TUTORIAL 4

INDIAN INSTITUTE OF TECHNOLOGY PATNA

COURSE CODE: PH103 COURSE TITLE: PHYSICS-I

1. **Physical interpretation of $\nabla\times$:** Consider the motion of a rigid body rotating about a fixed axis through O. If $\vec{\Omega}$ be its angular velocity then the velocity $\vec{v}$ of any particle P($\vec{r}$) of the body is given by: $\vec{v} = \vec{\Omega}\times\vec{r}$. Show that $\vec{\Omega} = \frac{1}{2}\vec{\nabla}\times\vec{v}$

2. If $\vec{F} = (x+y+1)\hat{\imath} + \hat{\jmath} - (x+y)\hat{k}$, show that $\vec{F}.\left(\vec{\nabla}\times\vec{F}\right) = 0$.

3. Find the work done in moving a particle from (0,0) to (1,2) in the force field $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$ along the curve C in the xy-plane defined by equation $y = 2x^2$.

4. Verify the Stoke's Theorem for $\vec{F} = (x^2+y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken around the rectangle bounded by the lines: $x = \pm a$, $y = 0$, $y = b$.

5. r_{min} and r_{max} for a Earth's satellite are 10000 km and 6000 km, respectively. The mass of satellite is 2000 kg. Compute the eccentricity, energy, angular momentum and minimum and maximum speed of satellite.

6. A particle moving under the influence of potential $U(r) = k/r^2$ with k > 0. Derive the trajectory of the particle.

7. **Forced Oscillations of LCR Circuit:** Consider a series LCR circuit which is being driven by a sinusoidal voltage source $V_o \sin\omega t$. Assume that the capacitor is totally uncharged and the inductor is totally demagnetized at t = 0 when the switch in between the voltage source and series-LCR-combination is closed. Obtain the expressions for instantaneous current $i(t)$ and instantaneous charge $q(t)$ on the capacitor plates.
   [**Hint:** Start writing the KVL ⇒ Convert it to a linear differential equation of second order in $i(t)$ ⇒ Obtain the Auxuliary equation and solve it to obtain its roots ⇒ Depending on the nature of roots, write the Complimentary Function (C.F.) and particular integral (P.I.) ⇒ $i(t)$ = C.F. + P.I.; it will have two arbitrary constants, say c_1 and c_2 ⇒ Obtain $\frac{di}{dt}$ and from the expressions for $i(t)$ and $\frac{di}{dt}$, obtain the expression for $q(t)$ using KVL ⇒ Using the initial conditions upon $i(t)$ and $q(t)$, obtain two linear equation in c_1 and c_2 ⇒ Substitute it back in expressions for $i(t)$ and $q(t)$ and that's it.]