MA101 Real Analysis
Max-Min, Lagrange Multiplier

1. Draw and realize the following 3D figures:
   - The Cone: $z^2 = x^2 + y^2$.
   - The Paraboloid: $z = x^2 + y^2$.
   - The Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
   - The Spheroid: $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$.
   - Cylinder: $x^2 + y^2 = 1$. (How to define a Cylinder?)
   - Cylinder: $y^2 + z^2 = 1$.
   - Cylinder: $x^2 + 2y^2 = 1$.

2. Find the maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

3. Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$.

4. Show that if $f(x, y) = 2x^4 - 3x^2y + y^2$ then $f_{xx}f_{xy} - (f_{xy})^2 = 0$ at $(0, 0)$ but f has neither a maximum nor a minimum value at $(0, 0)$.

5. (a)Use Lagrange multipliers to find all the critical points of f on the surfaces(curves), given below. (b) Determine also the maxima and minima of f on the surfaces (or curves) by evaluating f at the critical values:

   (i) The function $f(x, y, z) = x + y + 2z$ on the surface $x^2 + y^2 + z^2 = 3$.

   (ii) The function $f(x, y) = xy$ on the curve $3x^2 + y^2 = 6$.

   (iii) The function $f(x, y, z) = x^2y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$. (Make sure you find all the critical points!).

6. Use Lagrange multipliers to show that $f(x, y, z) = z^2$ has only one critical point on the surface $x^2 + y^2 - z = 0$. Show that the one critical point is a minimum.

7. Show that the maximum and minimum value of $r^2 = a^2x^2 + b^2y^2 + c^2z^2$, where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$ are given by

$$\frac{l^2}{a^2-r^2} + \frac{m^2}{b^2-r^2} + \frac{n^2}{c^2-r^2}$$

8. Find the absolute maxima and minima of the following function on the given domain

$$f(x,y) = \left(4x - x^2\right)\cos y, \quad 1 \le x \le 3, \quad -\frac{\pi}{4} \le y \le \frac{\pi}{4}.$$

How the surface will look like (try to sketch)?

9. Use Lagrange's Multiplies to Maximize $f(x,y) = x + y$ subject to $g(x,y) = xy - 16 = 0$. What is the conclusion?

10. The plane $x + y + x = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

11. Minimize $f(x,y,z) = xy + yz$ subject to the constraints $x^2 + y^2 - 2 = 0$ and $x^2 + z^2 - 2 = 0$.