

## SOLUTION (Tutorial-1)

1) Ans  
(a)

$$k = 8.617 \times 10^{-5} \text{ eV/K}$$

$$n_i (T=300\text{K}) = 1.66 \times 10^{15} (300\text{K})^{3/2} \exp \left[ -\frac{0.66 \text{ eV}}{2(8.617 \times 10^{-5} \text{ eV/K})(300\text{K})} \right]$$

$$= \boxed{2.465 \times 10^{13} \text{ cm}^{-3}}$$

$$n_i (T=600\text{K}) = 1.66 \times 10^{15} (600\text{K})^{3/2} \exp \left[ -\frac{0.66 \text{ eV}}{2(8.617 \times 10^{-5} \text{ eV/K})(600\text{K})} \right]$$

$$= \boxed{4.124 \times 10^{16} \text{ cm}^{-3}}$$

Compared to the values obtained in example 2.1, we can see that the intrinsic carrier concentration in Ge at  $T=300\text{K}$  is  $\frac{2.465 \times 10^{13}}{1.08 \times 10^{10}} = 2282$  times higher

than the intrinsic carrier concentration in Si at  $T=300\text{K}$ . Similarly, at  $T=600\text{K}$ , the intrinsic carrier concentration in Ge is  $\frac{4.124 \times 10^{16}}{1.54 \times 10^{15}} = 26.8$  times higher than that in Si.

(b). Since phosphorus is a group V element, it is a donor, meaning  $N_D = 5 \times 10^{16} \text{ cm}^{-3}$ . For an n-type material, we have

$$n = N_D = \boxed{5 \times 10^{16} \text{ cm}^{-3}}$$

$$p(T=300\text{K}) = \frac{[n_i(T=300\text{K})]^2}{n} = \boxed{1.215 \times 10^{10} \text{ cm}^{-3}}$$

$$p(T=600\text{K}) = \frac{[n_i(T=600\text{K})]^2}{n} = \boxed{3.401 \times 10^{16} \text{ cm}^{-3}}$$

2) Ans →

(a). mobility of  $e^-$  in Si =  $1350 \text{ cm}^2/\text{V-s}$

mobility of holes in Si =  $480 \text{ cm}^2/\text{V-s}$

$$\text{velocity of } e^- = \mu_n E = (1350 \text{ cm}^2/\text{V-s}) \times (0.1 \text{ V}/\mu\text{m})$$
$$= \boxed{1.35 \times 10^4 \text{ m/s}}$$

$$\text{velocity of holes} = \mu_p E = (480 \text{ cm}^2/\text{V-s}) \times (0.1 \text{ V}/\mu\text{m})$$
$$= \boxed{4.8 \times 10^3 \text{ m/s}}$$

b). Given  $E = 0.1 \text{ V}/\mu\text{m}$ , Hole current is negligible,  
 $\mu_n = 1350 \text{ cm}^2/\text{V-s}$ ,  $\mu_p = 480 \text{ cm}^2/\text{V-s}$

$$J_{\text{tot}} = 1 \text{ mA}/\mu\text{m}^2 = q [\mu_n n E + \mu_p p E] \approx q \mu_n n E$$

$$\therefore n = \frac{J_{\text{tot}}}{q \mu_n E} = \frac{1 \text{ mA}/\mu\text{m}^2}{(1.6 \times 10^{-19} \text{ C}) (1350 \text{ cm}^2/\text{V-s}) (0.1 \text{ V}/\mu\text{m})}$$
$$= \boxed{4.6 \times 10^{17} \text{ cm}^{-3}}$$



3) Ans)

(a). From problem 1, we can calculate  $n_i$  for Ge.

$$n_i (T=300K) = 2.465 \times 10^{13} \text{ cm}^{-3}$$

$$I_{\text{tot}} = q(n\mu_n + p\mu_p)AE$$

$$n = 10^{17} \text{ cm}^{-3}$$

$$p = \frac{n_i^2}{n} = 6.076 \times 10^9 \text{ cm}^{-3}$$

$$\mu_n = 3900 \text{ cm}^2/\text{V}\cdot\text{s}, \mu_p = 1900 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$E = V/d = \frac{1\text{V}}{0.1\mu\text{m}} = \boxed{10^5 \text{ V/cm}}$$

$$A = 0.05\mu\text{m} \times 0.05\mu\text{m} = \boxed{2.5 \times 10^{-11} \text{ cm}^2}$$

Since  $n\mu_n \gg p\mu_p$ , we can write

$$I_{\text{tot}} \approx qn\mu_nAE = 1.6 \times 10^{-19} \times 10^{17} \times 3900 \times 2.5 \times 10^{-11} \times 10^5$$
$$= \boxed{156 \mu\text{A}}$$

(b). All of the parameters are the same except  $n_i$ , which means we must re-calculate  $p$ .

$$n_i (T=400K) = 9.230 \times 10^{14} \text{ cm}^{-3}$$

$$p = \frac{n_i^2}{n} = 8.520 \times 10^{12} \text{ cm}^{-3}$$

Since  $n\mu_n \gg p\mu_p$  still holds (note that  $n$  is 5 orders of magnitude larger than  $p$ ), the hole concentration once again drops out of the equation and we have

$$I_{\text{tot}} \approx qn\mu_nAE$$

$$= \boxed{156 \mu\text{A}}$$

4) Ans →

$$(a). \quad n_n = N_D = \boxed{5 \times 10^{17} \text{ cm}^{-3}}$$

$$P_n = \frac{n_i^2}{n_n} = \boxed{233 \text{ cm}^{-3}}$$

$$P_p = N_A = \boxed{4 \times 10^{16} \text{ cm}^{-3}}$$

$$n_p = \frac{n_i^2}{P_p} = \boxed{2916 \text{ cm}^{-3}}$$

(b). We can express the formula for  $V_0$  in its full form, showing its temperature dependence:

$$V_0(T) = \frac{kT}{q} \ln \left[ \frac{N_A N_D}{(5.2 \times 10^{15})^2 T^3 e^{-(E_g/kT)}} \right]$$

$$V_0(T=250\text{K}) = \boxed{906 \text{ mV}}, \quad V_0(T=300\text{K}) = \boxed{849 \text{ mV}},$$

$$V_0(T=350\text{K}) = \boxed{789 \text{ mV}}$$

Looking at the expression for  $V_0(T)$ , we can expand it as follows:

$$V_0(T) = \frac{kT}{q} \left[ \ln(N_A) + \ln(N_D) - 2 \ln(5.2 \times 10^{15}) - 3 \ln(T) + \frac{E_g}{kT} \right]$$

Let's take the derivative of this expression to get a better idea of how  $V_0$  varies with temperature.

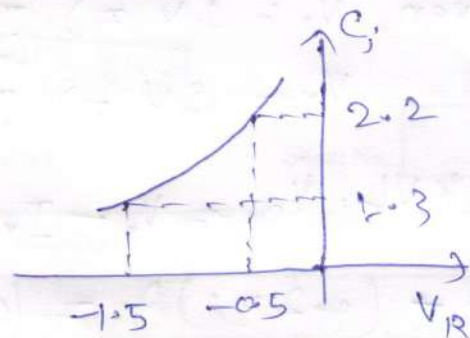
$$\frac{dV_0(T)}{dT} = \frac{k}{q} \left[ \ln(N_A) + \ln(N_D) - 2 \ln(5.2 \times 10^{15}) - 3 \ln(T) - 3 \right]$$

From this expression, we can see that if  $\ln(N_A) + \ln(N_D) < 2 \ln(5.2 \times 10^{15}) + 3 \ln(T) + 3$ , or equivalently, if  $\ln(N_A N_D) < \ln[(5.2 \times 10^{15})^2 T^3] - 3$ , then  $V_0$  will decrease with temperature, which we observe in this case. In order for this not to be true (i.e., in order for  $V_0$  to increase with temperature), we must have either very high doping concentration or very low temp



5) Ans)

$$\frac{C_{j0}}{\sqrt{1 + \frac{0.5}{V_0}}} = 2.2 \quad \text{--- (1)}$$



$$\frac{C_{j0}}{\sqrt{1 + \frac{1.5}{V_0}}} = 1.3 \quad \text{--- (2)}$$

(1) divided by (2)

$$\frac{1 + \frac{1.5}{V_0}}{1 + \frac{0.5}{V_0}} = \left( \frac{2.2}{1.3} \right)^2 \Rightarrow V_0 = 0.0365 \text{ V}$$

substitute  $V_0$  into (1):

$$C_{j0} = 2.2 \times \sqrt{1 + \frac{0.5}{V_0}} \approx 8.43 \text{ fF}/\mu\text{m}^2$$

$$\Rightarrow \frac{N_A N_D}{N_A + N_D} = \frac{(C_{j0})^2 V_0 \cdot 2}{\epsilon_{\text{Si}} q} = \frac{(8.43 \text{ fF}/\mu\text{m}^2)^2 \times (0.0365 \text{ V}) \times 2}{\epsilon_{\text{Si}} q} \approx 3.13 \times 10^{17} \text{ cm}^{-3}$$

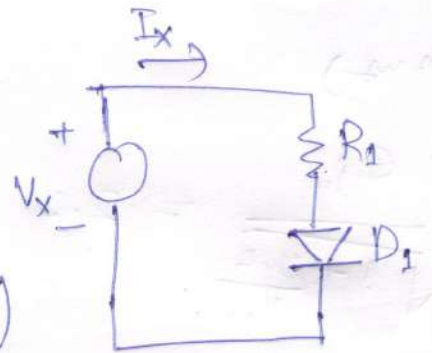
Fix a value for  $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq \gamma$

$$\begin{aligned} N_A = 2 \times 10^{18} \text{ cm}^{-3} &\Rightarrow N_D = \frac{\gamma N_A}{N_A - \gamma} \\ &= \frac{(3.13 \times 10^{17} \text{ cm}^{-3}) (2 \times 10^{18} \text{ cm}^{-3})}{(2 \times 10^{18} - 3.13 \times 10^{17}) \text{ cm}^{-3}} \\ &\approx \boxed{3.71 \times 10^{17} \text{ cm}^{-3}} \end{aligned}$$

6) Ans

Given,  $V_x = 1V \Rightarrow I_x = 0.2mA$

$V_x = 2V \Rightarrow I_x = 0.5mA$



By KVL,  $V_{D1} = V_x - I_x R_1 = V_T \ln\left(\frac{I_x}{I_s}\right)$

$\Rightarrow 1 - (0.2mA)R_1 = (0.026V) \ln\left(\frac{0.2mA}{I_s}\right)$  — (1)

$2 - (0.5mA)R_1 = (0.026V) \ln\left(\frac{0.5mA}{I_s}\right)$  — (2)

subtract (1) from (2) :

$1 - (0.3mA)R_1 = (0.026V) \ln\left(\frac{0.5}{0.2}\right)$

$\Rightarrow R_1 = \frac{1 - (0.026)V}{0.3mA} = \boxed{3.25k\Omega}$

Substitute  $R_1$  into (1) :

$I_s = I_x \exp\left[-\frac{V_x - I_x R_1}{V_T}\right]$

$= (0.2mA) \exp\left[-\frac{1 - (0.2mA)(3.25k)}{0.026}\right] \approx \boxed{2.94 \times 10^{-10} A}$

$\therefore \boxed{R_1 \approx 3.25k\Omega}$

$\boxed{I_s \approx 2.94 \times 10^{-10} A}$

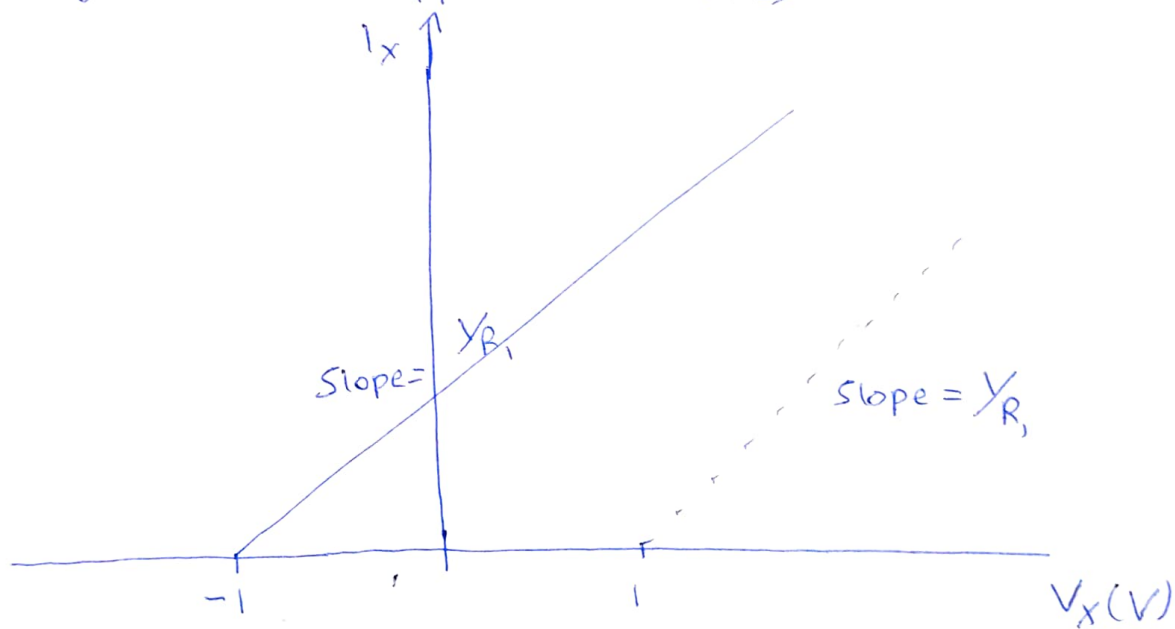
~~Soln 7~~

### Soln 7

- When  $V_B < 0$  the diode will be on even when  $V_X = 0$   
When  $V_B > 0$  the diode will be in ON state when  $V_X > V_B$

$$I_X = \begin{cases} 0 & V_X < V_B \\ \frac{V_X - V_B}{R_1} & V_X > V_B \end{cases}$$

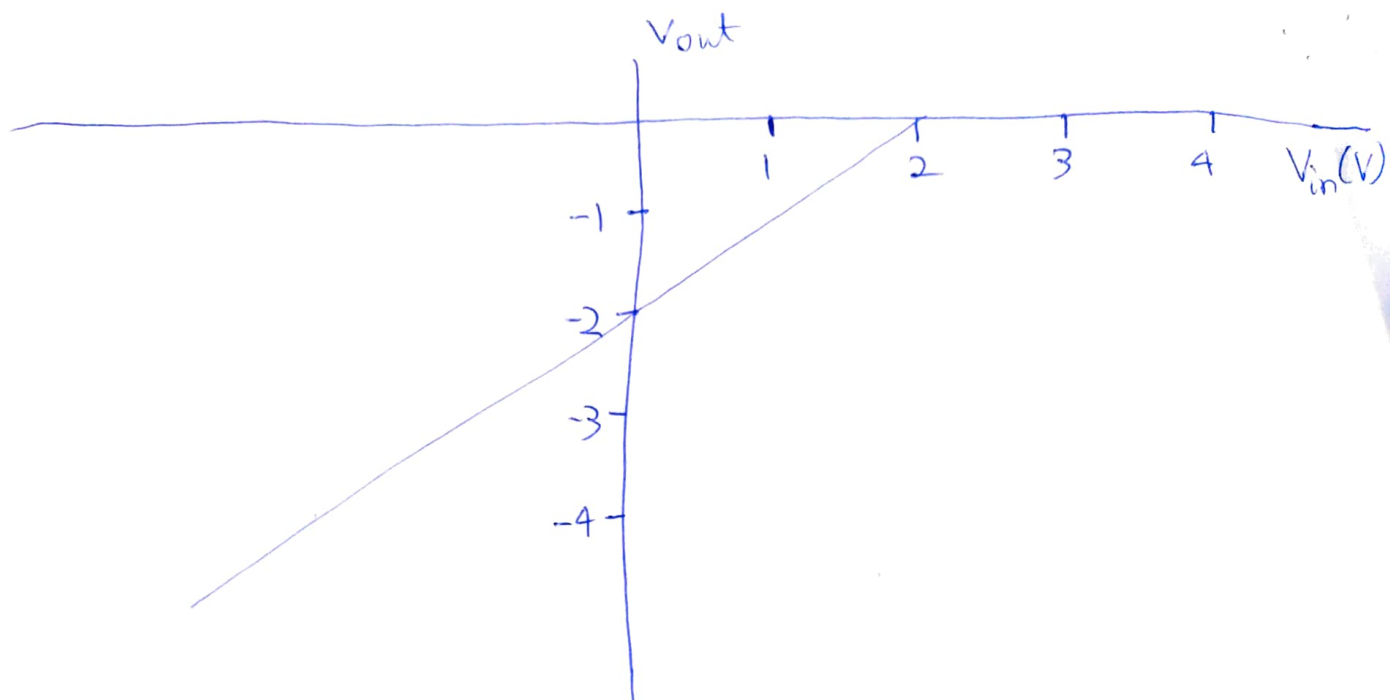
We will get two different curves



### Soln 8

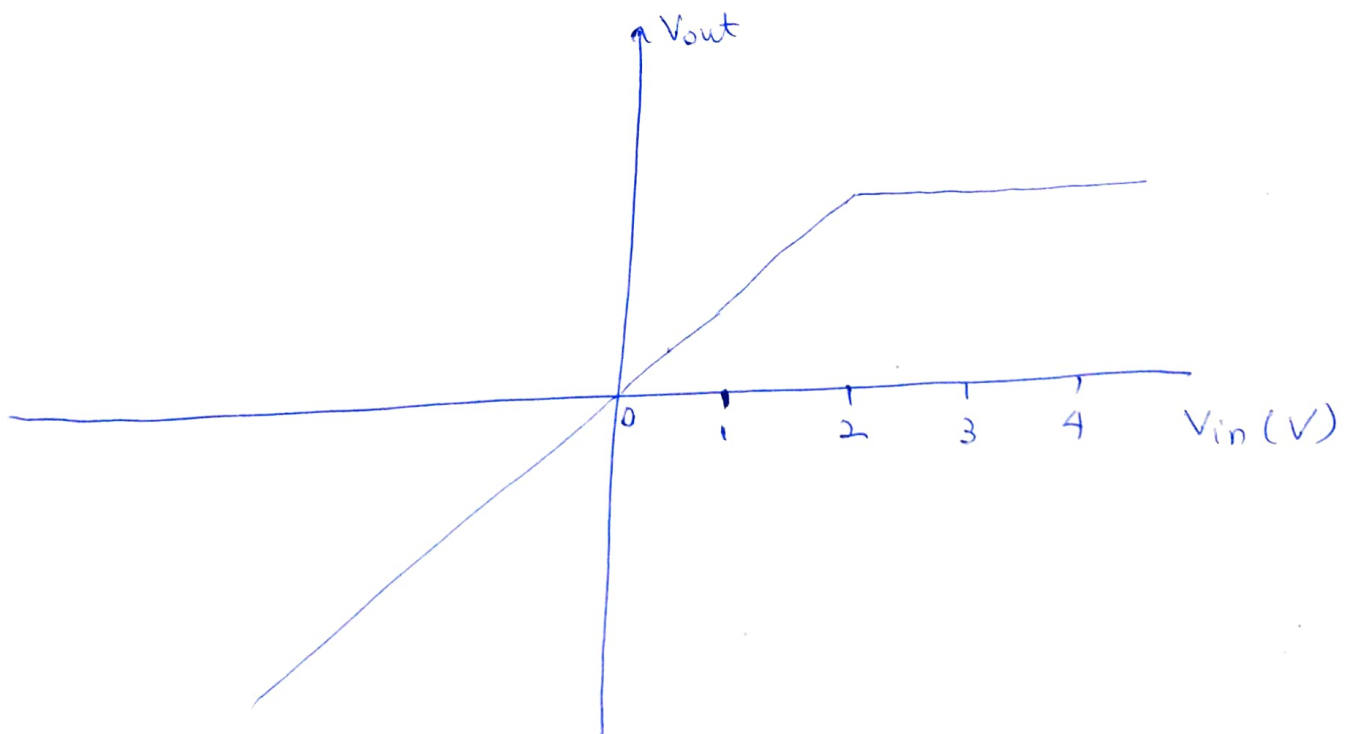
(a)  $V_{out}$  will be zero when diode is reversed biased

$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



(b) When diode is in OFF states  $V_{out} = V_B$  otherwise  $V_{out} = V_{in}$  .

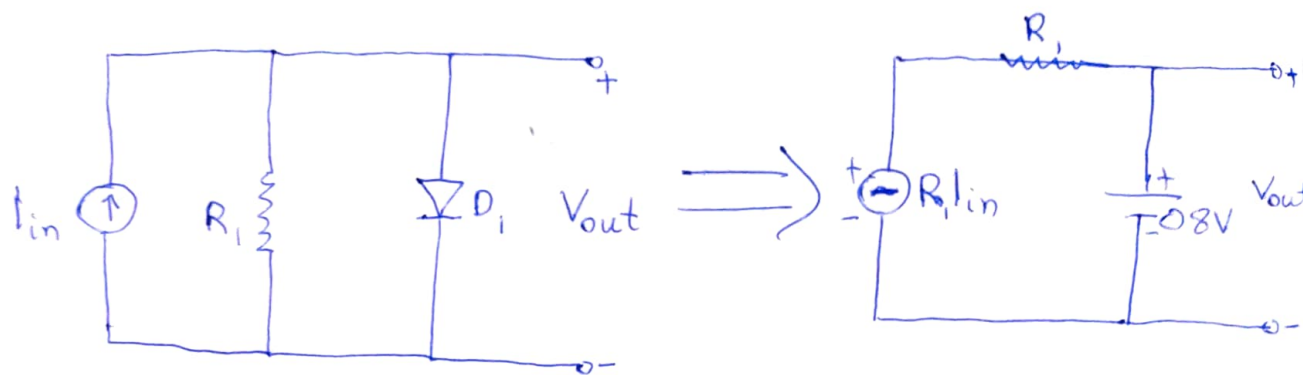
$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$





Soln Q9

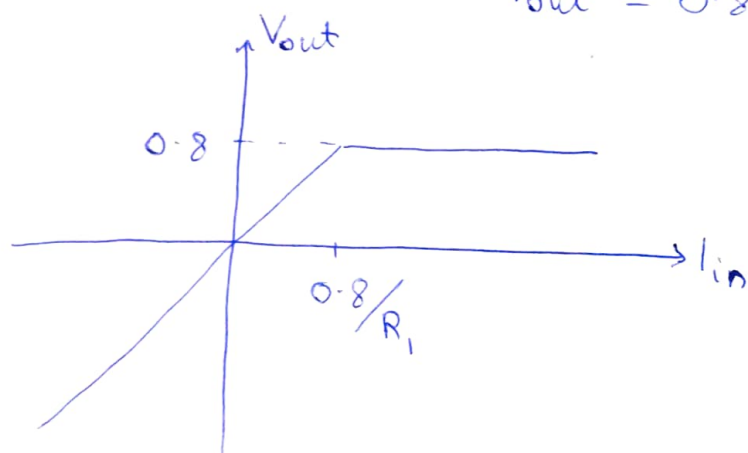
(a)



Converting current source to voltage source and applying constant voltage of diode  $0.8V$

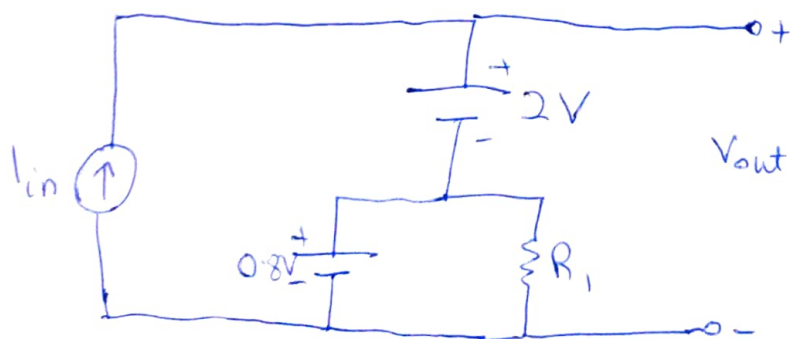
When diode is OFF  $V_{out} = R_1 I_{in}$

When diode is ON  $V_{out} = 0.8V$



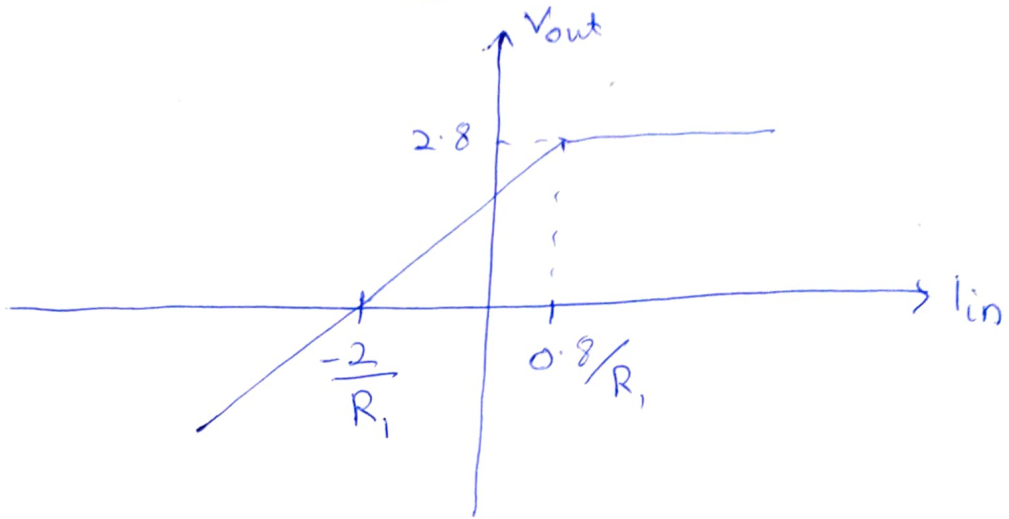
(b)

Considering  $V_B = 2V$  and constant voltage of diode of  $0.8V$



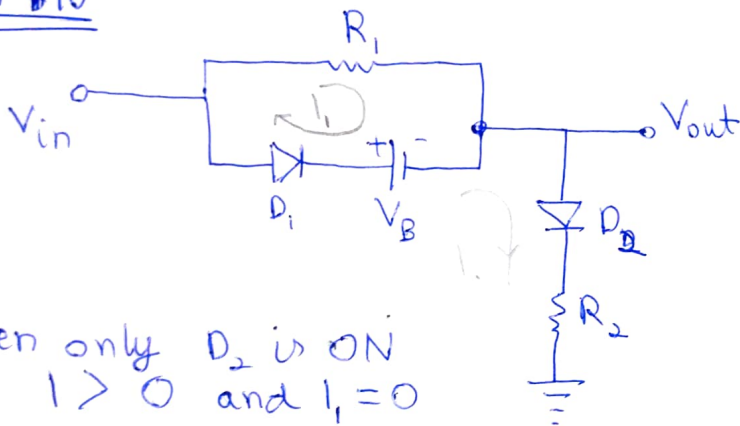
When diode is OFF  $V_{out} = 2 + I_{in} R_1$

When diode is ON  $V_{out} = 2 + 0.8 = 2.8$



Soln 10

(a)



$V_{on}$  is constant voltage of diode

When only  $D_2$  is ON  
 $I > 0$  and  $I_1 = 0$

$$V_{in} = I(R_1 + R_2) + V_{on} \quad - (1)$$

$$I = \frac{V_{in} - V_{on}}{R_1 + R_2} \Rightarrow \frac{V_{in} - V_{on}}{R_1 + R_2} > 0$$

$$V_{in} > V_{on}$$

For diode  $D_1$  to be in ON state

$$V_{in} - V_{out} > V_B + V_{on}$$

$$V_{out} = I R_2 + V_{on} \quad - (2)$$

$$V_{in} = I_1 R_1 + V_{on} + I R_2$$

$$I = \frac{V_{in} - V_{on}}{R_1 + R_2} \rightarrow \text{from ①}$$

$$V_{in} - V_{out} > V_B + V_{on}$$

$$V_{in} - (I R_2 - V_{on}) > V_B - V_{on} \rightarrow \text{by substituting eq ②}$$

$$V_{in} - \left( \frac{V_{in} - V_{on}}{R_1 + R_2} R_2 + V_{on} \right) > V_{on} + V_B \rightarrow \text{substituting value of } I$$

Solving the above in equation we will get

$$V_{in} > \frac{2 V_{on} R_1 + V_B (R_1 + R_2) + V_{on} R_2}{R_1}$$

$$\left[ V_{in} > V_{on} + \frac{R_1 + R_2}{R_1} (V_{on} + V_B) \right]$$

condition for both diodes to be in ON state

When ~~both~~ only  $D_2$  is in ON state

$$V_{in} = I R_1 + V_{on} + I R_2$$

$$I = \frac{V_{in} - V_{on}}{R_1 + R_2}$$

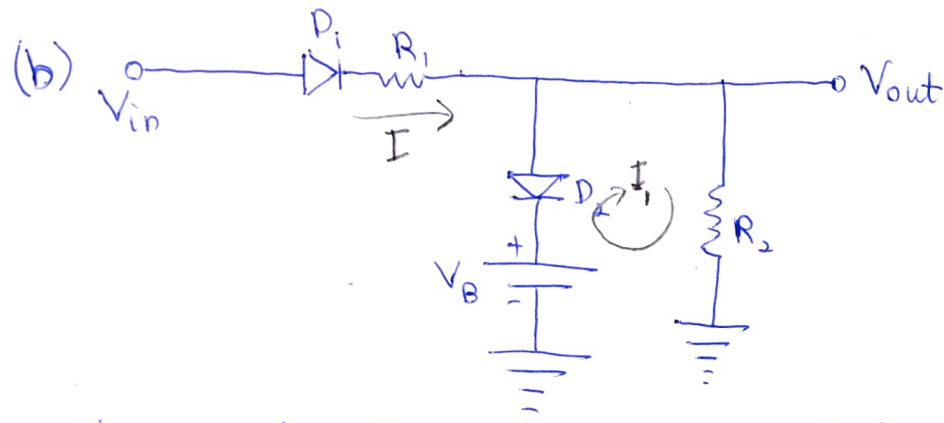
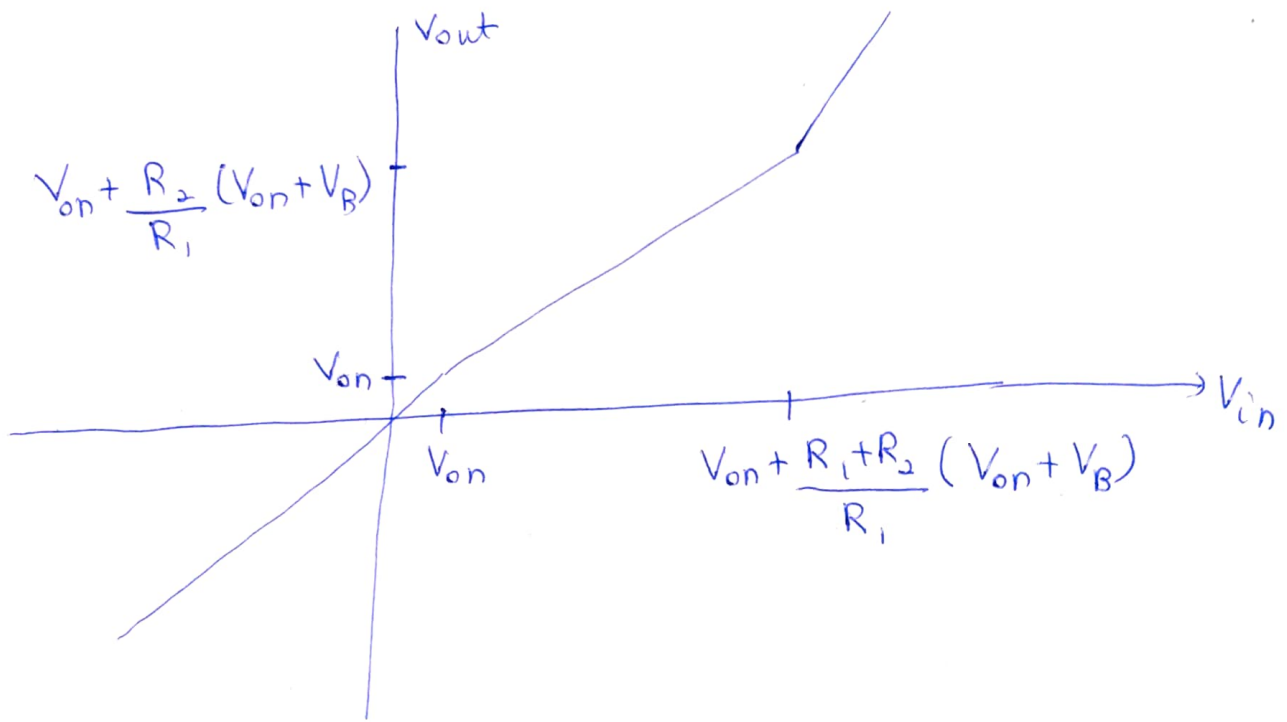
$$V_{out} = V_{on} + \frac{(V_{in} - V_{on}) R_2}{R_1 + R_2}$$

When both diodes are in ON state

$$V_{in} = V_{on} + V_B + V_{out}$$

$$V_{out} = V_{in} - V_{on} - V_B$$





When both diode in OFF state  
 $V_{out} = 0$

$D_1$  ON     $D_2$  OFF     $\rightarrow$  condition  $I > 0$

$$V_{in} = V_{on} + IR_1 + IR_2$$

$$I = \frac{V_{in} - V_{on}}{R_1 + R_2} \Rightarrow \frac{V_{in} - V_{on}}{R_1 + R_2} > 0$$

$V_{in} > V_{on}$   $\Rightarrow$  condition for  $D_1$  in ON state only

$D_1$  ON     $D_2$  ON

$$V_{in} = V_{on} + IR_1 + V_{on} + V_B$$

$$V_{in} = V_{on} + IR_1 + IR_1$$

$$I = \frac{V_{in} - 2V_{on} - V_B}{R_1}$$

For  $D_2$  in ON state  $V_{out} > V_B + V_{on}$

$$V_{out} = I R_2$$

$$\therefore \frac{R_2}{R_1} (V_{in} - 2V_{on} - V_B) > V_B + V_{on}$$

$$R_2 (V_{in} - 2V_{on} - V_B) > R_1 (V_B + V_{on})$$

$$R_2 V_{in} > R_1 (V_B + V_{on}) + (2V_{on} + V_B) R_2$$

$$V_{in} > V_{on} + \frac{R_1 + R_2}{R_2} (V_B + V_{on})$$

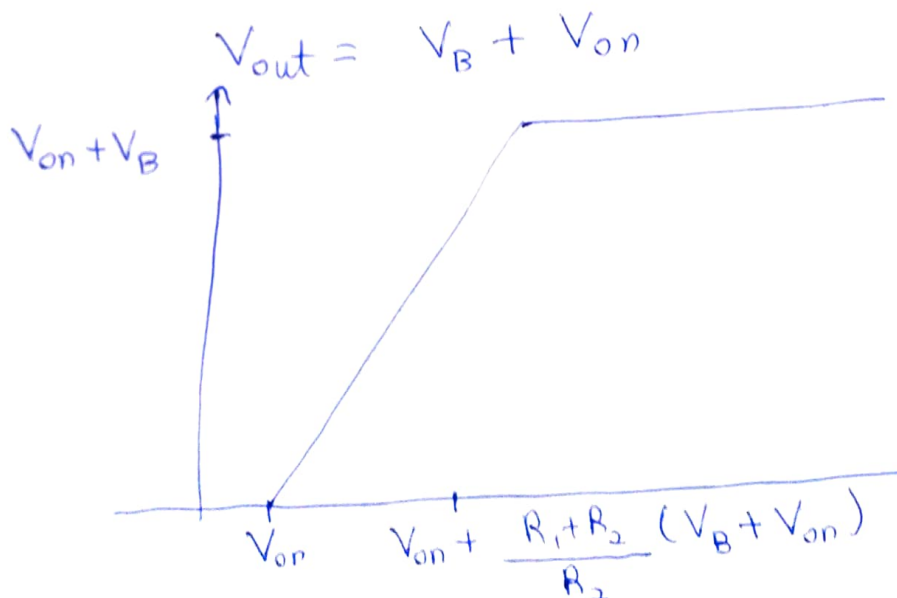


condition for  $D_1$  ON and  $D_2$  ON

For  $D_1$  in ON state and  $D_2$  OFF

$$V_{out} = \frac{R_2 (V_{in} - V_{on})}{R_1 + R_2}$$

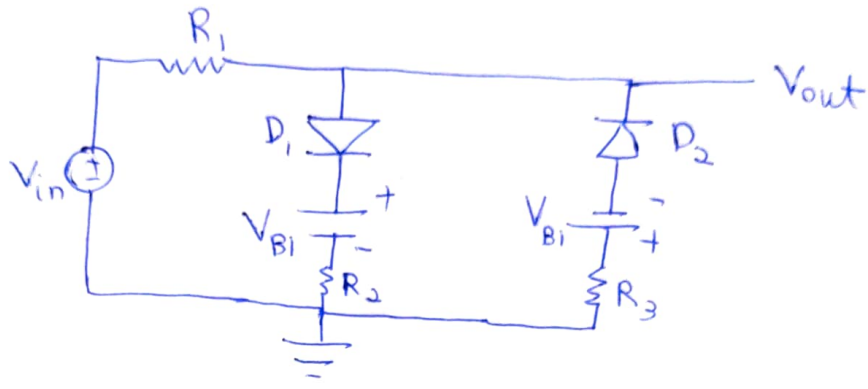
For  $D_1$  ON and  $D_2$  ON



Q4

Soln

The required circuit is



$$V_{B1} = V_{B2} = 2 - 0.8 = 1.2 \text{ V}$$

For  $V_{in} > 2 \text{ V}$

$\frac{V_{out}}{V_{in}}$  has a value of 0.5

$$\therefore R_2 = R_1$$

Similarly  $R_3 = R_1$

$$\therefore R_1 = R_2 = R_3 = 1 \text{ k}\Omega$$