40(1-e^{-1})\mu J = \underline{25.28\mu J}$$

5.

Chapter 7, Problem 16.

Determine the time constant for each of the circuits in Fig. 7.96.

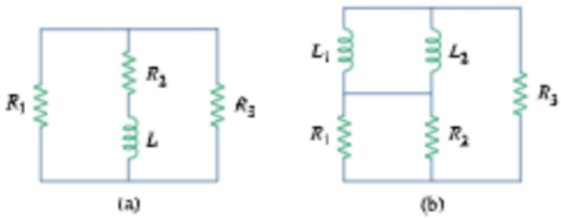


Figure 7.96
For Prob. 7.16.

Chapter 7, Solution 16.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \frac{R_1R_3}{R_1+R_3} = \frac{R_2(R_1+R_3)+R_1R_3}{R_1+R_3}$

$$\underline{\tau = \frac{\mathbf{L(R_1+R_3)}}{\mathbf{R_2(R_1+R_3)+R_1R_3}}}$$

(b) where $L_{eq} = \frac{L_1L_2}{L_1+L_2}$ and $R_{eq} = R_3 + \frac{R_1R_2}{R_1+R_2} = \frac{R_3(R_1+R_2)+R_1R_2}{R_1+R_2}$

$$\underline{\tau = \frac{\mathbf{L_1L_2(R_1+R_2)}}{\mathbf{(L_1+L_2)(R_3(R_1+R_2)+R_1R_2)}}}$$

6.

Chapter 7, Problem 19.

In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 2$ A.

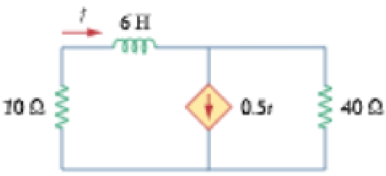
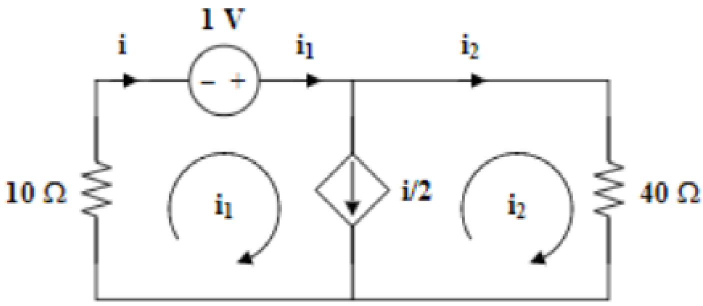


Figure 7.99
For Prob. 7.19.

Chapter 7, Solution 19.



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$$10i_1 - 1 + 40i_2 = 0$$

But $\quad i = i_2 + i/2 \quad$ and $\quad i = i_1$

i.e. $\quad i_1 = 2i_2 = i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2\text{ s}$$

$$i(t) = \underline{2e^{-5t}u(t)\text{ A}}$$

7.

Chapter 7, Solution 39.

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \underline{\mathbf{4\ V}}$$

After t = 0,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)e^{-t/8}$$

$$v(t) = \underline{20 - 16e^{-t/8}\ \text{V}}$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12\ \text{V}$$

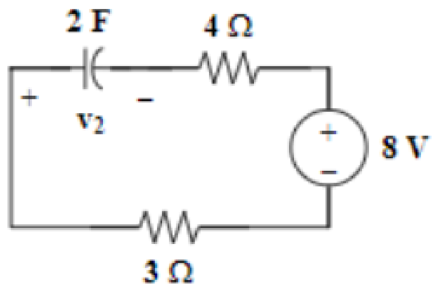
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = -8\ \text{V}$$

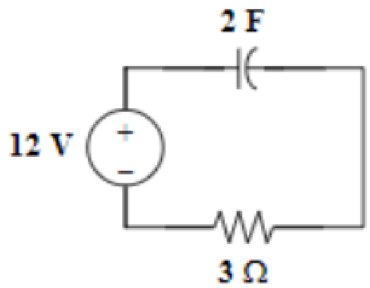
Thus,

$$v = 12 - 8 = \underline{\mathbf{4\ V}}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a)



(b)

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)e^{-t/6}$$

$$v(t) = \underline{\mathbf{12 - 8e^{-t/6}\ V}}$$

8.

Chapter 7, Problem 55.

Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

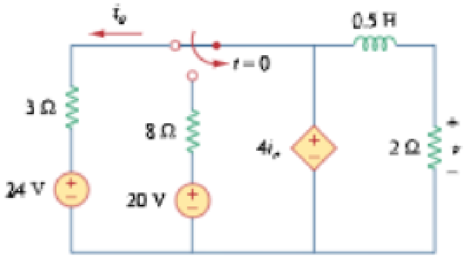
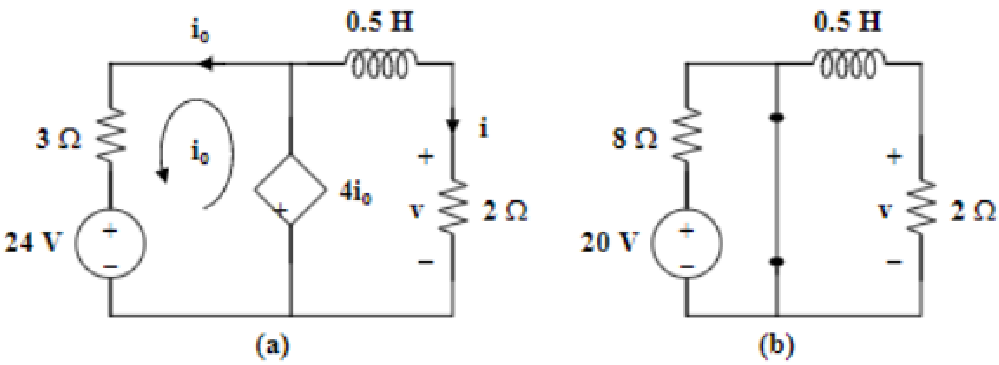


Figure 7.121
For Prob. 7.55.

Chapter 7, Solution 55.

For t < 0, consider the circuit shown in Fig. (a).



$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$\underline{v(t) = 4i_o = \mathbf{96\ V}} \qquad i = \frac{v}{2} = 48\text{ A}$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)e^{-4t}$$

$$v(t) = 2i(t) = \underline{96e^{-4t}\ u(t)\text{V}}$$

9.

Chapter 7, Problem 56.

For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

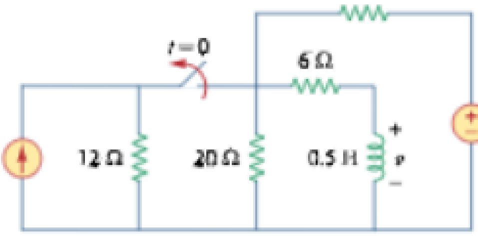


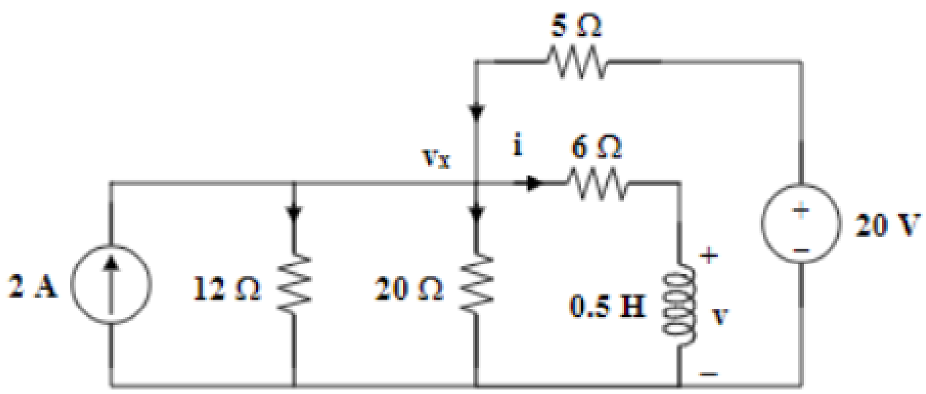
Figure 7.122
For Prob. 7.56.

Chapter 7, Solution 56.

$$R_{eq} = 6 + 20 \,\|\, 5 = 10\ \Omega, \qquad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2\ \text{A}$$

Since $20 \,\|\, 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$

$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

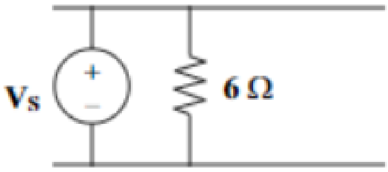
$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$

$$v(t) = \underline{\mathbf{-\,4e^{-20t}\ V}}$$

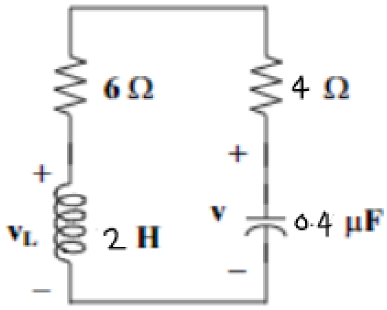
10.

Chapter 8, Solution 1.

(a) At $t = 0$-, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A, \ v(0-) = 12V$$

$$\text{At } t = 0+, \ i(0+) = i(0-) = \underline{\mathbf{2A}}, \ v(0+) = v(0-) = \underline{\mathbf{12V}}$$

(b) For $t > 0$, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \ \text{ or } \ di/dt = v_L/L$$

Applying KVL at $t = 0+$, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \ \text{ or } \ v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 = \underline{\mathbf{-4\ A/s}}$

Similarly, $i_C = Cdv/dt, \ \text{ or } \ dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

$$dv(0+)/dt = -2/0.4 = \underline{\mathbf{-5\ V/s}}$$

(c) As t approaches infinity, the circuit reaches steady state.

$$i(\infty) = \underline{\mathbf{0\ A}}, \ v(\infty) = \underline{\mathbf{0\ V}}$$

11.

Chapter 8, Problem 14.

The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Find $v(t)$ for $t > 0$

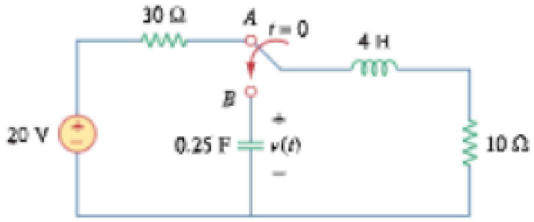


Figure 8.69
For Prob. 8.14.

Chapter 8, Solution 14.

When the switch is in position A, $v(0^-) = 0$ and $i_L(0) = \frac{20}{40} = 0.5A$. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.5625 - 1} = -0.5 \text{ and } -2$$

$$v(t) = Ae^{-2t} + Be^{-0.5t} \qquad (1)$$

$$v(0) = 0 = A + B \qquad (2)$$

$$i_C(0) = C\frac{dv(0)}{dt} = 0.5 \longrightarrow \frac{dv(0)}{dt} = \frac{0.5}{C} = 2$$

But $\frac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$

$$\frac{dv(0)}{dt} = -2A - 0.5B = 2 \qquad (3)$$

Solving (2) and (3) gives A= –1.3333 and B = 1.3333

$$v(t) = \underline{-1.3333e^{-2t} + 1.3333e^{-0.5t}\ V}.$$

12.

Chapter 8, Problem 24.

The switch in Fig. 8.77 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Determine $i(t)$ for $t > 0$

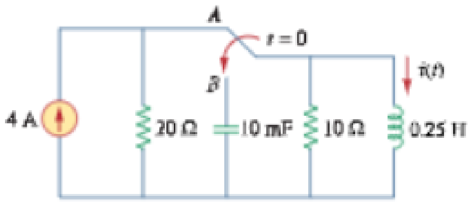


Figure 8.77
For Prob. 8.24.

Chapter 8, Solution 24.

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25 - 400} = -5 + j19.365$$

$$i(t) = e^{-5t}(A_1 \cos 19.365t + A_2 \sin 19.365t)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t)$$

$$0 = \frac{di(0)}{dt} = -5A_1 + 19.365A_2 \longrightarrow A_2 = \frac{5A_1}{19.365} = 1.033$$

$$i(t) = \underline{e^{-5t}(4\cos 19.365t + 1.033 \sin 19.365t)}$$