MA101 Real Analysis
Multiple Integral

1. State Fubini's theorem. What is application of Fubini's theorem?

2. Evaluate the iterated integrals.

   (a) $\int_1^2 \int_0^4 2xydxdy$,

   (b) $\int_0^1 \int_0^1 \left(1 - \frac{x^2+y^2}{2}\right) dxdy$,

   (c) $\int_0^1 \int_1^2 xye^x dydx$,

   (d) $\int_\pi^{2\pi} \int_0^\pi xye^x dydx$,

3. Evaluate the double integral over the given region R.

   (a) $\iint_R (6y^2 - 2x)dA,\ R : 0 \le x \le 1,\ 0 \le y \le 2$,

   (b) $\iint_R (6y^2 - 2x)dA,\ R : 0 \le x \le 1,\ 1 \le y \le 2$.

4. Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

   (a) $\int_0^2 \int_0^{4-y^2} ydxdy$,

   (b) $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} ydxdy$,

   (c) $\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2 dydx$,

   (d) $\int_{y=0}^{y=\sqrt{3}} \int_{x=0}^{x=\tan^{-1} y} \sqrt{xy}dxdy$.

5. Volume beneath a surface:

   (a) Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by square $|x| \le 1$, $|y| \le 1$.

   (b) Find the volume of the region bounded above by the ellipitical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R : 0 \le x \le 2, 0 \le y \le 2$.

   (c) Find the volume of the region bounded above by the surface (Cylinder) $z = 4 - y^2$ and below by the rectangle $R : 0 \le x \le 1, 0 \le y \le 2$.

   (d) Find the volume of the solid that is hounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane.

6. Integrate $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$ and $x = 2$.

7. Minimizing Double Integral: What region R in the xy-plane minimizes the value of

$$\iint_R (x^2 + y^2 - 9)dA?$$

   Give reasons for your answer.

8. Sketch each region, label each bounding curve with its equation and give the coordinates of the points where the curves intersect Then find the area of the region.

   (a) $\int_0^{\pi/4} \int_{\sin x}^{\cos x} dydx$,

   (b) $\int_{-1}^{2} \int_{y^2}^{y+2} dxdy$,

   (c) $\int_{-1}^{0} \int_{-2x}^{1-x} dydx + \int_0^2 \int_{-x/2}^{1-x} dydx$.

9. Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral

   (a) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)dxdy$,

   (b) $\int_0^2 \int_0^x ydydx$,

   (c) $\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{(x^2+y^2)^2} dydx$,

   (d) $\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2} dydx$.

10. Triple Integral

    (a) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(b) The region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos \pi x/2$, $0 \le x \le 1$.

(c) Integrate the following integral in all possible orders (total 6):

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dzdydx.$$

(d) Find the volume common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

11. Transform and evaluate the integral

$$\iint_R (3x^2 + 14xy + 8y^2)dxdy$$

for the region R in the first quadrant bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$, $y = -(1/4)x$, and $y = -(1/4)x + 1$.

12. Give the limits of integration for evaluating the integral

$$\iiint f(r, \theta, z)dzrdrd\theta$$

as iterated integral over the region that is bounded below by the plane $z = 0$, on the side by the cylinder $r = \cos \theta$, and on top by the paraboloid $z = 3r^2$.

13. Convert the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x} (x^2 + y^2)dzdxdy$$

to an equivalent integral in cylindrical coordinates and evaluate result.

14. D is the solid right cylinder whose base is the region in xy-plane that lies inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 1$ and whose top lies in the plane $z = 4$. Use cylindrical system to find the volume of the region D.

15. Find the volume of the solid between the sphere $\rho = \cos \phi$ and the hemisphere $\rho = 2, z \ge 0$ using spherical system.

16. Find the volume of the solid bounded below by the hemisphere $\rho = 1, z \ge 0$, and above by the Cardioid of revolution $\rho = 1 + \cos \phi$.

17. Sphere and cones: Find the volume of the solid sphere $\rho \le a$ that lies between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{2\pi}{3}$.

18. Evaluate the integral by using a suitable transformation to uv-system

$$\int_{0}^{4}\int_{x=y/2}^{x=(y/2)+1} \frac{2x - y}{2} dxdy.$$

19. Evaluate

$$\iiint |xyz|dxdydz,$$

over the solid ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1,$$

by using suitable transformations. Hint: 2 transformations may be required for easy evaluation.

20. Polar moment of inertia of an elliptical plate: A thin plate of constant density covers the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = I$, $a > 0$, $b > 0$, in the $xy-$plane. Find the first moment of the plate about the origin. (Hint: Use the transformation $x = ar \cos \theta$, $y = br \sin \theta$.)