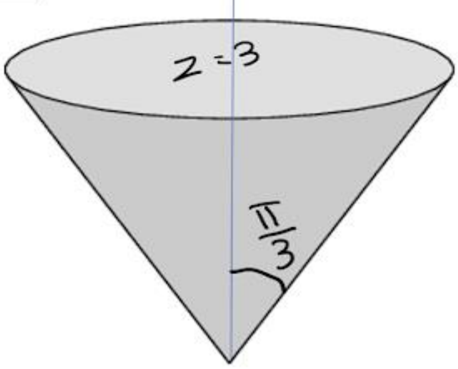
1 Determine the flux of $\vec{F} = \rho^2 \cos^2\phi\,\hat{e}_\rho + z\sin\phi\,\hat{e}_\phi$ over the closed cylinder $0 \le z \le 1$, ρ=4. Show that $\iiint\limits_V \vec{\nabla}.\vec{F}dV = \iint\limits_S \vec{F}.\vec{dS}$. All relevant steps carry marks

4 marks

2 Find the volume of a cone whose angle is $\frac{\pi}{3}$ and below the plane $z = 3$ using the **<u>spherical polar coordinate system.</u>**



4 marks

3 Express the below mentioned integral into cylindrical polar co-ordinate system. Note: No need to work out the integral, but simply express the integral.

$$\int_{y=-1}^{y=1}\int_{x=0}^{x=\sqrt{1-y^2}}\int_{z=x^2+y^2}^{z=\sqrt{x^2+y^2}} xyzdzdxdy$$

The range of limits are

$$-1 \le y \le 1$$

$$0 \le x \le \sqrt{1-y^2}$$

$$x^2+y^2 \le z \le \sqrt{x^2+y^2}$$

4 marks

4 A force is described by

$\vec{F} = -\hat{e}_x \frac{y}{x^2+y^2} + \hat{e}_y \frac{x}{x^2+y^2}$

(a) Express $\vec{F}$ in cylindrical polar co-ordinates

(b) Calculate curl of $\vec{F}$ in cylindrical polar co-ordinates

3 marks

Cylindrical Coordinates

$q_1 = \rho,\quad q_2 = \varphi,\quad q_3 = z;\quad h_1 = h_\rho = 1,\quad h_2 = h_\varphi = \rho,\quad h_3 = h_z = 1,$

Spherical Polar Coordinates

$q_1 = r,\quad q_2 = \theta,\quad q_3 = \varphi;\quad h_1 = h_r = 1,\quad h_2 = h_\theta = r,\quad h_3 = h_\varphi = r\sin\theta,$

$$\nabla \cdot \mathbf{F} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(F_1h_2h_3) + \frac{\partial}{\partial q_2}(F_2h_3h_1) + \frac{\partial}{\partial q_3}(F_3h_1h_2)\right]$$

$$\nabla^2 V = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial V}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_3h_1}{h_2}\frac{\partial V}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1h_2}{h_3}\frac{\partial V}{\partial q_3}\right)\right]$$

$$\nabla \times \mathbf{F} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\,\hat{\mathbf{q}}_1 & h_2\,\hat{\mathbf{q}}_2 & h_3\,\hat{\mathbf{q}}_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1F_1 & h_2F_2 & h_3F_3 \end{vmatrix}$$