19 Dec 2022

EE101 Tutorial 5 Solution Sheet

1.

Given,

2\. $V_{out} = \alpha \tanh[\beta(V_{in_1} - V_{in_2})]$

For finding small-signal gain,

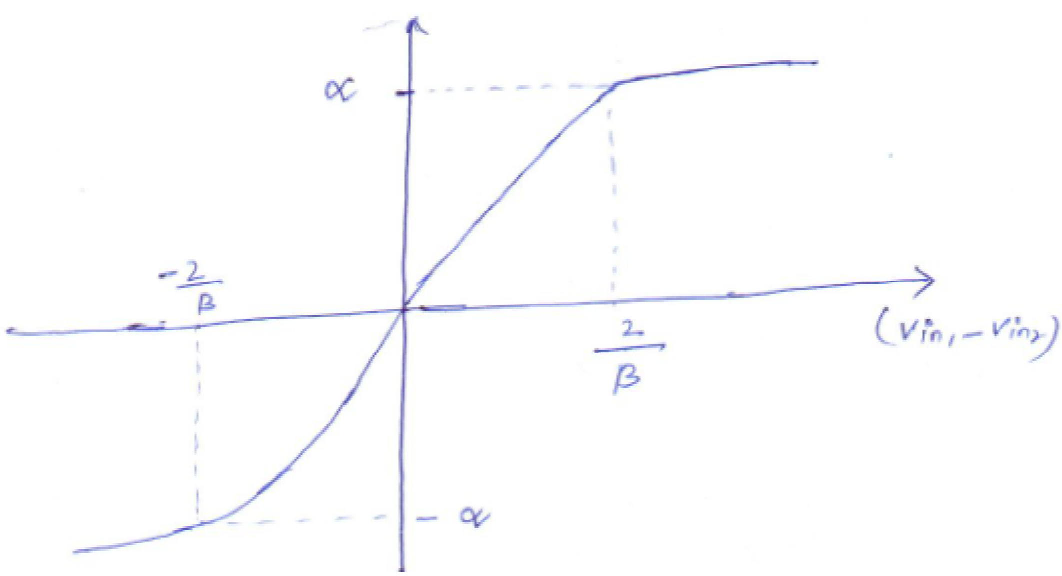
$$\frac{dV_{out}}{d(V_{in_1} - V_{in_2})} = \frac{d}{d(V_{in_1} - V_{in_2})}\left[\alpha \tanh[\beta(V_{in_1} - V_{in_2})]\right]$$

$$= \alpha \frac{d}{d(V_{in_1} - V_{in_2})}\left[\tanh[\beta(V_{in_1} - V_{in_2})]\right]$$

$$= \alpha \frac{d}{d(V_{in_1} - V_{in_2})}\left[\beta(V_{in_1} - V_{in_2}) - \frac{1}{3}(\beta(V_{in_1} - V_{in_2}))^3 + \dots\right]$$

$$\left(\because \tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots\right)$$

$$= \alpha\beta$$



2.

$$
\begin{aligned}
V_- &= V_+ = V_{in} \\
V_- &= \frac{R_4 \parallel (R_2+R_3)}{R_1 + R_4 \parallel (R_2+R_3)} \frac{R_2}{R_2+R_3} V_{out} = V_{in} \\
\frac{V_{out}}{V_{in}} &= \left[\frac{R_4 \parallel (R_2+R_3)}{R_1 + R_4 \parallel (R_2+R_3)} \frac{R_2}{R_2+R_3} \right]^{-1} \\
&= \boxed{\frac{(R_2+R_3)\left[R_1 + R_4 \parallel (R_2+R_3)\right]}{R_2\left[R_4 \parallel (R_2+R_3)\right]}}
\end{aligned}
$$

If $R_1 \to 0$, we expect the result to be:

$$
\begin{aligned}
V_{in} &= \frac{R_2}{R_2+R_3} V_{out} \\
\left.\frac{V_{out}}{V_{in}}\right|_{R_1=0} &= \frac{R_2+R_3}{R_2} = 1 + \frac{R_3}{R_2}
\end{aligned}
$$

Taking limit of the original expression as $R_1 \to 0$, we have:

$$
\begin{aligned}
\lim_{R_1 \to 0} \frac{(R_2+R_3)\left[R_1 + R_4 \parallel (R_2+R_3)\right]}{R_2\left[R_4 \parallel (R_2+R_3)\right]} &= \frac{(R_2+R_3)\left[R_4 \parallel (R_2+R_3)\right]}{R_2\left[R_4 \parallel (R_2+R_3)\right]} \\
&= 1 + \frac{R_3}{R_2}
\end{aligned}
$$

This agrees with the expected result. Likewise, if $R_3 \to 0$, we expect the result to be:

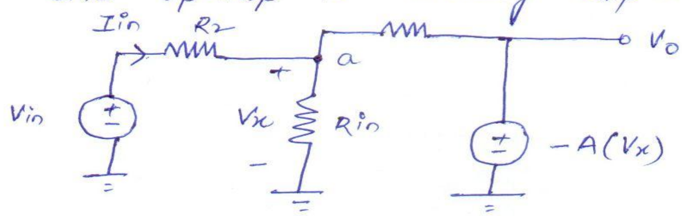
$$
\begin{aligned}
V_{in} &= \frac{R_2 \parallel R_4}{R_1 + R_2 \parallel R_4} V_{out} \\
\left.\frac{V_{out}}{V_{in}}\right|_{R_3=0} &= \frac{R_1 + R_2 \parallel R_4}{R_2 \parallel R_4} \\
&= 1 + \frac{R_1}{R_2 \parallel R_4}
\end{aligned}
$$

Taking the limit of the original expression as $R_3 \to 0$, we have:

$$
\begin{aligned}
\lim_{R_3 \to 0} \frac{(R_2+R_3)\left[R_1 + R_4 \parallel (R_2+R_3)\right]}{R_2\left[R_4 \parallel (R_2+R_3)\right]} &= \frac{R_2\,(R_1 + R_2 \parallel R_4)}{R_2\,(R_2 \parallel R_4)} \\
&= \frac{R_1 + R_2 \parallel R_4}{R_2 \parallel R_4} \\
&= 1 + \frac{R_1}{R_2 \parallel R_4}
\end{aligned}
$$

3.

3. Since op-amp is inverting ampr.:



Also, $V_0 = -AV_x$

$$\frac{V_{in} - V_x}{R_2} + \frac{V_0 - V_x}{R_1} = \frac{V_x}{R_{in}}$$

Putting $V_x = \frac{V_0}{-A}$

$$\frac{V_{in} - \left(\frac{V_0}{-A}\right)}{R_2} + \frac{V_0 - \left(\frac{V_0}{-A}\right)}{R_1} = \frac{\left(\frac{V_0}{-A}\right)}{R_{in}}$$

$$\Rightarrow \frac{V_{in}}{R_2} = -\frac{V_0}{A}\frac{1}{R_2} - \frac{V_0}{R_1} - \frac{V_0}{AR_1} - \frac{V_0}{AR_{in}}$$

$$\Rightarrow \frac{V_{in}}{R_2} = -\frac{V_0}{A}\left[\frac{1}{R_2} - \frac{A}{R_1} - \frac{1}{R_1} - \frac{1}{R_{in}}\right]$$

$$\Rightarrow \boxed{\frac{V_0}{V_{in}} = \frac{-A R_{in} R_1}{R_1 R_{in} + R_2 R_{in} A + R_2 R_{in} + R_1 R_2}}$$

Input Impedance (Z_{in}) $= \frac{V_{in}}{I_{in}}$

Applying KCL at node a: $I_{in} - \frac{V_x}{R_{in}} + \frac{(-AV_x - V_x)}{R_1} = 0$

$$\Rightarrow \boxed{I_{in} = V_x\left[\frac{1}{R_{in}} + \frac{(A+1)}{R_1}\right]}$$

Also, $V_x = V_{in} - I_{in} R_2$

$$I_{in} = (V_{in} - I_{in}R_2)\left[\frac{1}{R_{in}} + \frac{(A+1)}{R_1}\right] = \frac{\cancel{V_{in}}}{R_{in}} V_{in}\left(\frac{1}{R_{in}} + \frac{(A+1)}{R_1}\right) - I_{in}R_2\left(\frac{1}{R_{in}} + \frac{(A+1)}{R_1}\right)$$

$$\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1)}{1 + A+1}$$

//

4.

$$
\begin{aligned}
V_+ &= V_- \text{ (since } A_0 = \infty) \\
V_X &= \frac{R_3}{R_3+R_4} V_{out} = \frac{R_2}{R_1+R_2}(V_{out}-V_{in}) + V_{in} \\
V_{out}\left(\frac{R_3}{R_3+R_4} - \frac{R_2}{R_1+R_2}\right) &= V_{in}\left(1 - \frac{R_2}{R_1+R_2}\right) \\
V_{out}\left[\frac{R_3(R_1+R_2) - R_2(R_3+R_4)}{(R_1+R_2)(R_3+R_4)}\right] &= V_{in}\left(\frac{R_1}{R_1+R_2}\right) \\
\frac{V_{out}}{V_{in}} &= \boxed{\frac{R_1(R_3+R_4)}{R_3(R_1+R_2) - R_2(R_3+R_4)}}
\end{aligned}
$$

5.

5. $V_{out} = -\frac{1}{R_1 C_1} \int V_{in}\, dt$

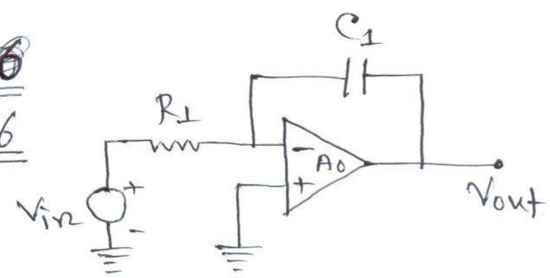
$= -\frac{1}{R_1 C_1} \int V_0 \sin \omega t \, dt$

$= -\frac{V_0}{R_1 C_1 \omega} \times \cos \omega t$

$\therefore$ $\boxed{\text{Amplitude of output signal} = \frac{V_0}{R_1 C_1 \omega}}$

6.

8.6



Given!

$A = \text{Gain} = 10$

$A_0 \to \infty$ & $R_1C_1 = 10ns$

$f = ??$

Output of inverting amplifier,

$$V_{out} = -\frac{X_c}{R_1} V_{in} = -\frac{(1/sC_1)}{R_1} V_{in}$$

$$= -\frac{1}{sR_1C_1} V_{in}$$

$$= -\frac{1}{(j\omega)R_1C_1} V_{in}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = A = \frac{1}{\omega R_1 C_1} = 10 \quad (\text{Given})$$

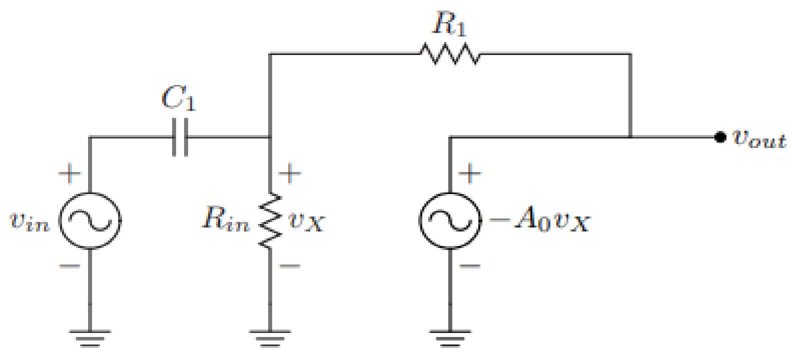
$$\therefore \quad 2\pi f R_1 C_1 = \frac{1}{10}$$

$$\Rightarrow f = \frac{1}{10 \times 2\pi \times 10 \times 10^{-9}}$$

$$= 10^7/2\pi$$

$$\boxed{f = 1.59 \text{ MHz}}$$

7.



$$v_{out} = -A_0 v_X$$

$$v_X = \left[(v_{in} - v_X)\, sC_1 - \frac{v_X - v_{out}}{R_1} \right] R_{in}$$

$$v_X \left[1 + sR_{in}C_1 + \frac{R_{in}}{R_1} \right] = v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}$$

$$v_X = \frac{v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out} = -A_0 \frac{v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out} \left[1 + \frac{A_0 \frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}} \right] = -v_{in} \frac{sR_{in}C_1 A_0}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out} \left[\frac{1 + sR_{in}C_1 + (1 + A_0)\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}} \right] = -v_{in} \frac{sR_{in}C_1 A_0}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out} \left[1 + sR_{in}C_1 + (1 + A_0)\frac{R_{in}}{R_1} \right] = -v_{in} sR_{in}C_1 A_0$$

$$\frac{v_{out}}{v_{in}} = \boxed{-\frac{sR_1R_{in}C_1A_0}{R_1 + sR_1R_{in}C_1 + (1 + A_0)\,R_{in}}}$$

$$\lim_{A_0 \to \infty} \frac{v_{out}}{v_{in}} = -sR_1C_1$$

8.

$$v_{out} = -A_0 v_-$$

$$v_- = v_{in} + (v_{out} - v_{in}) \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)}$$

$$v_{out} = -A_0 \left[v_{in} + (v_{out} - v_{in}) \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right]$$

$$v_{out} \left[1 + A_0 \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right] = -v_{in} A_0 \left[1 - \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right]$$

$$v_{out} \frac{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) + A_0 \left(\frac{1}{sC_1} \parallel R_1\right)}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} = -v_{in} A_0 \frac{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) - \left(\frac{1}{sC_1} \parallel R_1\right)}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)}$$

$$v_{out} \left\{ (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1 \right) + \left(\frac{1}{sC_2} \parallel R_2 \right) \right\} = -v_{in} A_0 \left(\frac{1}{sC_2} \parallel R_2 \right)$$

$$\frac{v_{out}}{v_{in}} = \boxed{-A_0 \frac{\frac{1}{sC_2} \parallel R_2}{(1 + A_0)\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)}}$$

Unity gain occurs when the numerator and denominator are the same (note that we can drop the negative sign since we only care about the magnitude of the gain):

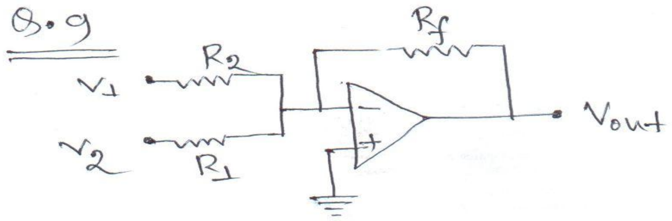
$$A_0 \left(\frac{1}{sC_2} \parallel R_2 \right) = (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1 \right) + \left(\frac{1}{sC_2} \parallel R_2 \right)$$

$$(A_0 - 1) \left(\frac{1}{sC_2} \parallel R_2 \right) = (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1 \right)$$

$$\frac{\left(\frac{1}{sC_2} \parallel R_2\right)}{\left(\frac{1}{sC_1} \parallel R_1\right)} \equiv \frac{A_0 + 1}{A_0 - 1}$$

9.

Q.9



Given:
$V_1 = V_0 \sin\omega t$
$V_2 = V_0 \sin(3\omega t)$
$R_1 = R_2 = R$ (say)
$A_0 = \infty$

we know the output of adder ckt,

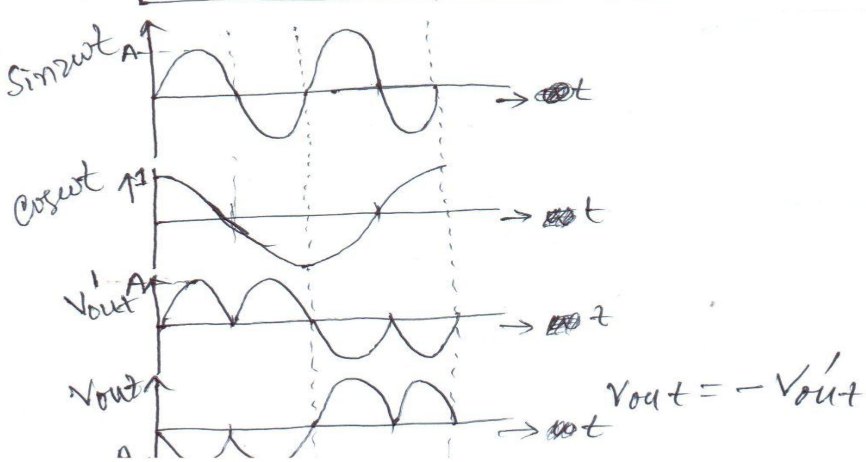
$$V_{out} = -R_f\left(\frac{V_1}{R_2} + \frac{V_2}{R_1}\right)$$

$$= -\frac{R_f}{R}(V_1 + V_2) \qquad [\because R_1 = R_2 = R]$$

$$= -\frac{R_f V_0}{R}(\sin\omega t + \sin 3\omega t)$$

$$= -\frac{2R_f V_0}{R}\left(\sin\left(\frac{3\omega t + \omega t}{2}\right) \times \cos\left(\frac{3\omega t - \omega t}{2}\right)\right)$$

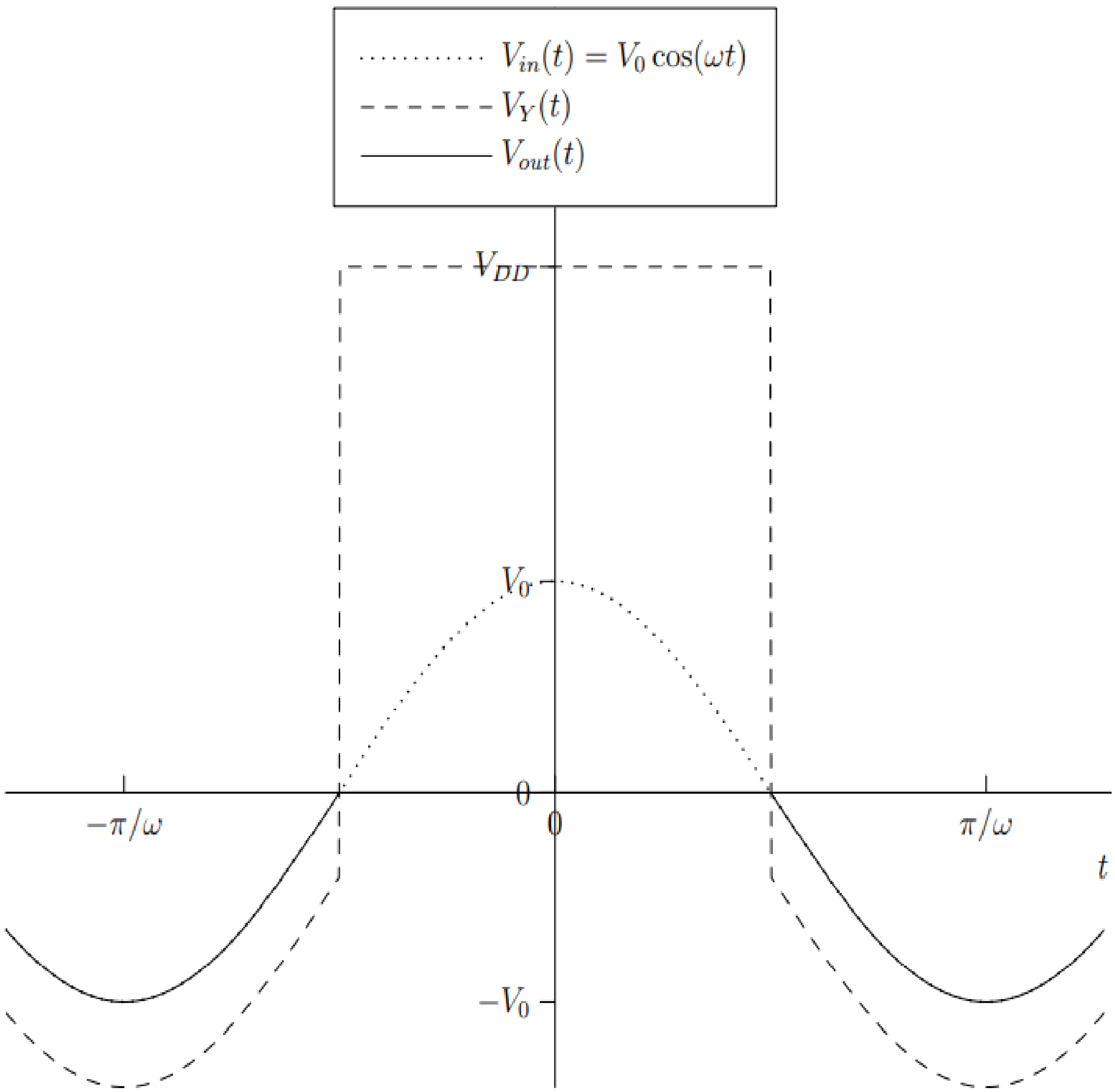
$$\boxed{V_{out} = -(A \sin 2\omega t) \cdot \cos\omega t = -V'_{out}} \; ; \; A = \frac{2R_f V_0}{R}$$

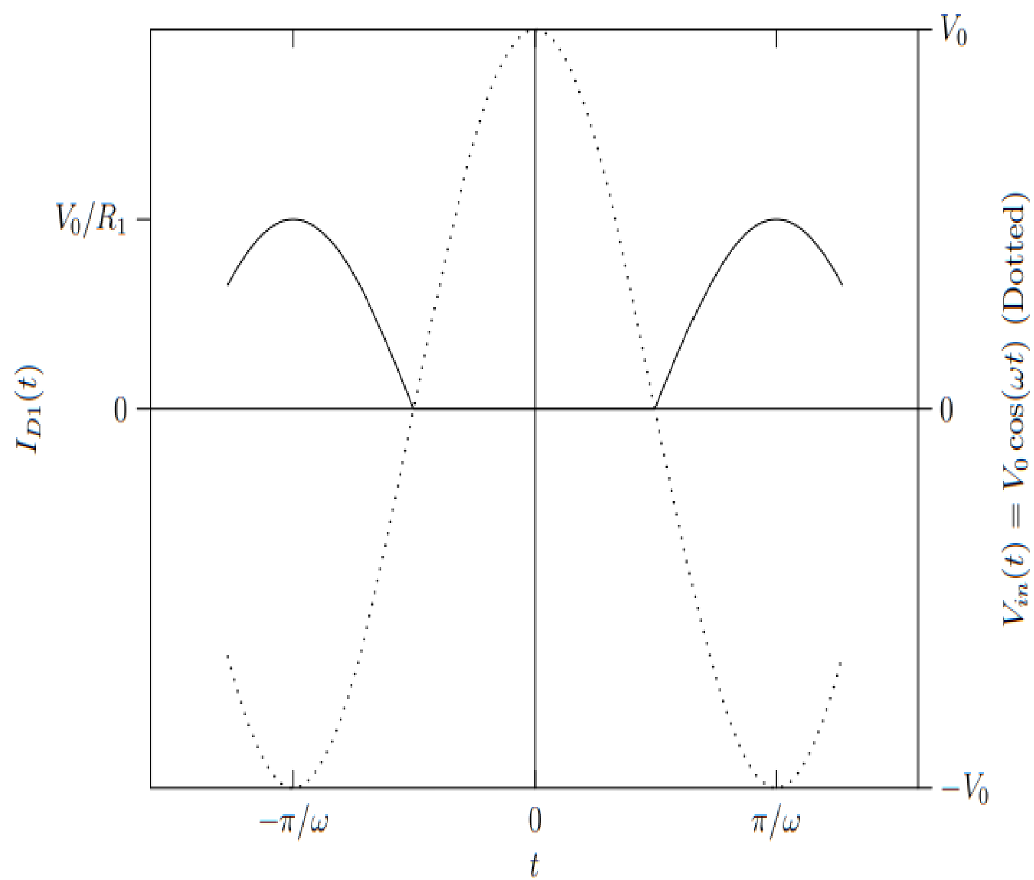


10.

$$V_Y = \begin{cases} V_{in} - V_{D,on} & V_{in} < 0 \\ V_{DD} & V_{in} > 0 \end{cases} V_{out} \qquad = \begin{cases} V_{in} & V_{in} < 0 \\ 0 & V_{in} > 0 \end{cases} I_{D1} = \begin{cases} \frac{V_{in}}{R_1} & V_{in} < 0 \\ 0 & V_{in} > 0 \end{cases}$$

Plotting $V_Y(t)$ and $V_{out}(t)$, we have




$I_{D1}(t)$
V_0/R_1
0
V_0
0
$-V_0$
$-\pi/\omega$
0
π/ω
t
$V_{in}(t) = V_0 \cos(\omega t)$ (Dotted)