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PH-101 Physics Tutorial - 1 15/11/2022

"A recap dimensional Analysis, coordinate Systems, Constraints Degrees of freedom"

Que-① ⓐ Consider a vibrating water drop whose frequency $\nu$ depends on its radius R, mass density $\rho$ and surface tension S. Using dimensional analysis, obtain the dependence of $\nu$ on R, $\rho$, and S? How is frequence changed if the radius of water drop is doubled

$$\nu \propto R^a \rho^b S^c$$

$$[T^{-1}] = [L]^a [ML^{-3}]^b \left[\frac{MLT^{-2}}{L}\right]^c$$

$$a + (-3)b = 0 \qquad b + c = 0 \qquad -2c = -1$$

$$a = 3b \qquad b = \boxed{-1/2} \qquad \boxed{c = 1/2}$$

$$\boxed{a = -\frac{3}{2}}$$

$$\nu \propto R^{-3/2} \rho^{-1/2} c^{1/2}$$

$$\nu = k\sqrt{\frac{c}{\rho R^3}}$$

$$\nu' = k\sqrt{\frac{c}{\rho(2R)^3}} = \frac{k}{2\sqrt{2}}\sqrt{\frac{c}{\rho R^3}} = \frac{\nu}{2\sqrt{2}}$$

$$\nu' = \frac{\nu}{2\sqrt{2}}$$

frequency would be $\frac{1}{2\sqrt{2}}$ times the original frequency.

ⓑ Using dimensional analysis, construct the expression for Planck's mass Mp in terms of $\hbar$, c and G (reduced Planck's constant, speed of light in vaccum, and Gravitational constt.)

$\hbar = \frac{h}{2\pi}$

$$M_p \propto \hbar^a c^b G^c \qquad [\hbar] = [ML^2T^{-1}]$$

$$[M_p] = [M] = [ML^2T^{-1}]^a [LT^{-1}]^b [M^{-1}L^3T^{-2}]^c \qquad [c] = [LT^{-1}] \qquad [G] = [M^{-1}L^3T^{-2}]$$

$\frac{MLT^{-2}L^2}{M^2}$

$M^{-1}L^3T^{-2}$

$$[M] = [M^{a-c} L^{2a+b+3c} T^{-a-b-2c}]$$

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$a - c = 1$ $\quad 2a + b + 3c = 0$ $\quad -a - b - 2c = 0$

$-c - c = 1$ $\quad (a + b + 2c) + a + c = 0$ $\quad a + b + 2c = 0$

$\boxed{c = \frac{-1}{2}}$ $\quad \boxed{a = -c}$ $\quad \frac{1}{2} + b + \frac{-1}{2}(2) = 0$

$\boxed{a = \frac{1}{2}}$ $\quad \boxed{b = \frac{1}{2}}$

$$M_p = k\,\hbar^{1/2} c^{1/2} G^{-1/2}$$
$$= k\sqrt{\frac{\hbar c}{G}}$$

(c) Obtain the expression for plancks time $T_p$ following a method similar to (b) above.

$$T_p \propto [T] \propto [ML^2T^{-1}]^a\,[LT^{-1}]^b\,[M^{-1}L^3T^{-2}]^c$$

$a + (-c) = 0$ $\quad 2a + b + 3c = 0$ $\quad -a - b - 2c = 1$

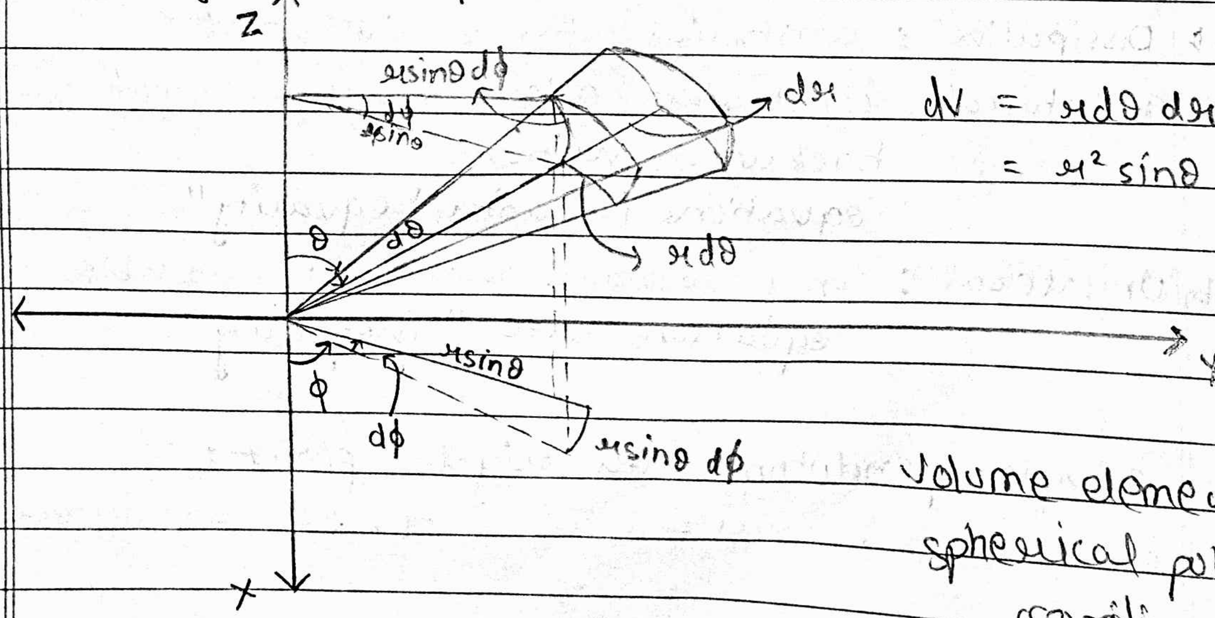
$\boxed{a = c}$ $\quad 5c = -b$ $\quad -c + 5c - 2c = 1$

$\boxed{a = \frac{1}{2}}$ $\quad b = -5c$ $\quad 2c = 1$

$\boxed{b = -\frac{5}{2}}$ $\quad \boxed{c = \frac{1}{2}}$

$$T_p = k\,\hbar^{1/2} c^{-5/2} G^{1/2} = k\sqrt{\frac{\hbar G}{c^5}}$$

Que
(2) Practice drawing volume element in spherical polar and cylindrical polar coordinates:




$$dV = r\,d\theta\; dr\; r\sin\theta\, d\phi$$
$$= r^2 \sin\theta\; d\theta\, d\phi\, dr$$

Volume element in spherical polar coordinate.

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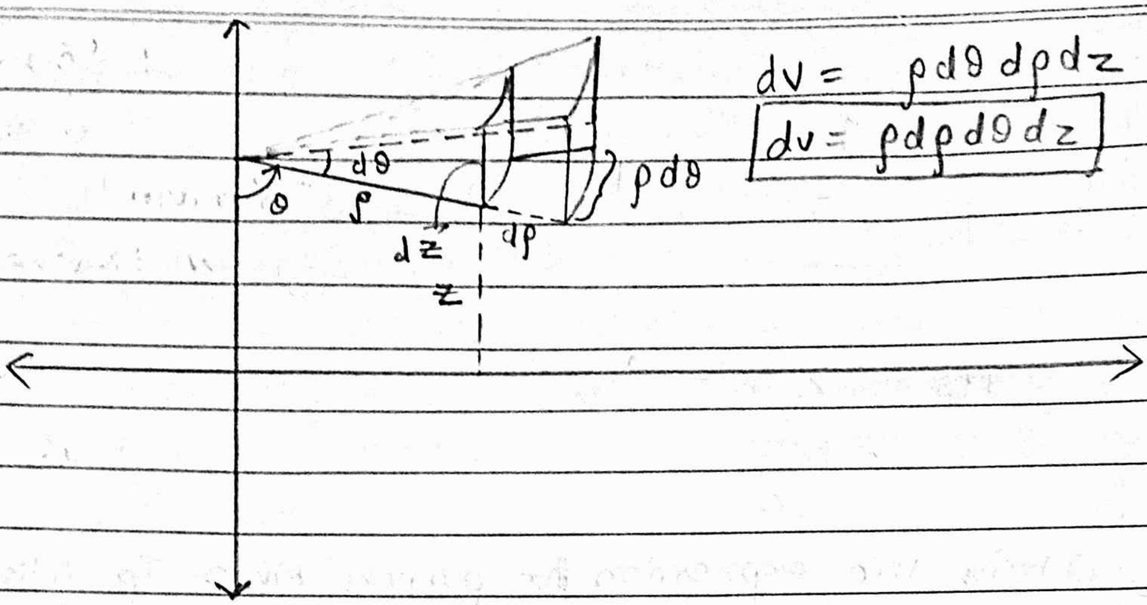



$dV = \rho\, d\theta\, d\rho\, dz$

$$\boxed{dv = \rho\, d\rho\, d\theta\, dz}$$

Que ③ what is the nature of constraint (among scleronomic, rheonomic, holonomic, non-holonomic, conservative dissipative) for :

- Scleronomic : constraint relation donot explicitely depends on time
- Rheonomic : constraint relation explicitely depends on time
- Holonomic : constraint relations can be made independent of velocities
- Non-Holonomic : constraint relations that are not Holonomic.
- Conservative : constraint force donot do any work.
- Dissipative : constraint force do work.
- Bilateral : constraint allow both forward and backward motion. equations in/with "equality"
- Unilateral : only forward motion is possible equations with "inequality"

(a) Simple pendulum with rigid support :

constraints : $r = l$ $\qquad \vec{T} \cdot \Delta \vec{R} = 0$ (work done)

$r = l$ → donot depend on time. "Scleronomic"
→ does not depend on velocity "Holonomic"
→ $eq^n$ in equality "Bilateral"
$\vec{T} \cdot \Delta\vec{R} = 0$ → work done = 0 "Conservative"

(b) Deformable Body : shape changes as time varies.

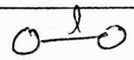
$|\vec{r} - \vec{r}'| = f(t)$ → depends on time "Rheonomic"
→ doesnot depend on velocity "Holonomic"
→ equation in equality "Bilateral"
→ work is done on the body → "Dissipative"

(c) An expanding / contracting spherical container of gas.

$r' \leq R(t)$ while contracting
$r' \geq R(t)$ while expanding.
time dependent → Rheonomic
does not depend on velocity → Holonomic
$eq^n$ in inequality → Unilateral.
work is done in expanding / contracting → Dissipative.

Que (4) Obtain degree of freedom for:

(a) A dombell in 2-D space. Degree of freedom = Total D.of − Constraints.

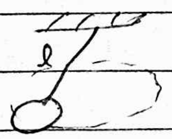



$l$ = const

$2 + 2 - 1$
$= 3$

(b) Bob of conical pendulum

$3 - 1$
$= 2$




$l$ = const

(c) Rigid Body fixed at a point.


$= 3 + 3 + 0 - (3)$
3rd point fixed

$= \boxed{3}$