

# TUTORIAL 2

## INDIAN INSTITUTE OF TECHNOLOGY PATNA

COURSE CODE: PH103

COURSE TITLE: PHYSICS-I

1. Let  $\vec{A}$  be an arbitrary vector and let  $\hat{n}$  be a unit vector in fixed direction. Show that

$$\vec{A} = (\vec{A} \cdot \hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n}$$

2. Evaluate the integral  $\iiint (x^2 + y^2 + z^2) dx dy dz$  over the volume  $V$  of a sphere having center at the origin and radius equal to  $a$ .
3. A particle moves in a plane with constant radial velocity 4 m/s. The angular velocity is constant and has a magnitude of 2 rad/s. When the particle is 3 m from origin, find the magnitude of (a) velocity and (b) acceleration.
4. A tire rolls in a straight line without slipping. Its center moves with constant speed  $V$ . A small pebble lodged in the tread of the tire touches the road at  $t = 0$ . Find the pebble's position, velocity and acceleration as functions of time.
5. A particle's trajectory is given by  $r = A\theta$ , where  $A = (1/\pi)$  m/rad. Additionally,  $\theta$  increases in time according to  $\theta = \alpha^2/2$ , where  $\alpha$  is a constant.
- (a) Sketch the motion, and indicate the approximate velocity and acceleration.
- (b) What value of  $\theta$  makes the radial acceleration zero.
- (c) At what angles do the radial and tangential accelerations have equal magnitudes?
6. The height of a certain hill is given by
- $$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$
- where  $y$  is the distance north and  $x$  is the distance east of South Hadley.
- (a) Where is the top of the hill located?
- (b) How high is the hill?
- (c) How steep is the slope at a point  $(x, y) \equiv (1, 1)$ ? In what direction, is the slope steepest, at that point?
7. Let  $\vec{r}$  be the separation vector from a fixed point  $(x', y', z')$  to the point  $(x, y, z)$  and let  $r$  be its length. Show that
- (a)  $\vec{\nabla}(r^2) = 2\vec{r}$
- (b)  $\vec{\nabla}(1/r) = -\hat{r}/r^2$
- (c) What is the general formula for  $\vec{\nabla}(r^n)$ ?