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1 (a) Let $\nu \propto R^a \rho^b S^c$

$\Rightarrow [\nu] = [R]^a [\rho]^b [S]^c$

$\therefore \; T^{-1} = L^a (ML^{-3})^b (MT^{-2})^c$.

$$\Rightarrow \begin{cases} -2c = -1 \\ a - 3b = 0 \\ b + c = 0 \end{cases}$$

Thus, $c = \frac{1}{2}$, $b = -\frac{1}{2}$, $a = -\frac{3}{2}$.

$$\therefore \; \nu \propto \sqrt{\frac{S}{\rho R^3}}.$$

[Note: Mass of the drop $m \propto \rho R^3$.

$\Rightarrow \nu \propto \sqrt{\frac{S}{m}}$.].

Thus if radius is doubled, frequency is reduced by a factor of $\frac{1}{2\sqrt{2}}$.

(b) $[\hbar] = ML^2T^{-1}$.

$[c] = LT^{-1}$.

$[G] = M^{-1}L^3T^{-2}$

Let $M_p = K_1 \hbar^a c^b G^c$

$[M_p] = [\hbar]^a [c]^b [G]^c$

$= [ML^2T^{-1}]^a [LT^{-1}]^b [M^{-1}L^3T^{-2}]^c$

$\therefore M^1 = M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$

$\Rightarrow a-c = 1$, $2a+b+3c = 0$

and $-a-b-2c = 0$.

$\Rightarrow a = -b-2c$. Thus, $-2b-4c+b+3c = 0$

$\therefore b = -c \Rightarrow a = -c$

$\therefore c = -\frac{1}{2}$, $a = \frac{1}{2}$, $b = \frac{1}{2}$.

$\therefore \boxed{M_p = K_1 \sqrt{\frac{\hbar c}{G}}}$.

(c) $T_p = K_2 \hbar^a c^b G^c$

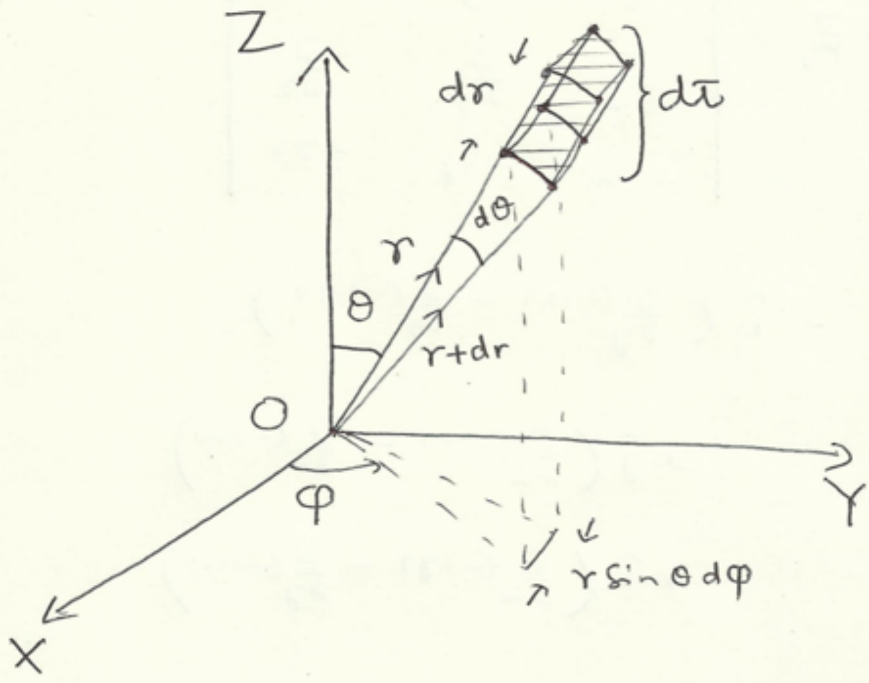
$\Rightarrow T^1 = M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$

$\therefore a = c$, $b = -5c$, $-c+5c-2c = 1$

$\therefore c = \frac{1}{2}$, $a = \frac{1}{2}$, $b = -\frac{5}{2}$.

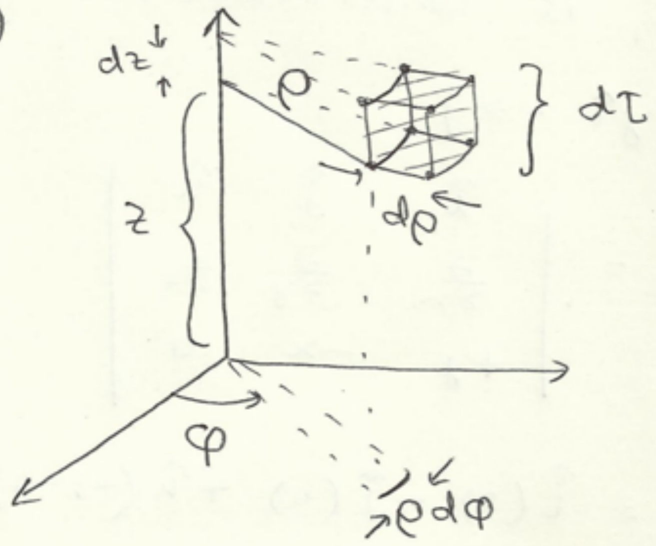
$\therefore \boxed{T_p = K_2 \sqrt{\frac{\hbar G}{c^5}}}$

2 (a.) (Done in class)




$d\tau = r^2 \sin\theta \, d\theta \, d\varphi \, dr$

(b.)



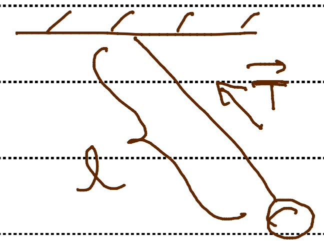

$d\tau = \rho \, d\rho \, d\varphi \, dz$

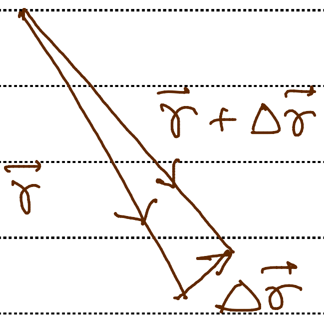
3 (a) Simple pendulum with rigid support.

Position of bob at any time must satisfy the equation

$|\vec{r}|^2 = l^2$, independent of time.

Also, tension $\vec{T} \parallel -\vec{r}$.




$\Delta\vec{r} \perp \vec{r}$.

$\Rightarrow \vec{T}\cdot\Delta\vec{r} = 0$.

$\therefore$ The constraint is scleronomic, holonomic, bilateral & conservative in nature.

(b) Deformable body

The constraint is given by an equation

$|\vec{r}_i - \vec{r}_k| = f(t)$ (time dependent)

Here $\Delta W \neq 0$.

$\therefore$ The constraint is rheonomic, holonomic, bilateral and dissipative in nature.

3(c.) An expanding/contracting spherical container of gas

Position of gas particles at any time will satisfy

$$|\vec{r}| \leq R(t)$$

If the chamber is expanding/contracting, the kinetic energy of the bouncing particle decreases/increases at each bounce.

The constraint is therefore rheonomic, holonomic, dissipative and unilateral.

4.(a.) DOF (dumbbell in 2d) = 3.

(b.) DOF (bob of a conical pendulum) = 2.

(c.) DOF (rigid body fixed at a point) = 3.

END