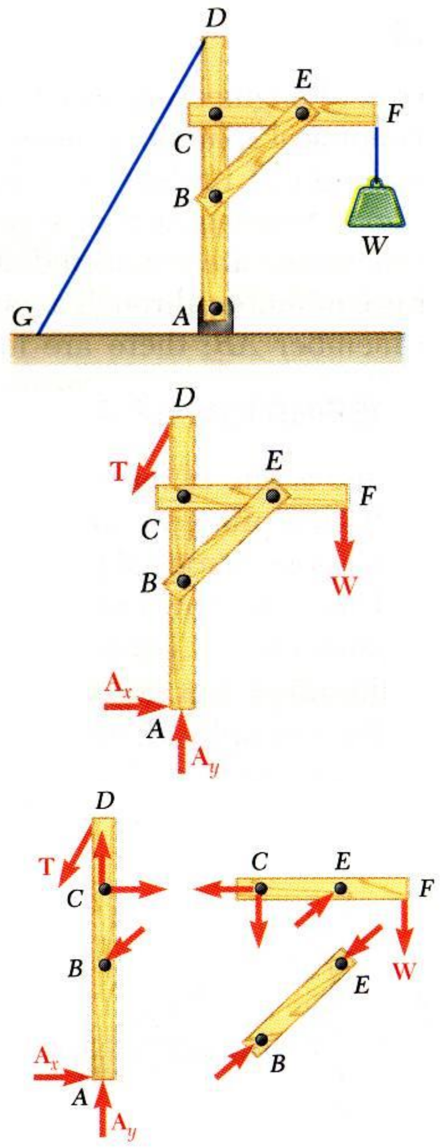
Engineering Mechanics (ME102)

Analysis of Structures

Introduction



- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.

- Three categories of engineering structures are considered:
  - a) *Trusses*: formed from *two-force members*, i.e., straight members with end point connections and forces that act only at these end points.
  - b) *Frames*: contain at least one multi-force member, i.e., member acted upon by 3 or more forces.
  - c) *Machines*: structures containing moving parts designed to transmit and modify forces.

Image Courtesy: https://www.waagnerbiro-bridgesystems.com/references/tag/Truss+Bridges

Image Courtesyhttps://boggsinspect.com/what-is-a-roof-truss/

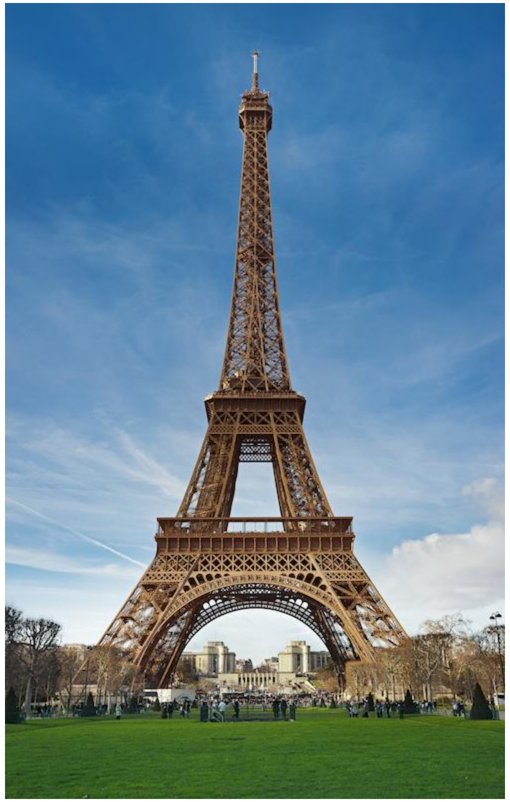
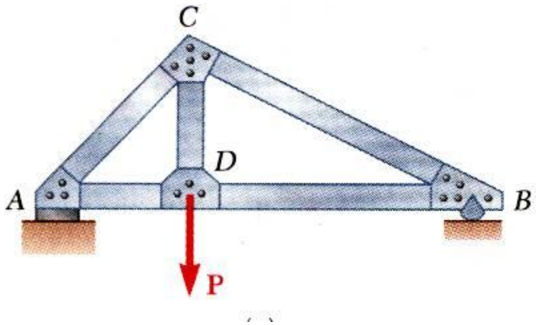


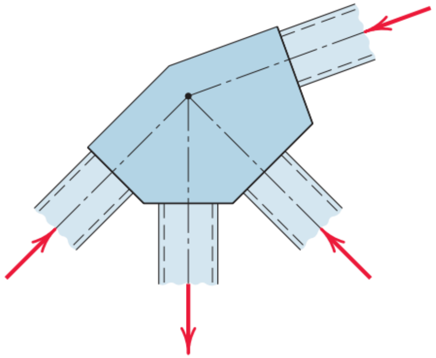
Image Courtesy: https://www.comsol.com/blogs/creating-an-app-to-prevent-buckling-in-a-truss-tower-design/

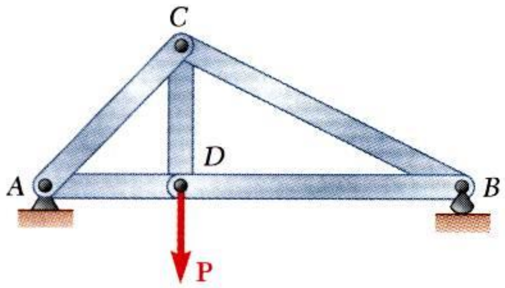
Image Courtesy: https://dafangcranez.com/truss-gantry-crane/

Definition of a Truss

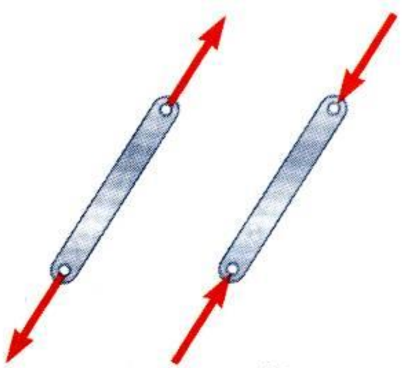


- A truss consists of straight members connected at joints. No member is continuous through a joint.

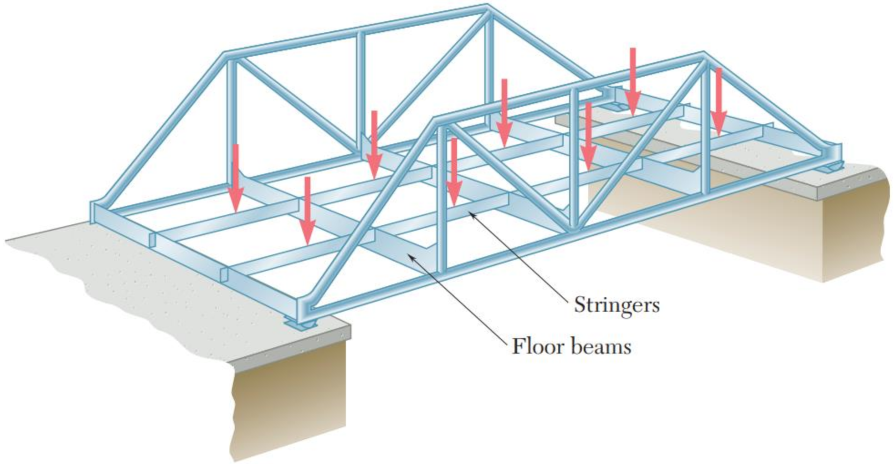




- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.

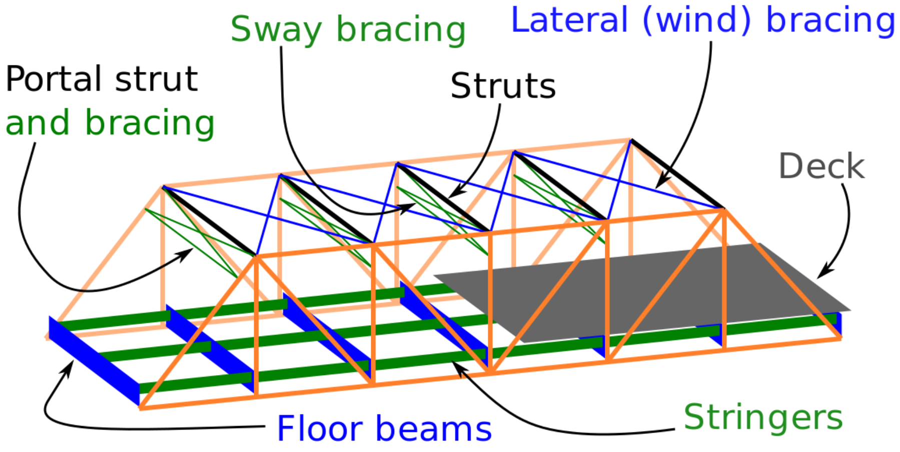


- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.



- Force is applied to the joints and *not on the members* themselves.

- For supporting concentrated loads, a floor system comprising of stringers and floor beams are used to transmit loads to the joints.



- Weight of the truss member is also assumed to be applied to the joints, half on each joint.

- Although the members are typically joined by means of welded, bolts, or are riveted together, it is customary to assume pin joints.

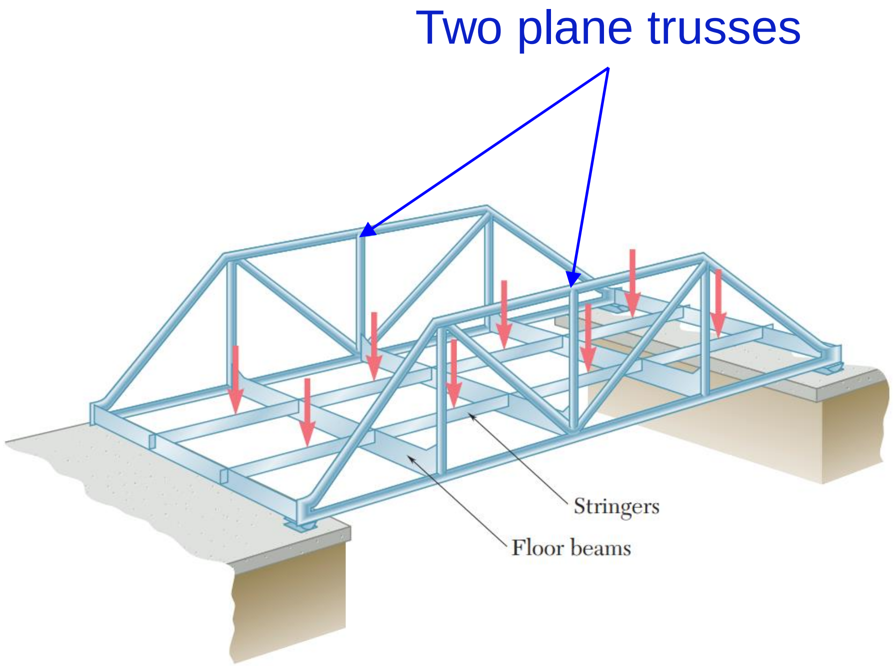
Ex: Digha-Sonpur Bridge

Stringers

Floor Beams

Deck

Plane Trusses

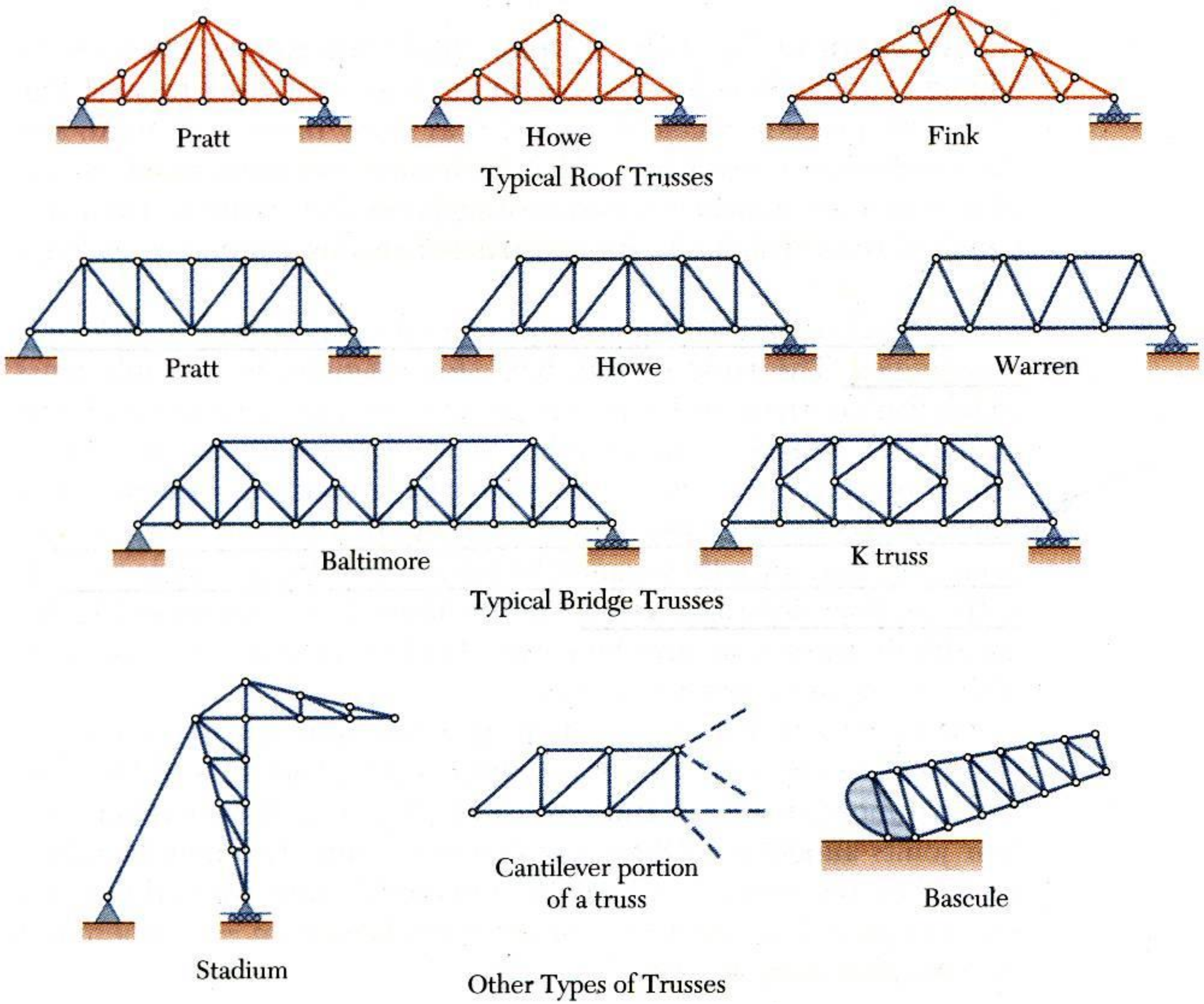


- When the members of the truss lie in a single plane, the truss is called a *plane truss*.

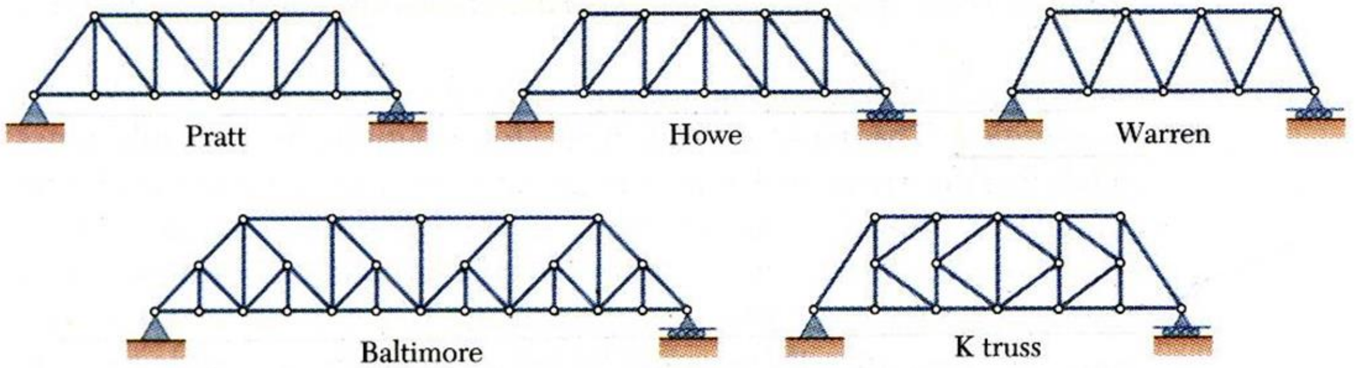
- The loads are transmitted from the floor beam to the two vertical sides of the structure.

- Each side of the truss carries loads acting in its plane, so we can consider it a two-dimensional structure for analysis.

Types of Truss



Bridge Truss



Typical Bridge Trusses

About 2,59,000 results (1.07 seconds)

The total length of construction, including approaches, is 20 km. It is a K-**truss bridge**. There are two rail tracks (up and down tracks) and a two lane road. Its south link road which is also called as AIIMS- **Digha** elevated road is expected to complete in February 2020.

Design: Warren truss and double Warren truss

Longest span: 123 metres (404 ft)

Total length: 4,556 metres (14,948 ft)

Construction end: August 2015

en.wikipedia.org › wiki › Digha–Sonpur_Bridge

Digha–Sonpur Bridge - Wikipedia

Digha–Sonpur Bridge

Directions Save

4.4 ★★★★★ 1,426 Google reviews

Truss bridge in Patna, Bihar

Roof Truss

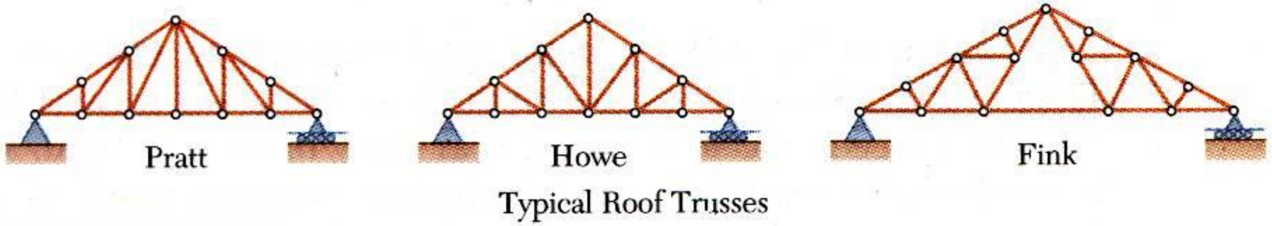
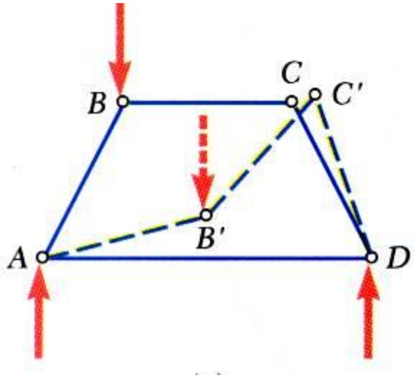
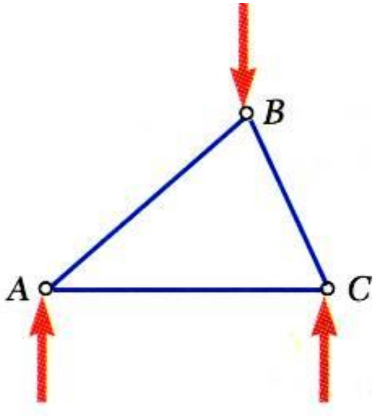


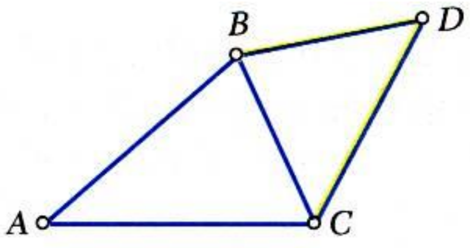
Image Courtesy: https://www.exportersindia.com/adishilpi-infracon/other-products.htm#5913046

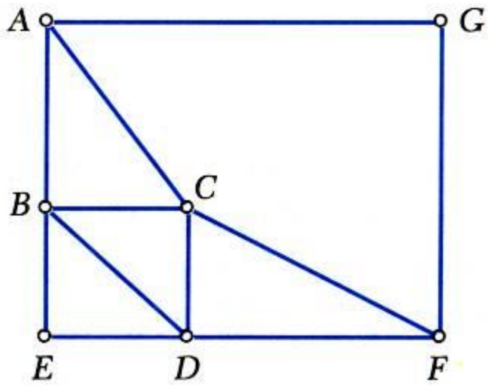
Simple Trusses





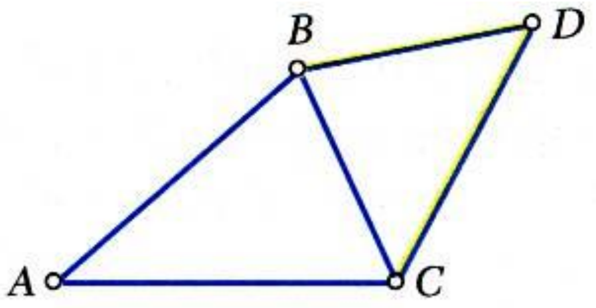
- A *rigid truss* will not collapse under the application of a load.

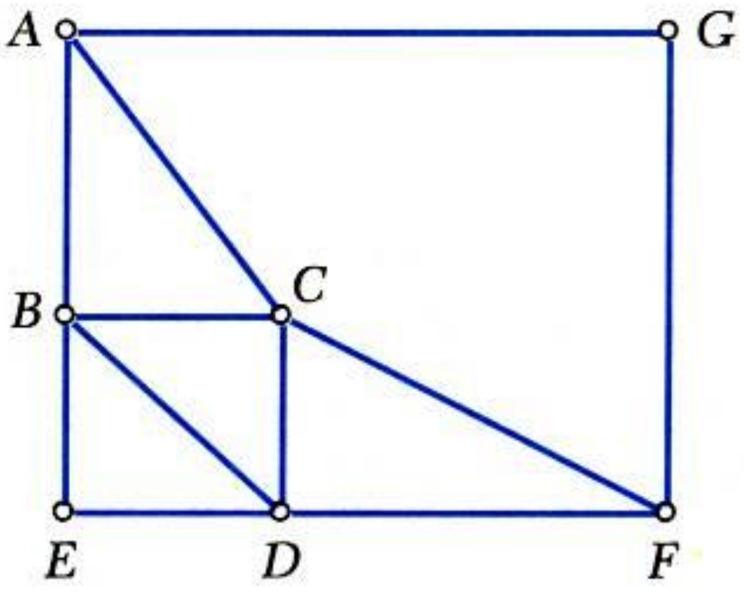




- A *simple truss* is constructed by successively adding two members and one connection to the *basic triangular truss*. This process can be repeated to get larger trusses.

Relation for a Simple Trusses



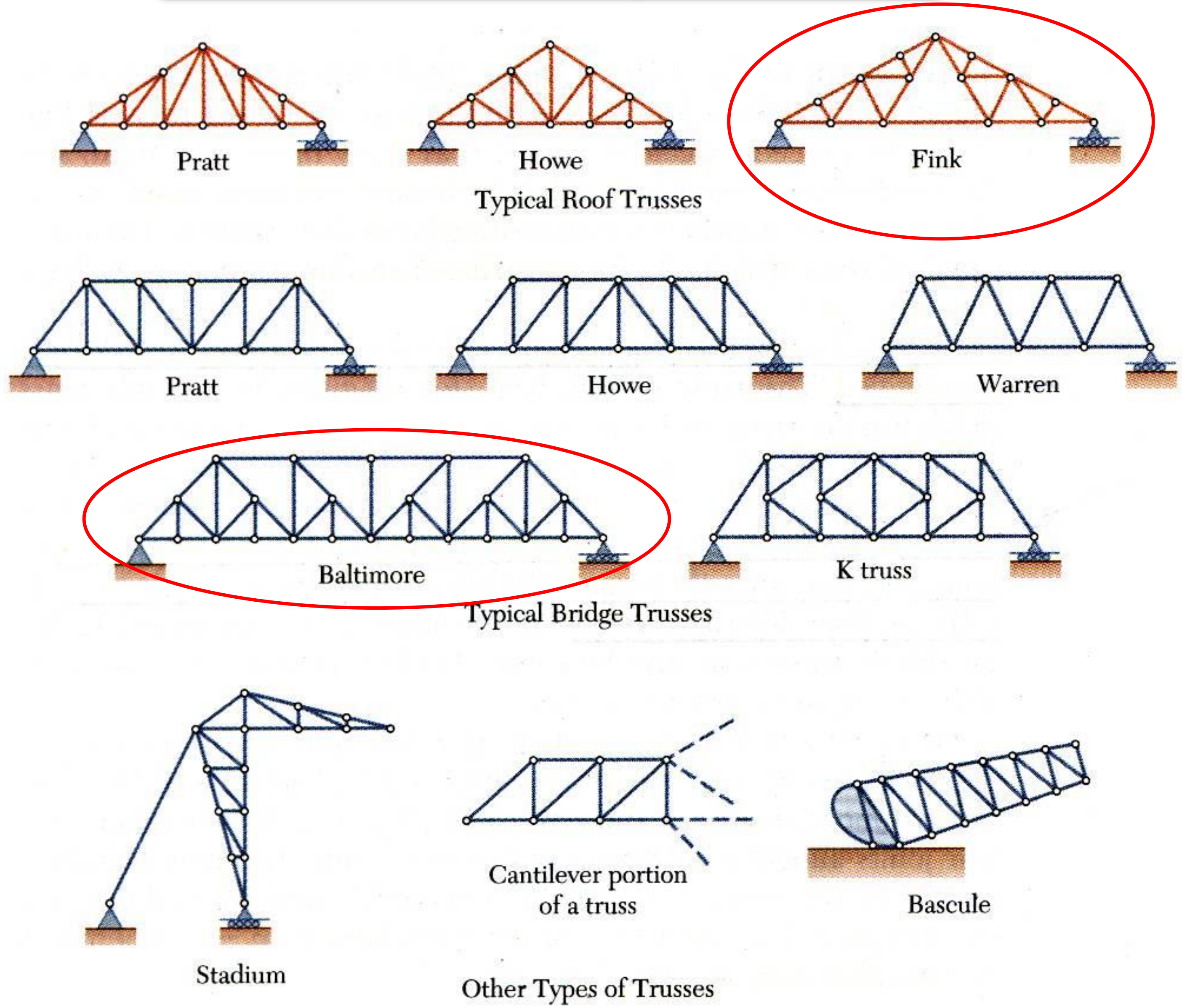


- A *simple truss* is constructed by successively adding two members and one connection to the *basic triangular truss*. This process can be repeated to get larger trusses.

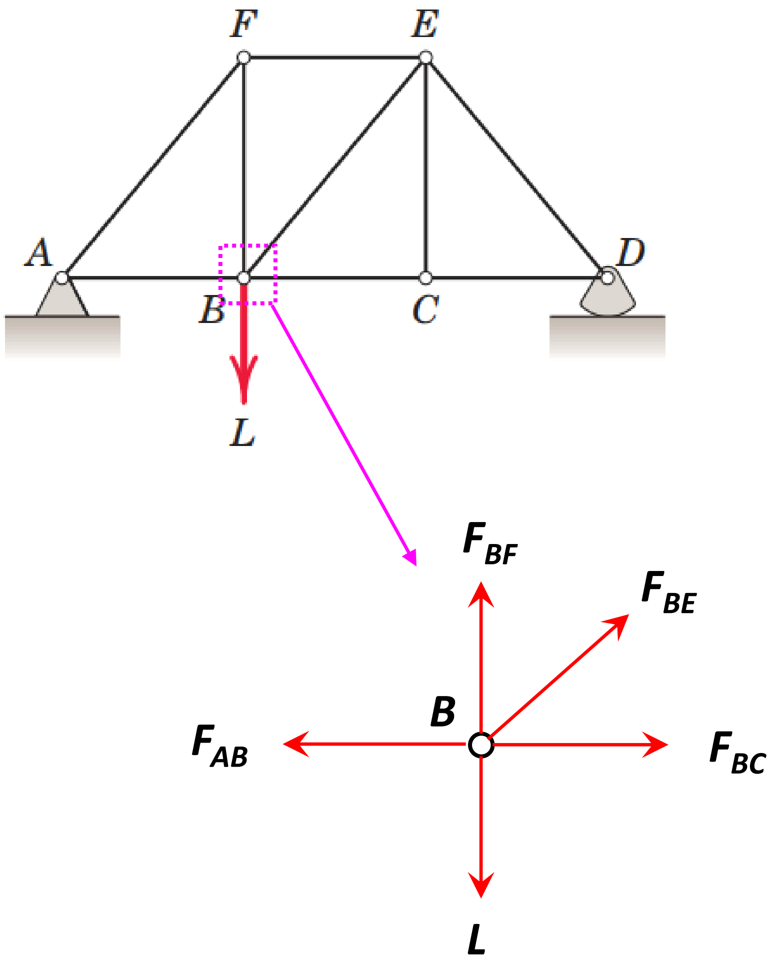
	Members (*m*)	Joints (*n*)	Total *m*	Total *n*
Simple Truss	3	3	3	3
	2	1	5	4
	2	1	7	5
	2	1	9	6
	2	1	11	7
	2	1	13	8
	2	1	15	9

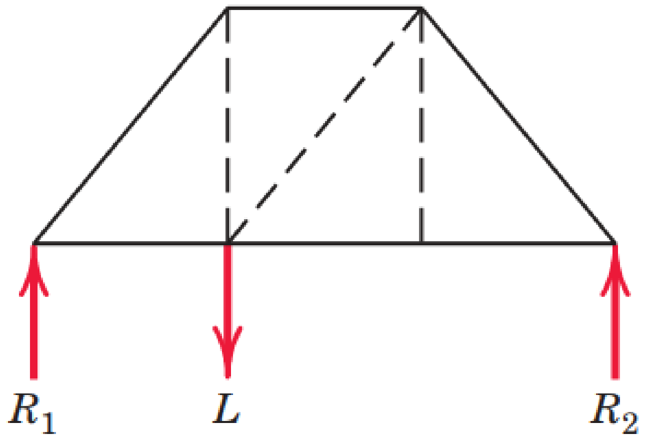
- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

Relation for a Simple Trusses



Determinacy for Planer Trusses





- Two equations of equilibrium are available per truss joints to find unknows

$$\sum F_x = 0 \qquad \sum F_y = 0$$

m = Total number of the truss members

n = Total number of truss joints

r = Total number of truss reactive forces

Determinacy for Planer Trusses

$m+r$ = Total unknown

$2n$ = Total equations

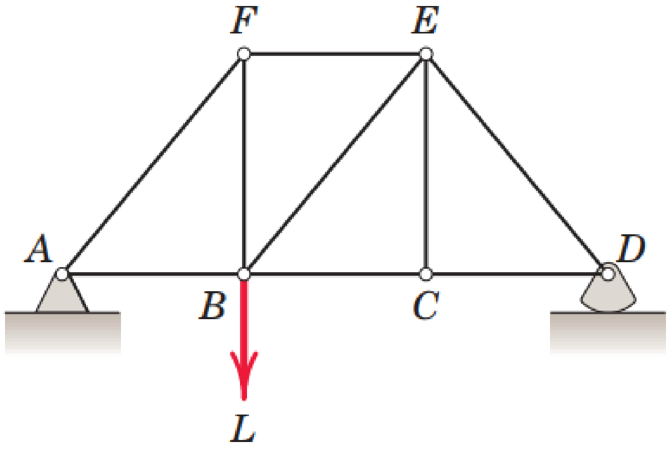
- For stable planer truss

$m+r = 2n$ → *Statically determinate*

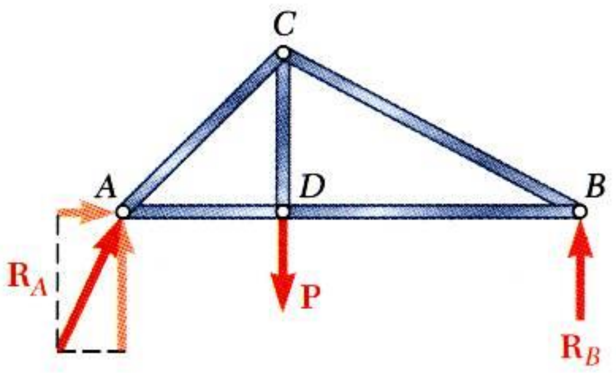
$m+r > 2n$ → *Statically indeterminate*

- For statically indeterminate truss

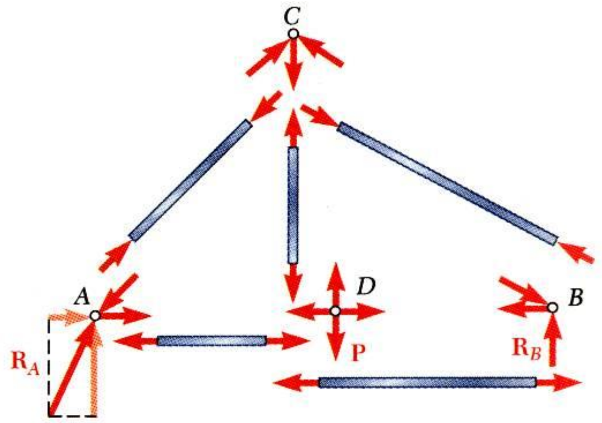
$(m+r) - 2n$ → *Degree of indeterminacy*



Analysis of Trusses (Method of Joints)

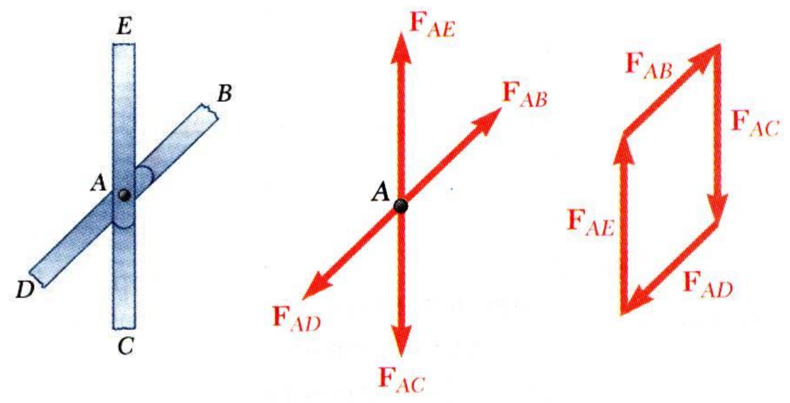


- Draw the free-body diagram (FBD) and solve the reaction forces.
- Check whether the truss is statically determinate or not.
- Dismember the truss and create an FBD for each member and joint.
- The two forces exerted on each member are equal, have the same line of action, and opposite senses.

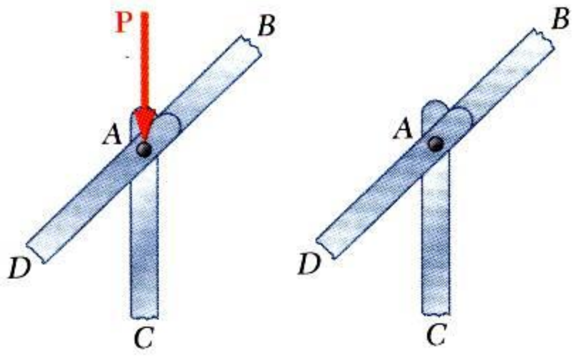


- Assume all members are in tension, so we draw the internal forces as pointing away from the joint.
- Conditions of equilibrium are used to solve for 2 unknown forces at each pin (or joint), giving a total of 2n solutions, where n=number of joints. Forces are found by solving for unknown forces while moving from joint to joint sequentially.

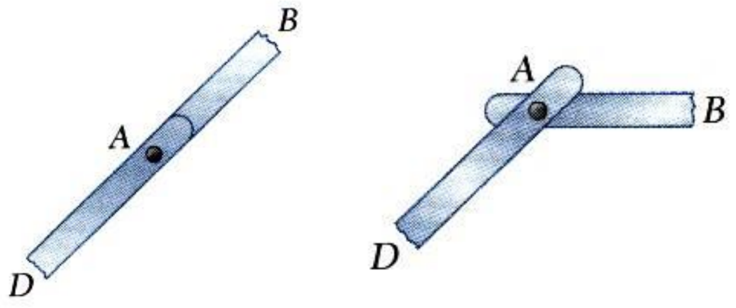
Joints under Special Loading Conditions



- Forces in opposite members intersecting in two straight lines at a joint are equal.



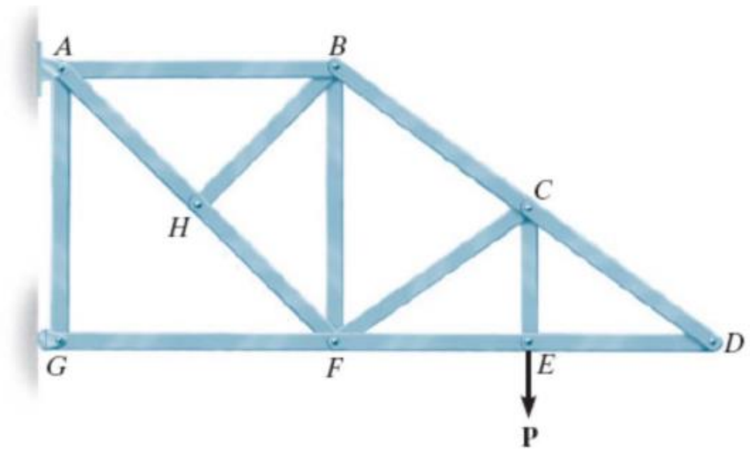
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).

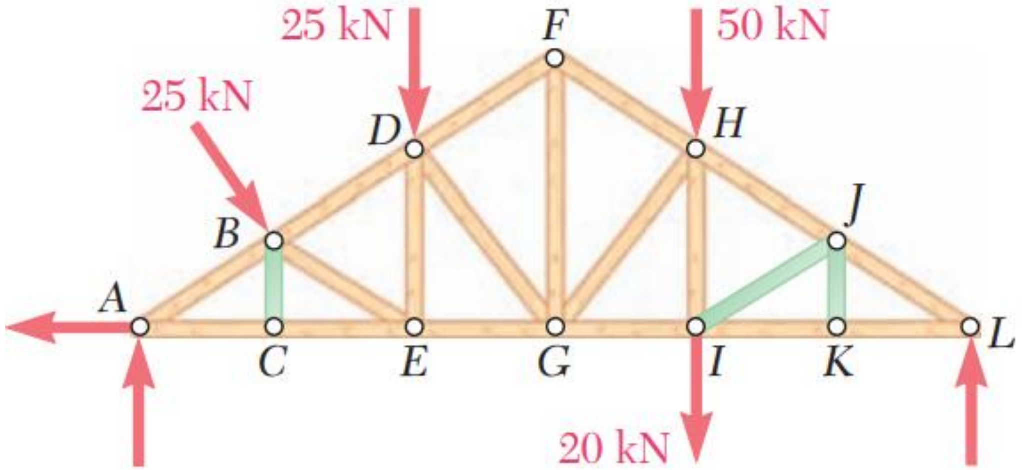


- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.

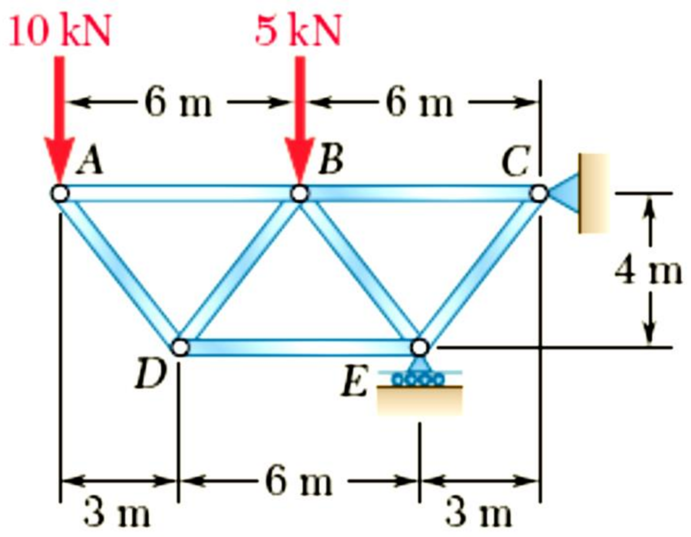
Joints under Special Loading Conditions

Indicate the zero-force members



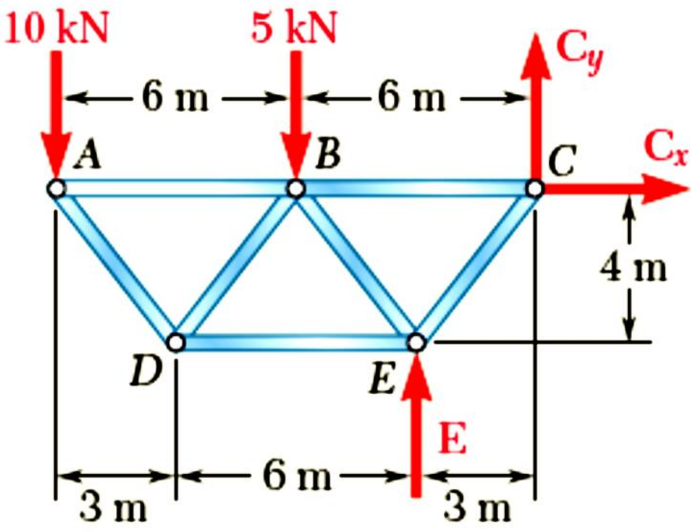


Sample Problems 6.1:



Using the method of joints, determine the force in each member of the truss.

Sample Problems 6.1:



SOLUTION:

- Based on a free body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Looking at the FBD, which "sum of moments" equation could you apply in order to find one of the unknown reactions with just this one equation?

$$\sum M_C = 0$$
$$= (10\text{ kN})(12\text{ m}) + (5\text{ kN})(6\text{ m}) - E(3\text{ m})$$

$$\boxed{E = 50\,\text{kN}}$$

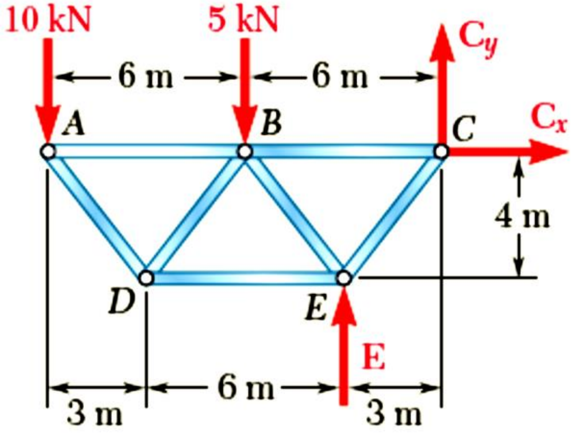
- Next, apply the remaining equilibrium conditions to find the remaining 2 support reactions.

$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -10\text{ kN} - 5\text{ kN} + 50\text{ kN} + C_y$$

$$\boxed{C_y = -35\text{ kN}}$$

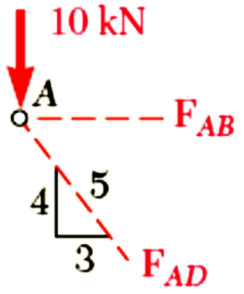
Sample Problems 6.1:

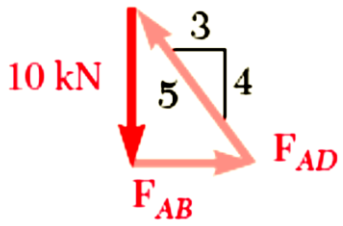


- We now solve the problem by moving sequentially from joint to joint and solving the associated FBD for the unknown forces.

- Which joint should you start with, and why?

- Joints *A* or *C* are equally good because each has only 2 unknown forces. Use joint *A* and draw its FBD and find the unknown forces.



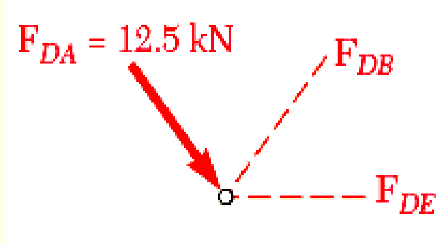


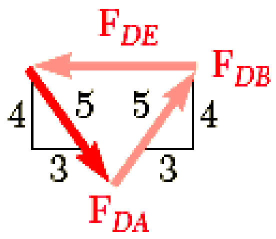
$$\frac{10\text{ kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\boxed{\begin{aligned} F_{AB} &= 7.5\text{ kN } \ T \\ F_{AD} &= 12.5\text{ kN } \ C \end{aligned}}$$

- Which joint should you move to next, and why?

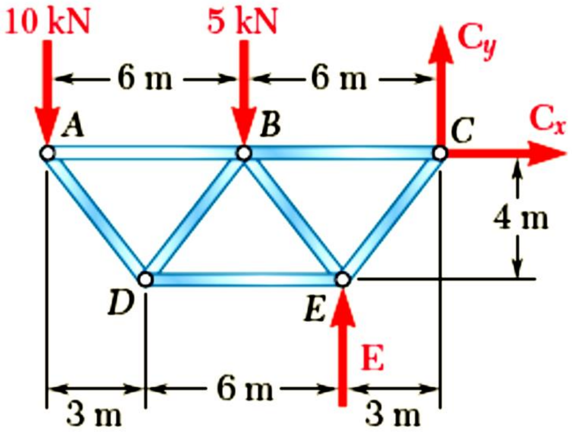
- Joint *D*, since it has 2 unknowns remaining (joint *B* has 3). Draw the FBD and solve.

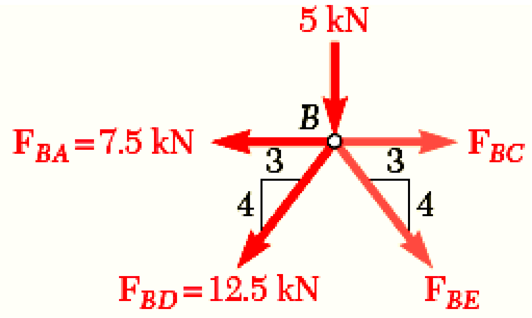


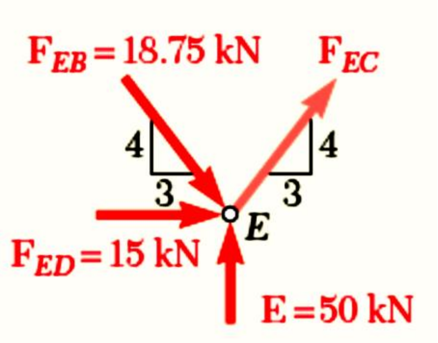


$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 12.5\text{ kN } \ T \\ F_{DE} &= 15\text{ kN } \ C \end{aligned}}$$







- There are now only two unknown member forces at joint *B*. Assume both are in tension.

$$\sum F_y = 0 = -5\,\text{kN} - \tfrac{4}{5}(12.5\,\text{kN}) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -18.75\,\text{kN}$$

$$\boxed{F_{BE} = 18.75\,\text{kN} \;\; C}$$

$$\sum F_x = 0 = F_{BC} - 7.5\,\text{kN} - \tfrac{3}{5}(12.5\,\text{kN}) - \tfrac{3}{5}(18.75\,\text{kN})$$

$$F_{BC} = +26.25\,\text{kN}$$

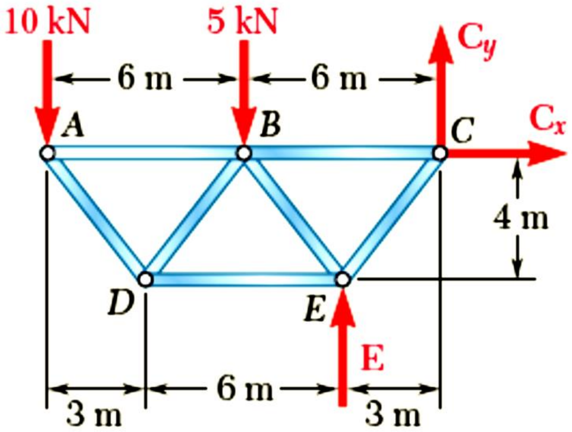
$$\boxed{F_{BC} = 26.25\,\text{kN} \;\; T}$$

- There is one remaining unknown member force at joint *E* (or *C*). Use joint *E* and assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 15\,\text{kN} + \tfrac{3}{5}(18.75\,\text{kN})$$

$$F_{EC} = -43.75\,\text{kN}$$

$$\boxed{F_{EC} = 43.75\,\text{kN} \;\; C}$$

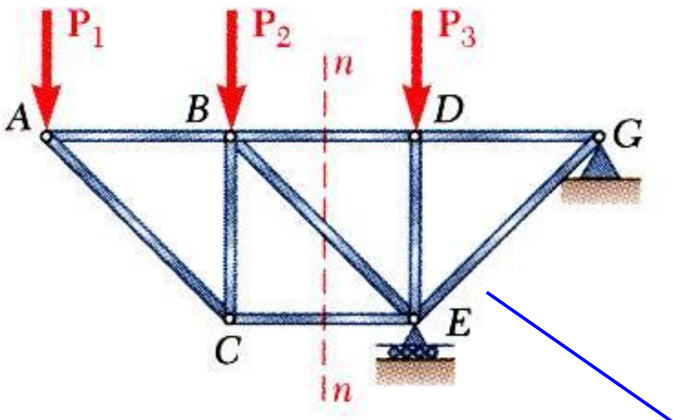


- All member forces and support reactions are known at joint C. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -26.25\,\text{kN} + \tfrac{3}{5}(43.75)\,\text{kN} = 0 \quad (\text{checks})$$

$$\sum F_y = -35\,\text{kN} + \tfrac{4}{5}(43.75)\,\text{kN} = 0 \quad (\text{checks})$$

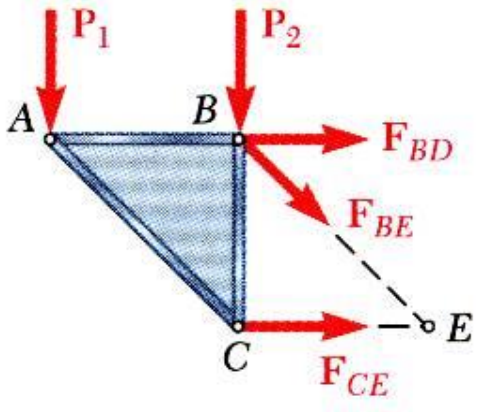
Analysis of Trusses (Method of Sections)



- When the force in only one member or the forces in very few members are desired, the *method of sections* works well.

- To determine the force in member *BD*, form a *section* by "cutting" the truss at *n-n* and create a free-body diagram for the left side.

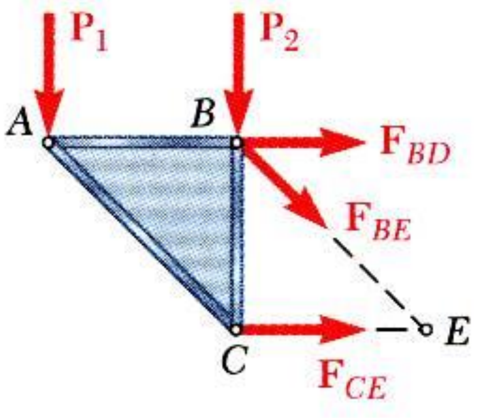
❖ A FBD could have also been created for the right side, but why is this a less desirable choice?



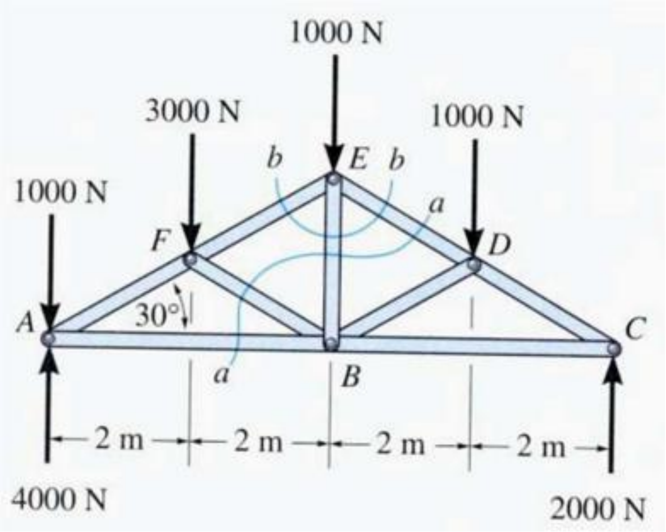
- Notice that the exposed internal forces are all *assumed* to be in tension.

- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including $\mathbf{F}_{BD}$.

Analysis of Trusses (Method of Sections)



- Using the left-side FBD, write one equilibrium equation that can be solved to find $\mathbf{F}_{BD}$.

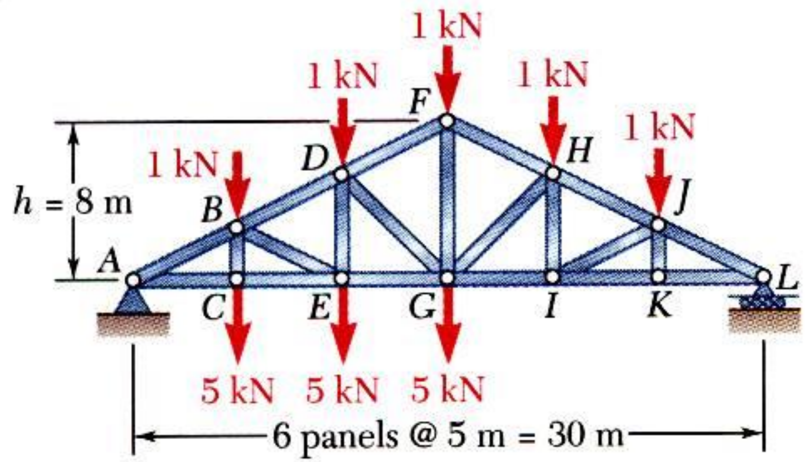


Q.1 Can we determine the force in member ED by making the cut at section a-a? Explain.

Q.2 If we know the force in member ED, how we will determine the force in member EB? Explain

Analysis of Trusses (Method of Sections)

Sample Problems 6.3:



Determine the force in members *FH*, *GH*, and *GI*.

SOLUTION:

- Take the FBD of the entire truss. Apply the conditions for static equilibrium to solve for the reactions at ***A*** and ***L***.

$$\sum M_A = 0 = -(5\,\text{m})(6\,\text{kN}) - (10\,\text{m})(6\,\text{kN}) - (15\,\text{m})(6\,\text{kN}) - (20\,\text{m})(1\,\text{kN}) - (25\,\text{m})(1\,\text{kN}) + (25\,\text{m})L$$

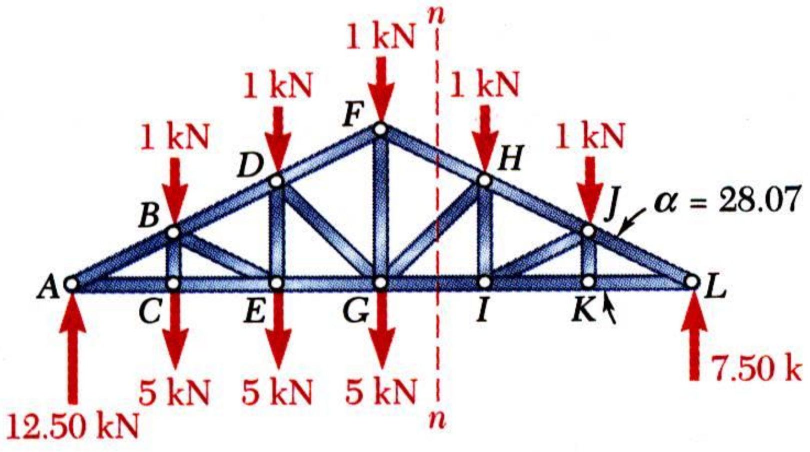
$$L = 7.5\,\text{kN} \uparrow$$

$$\sum F_y = 0 = -20\,\text{kN} + L + A$$

$$A = 12.5\,\text{kN} \uparrow$$

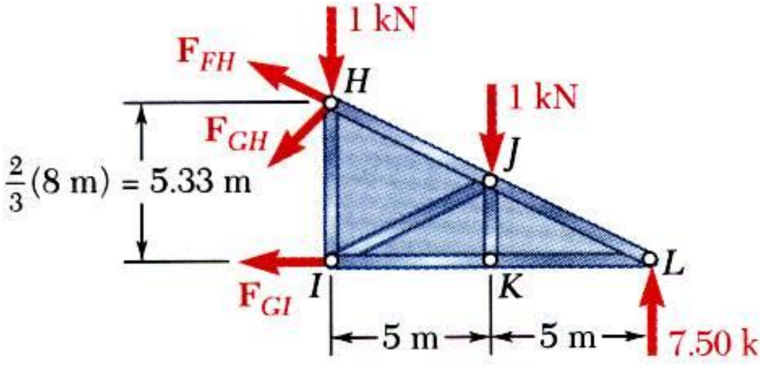
Analysis of Trusses (Method of Sections)

Sample Problems 6.3:



- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.

- Apply the conditions for static equilibrium to determine the desired member forces.



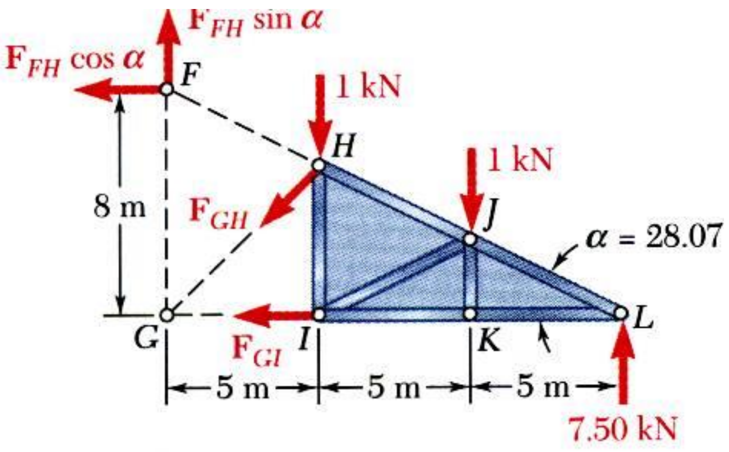
$$\sum M_H = 0$$

$$(7.50\,\text{kN})(10\,\text{m}) - (1\,\text{kN})(5\,\text{m}) - F_{GI}(5.33\,\text{m}) = 0$$

$$F_{GI} = +13.13\,\text{kN}$$

$$\boxed{F_{GI} = 13.13\,\text{kN} \;\; T}$$

Analysis of Trusses (Method of Sections)



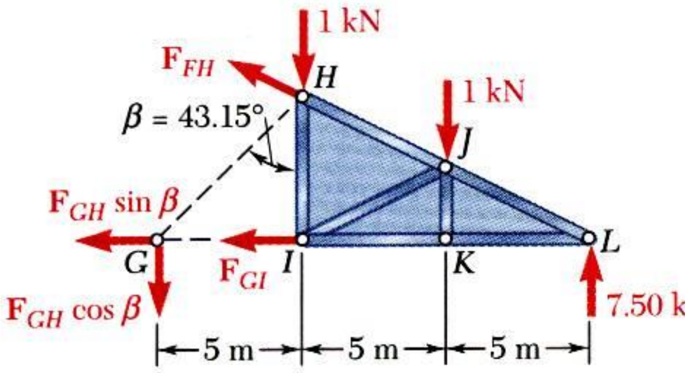
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN}\ \ C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

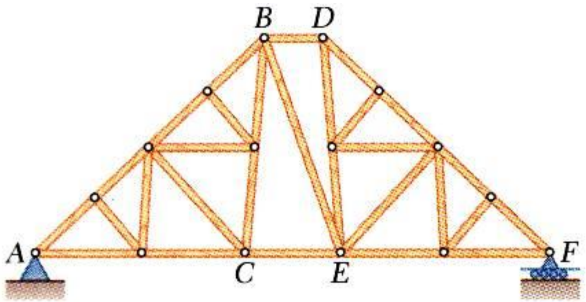
$$\sum M_L = 0$$

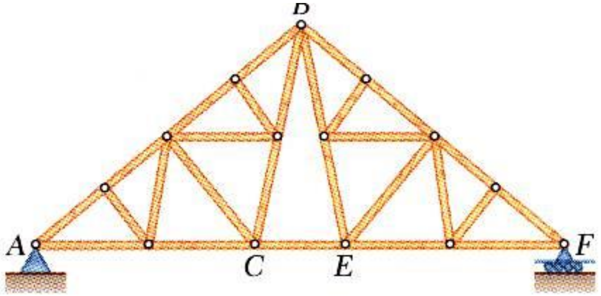
$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

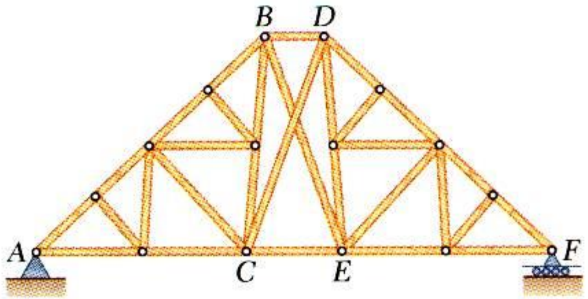
$$F_{GH} = -1.371\text{ kN}$$

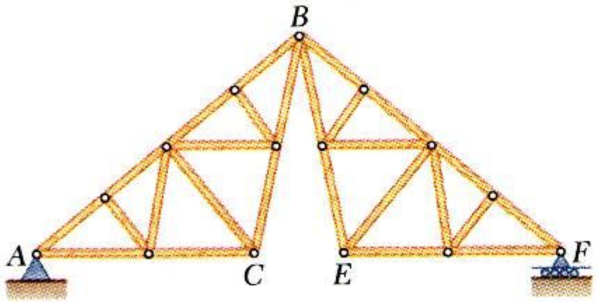
$$\boxed{F_{GH} = 1.371\text{ kN}\ \ C}$$

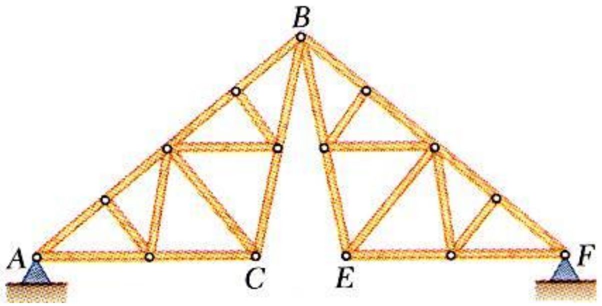
Trusses Made of Several Simple Trusses











- *Compound trusses* are statically determinate, rigid, and completely constrained.

$$m+r = 2n$$

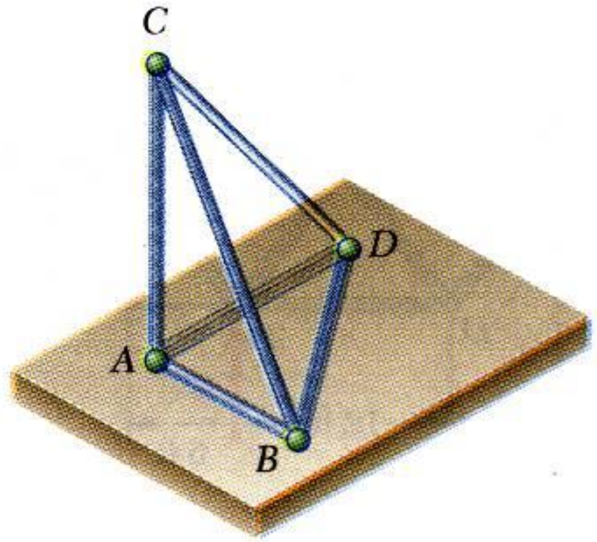
- Truss contains a *redundant member* (over-rigid) and is *statically indeterminate*.

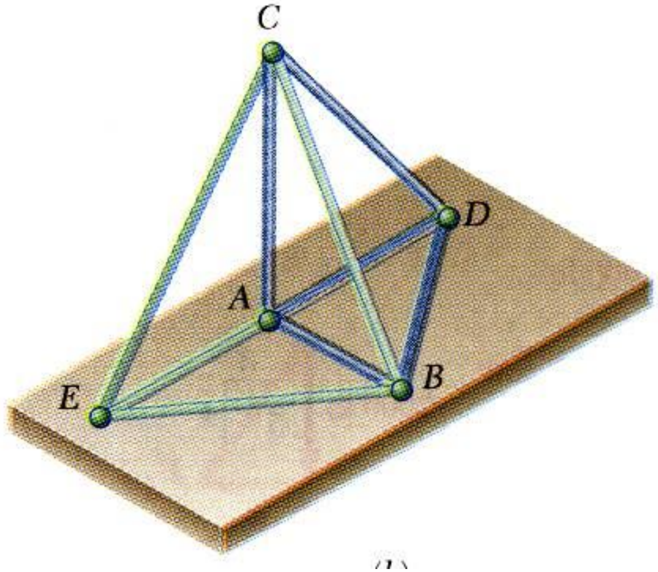
$$m+r > 2n$$

- *Necessary but not sufficient condition* for a compound truss to be statically determinate, rigid, and completely constrained,

$$m+r = 2n$$

Space Trusses





- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.

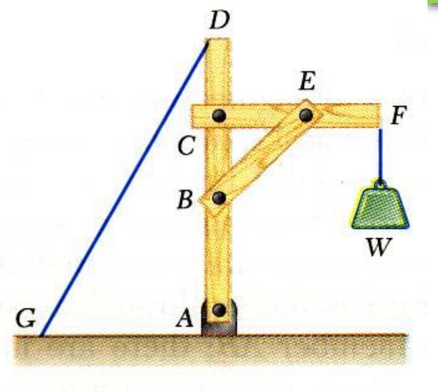
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.

- In a simple space truss, $m = 3n - \mathbf{6}$ where m is the number of members and n is the number of joints.

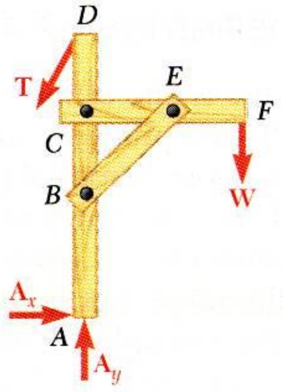
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.

- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

Analysis of a Frame



- *Frames* and *machines* are structures with at least one *multi-force* (>2 forces) member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and transmit and modify forces.

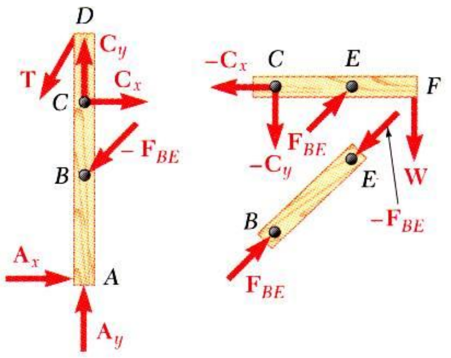


➢ Analysis of the frame involves determining,

I. External reactions

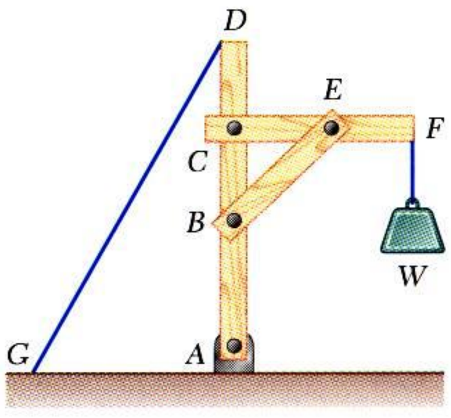
II. Internal forces

- A FBD of the complete frame is used to determine the external forces acting on the frame.



- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.

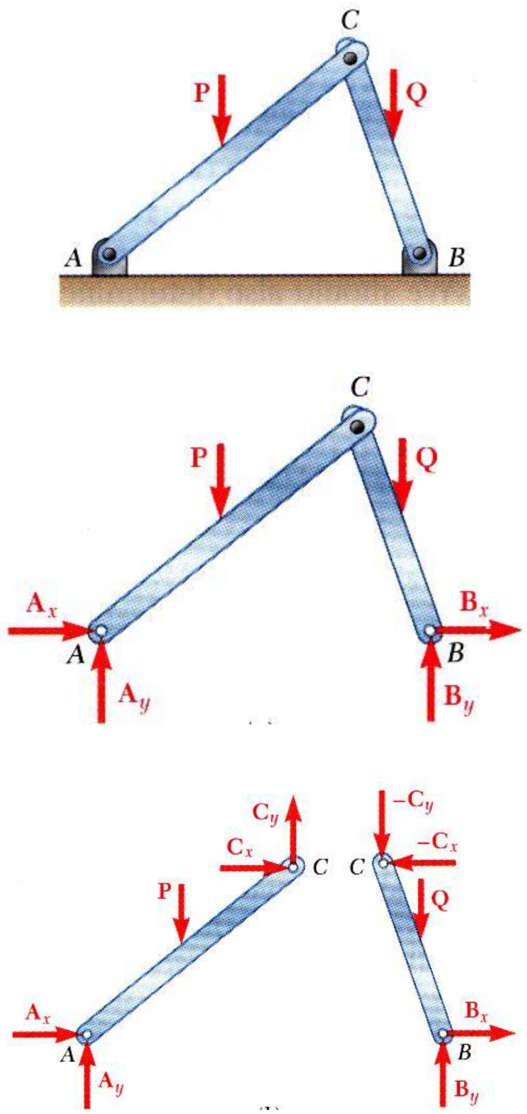
Analysis of a Frame



Q. What will happen if support A is fixed support?

- The system will be over-rigid
- The cable GD will not carry any load and will be redundant

Analysis of a Frame



- Some frames may collapse if removed from their supports. Such frames cannot be treated as rigid bodies.

- A free-body diagram of the complete frame indicates four unknown force components that cannot be determined from the three equilibrium conditions (statically indeterminate).

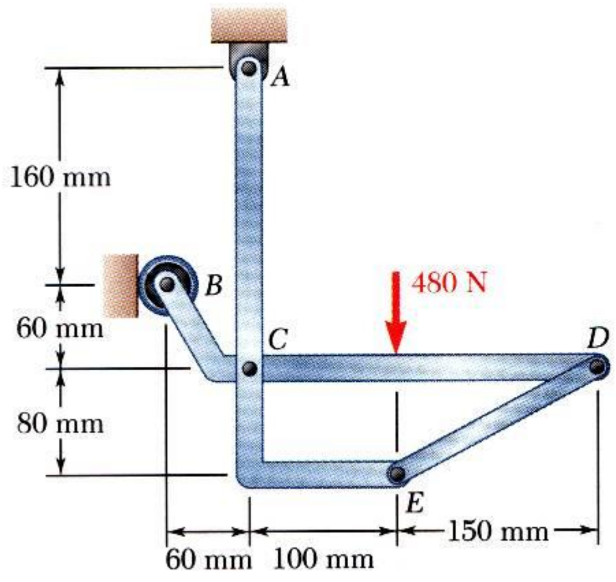
- The frame must be considered as two distinct, but related, rigid bodies.

- With equal and opposite reactions at the contact point between members, the two free-body diagrams show 6 unknown force components.

- Equilibrium requirements for the two rigid bodies yield 6 independent equations. Thus, taking the frame apart made the problem solvable.

Analysis of a Frame

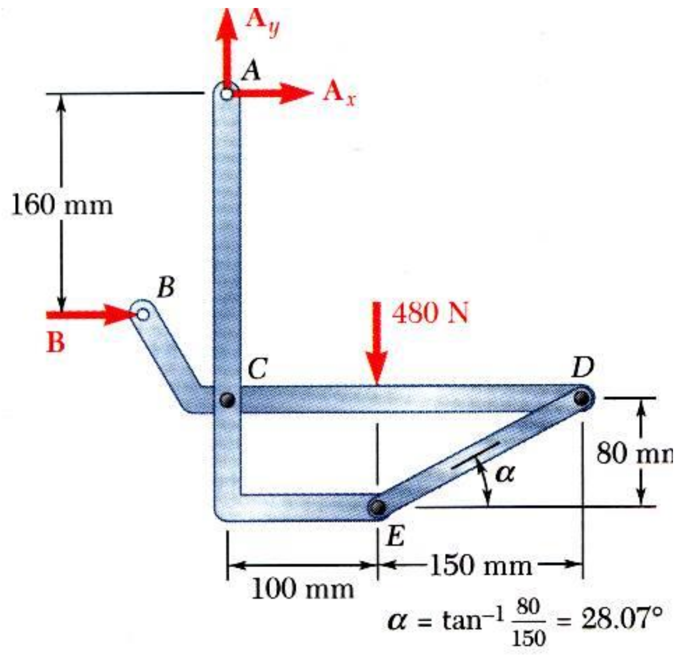
Sample Problems 6.4:



Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

Analysis of a Frame

Sample Problems 6.4:



SOLUTION:

1. Create a free-body diagram for the complete frame and solve for the support reactions.

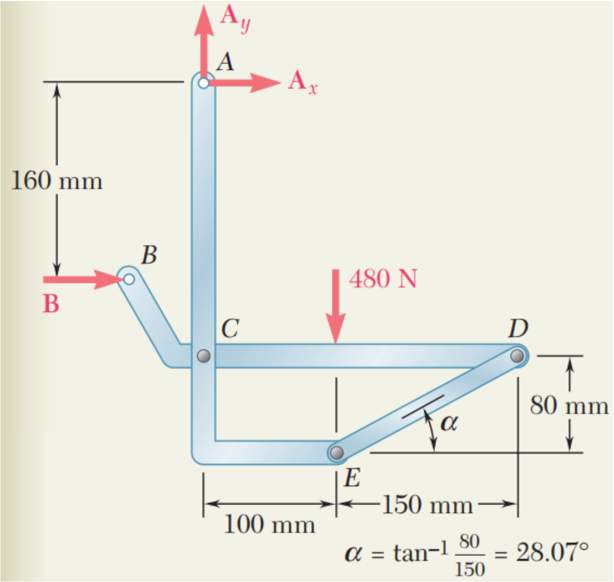
$$\sum F_y = 0 = A_y - 480 \text{ N} \qquad \boxed{A_y = 480 \text{ N}}$$

$$\sum M_A = 0 = -(480 \text{ N})(100 \text{ mm}) + B(160 \text{ mm})$$

$$\boxed{B = 300 \text{ N}}$$

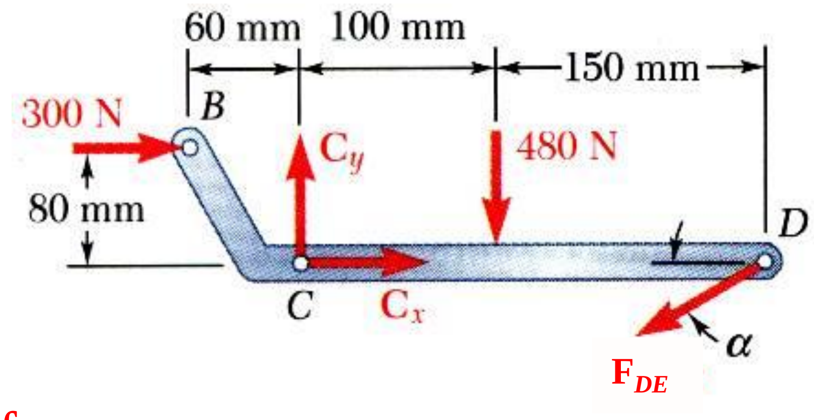
$$\sum F_x = 0 = B + A_x$$

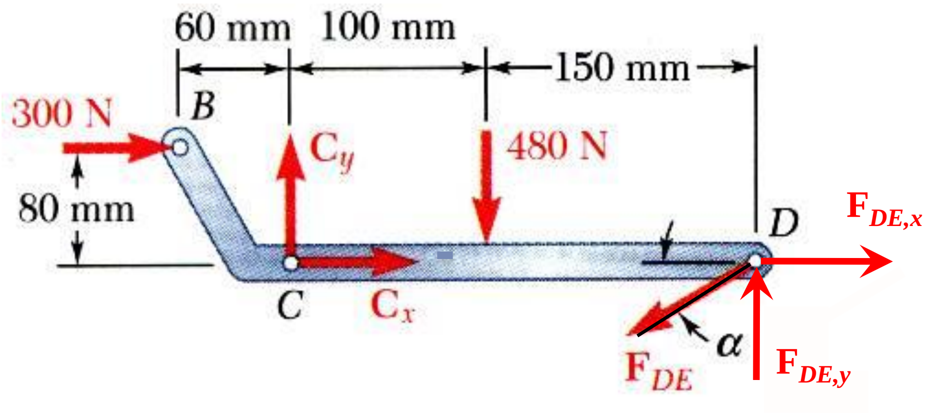
$$A_x = -300 \text{ N} \qquad \boxed{A_x = -300 \text{ N}}$$



SOLUTION (cont.):

2. Create an FBD for member *BCD* (since the problem asked for forces on this body).



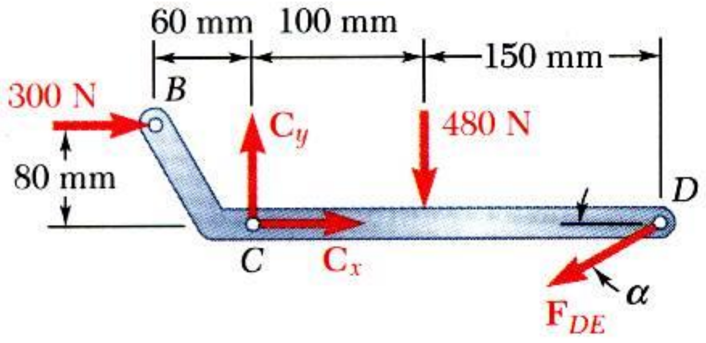


3. Using the best FBD for member *BCD*, what is the one equilibrium equation that can directly find $\mathbf{F}_{DE}$? Please discuss.

$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250\ \text{mm}) + (300\ \text{N})(60\ \text{mm}) + (480\ \text{N})(100\ \text{mm})$$

$$F_{DE} = -561\ \text{N}$$

$$\boxed{F_{DE} = 561\ \text{N}\ \ C}$$



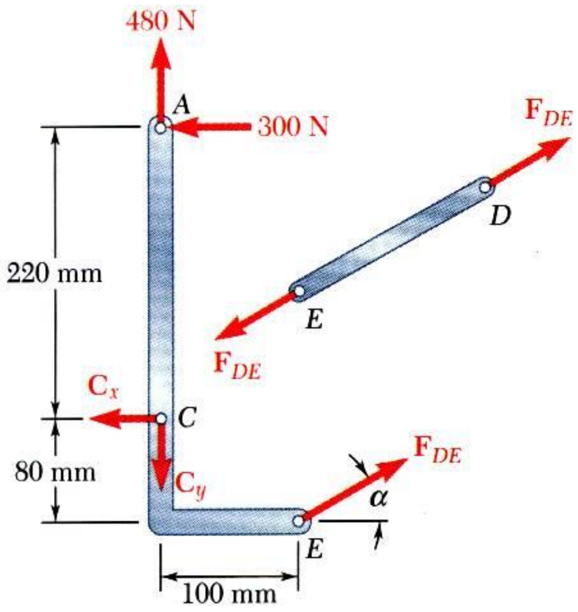
- Sum of forces in the x and y directions may be used to find the force components at C.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$
$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

$$\boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$
$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N}$$

$$\boxed{C_y = 216\text{ N}}$$

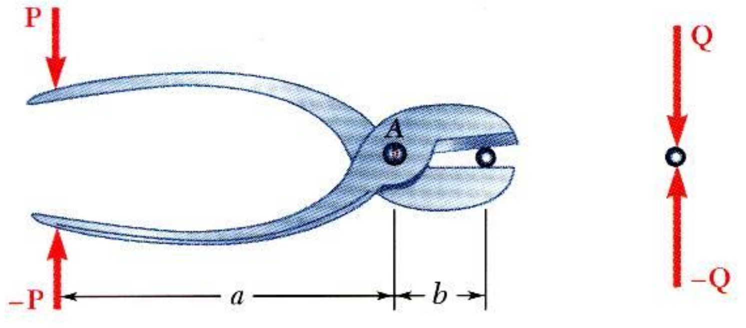


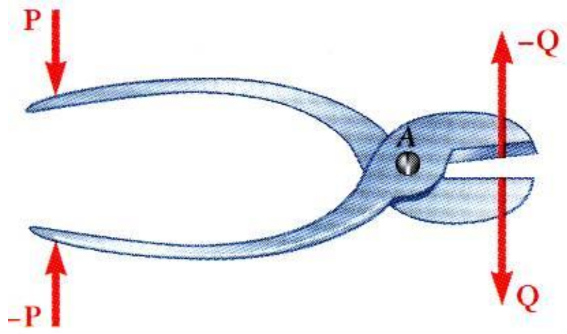
- With member ACE as a free body with no additional unknown forces, check the solution by summing moments about A.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

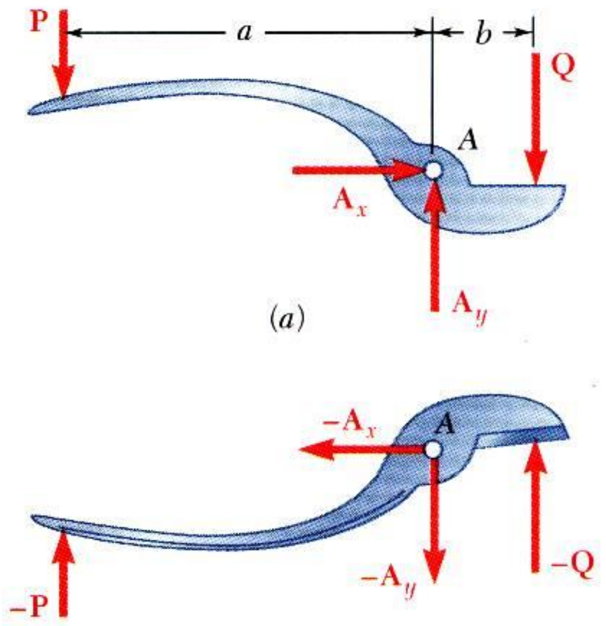
(checks)

Machines





- Machines are structures designed to transmit and modify forces. Typically, they transform *input forces* (**P**) into *output forces* (**Q**).



- Given the magnitude of **P**, determine the magnitude of **Q**.
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a non-rigid structure. Use one of the components as a free-body.
- Sum moments about *A*,

$$\sum M_A = 0 = aP - bQ \qquad Q = \frac{a}{b}P$$