Potential energy experienced by an electron having mass m_e in the coulomb field of a proton inside the nucleus is written in spherical polar coordinate system as

$$V(r) = \left(-\frac{B}{r} + \frac{C}{r^2} \right),$$

where r is the radial distance from the nucleus, and B and C are positive constants.

(a) Plot the potential energy as a function of r. Indicate if any zero crossing, maxima, minima points are there (Mention Specifically the values of r at these locations in the plot)

(b) Comment if the force field derived from this potential energy is conservative or not. Justify your answer

(c) Find the force experienced by the electron if the force field is conservative.

(d) Find the work done to move the electron from the stable equilibrium (if any) to infinite distance. Justify your answer by connecting it with the value of binding energy (potential energy at the equilibrium).

(e) Consider electron as a classical particle (hypothetically) and obtain the frequency of oscillation about the equilibrium point.

(a) $$V(r) = \left(-\frac{B}{r} + \frac{C}{r^2} \right),$$

Zero crossing point is r=C/B

Maxima / minima is at: r=2C/B

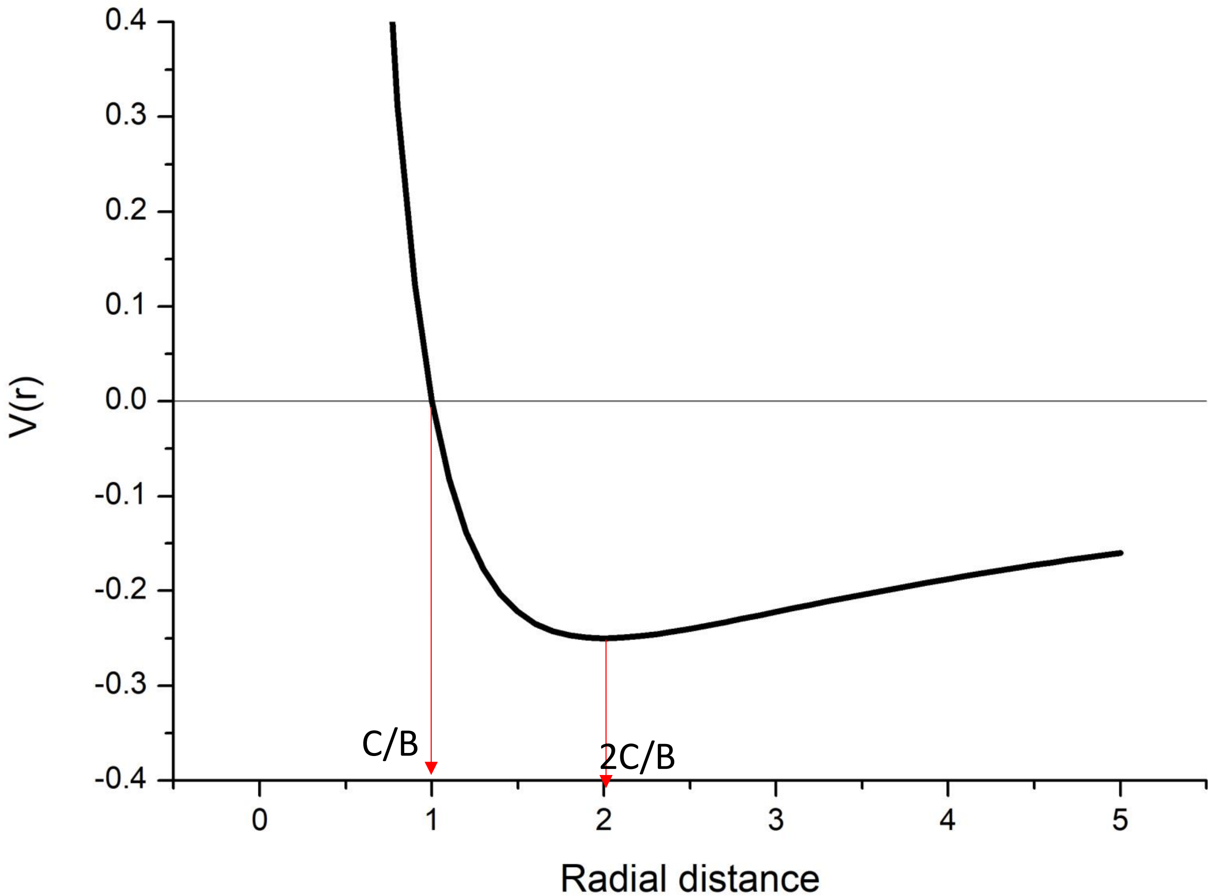
Find whether r=2C/B is a maximum / minimum by finding whether V''(r)<0 or >0

$$V''(r) = \frac{-2B}{r^3} + \frac{6C}{r^4}$$

Putting r=2C/B in the above equation, we get $$V''\left(\frac{2C}{B}\right) = \frac{B^4}{C^3}\left[\frac{1}{8}\right] > 0$$

R=2C/B is a minimum

-------------------- (1 Mark for finding the results on this slide)



-------------------- (1 Mark for plotting the graph with correct markings)

(b) The force field is conservative as the potential is of 'central' nature, hence conservative

-------------------- (2 Marks)

(c) The force field is :

$$\vec{F}(r) = \left[-\frac{B}{r^2} + \frac{2C}{r^3} \right] \hat{e}_r$$

-------------------- (2 Marks)

(d) Find the work done to move the electron from the stable equilibrium (if any) to infinite distance. Justify your answer by connecting it with the value of binding energy (potential energy at the equilibrium).

$$Work\ done = \int_{2C/B}^{\infty} \left[-\frac{B}{r^2} + \frac{2C}{r^3} \right] \hat{e}_r \bullet dr\hat{e}_r = -\frac{B^2}{4C}$$

The work done is same as that of the value of the potential energy at the equilibrium. This means that the work needed to displace an electron from equilibrium to infinity is the binding energy or the work function, like in the photoelectric effect. -------------------- (2 Marks)

(e) Consider electron as a classical particle (hypothetically) and obtain the frequency of oscillation about the equilibrium point.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(\left. \frac{d^2V}{dr^2} \right|_{r=2C/B} \right)}{m}}$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{B}{2C}\sqrt{\frac{1}{2Cm_e}}$$

--------------------- (2 Marks)