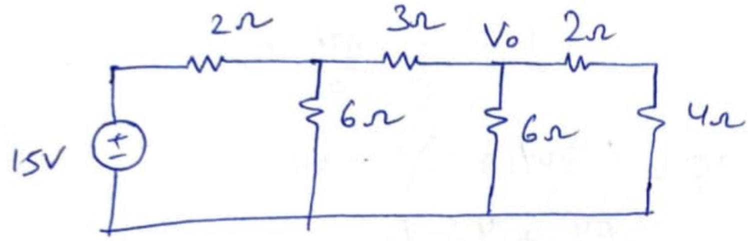
06 Feb 2023

EE101 Tutorial 11 Solution Sheet

1.

1.)



If $V_o = 1V$, $V_1 = \frac{1}{3} + 1 = 2V$

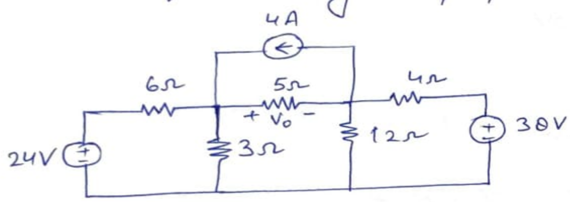
$$V_s = 2\left(\frac{2}{3}\right) + V_1 = \frac{10}{3}.$$

If $V_s = \frac{10}{3} \rightarrow V_o = 1$

Then $V_s = 15 \longrightarrow V_o = \frac{3}{10} \times 15 = 4.5V$

2.

2.) $V_o = ?$ using superposition principle.



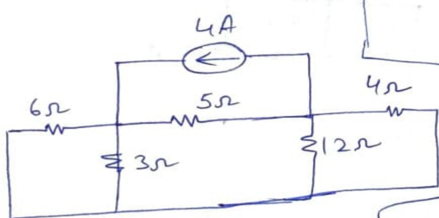
$V_o = V_{o1} + V_{o2} + V_{o3}$

$V_{o1} \rightarrow 4A$

$V_{o2} \rightarrow 24V$

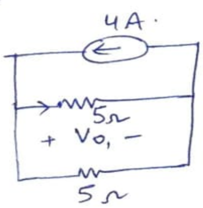
$V_{o3} \rightarrow 38V$

For $V_{o1} \rightarrow$



$3 \| 6 = 2\,\Omega$

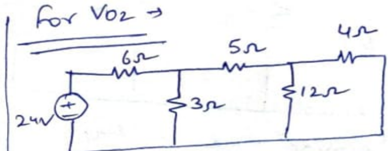
$4 \| 12 = 3\,\Omega$.



$V_{o1} = 5 \times 2$

$\boxed{V_{o1} = 10V.}$

For $V_{o2} \rightarrow$



$4 \| 12 = 3\,\Omega$

$3 + 5 = 8\,\Omega$

$3 \| 8 = \frac{24}{11}$.

$V_1 = \left(\frac{24/11}{6 + 24/11}\right) \times 24$

$= \frac{\cancel{24}^{4}/\cancel{11}}{\cancel{96}/\cancel{11}} \times \cancel{24}^{8}$

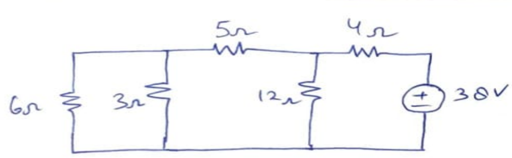
$V_1 = 32/5$

$V_{o2} = \frac{5}{8} \times V_1 = \frac{\cancel{5}}{\cancel{8}} \times \frac{\cancel{32}^{4}}{\cancel{5}}$

$\boxed{V_{o2} = 4V}$.

For $V_{o3} \rightarrow$

PTO



$6 \| 3 = 2\,\Omega$

$2 + 5 = 7\,\Omega$

$7 \| 12 = \frac{84}{19}\,\Omega$

$V_2 = \left(\frac{84/19}{\frac{84}{19} + 4}\right) \times 38$

$= \frac{84/\cancel{19}}{160/\cancel{19}} \times 38 = 19.95$.

$V_{o3} = -\frac{5}{7} \times 19.95$

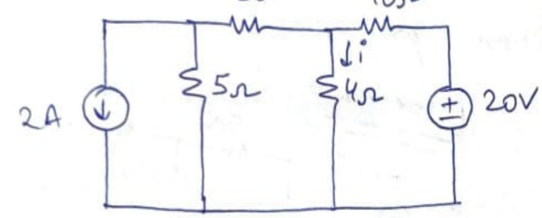
$\boxed{V_{o3} = -14.25\ V}$

$V_o = 10 + 4 - 14.25 = -0.25V$

$\boxed{V_o = -250\,mV}$

3.

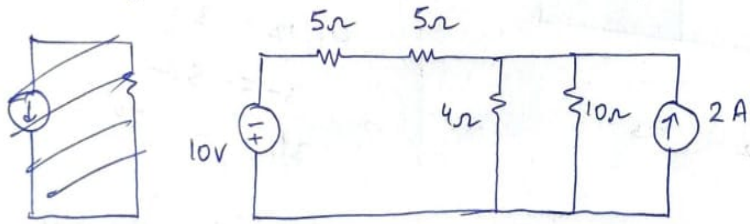
3.)

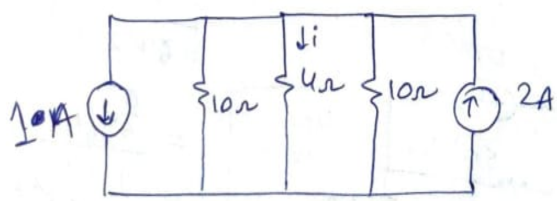


Using Source transformation

$i = ?$

Transforming the voltage source and current source





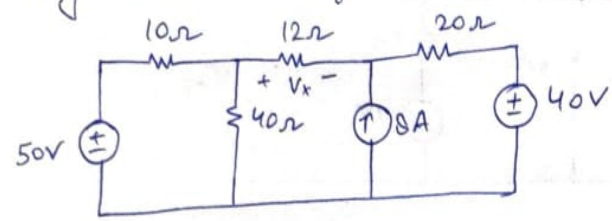
$10 \| 10 = 5\Omega$

$i = \frac{5}{5+4} \times (2-1) = \frac{5}{9} = 555.5\ mA.$

$\boxed{i = 555.5 mA}$

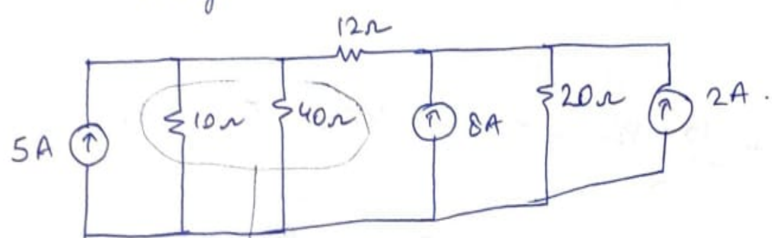
4.

4) Using Source transformation, find V_x



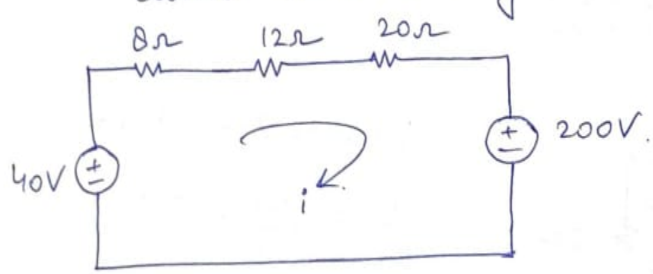
⇓

Voltage source → current source.



$10 \| 40 = 8\Omega$.

current source → vlg source.



Applying KVL.

$$-40 + (8+12+20)\, i + 200 = 0.$$

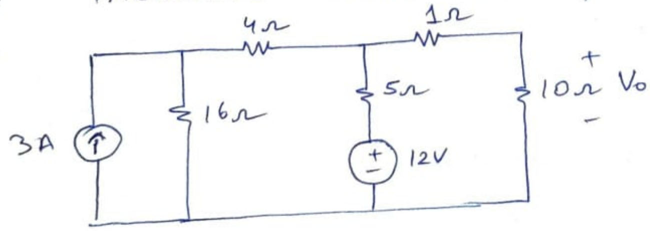
$$\boxed{i = -4\,A}.$$

$$V_x = 12i = 12 \times (-4)$$

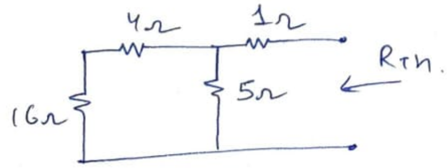
$$\boxed{V_x = -48V}.$$

5.

5.) Thevenin's theorem → find V_o.



for R_{Th} →

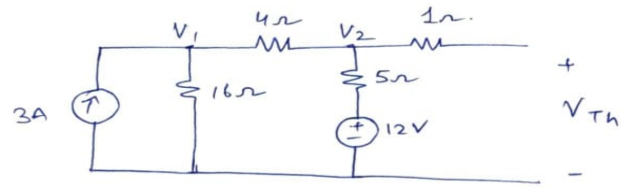


$$(16+4)\,||\,5 + 1 = R_{Th}$$

$$R_{Th} = \frac{\cancel{20}^{4} \times \cancel{5}}{\cancel{25}} + 1$$

$$\boxed{R_{Th} = 5\,\Omega}$$

For V_{Th} →



At node 1

$$3 = \frac{V_1}{16} + \frac{V_1 - V_2}{4} \quad = \quad 48 = 5V_1 - 4V_2 \quad \text{— (1)}$$

At node 2

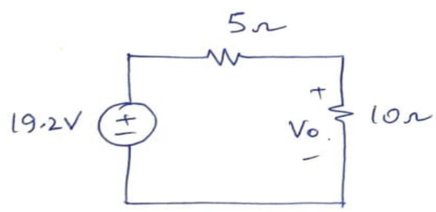
$$\frac{V_1 - V_2}{4} + \frac{12 - V_2}{5} = 0 \quad = \quad 48 = -5V_1 + 9V_2 \quad \text{— (2)}$$

$$5V_2 = 96$$

$$\boxed{V_2 = 19.2\,V}$$

$$\boxed{V_{Th} = V_2 = 19.2\,V}$$

Now,



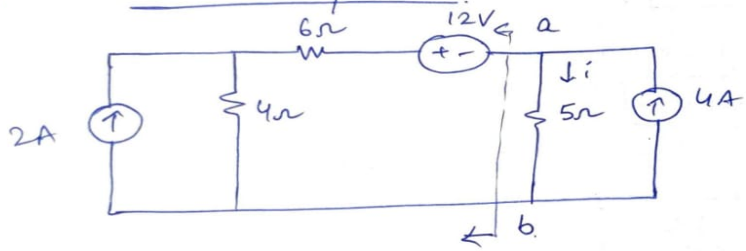
Using Voltage division

$$V_o = \frac{10}{15} \times 19.2$$

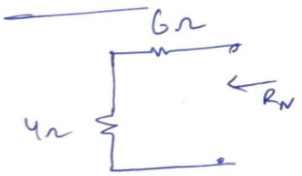
$$\boxed{V_o = 12.8\,V}$$

6.

6.) Norton Equivalent → find current i.

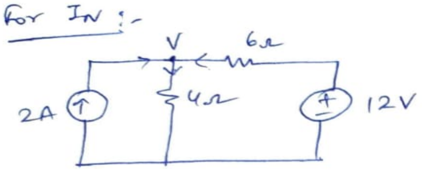


For R_N :-



$R_N = 6 + 4 = 10\Omega$

For I_N :-



$$2 + \frac{12-V}{6} = \frac{V}{4}$$

$$\frac{12-V}{6} - \frac{V}{4} = -2$$

$$\frac{2(12-V) - 3V}{12} = -2$$

$$\frac{24 - 2V - 3V}{12} = -2$$

$$24 - 5V = -24$$

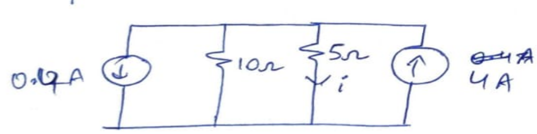
$$24 + 24 = 5V$$

$$\boxed{V = 9.6V}$$

$$-I_N = \frac{12-V}{6} = \frac{12-9.6}{6} = 0.4$$

$$\boxed{I_N = -0.4A}$$

Eq. ckt.



$$i = \frac{\cancel{10}^{2}}{\cancel{15}_{3}} \times (4 - 0.4)$$

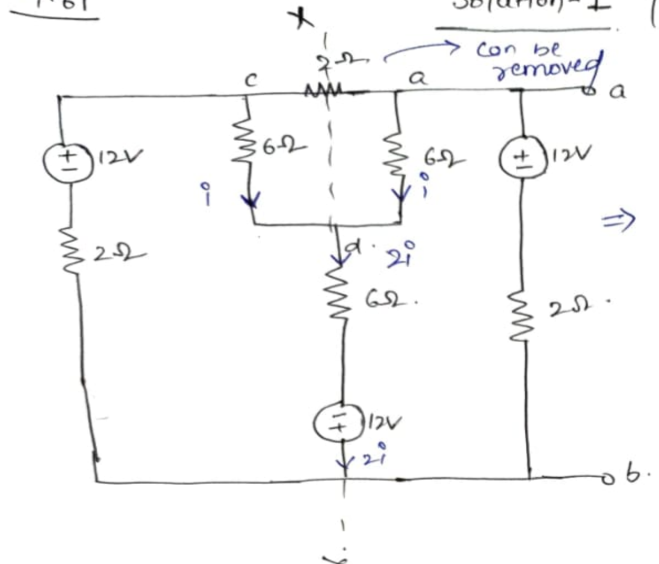
$$= \frac{2}{3} \times 3.6$$

$$\boxed{i = 2.4A}$$

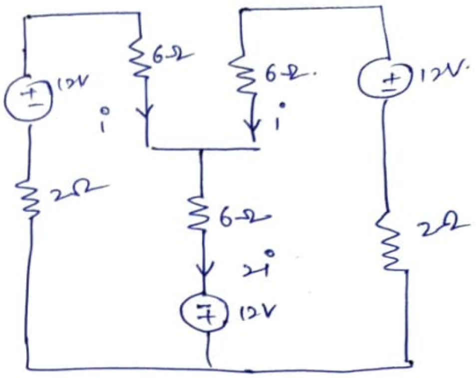
7. Solution-1

4.61

Solution-1 (solution using symmetry)



$\Rightarrow$



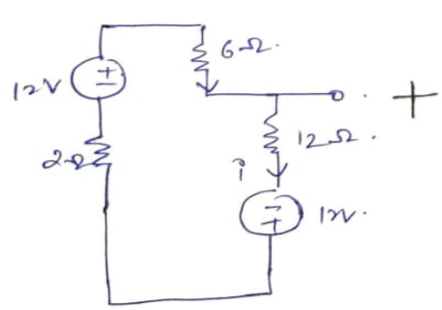
∵ Circuit is symmetrical at XY

– $V_a = V_c$ as 2Ω can be removed

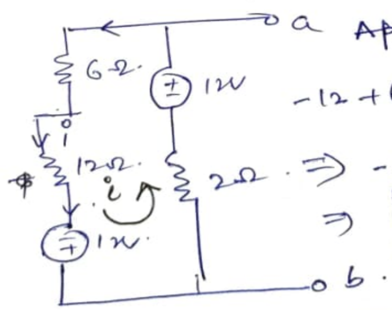
– Currents in branches c-d and a-d will be same

Let i

splitting above circuit:-



+



Applying KVL:-

$$-12+6i+12i-12+2i=0$$

$$\Rightarrow -24+20i=0$$

$$\Rightarrow \boxed{i = 1.2A}$$

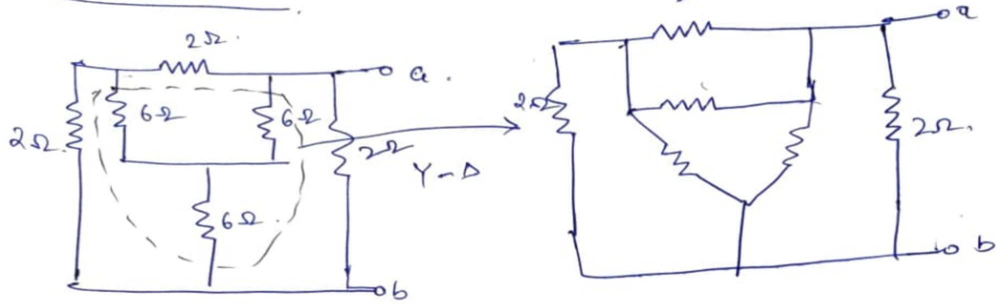
Also,

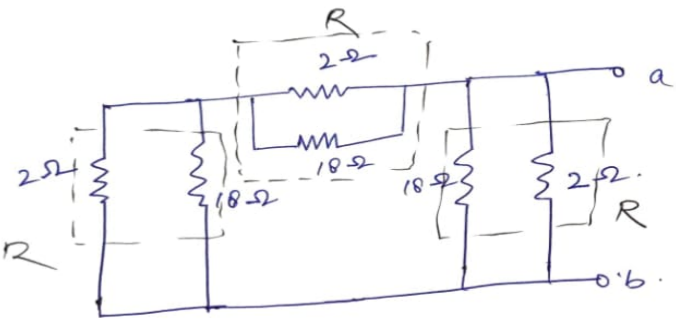
$$\boxed{\begin{aligned} V_{ab} &= 12-2i \\ &= 9.6V \end{aligned}}$$

Solution-2

4.61 Solution-2 (Y–Δ conversion)

To find R_{Th}:-

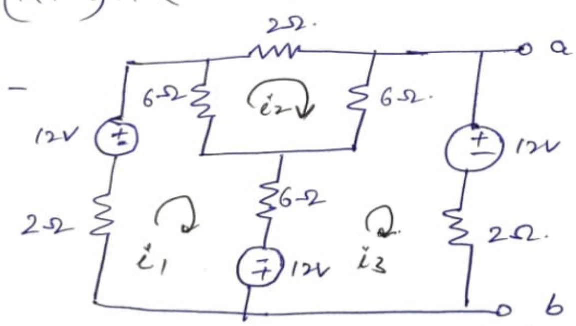




$R = 2 \| 18$
$= 1.08\,\Omega$

$R_{Th} = (R+R) \| R = 1.2\,\Omega$.

To find V_{Th}:-



Applying Mesh analysis:- $2i_1 - 12 + 6(i_1 - i_2) + 6(i_1 - i_3) - 12 = 0$

Mesh 1:- $14i_1 - 6i_2 - 6i_3 - 24 = 0$ — (I)

Mesh 2:- $14i_2 - 6i_1 - 6i_3 = 0$ — (II)

Mesh 3:- $-3i_1 - 7i_2 - 3i_3 = -12$ — (III)

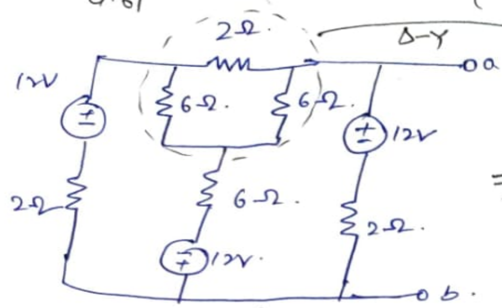
Solving (I), (II) & (III) $i_3 = -1.2A$

$V_{Th} = 12 + 2i_2 = 9.6V$ / $I_N = \frac{V_{Th}}{R_{Th}} = 8A$

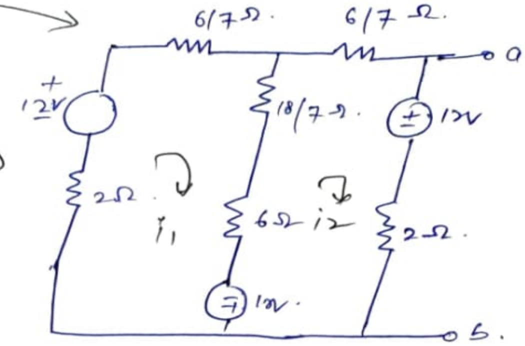
Solution-3

Solution-3 (Δ-Y conversion)

4.61

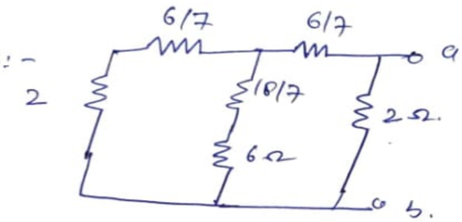


(1)



(2)

Rth:-



$$R_{Th} = \left(\left[\left(2+6/7\right) \parallel \left(\frac{18}{7}+6\right)\right] + 6/7\right) \parallel 2$$

$$= 1.2\Omega$$

Vth:- Applying Mesh analysis in fig. (2)

$$-12 + 2i_1 + 6/7\, i_1 + 18/7\, i_1 + 6 i_1 - 12 - 18/7\, i_2 - 6 i_2 = 0$$

$$-24 + \left(2 + \frac{6}{7} + \frac{18}{7} + 6\right) i_1 - \left(6 + \frac{18}{7}\right) i_2 = 0 \quad \text{— (I)}$$

$$12 + 6(i_2 - i_1) + 18/7\,(i_2 - i_1) + 6/7\, i_2 + 12 + 2 i_2 = 0$$

$$24 + \left(6 + \frac{18}{7} + \frac{6}{7} + 2\right) i_2 - \left(6 + \frac{18}{7}\right) i_1 = 0 \quad \text{— (II)}$$

Solving (I) & (II)

$V_{Th} = 9.6$ V / $I_N = \dfrac{V_{Th}}{R_{Th}} = 8A$

8.

To find $V_{Th}=V_x$, consider the left loop.

$$-3+1000i_o+2V_x=0 \longrightarrow 3=1000i_o+2V_x \qquad (1)$$

For the right loop,

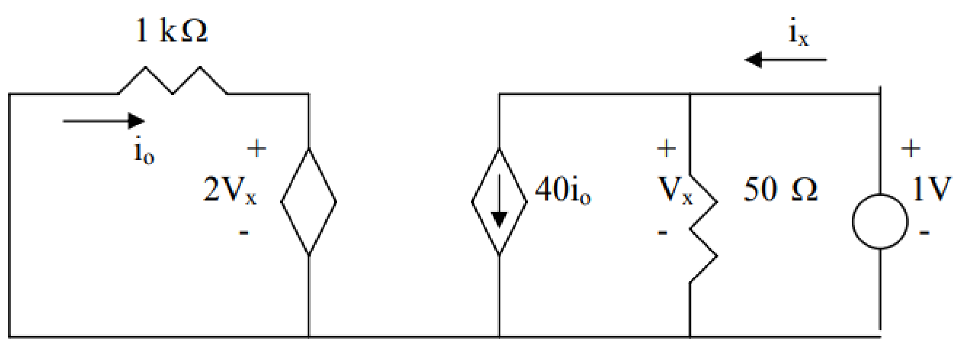
$$V_x=-50x40i_o=-2000i_o \qquad (2)$$

Combining (1) and (2),

$$3=1000i_o-4000i_o=-3000i_o \longrightarrow i_o=-1\text{mA}$$

$$V_x=-2000i_o=2 \longrightarrow \underline{V_{Th}=2}$$

To find R_{Th}, insert a 1-V source at terminals a-b and remove the 3-V independent source, as shown below.



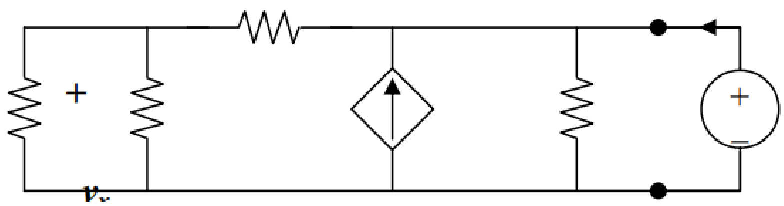
$$V_x=1, \quad i_o=-\frac{2V_x}{1000}=-2\text{mA}$$

$$i_x=40i_o+\frac{V_x}{50}=-80\text{mA}+\frac{1}{50}\text{A}=\text{-60mA}$$

$$R_{Th}=\frac{1}{i_x}=-1/0.060=\underline{-16.67\Omega}$$

9.

To find R_{Th}, remove the 50V source and insert a 1-V source at a – b, as shown in Fig. (a).



We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2, \text{ or } i + v_x = 0.6 \qquad (1)$$

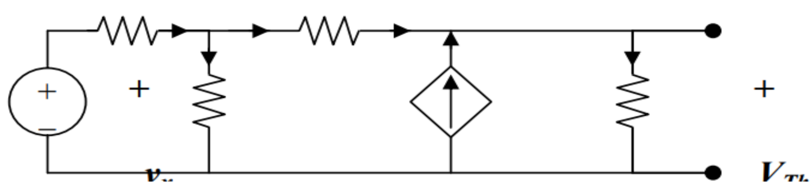
At node B,

$$(1 - v_o)/2 = (v_x/3) + (v_x/6), \text{ and } v_x = 0.5 \qquad (2)$$

From (1) and (2), $i = 0.1$ and

$$R_{Th} = 1/i = \underline{\mathbf{10\ ohms}}$$

To get V_{Th}, consider the circuit in Fig. (b).



At node 1, $(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2$, or $100 = 6v_1 - 3v_2$ (3)

At node 2, $0.5v_x + (v_1 - v_2)/2 = v_2/10$, $v_x = v_1$, and $v_1 = 0.6v_2$ (4)

From (3) and (4),

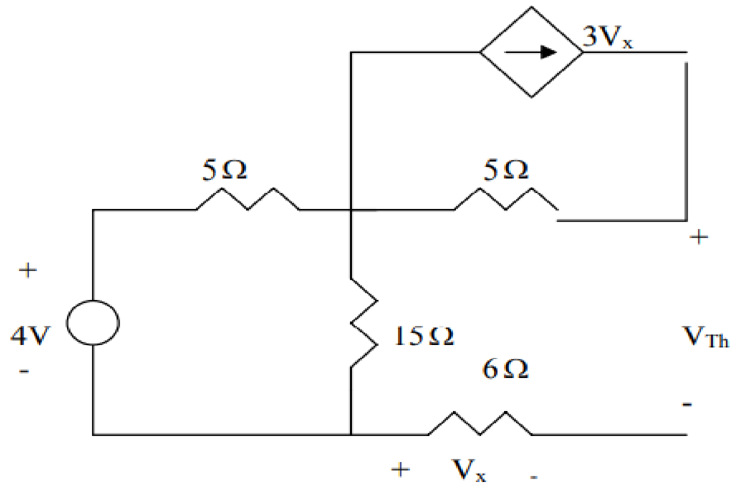
$$v_2 = V_{Th} = \underline{\mathbf{166.67\ V}}$$

$$I_N = V_{Th}/R_{Th} = \underline{\mathbf{16.667\ A}}$$

$$R_N = R_{Th} = \underline{\mathbf{10\ ohms}}$$

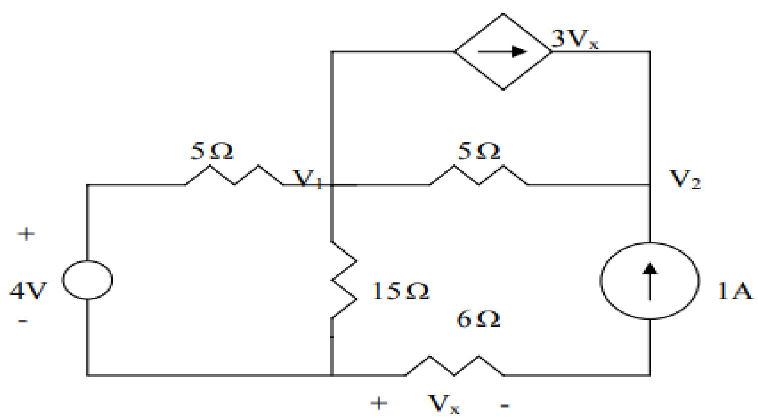
10.

We find the Thevenin equivalent across the 10-ohm resistor. To find V_{Th}, consider the circuit below.



$$V_x = 0, \qquad V_{Th} = \frac{15}{15+5}(4) = 3\text{V}$$

To find R_{Th}, consider the circuit below:



At node 1,

$$\frac{4-V_1}{5} = 3V_x + \frac{V_1}{15} + \frac{V_1 - V_2}{5}, \qquad V_x = 6x1 = 6 \qquad \longrightarrow \qquad 258 = 3V_2 - 7V_1 \qquad (1)$$

At node 2,

$$1 + 3V_x + \frac{V_1 - V_2}{5} = 0 \qquad \longrightarrow \qquad V_1 = V_2 - 95 \qquad (2)$$

Solving (1) and (2) leads to $V_2 = 101.75$ V

$$R_{Th} = \frac{V_2}{1} = 101.75\Omega, \qquad p_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{9}{4x101.75} = \underline{22.11\text{ mW}}$$