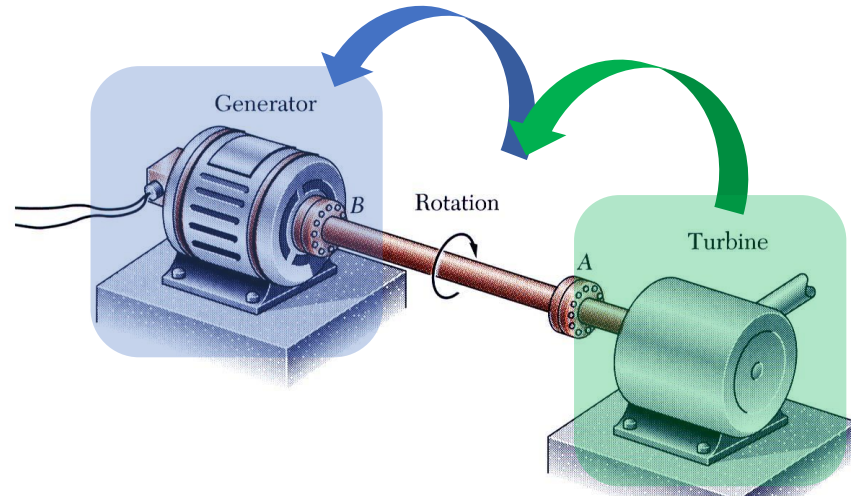


# **Engineering Mechanics (ME102)**

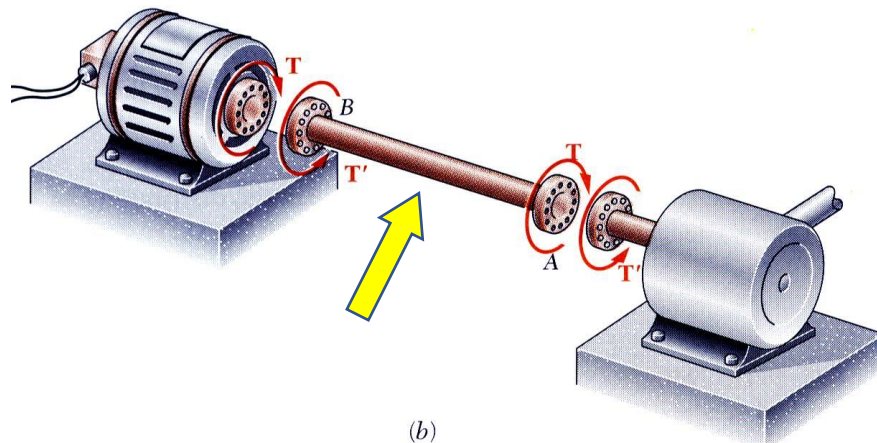
## **Torsion**

## Introduction

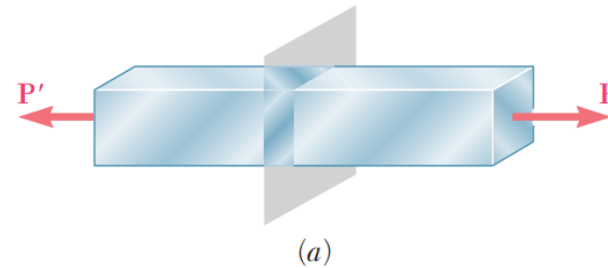
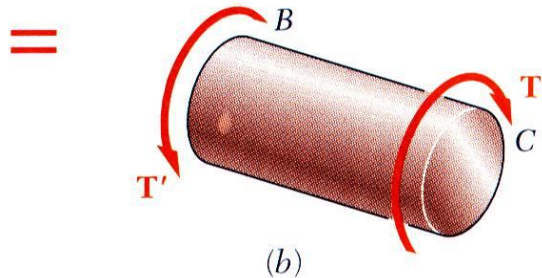
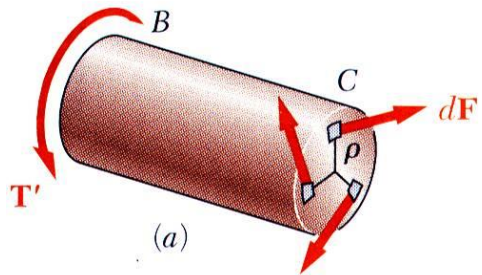
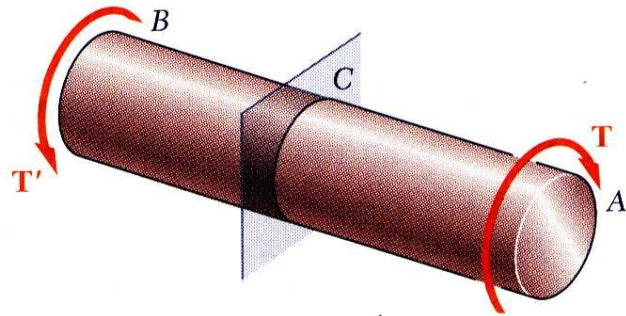
**Torsion:** Twisting of a structural member, when it is loaded by **couples (torques)** that produce rotation about a **longitudinal axis**.



- Turbine exerts torque  $T$  on the shaft
- Shaft transmits the torque to the generator
- Generator creates an equal and opposite torque  $T'$
- Let us consider the shaft in the next slide



## Introduction



- Net of the internal shearing stresses is an internal torque, equal and opposite to the applied torque,

$$T = \int \rho dF = \int \rho (\tau dA)$$

- Although the net torque due to the shearing stresses is known, the distribution of the stresses is not known yet.
- Distribution of shearing stresses is obtained by considering deformations of the shaft.
- Unlike the normal stress due to axial loads, the *distribution of shearing stresses due to torsional loads can not be assumed uniform.*

## State of Stress

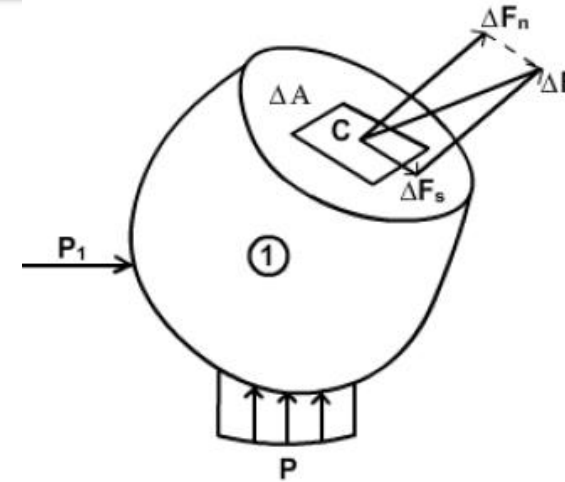
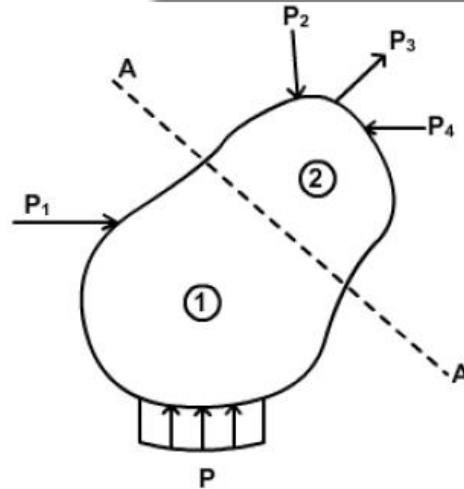
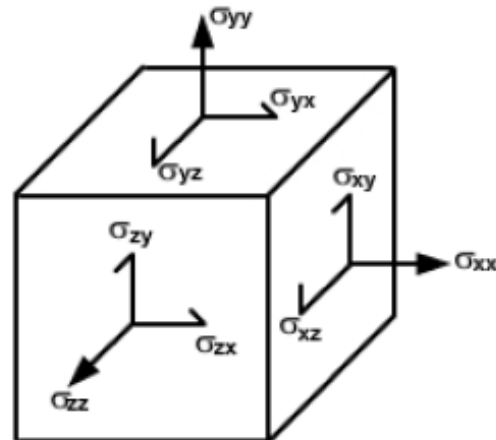


Image Courtesy: [http://nptel.ac.in/courses/IITMADRAS/Strength\\_of\\_Materials/Pdfs/2\\_1.pdf](http://nptel.ac.in/courses/IITMADRAS/Strength_of_Materials/Pdfs/2_1.pdf)

$$\sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

$$\sigma_n = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_n}{\Delta A}$$

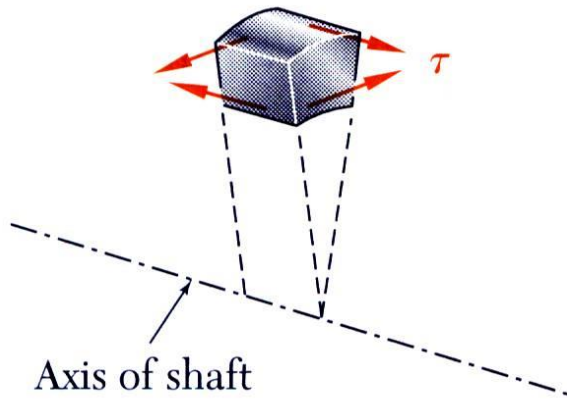
$$\sigma_s = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_s}{\Delta A}$$



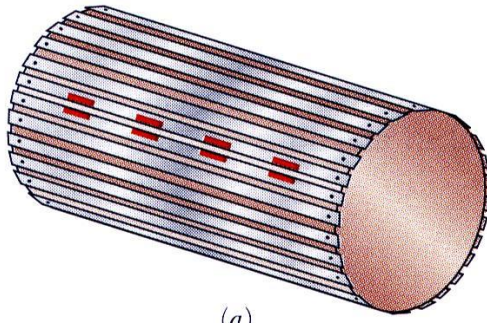
$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

**3D Stress Tensor**

## Axial Shear Components

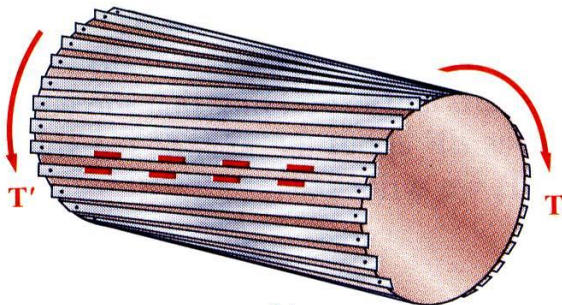


- Torque applied to shaft produces shearing stresses on the faces perpendicular to the axis.
- *Conditions of equilibrium require the existence of equal stresses on the faces of the two planes containing the axis of the shaft.*

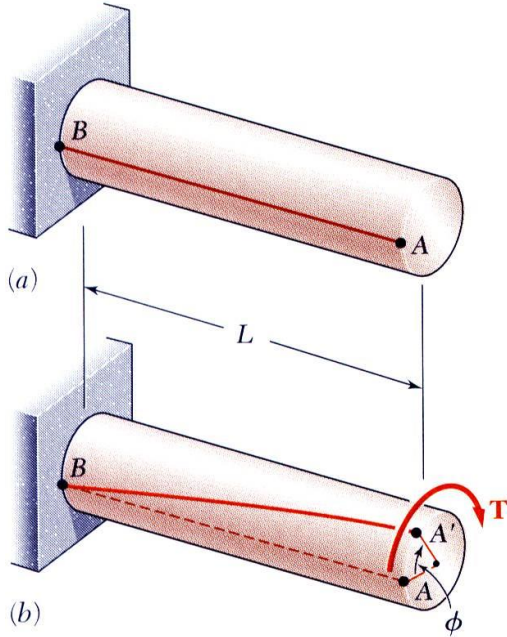


- The existence of the axial shear components is demonstrated by considering a shaft made up of axial slats.

The slats slide with respect to each other when equal and opposite torques are applied to the ends of the shaft.



## Deformation in a Circular Shaft

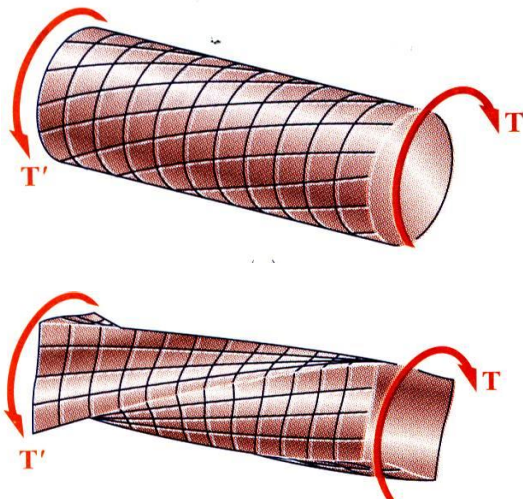


- Will see that angle of twist of shaft is proportional to applied torque and to shaft length.

$$\phi \propto T$$

$$\phi \propto L$$

- When subjected to torsion, every cross-section of circular (solid or hollow) shaft remains plane and undistorted. This is due to the axisymmetric of cross section.



- Cross-sections of noncircular (hence non-axisymmetric) shafts are distorted when subjected to torsion.



## Shearing Strain (Recap)

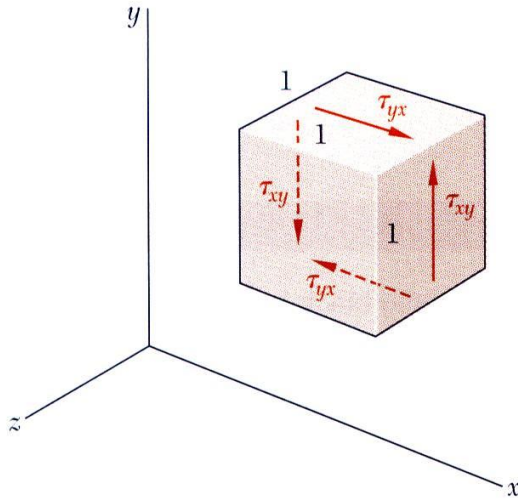


Fig. 2.46

- A cubic element subjected to shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

$$\tau_{xy} = f(\gamma_{xy})$$

- A plot of shear stress vs. shear strain is similar the plots of normal stress vs. normal strain except that the **strength values are approximately half**. For small strains,

$$\tau_{xy} = G \gamma_{xy} \quad \tau_{yz} = G \gamma_{yz} \quad \tau_{zx} = G \gamma_{zx}$$

where  $G$  is the modulus of rigidity or shear modulus.

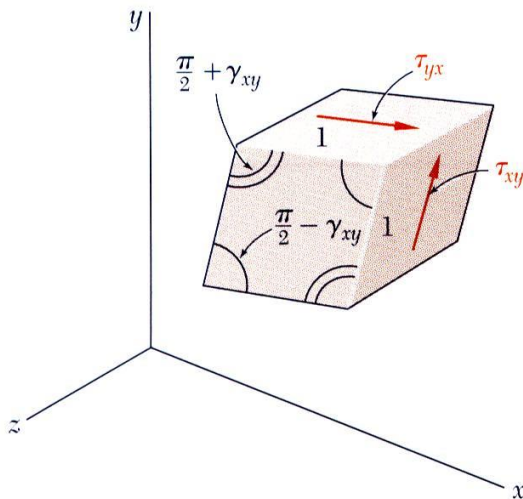
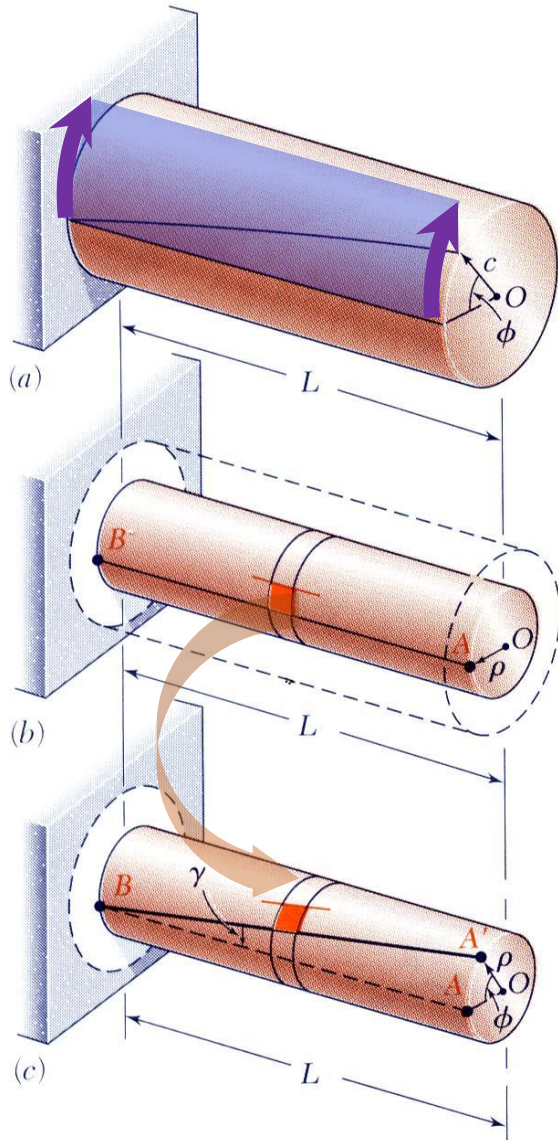


Fig. 2.47

## Shearing Strain



- Consider an interior section of the shaft. As a torsional load is applied, an element on the interior **cylinder** deforms into a **rhombus**.
- Since the ends of the element remain planar, the **shear strain**  $\gamma$  is proportional to **angle of twist**  $\phi$ .
- It follows that

$$AA' = L\gamma = \rho\phi \quad \text{or} \quad \gamma = \frac{\rho\phi}{L}$$

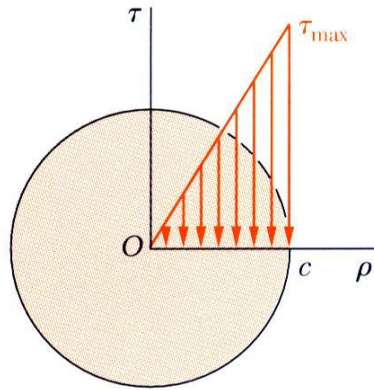
- Shear strain is proportional to twist and radius

$$\gamma_{\max} = \frac{c\phi}{L} \quad \text{and} \quad \gamma = \frac{\rho}{c}\gamma_{\max}$$

- Shear strain in a shaft varies linearly with the distance from the axis of the shaft



## Stresses in Elastic Range



$$J = \frac{1}{2} \pi c^4$$

- Multiplying the previous equation by the shear modulus,

$$G\gamma = \frac{\rho}{c} G\gamma_{\max}$$

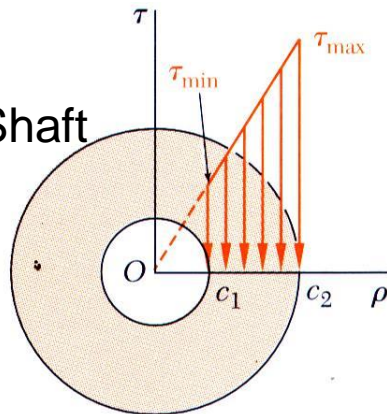
From Hooke's Law,  $\tau = G\gamma$ , so

$$\tau = \frac{\rho}{c} \tau_{\max}$$

- Recall that the sum of the moments from the internal stress distribution is equal to the torque on the shaft at the section,

$$T = \int \rho \tau dA = \frac{\tau_{\max}}{c} \int \rho^2 dA = \frac{\tau_{\max}}{c} J$$

Hollow Shaft

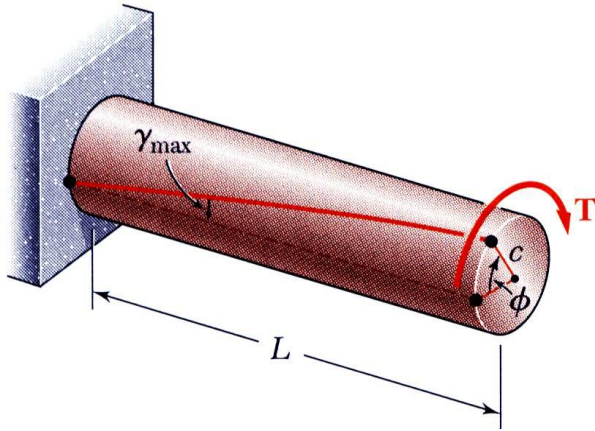


$$J = \frac{1}{2} \pi (c_2^4 - c_1^4)$$

- The results are known as the elastic torsion formulas,

$$\tau_{\max} = \frac{Tc}{J} \quad \text{and} \quad \tau = \frac{T\rho}{J}$$

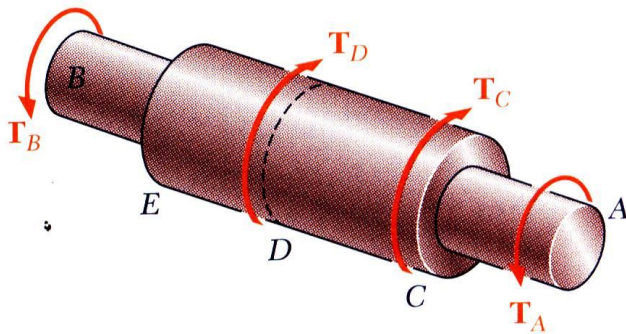
## Stresses in Elastic Range



$$\tau = \frac{T\rho}{J} \quad ; \quad \phi = \frac{TL}{GJ}$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$\text{so } \frac{\phi}{L} = \alpha = \frac{T}{GJ}, \text{ a constant if } T \text{ constant}$$

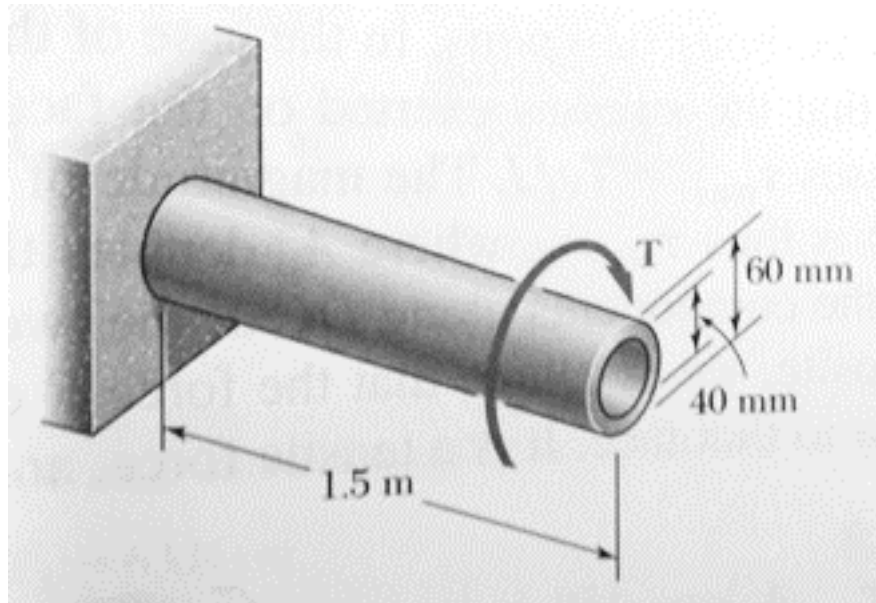


- If torsional loading or shaft cross-section changes (discretely) along length, the angle of rotation is found as sum of segment rotations

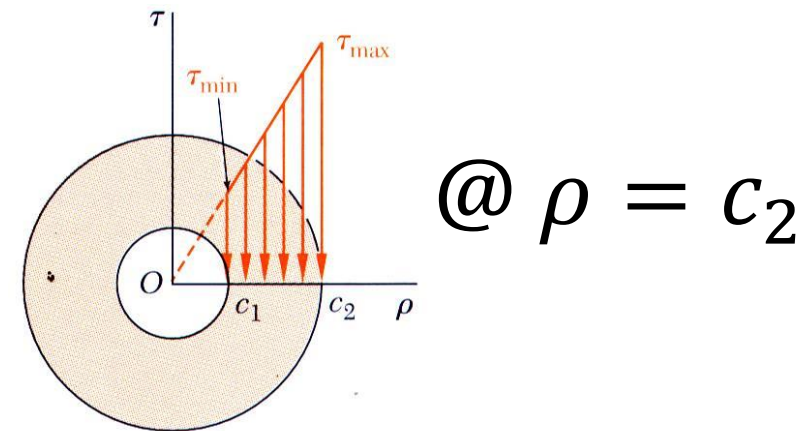
$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

**Problem 1:**

A hollow cylindrical shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm. (a) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (b) What is the corresponding minimum value of shearing stress in the shaft?

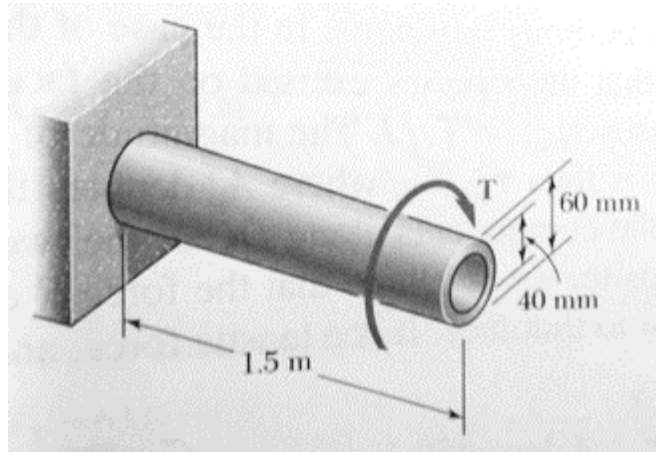


Where will the shaft experience the largest shearing stress?



$$\tau_{max} = 120 \text{ MPa}$$

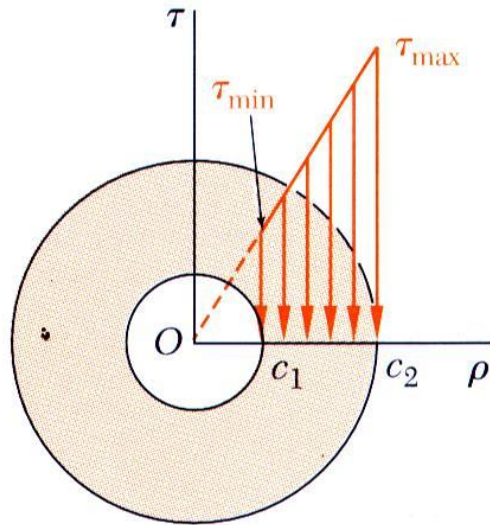
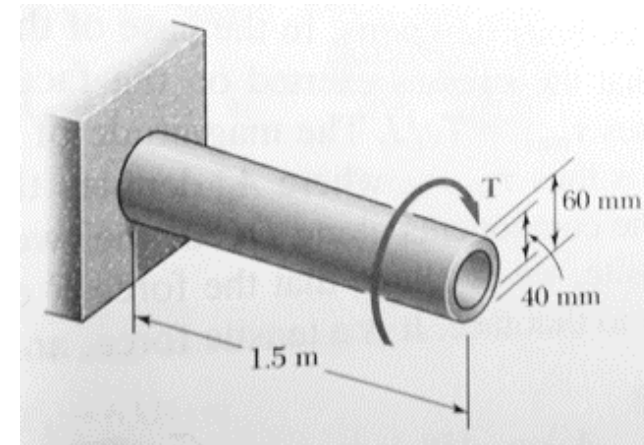
## SOLUTION:



*(a) Largest Permissible Torque.* The largest torque  $T$  that can be applied to the shaft is the torque for which  $\tau_{\max} = 120$  MPa. Since this value is less than the yield strength for steel, we can use Eq. (3.9). Solving this equation for  $T$ , we have

$$T = \frac{J\tau_{\max}}{c} \quad (3.12)$$

Where will the shaft experience minimum shearing stress?



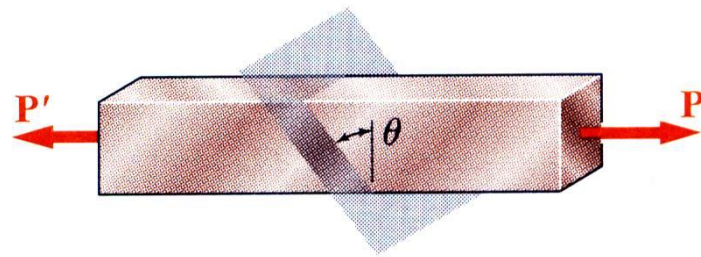
@  $\rho = c_1$

*(b) Minimum Shearing Stress.* The minimum value of the shearing stress occurs on the inner surface of the shaft. It is obtained from Eq. (3.7), which expresses that  $\tau_{\min}$  and  $\tau_{\max}$  are respectively proportional to  $c_1$  and  $c_2$ :

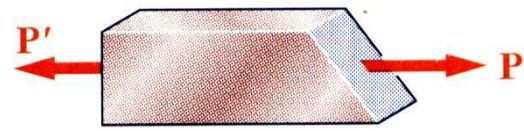
$$\tau_{\min} = \frac{c_1}{c_2} \tau_{\max} = \frac{0.02 \text{ m}}{0.03 \text{ m}} (120 \text{ MPa}) = 80 \text{ MPa}$$



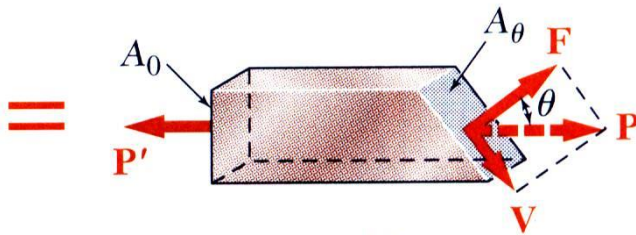
## Axial Load - Recap



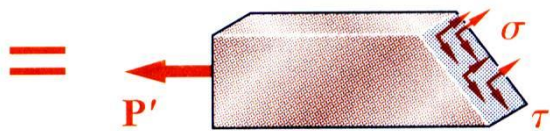
(a)



(b)



(c)



(d)

- Pass a section through the member forming an angle  $\theta$  with the normal plane.
- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force  $P$ .
- Resolve  $P$  into components normal and tangential to the oblique section,

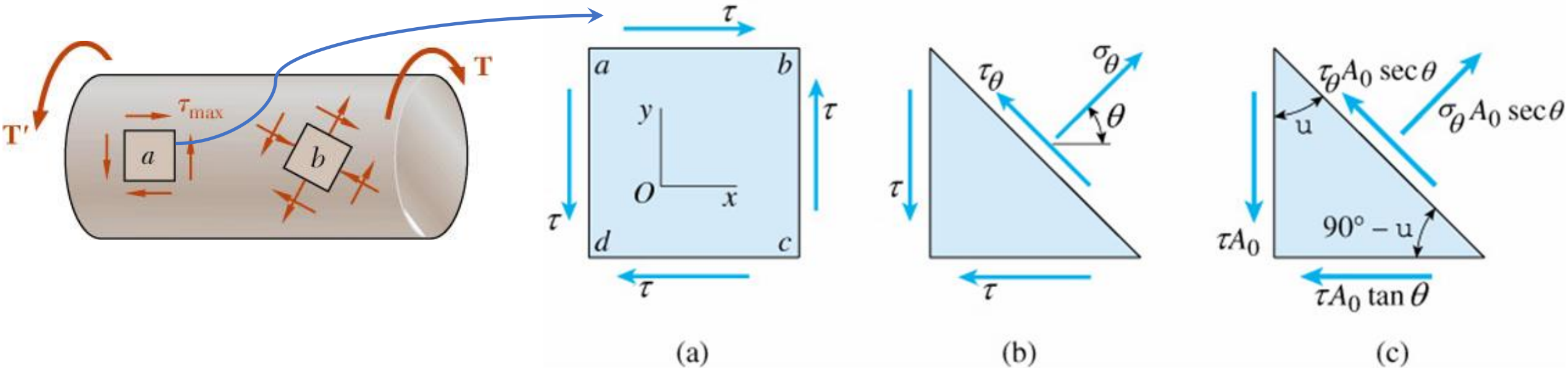
$$F = P \cos \theta \quad V = P \sin \theta$$

- The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P \cos \theta}{\frac{A_0}{\cos \theta}} = \frac{P}{A_0} \cos^2 \theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P \sin \theta}{\frac{A_0}{\cos \theta}} = \frac{P}{A_0} \sin \theta \cos \theta$$

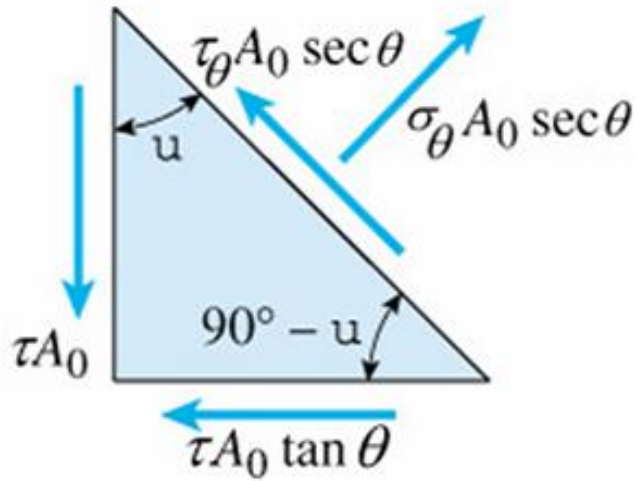
## Stresses on Inclined Plane



- (a) An element in *pure shear* generated due to *applied torque*,  
 (b) stresses acting on an inclined plane of a triangular stress element,  
 (c) forces acting on the triangular stress element (FBD).

**Sign convention** for stresses on an inclined plane  
 (Normal stresses tensile positive, shear stresses producing counterclockwise rotation positive.)

## Stresses on Inclined Plane



$$u \equiv \theta \quad (c)$$

Equilibrium normal to plane,

$$\sigma_{\theta} A_0 \sec \theta = \tau A_0 \sin \theta + \tau A_0 \tan \theta \cos \theta \quad (1)$$

$$\sigma_{\theta} = \tau \sin 2\theta$$

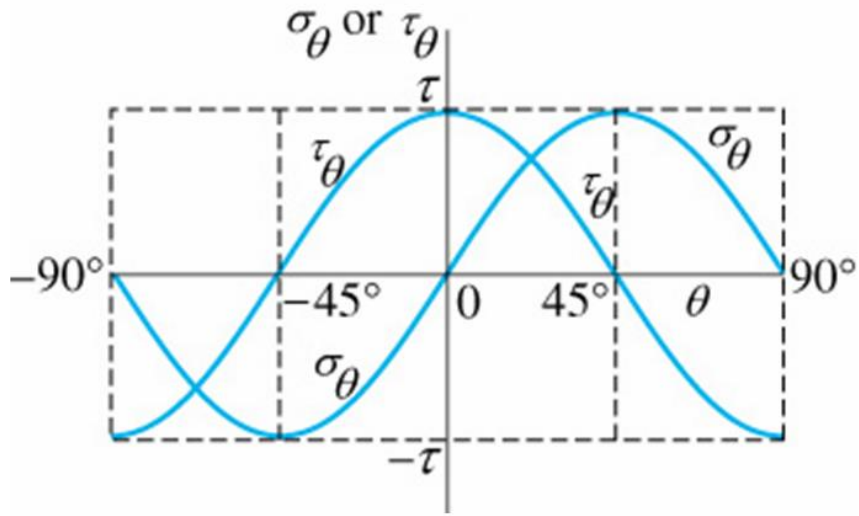
Equilibrium along the plane,

$$\tau_{\theta} A_0 \sec \theta = \tau A_0 \cos \theta - \tau A_0 \tan \theta \sin \theta \quad (2)$$

$$\tau_{\theta} = \tau \cos 2\theta$$

## Stresses on Inclined Plane

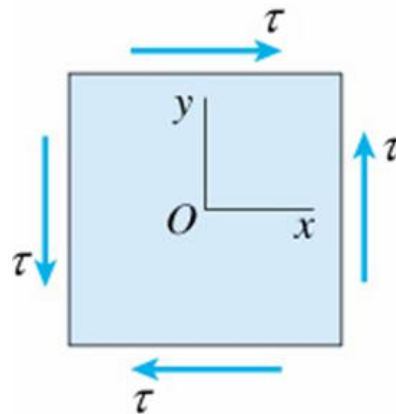
Graph of  $\sigma_\theta$  and  $\tau_\theta$  versus  $\theta$ .



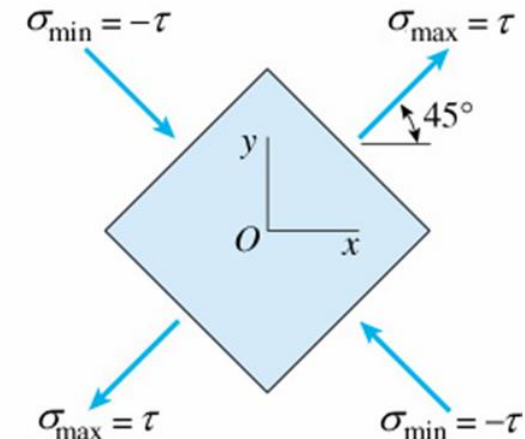
$$\sigma_\theta = \tau \sin 2\theta$$

$$\tau_\theta = \tau \cos 2\theta$$

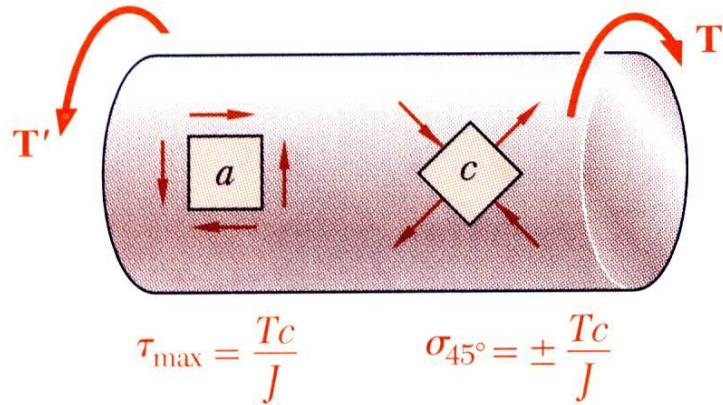
Maximum/Minimum shear stress occurs at  $\theta = 0^\circ$  or  $90^\circ$  plane  $\tau_{\max} = +\tau$ ;  $\tau_{\min} = -\tau$



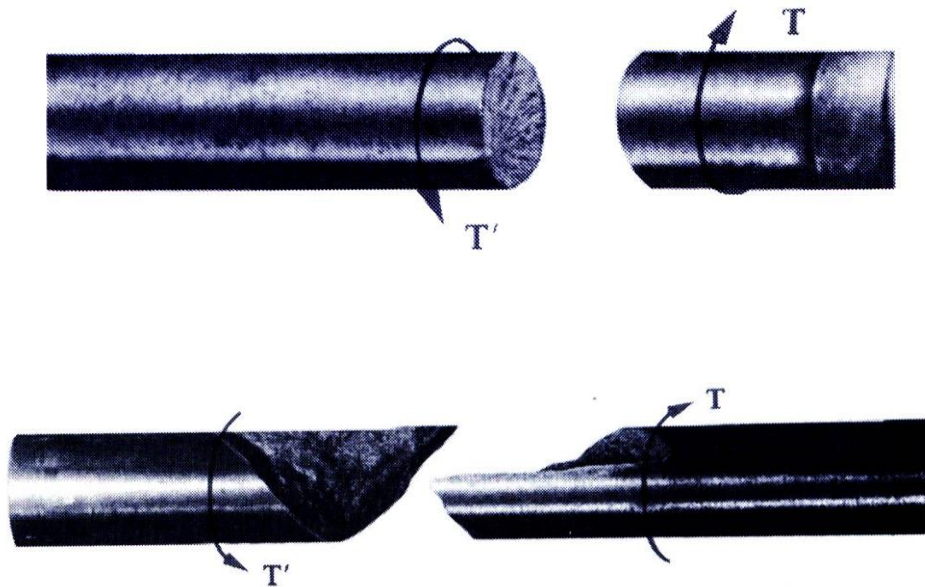
Maximum/minimum normal stress occurs at  $\theta = +45^\circ$  or  $-45^\circ$  plane  $\sigma_{\max} = +\tau$ ;  $\sigma_{\min} = -\tau$



## Torsional Failure Modes

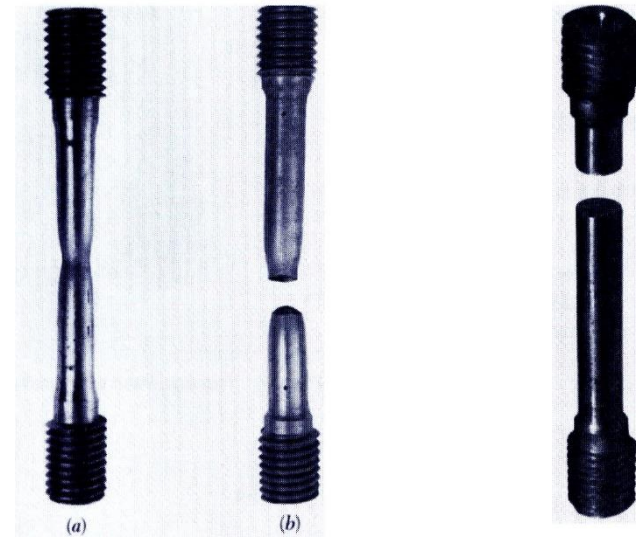


- Ductile materials generally fail in shear. Brittle materials are weaker in tension than shear.
- When subjected to torsion, a ductile specimen breaks along a plane of maximum shear, i.e., a plane perpendicular to the shaft axis.
- When subjected to torsion, a brittle specimen breaks along planes perpendicular to the direction in which tension is a maximum, i.e., along surfaces at 45° to the shaft axis.



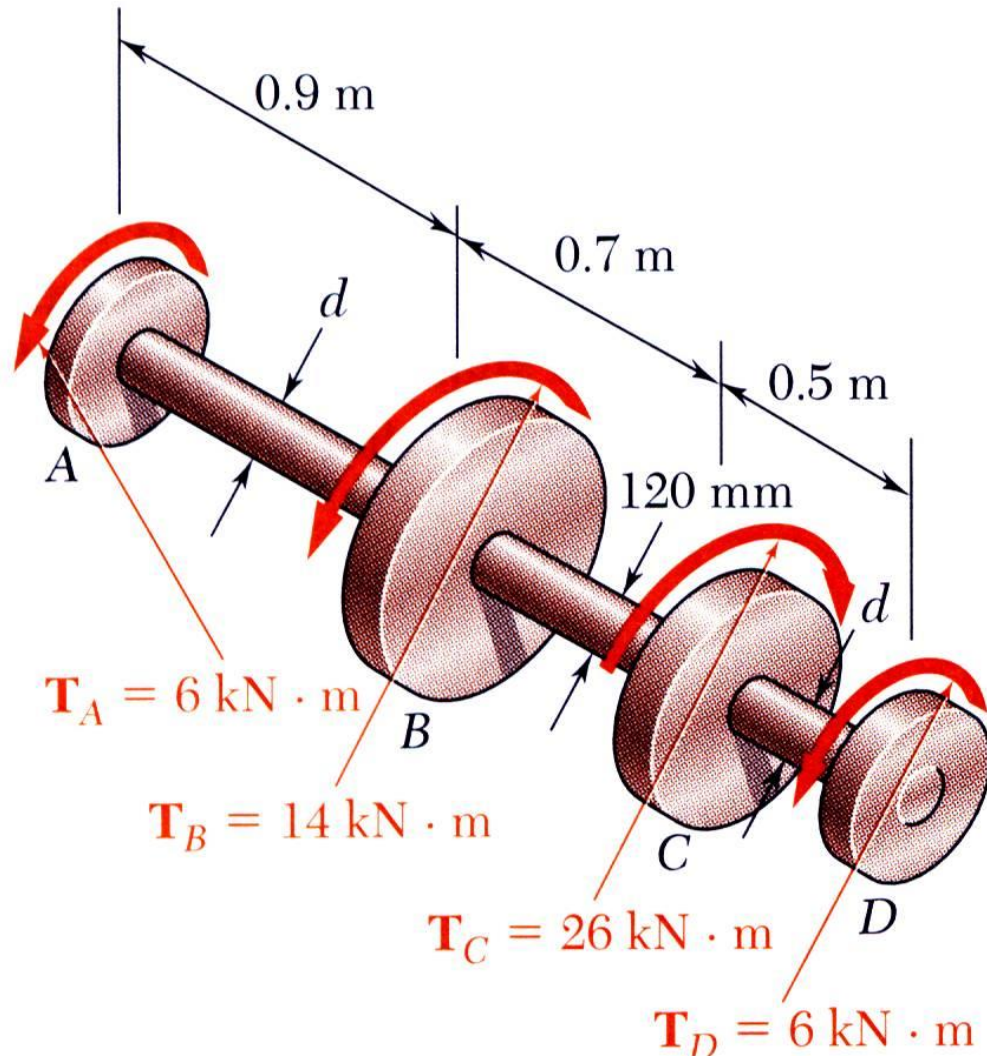
*Ductile*

*Brittle*





## Problem 2:

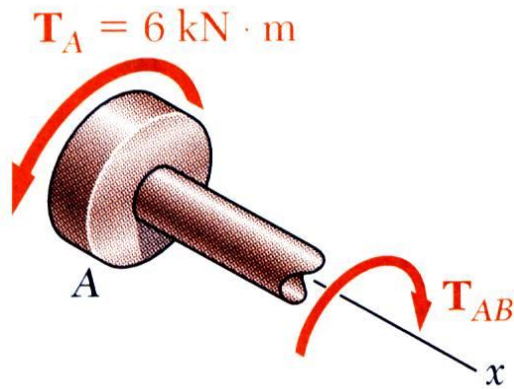


Shaft  $BC$  is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts  $AB$  and  $CD$  are solid of diameter  $d$ . For the loading shown, determine

- the minimum and maximum shearing stress in shaft  $BC$ ,
- the required diameter  $d$  of shafts  $AB$  and  $CD$  if the allowable shearing stress in these shafts is 65 MPa.

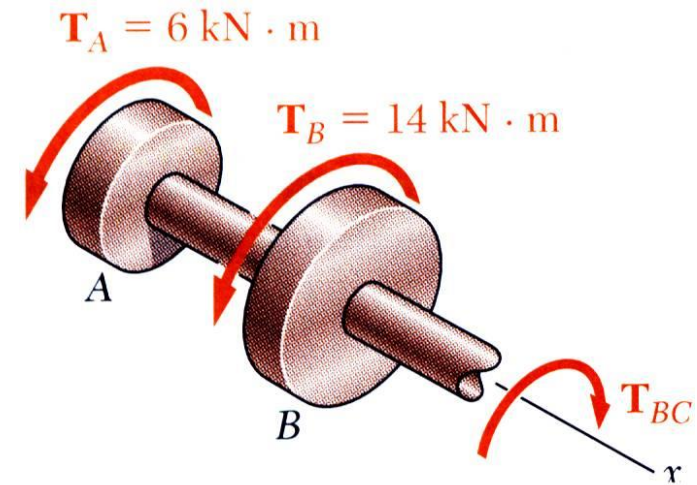
## SOLUTION:

- Cut sections through shafts  $AB$  and  $BC$  and perform static equilibrium analysis to find torque loadings



$$\sum M_x = 0 = (6\text{ kN}\cdot\text{m}) - T_{AB}$$

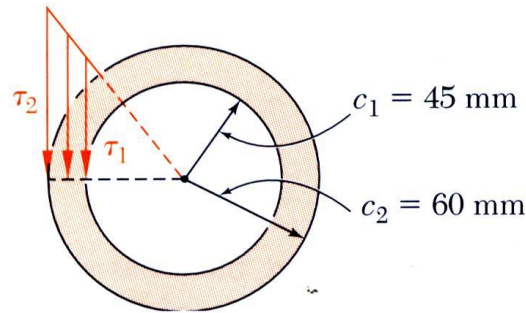
$$T_{AB} = 6\text{ kN}\cdot\text{m} = T_{CD}$$



$$\sum M_x = 0 = (6\text{ kN}\cdot\text{m}) + (14\text{ kN}\cdot\text{m}) - T_{BC}$$

$$T_{BC} = 20\text{ kN}\cdot\text{m}$$

- Apply elastic torsion formulas to find minimum and maximum stress on shaft *BC*



$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} [(0.060)^4 - (0.045)^4]$$

$$= 13.92 \times 10^{-6} \text{ m}^4$$

$$\tau_{\max} = \tau_2 = \frac{T_{BC} c_2}{J} = \frac{(20 \text{ kN} \cdot \text{m})(0.060 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4}$$

$$= 86.2 \text{ MPa}$$

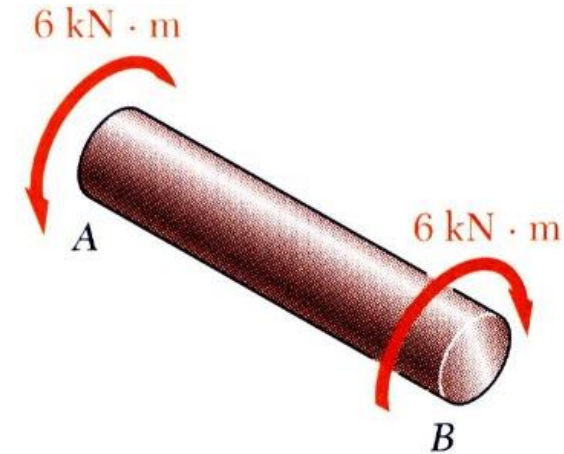
$$\frac{\tau_{\min}}{\tau_{\max}} = \frac{c_1}{c_2} \quad \frac{\tau_{\min}}{86.2 \text{ MPa}} = \frac{45 \text{ mm}}{60 \text{ mm}}$$

$$\tau_{\min} = 64.7 \text{ MPa}$$

$\tau_{\max} = 86.2 \text{ MPa}$

$\tau_{\min} = 64.7 \text{ MPa}$

- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter



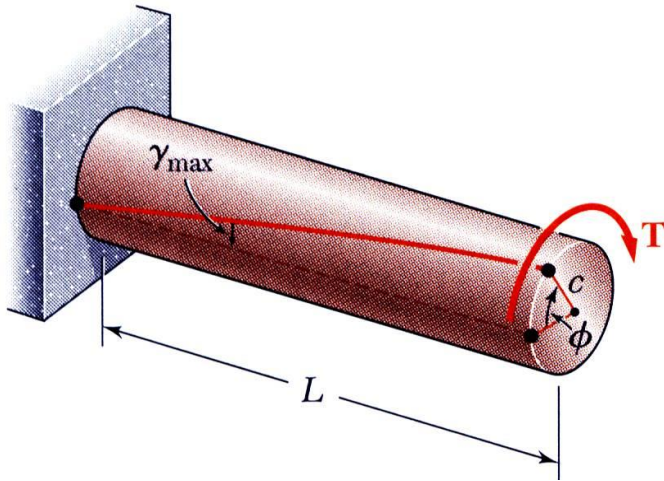
$$\tau_{\max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2} c^4} \quad 65 \text{ MPa} = \frac{6 \text{ kN} \cdot \text{m}}{\frac{\pi}{2} c^3}$$

$$c = 38.9 \times 10^{-3} \text{ m}$$

$d = 2c = 77.8 \text{ mm}$

What about shaft *CD*?

## Angle of Twist in Elastic Range



- Recall that the angle of twist and maximum shearing strain are related,

$$\gamma_{\max} = \frac{c\phi}{L}$$

- In the elastic range, the shearing strain and shear are related by Hooke's Law,

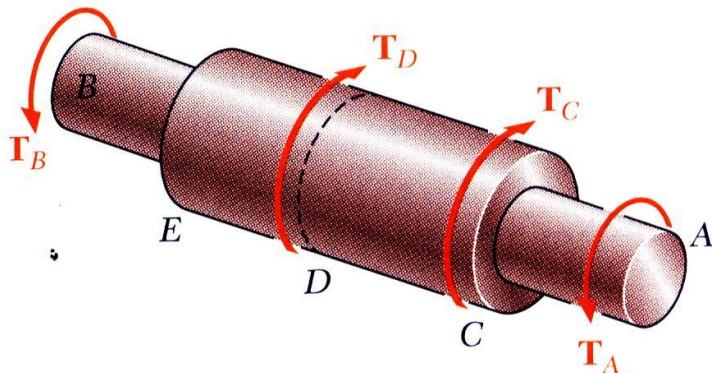
$$\gamma_{\max} = \frac{\tau_{\max}}{G} = \frac{Tc}{JG}$$

- Equating the expressions for shearing strain and solving for the angle of twist,

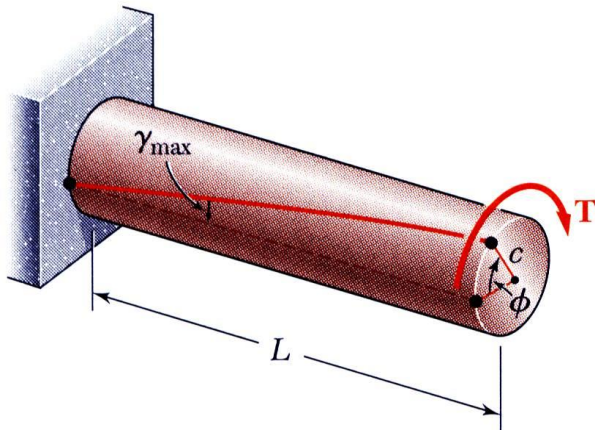
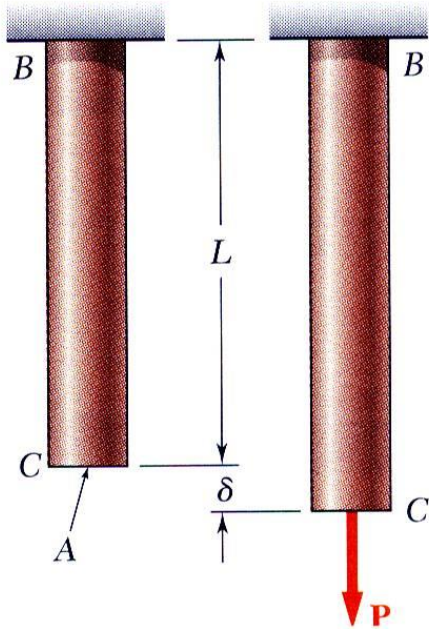
$$\phi = \frac{TL}{JG}$$

- If the torsional loading or shaft cross-section changes along the length, the angle of rotation is found as the sum of segment rotations

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$



## Comparison: Deformations under Axial and Torsional Loadings



- From Hooke's Law:

$$\sigma = E\varepsilon \quad \tau = G\gamma$$

- Deformation

$$\delta = \frac{PL}{AE} \quad \varphi = \frac{TL}{JG}$$

- With variations in loading, cross-section or material properties,

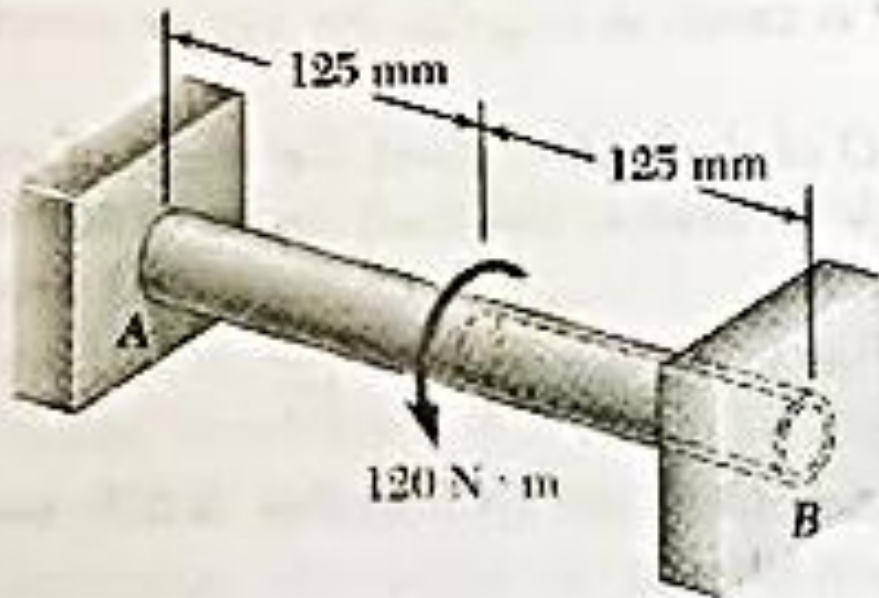
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} \quad \varphi = \sum_i \frac{T_i L_i}{J_i G_i}$$



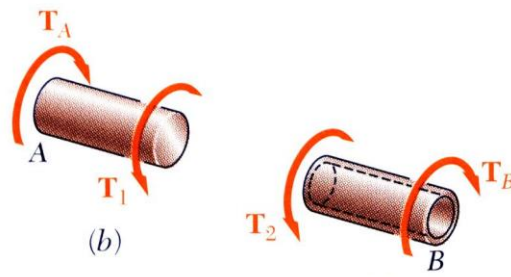
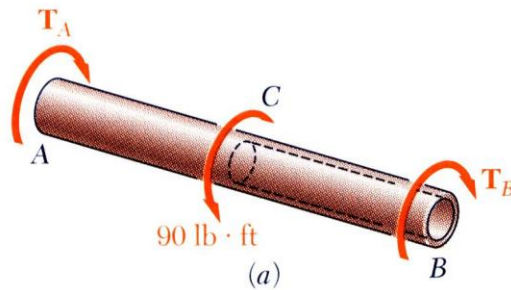
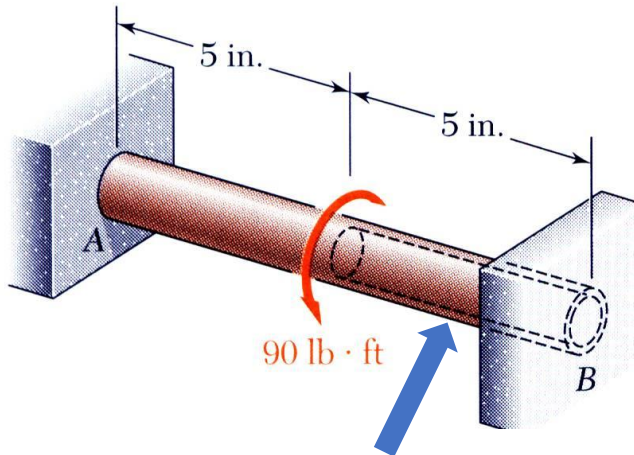
## Problem 3:

05

A circular shaft  $AB$  consists of a 250-mm-long, 22-mm-diameter steel cylinder, in which a 125-mm-long, 16-mm-diameter cavity has been drilled from end  $B$ . The shaft is attached to fixed supports at both ends, and a  $120\text{-N} \cdot \text{m}$  torque is applied at its midsection (Fig. 3.25). Determine the torque exerted on the shaft by each of the supports.



## Statically Indeterminate Shafts



Outer diameter = 22 mm  
Inner diameter = 16 mm

- Given the shaft dimensions and the applied torque, we would like to find the torque reactions at A and B.

- From a free-body analysis of the shaft,

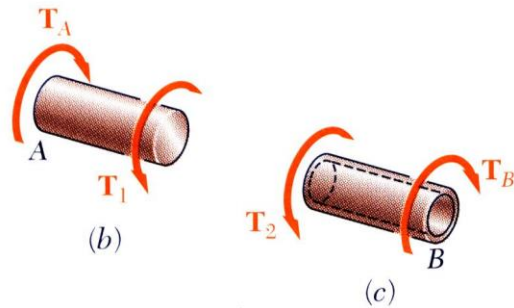
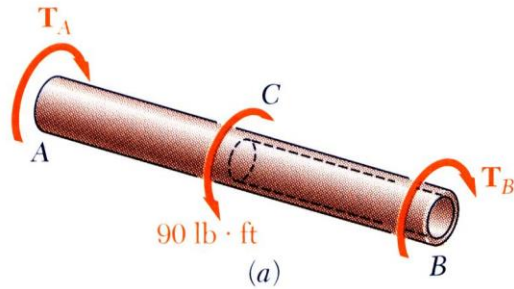
$$T_A + T_B = 120 \text{ N} \cdot \text{m}$$

which is not sufficient to find the end torques. The problem is statically indeterminate.

- Divide the shaft into two components which must have compatible deformations,

$$\phi = \phi_1 + \phi_2 = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0 \quad T_B = \frac{L_1 J_2}{L_2 J_1} T_A$$

## Statically Indeterminate Shafts



Outer diameter = 22 mm  
Inner diameter = 16 mm

- Divide the shaft into two components which must have compatible deformations,

$$\phi = \phi_1 + \phi_2 = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0 \quad T_B = \frac{L_1 J_2}{L_2 J_1} T_A$$

Substituting the numerical data

$$L_1 = L_2 = 125 \text{ mm}$$

$$J_1 = \frac{1}{2} \pi (0.011 \text{ m})^4 = 230 \times 10^{-6} \text{ m}^4$$

$$J_2 = \frac{1}{2} \pi [(0.011 \text{ m})^4 - (0.008 \text{ m})^4] = 165.6 \times 10^{-6} \text{ m}^4$$

we obtain

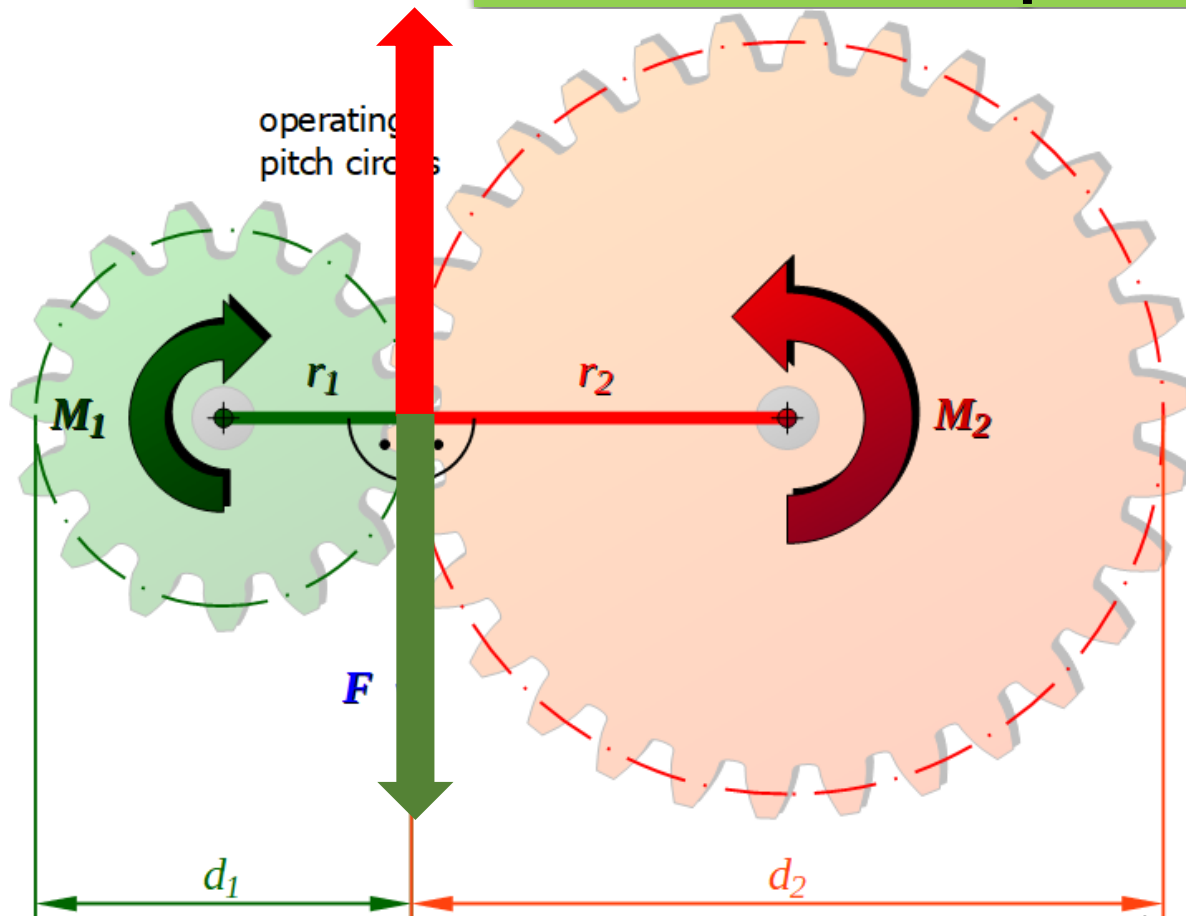
$$T_B = 0.72 T_A$$

- Substitute into the original equilibrium equation,

$$T_A + \frac{L_1 J_2}{L_2 J_1} T_A = 120 \text{ N} \cdot \text{m}$$

$$T_A = 69.8 \text{ N} \cdot \text{m} \quad T_B = 50.2 \text{ N} \cdot \text{m}$$

## Torque Transmission



Number of teeth interacted

$$F_1 = F_2 = F$$

$$\frac{T_1}{r_1} = \frac{T_2}{r_2}$$

Peripheral distance covered should be same

$$r_1 \theta_1 = r_2 \theta_2$$

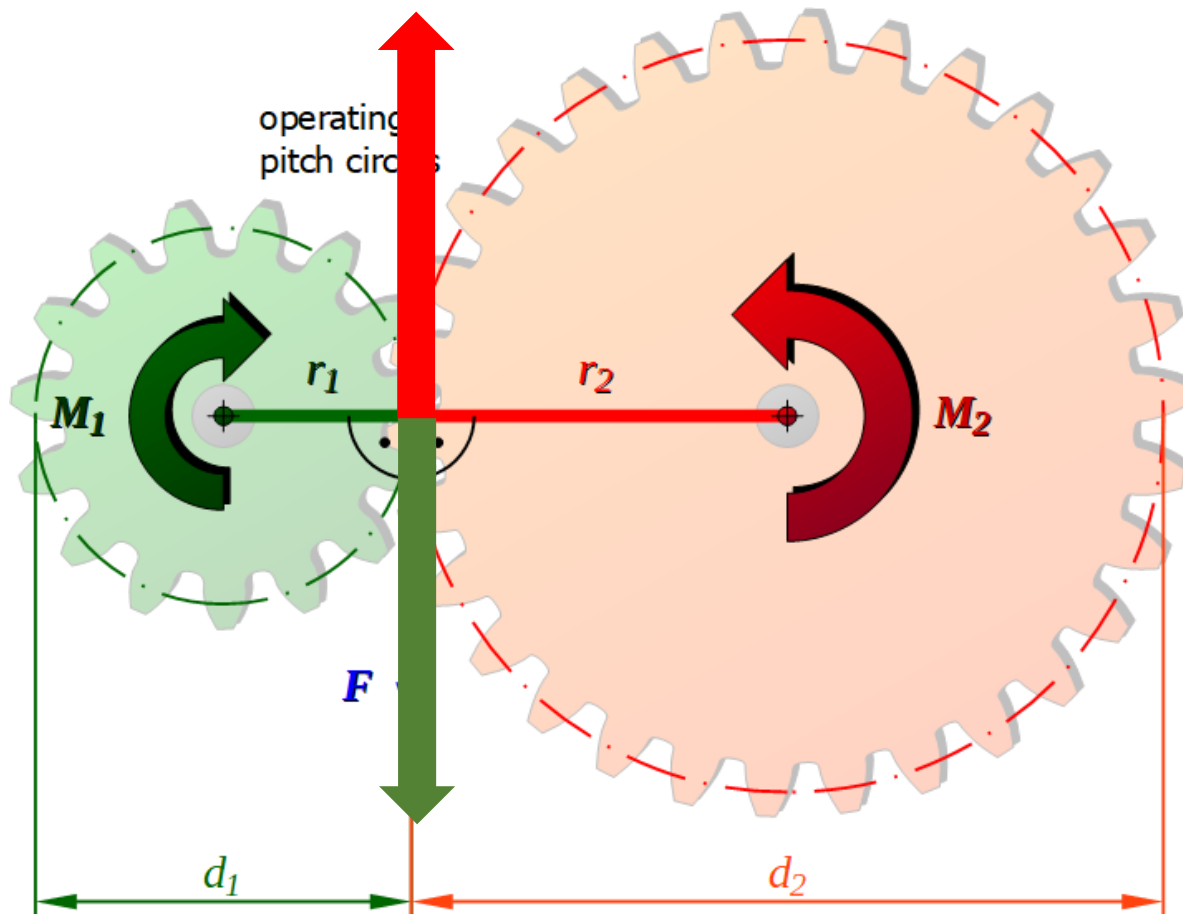
Peripheral velocity should be same

$$r_1 \omega_1 = r_2 \omega_2$$

$$\frac{\omega_1}{r_2} = \frac{\omega_2}{r_1}$$

$$r_1 \theta_1 = r_2 \theta_2 \rightarrow r_1 2\pi \frac{N}{N_1} = r_2 2\pi \frac{N}{N_2} \rightarrow \frac{r_1}{N_1} = \frac{r_2}{N_2}$$

## Torque Transmission



$$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{N_1}{N_2}$$



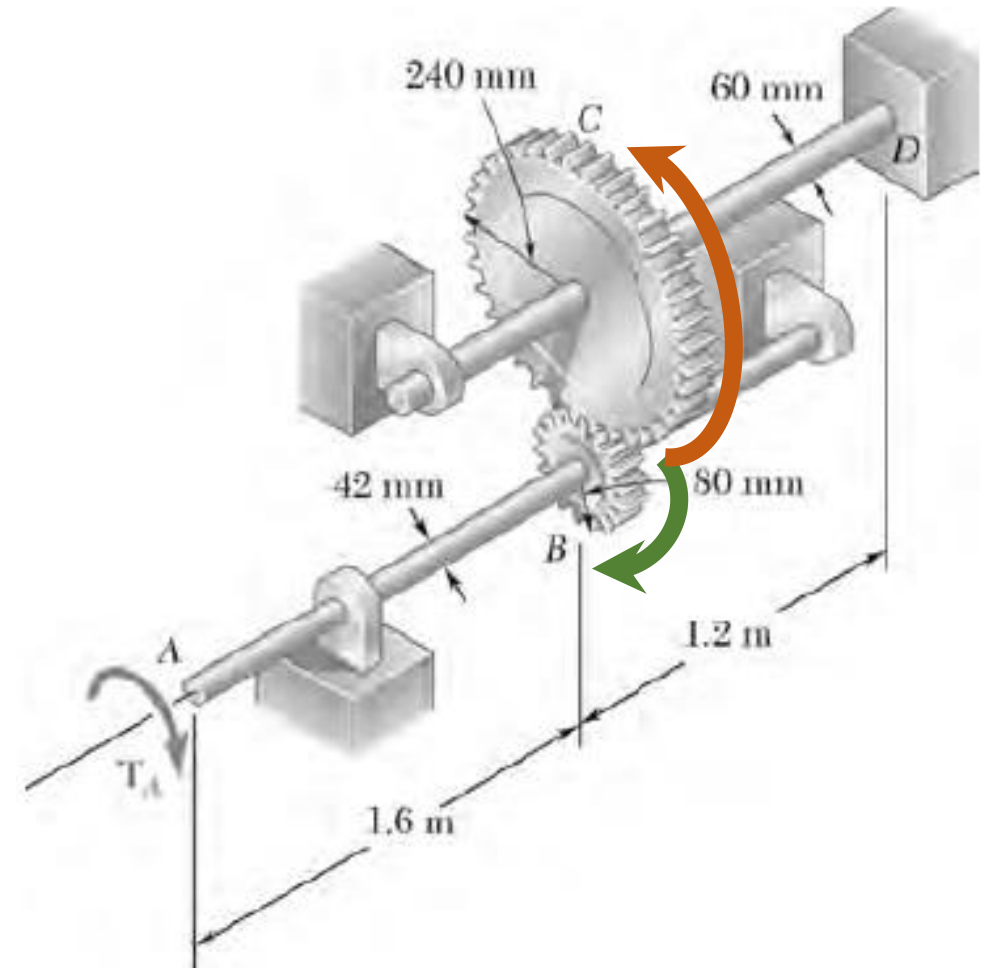
## Problem 4:

Two solid shafts are connected by gears as shown. Knowing that  $G = 77.2$  GPa for each shaft, determine the angle through which end A rotates when  $T_A = 1200\text{ N} \cdot \text{m}$ .

- Given the direction of torque at A, B will rotate **clockwise**, C will rotate **counter-clockwise**
- w.r.t. to D (fixed support), C **twists** counter-clockwise
- if C twists counter-clockwise, B **rotates** opposite, *i.e.* clockwise w.r.t. D
- w.r.t. to B, A **twists** clockwise

Hence, the twists  $\phi_{B/D}$  and  $\phi_{A/B}$  should be added to get  $\phi_{A/D}$

$$\phi_{A/D} = \phi_{A/B} + \phi_{B/D}$$



## SOLUTION:

- Given  $T_{AB} = 1200 \text{ Nm}$ , find  $T_{CD}$

### Calculation of torques:

Circumferential contact force between gears  $B$  and  $C$ .  $F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$

$$T_{AB} = 1200 \text{ N} \cdot \text{m} \quad T_{CD} = \frac{240}{80} (1200) = 3600 \text{ N} \cdot \text{m}$$

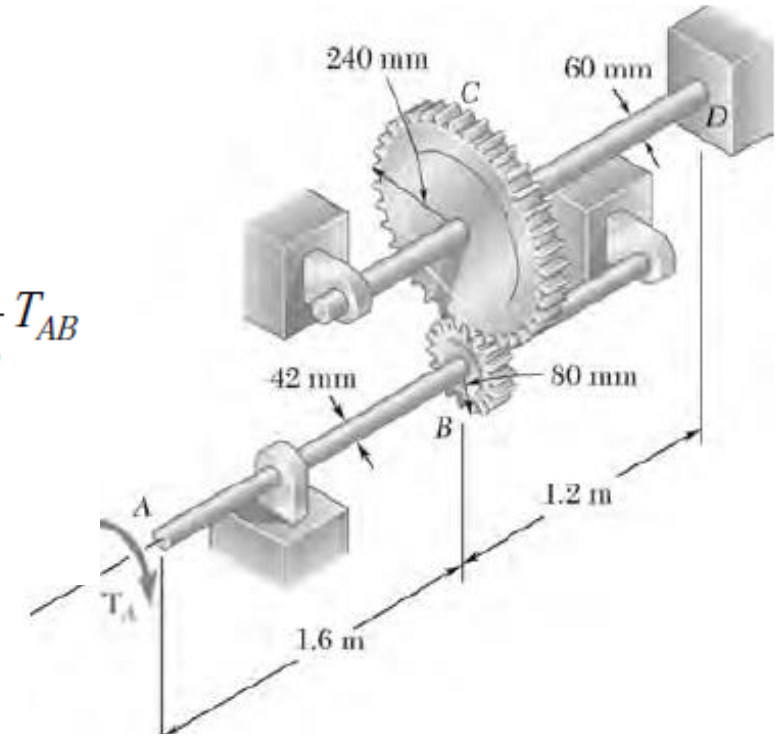
- Knowing  $T_{CD}$ , find  $\phi_{C/D}$

### Twist in shaft $CD$ :

$$c = \frac{1}{2} d = 0.030 \text{ m}, \quad L = 1.2 \text{ m}, \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-6})} = 43.981 \times 10^{-3} \text{ rad}$$



## SOLUTION:

- Due to rotation of C, find rotation in B,  $\phi_{B/D}$

Rotation angle at C.  $\phi_C = \phi_{C/D} = 43.981 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C.  $\delta = r_C \phi_C = r_B \phi_B$

Rotation angle at B.  $\phi_B = \frac{r_C}{r_B} \phi_C = \frac{240}{80} (43.981 \times 10^{-3}) = 131.942 \times 10^{-3} \text{ rad}$

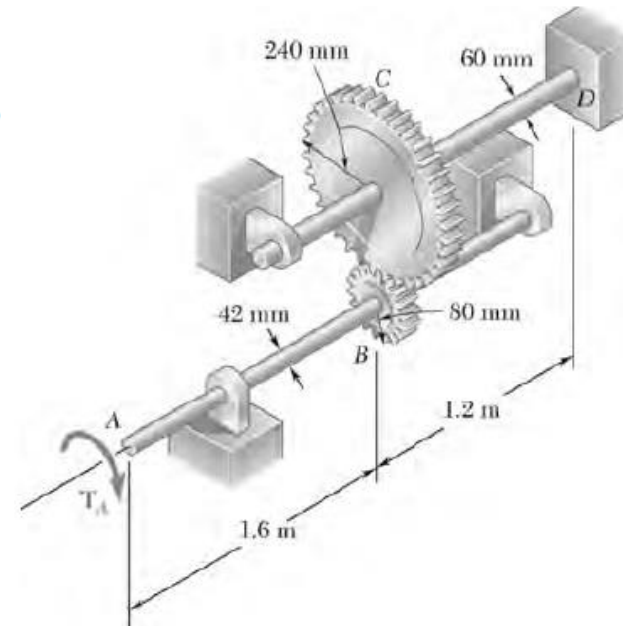
- Knowing  $T_{AB}$ , find  $\phi_{A/B}$

Twist in shaft AB:  $c = \frac{1}{2} d = 0.021 \text{ m}, \quad L = 1.6 \text{ m}, \quad G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

Rotation angle at A.  $\phi_A = \phi_B + \phi_{A/B} = 213.354 \times 10^{-3} \text{ rad} \qquad \phi_A = 12.22^\circ$



## Design of Transmission Shafts

- Principal transmission shaft performance specifications are:
  - power
  - speed
- Designer must select shaft material and cross-section to meet performance specifications **without exceeding allowable shearing stress.**

- Determine torque applied to shaft at specified power and speed,

$$P = T\omega = 2\pi fT$$

$$T = \frac{P}{\omega} = \frac{P}{2\pi f}$$

- Find shaft cross-section which will not exceed the maximum allowable shearing stress,

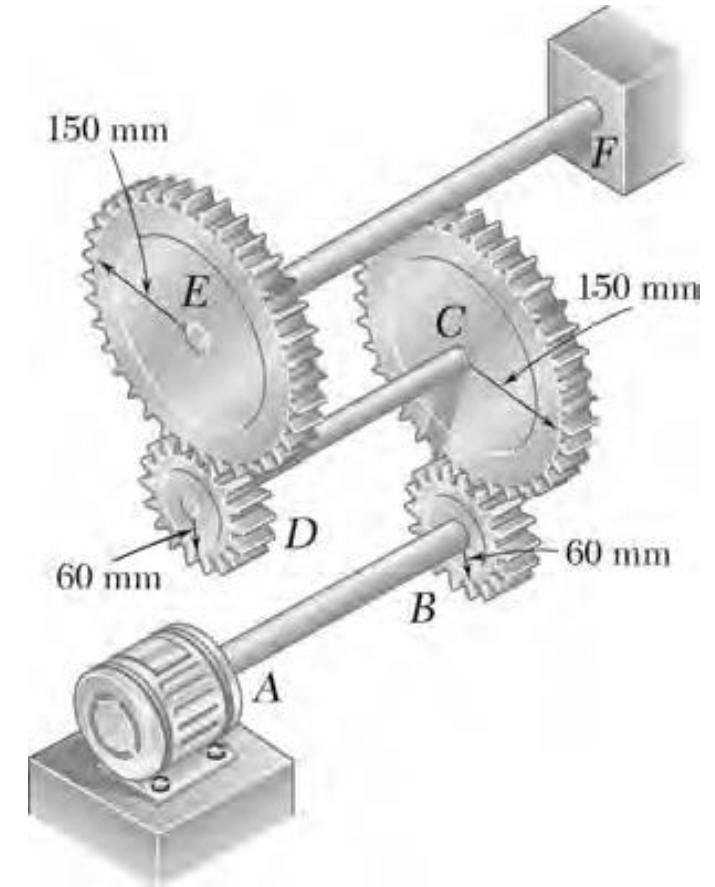
$$\tau_{\max} = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau_{\max}} \quad (\text{solid shafts})$$

$$\frac{J}{c_2} = \frac{\pi}{2c_2}(c_2^4 - c_1^4) = \frac{T}{\tau_{\max}} \quad (\text{hollow shafts})$$

**Problem 5:**

Three shafts and four gears are used to form a gear train that will transmit power from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows:  $d_{AB} = 16 \text{ mm}$ ,  $d_{CD} = 20 \text{ mm}$ ,  $d_{EF} = 28 \text{ mm}$ . Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.





## Solution:

- $\tau_{allowable, AB} = 75 \text{ MPa} \rightarrow T_{all/\max, AB} = ?? \rightarrow P_{all/\max, AB}$

$$\tau_{all} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$$

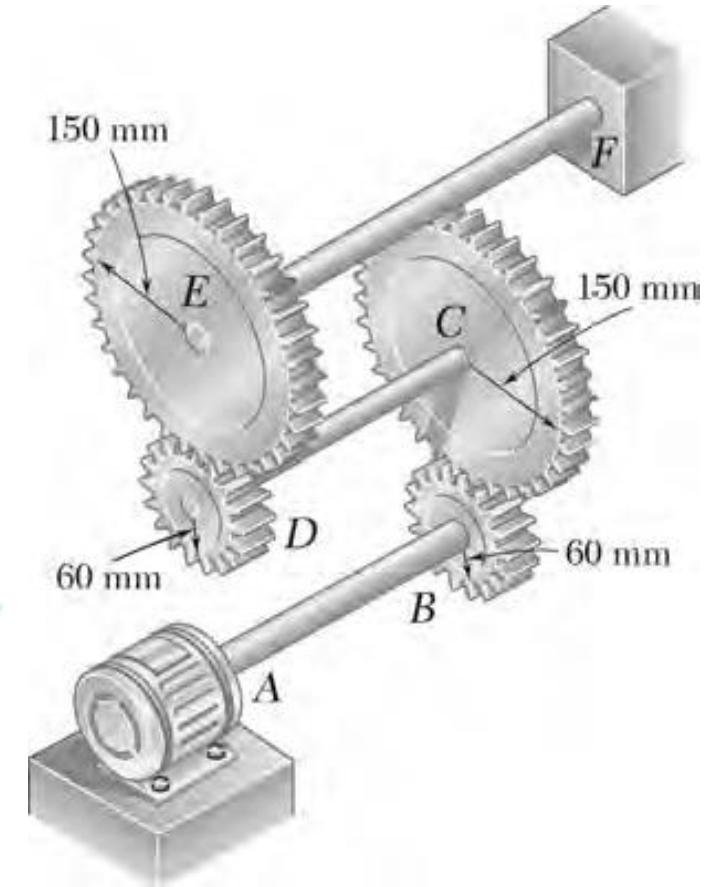
Shaft AB:

$$c_{AB} = \frac{1}{2} d_{AB} = 0.008 \text{ m} \quad \tau = \frac{T c_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3}$$

$$T_{all} = \frac{\pi}{2} c_{AB}^3 \tau_{all} = \frac{\pi}{2} (0.008)^3 (75 \times 10^6) = 60.319 \text{ N} \cdot \text{m}$$

$$f_{AB} = 24 \text{ Hz} \quad P_{all} = 2\pi f_{AB} T_{all} = 2\pi (24) (60.319) = 9.10 \times 10^3 \text{ W}$$

1



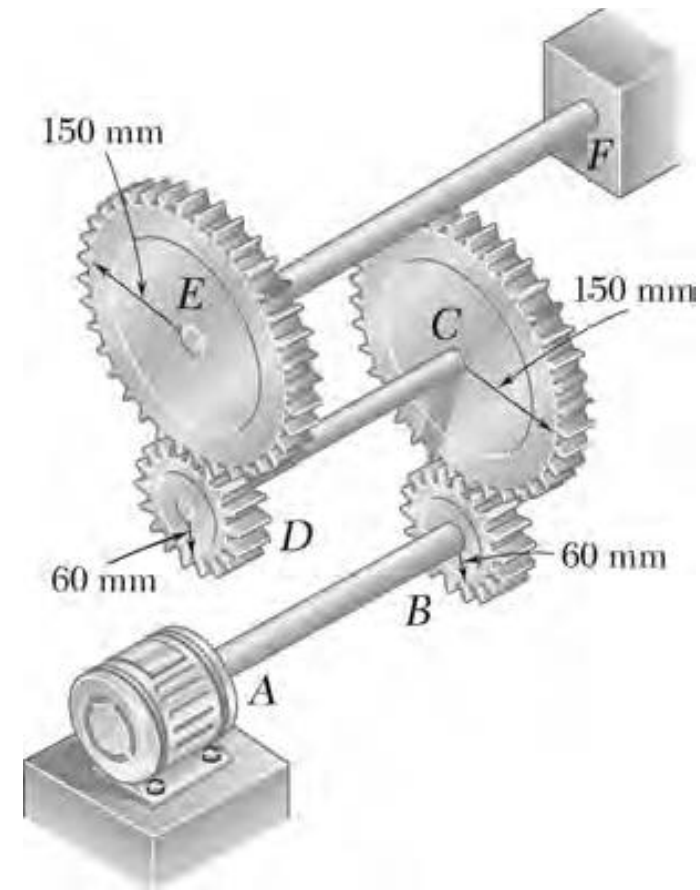
**Solution:**

$$\frac{\omega_1}{r_2} = \frac{\omega_2}{r_1} \rightarrow \frac{f_{AB}}{r_C} = \frac{f_{CD}}{r_B}$$

Shaft CD:  $c_{CD} = \frac{1}{2} d_{CD} = 0.010 \text{ m}$

$$\tau = \frac{T_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad T_{\text{all}} = \frac{\pi}{2} c_{CD}^3 \tau_{\text{all}} = \frac{\pi}{2} (0.010)^3 (75 \times 10^6) = 117.81 \text{ N} \cdot \text{m}$$

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150} (24) = 9.6 \text{ Hz} \quad P_{\text{all}} = 2\pi f_{CD} T_{\text{all}} = 2\pi (9.6) (117.81) = 7.11 \times 10^3 \text{ W}$$



**Solution:**

$$\frac{\omega_1}{r_2} = \frac{\omega_2}{r_1} \rightarrow \frac{f_{EF}}{r_D} = \frac{f_{CD}}{r_E}$$

Shaft EF:

$$c_{EF} = \frac{1}{2} d_{EF} = 0.014 \text{ m}$$

$$T_{all} = \frac{\pi}{2} c_{EF}^3 \tau_{all} = \frac{\pi}{2} (0.014)^3 (75 \times 10^6) = 323.27 \text{ N} \cdot \text{m}$$

$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150} (9.6) = 3.84 \text{ Hz}$$

$$P_{all} = 2\pi f_{EF} T_{all} = 2\pi (3.84) (323.27) = 7.80 \times 10^3 \text{ W}$$

$$P_{all/\text{max}, AB} = 9.10 \text{ kW}$$

$$P_{all/\text{max}, CD} = 7.11 \text{ kW}$$

$$P_{all/\text{max}, EF} = 7.80 \text{ kW}$$

Which shaft will fail (first)?

Maximum power that can be transmitted is hence 7.11 kW

