



CS540 Intro to AI Uninformed Search

**University of Wisconsin-Madison
Fall 2023**

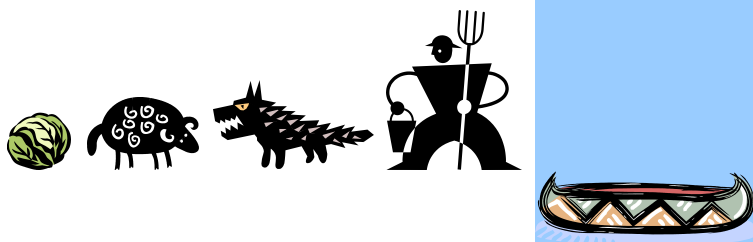
Many AI problems can be formulated as search.

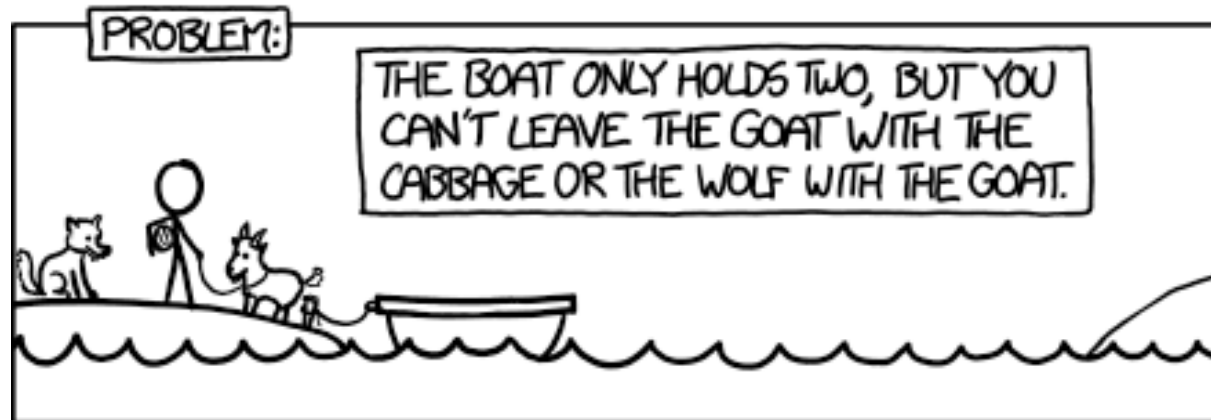
How to make a sequence of decisions to reach a desired goal.

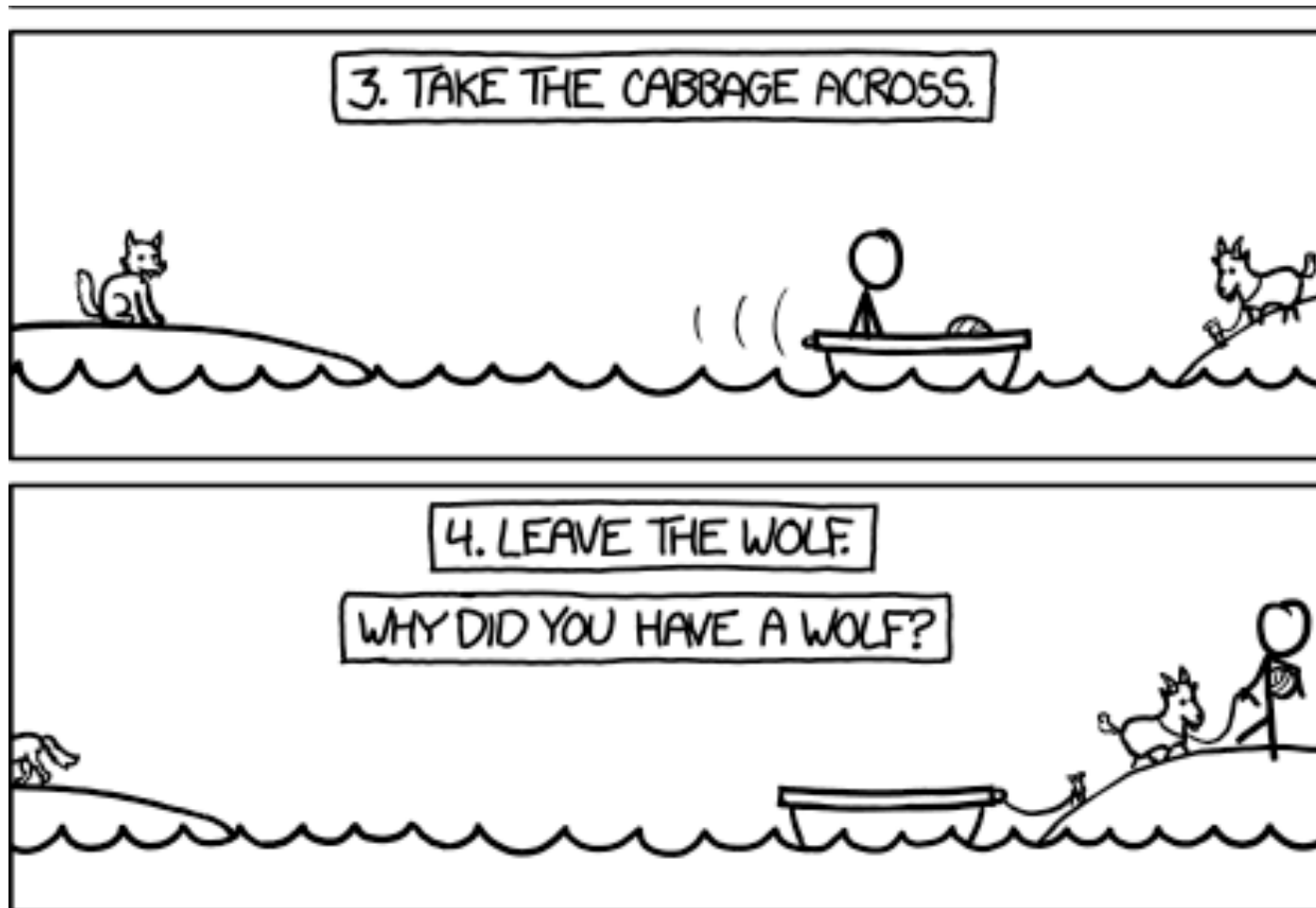
Leverage computation and a known model of world **dynamics** to make decisions.



“How the world changes in response to agent actions”

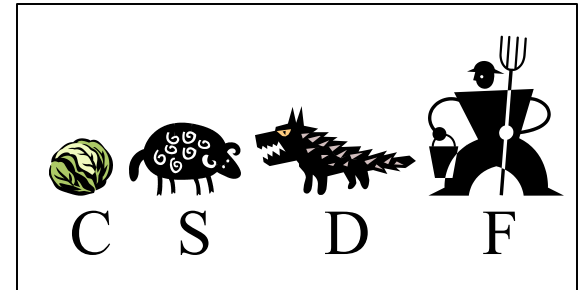




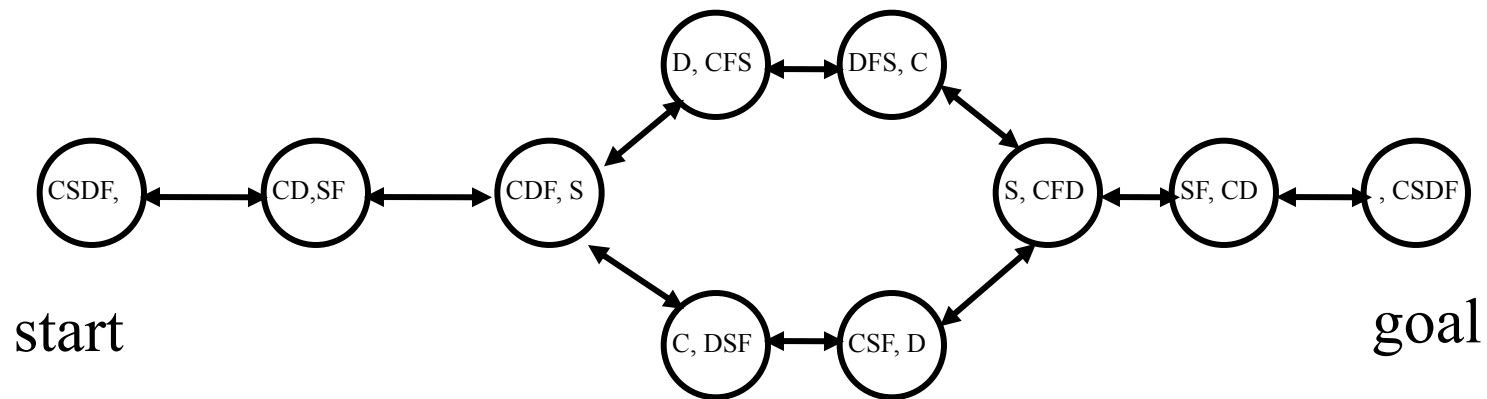
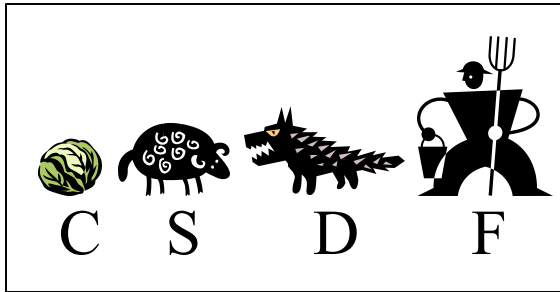


The search problem

- State space **S** : all valid configurations
- Initial state **I** = {(CSDF,)} \subseteq **S**
- Goal state **G** = {(,CSDF)} \subseteq **S**
- Successor function **succs**(s) \subseteq **S** : states reachable in one step from state s
 - **succs**((CSDF,)) = {(CD, SF)}
 - **succs**((CDF,S)) = {(CD,FS), (D,CFS), (C, DFS)}
- **Cost**(s,s')=1 for all steps. (weighted later)
- The search problem: find a solution path from a state in **I** to a state in **G**.
 - Optionally minimize the cost of the solution.



A directed graph in state space



Search examples

- 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- States = 3x3 array configurations
- Actions / Operators = up to 4 kinds of movement
- Cost = 1 for each move

Search examples

- Water jugs: how to get 1?



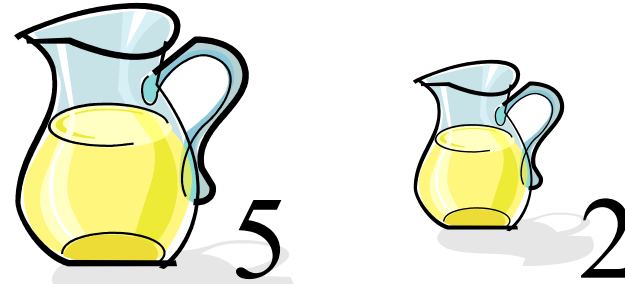
State = (x,y) , where x = number of gallons of water in the 5-gallon jug and y is gallons in the 2-gallon jug

Initial State = $(5,0)$

Goal State = $(*,1)$, where $*$ means any amount

Search examples

- Water jugs: how to get 1?



State = (x,y) , where x = number of gallons of water in the 5-gallon jug and y is gallons in the 2-gallon jug

Initial State = $(5,0)$

Goal State = $(*,1)$, where $*$ means any amount

Operators

$(x,y) \rightarrow (0,y)$; empty 5-gal jug

$(x,y) \rightarrow (x,0)$; empty 2-gal jug

$(x,2)$ and $x \leq 3 \rightarrow (x+2,0)$; pour 2-gal into 5-gal

$(x,0)$ and $x \geq 2 \rightarrow (x-2,2)$; pour 5-gal into 2-gal

$(1,0) \rightarrow (0,1)$; empty 5-gal into 2-gal

Search examples

- Route finding (State? Successors? Cost weighted)

Google Maps - from: 1210 W Dayton St, Madison, WI 53706 to: State St, Madison, WI 53703 - Mozilla Firefox

File Edit View Go Bookmarks Tools Help

Google Maps [Maps](#) [Local Search](#) [Directions](#)

1210 W Dayton St, Madison, WI 53706 Start address

State St, Madison, WI 53703 End address

Search Help

Maps

Map Satellite

Print Email Link to this page

Start address: 1210 W Dayton St
Madison, WI 53706

End address: State St
Madison, WI 53703

Distance: 1.2 mi (about 2 mins)

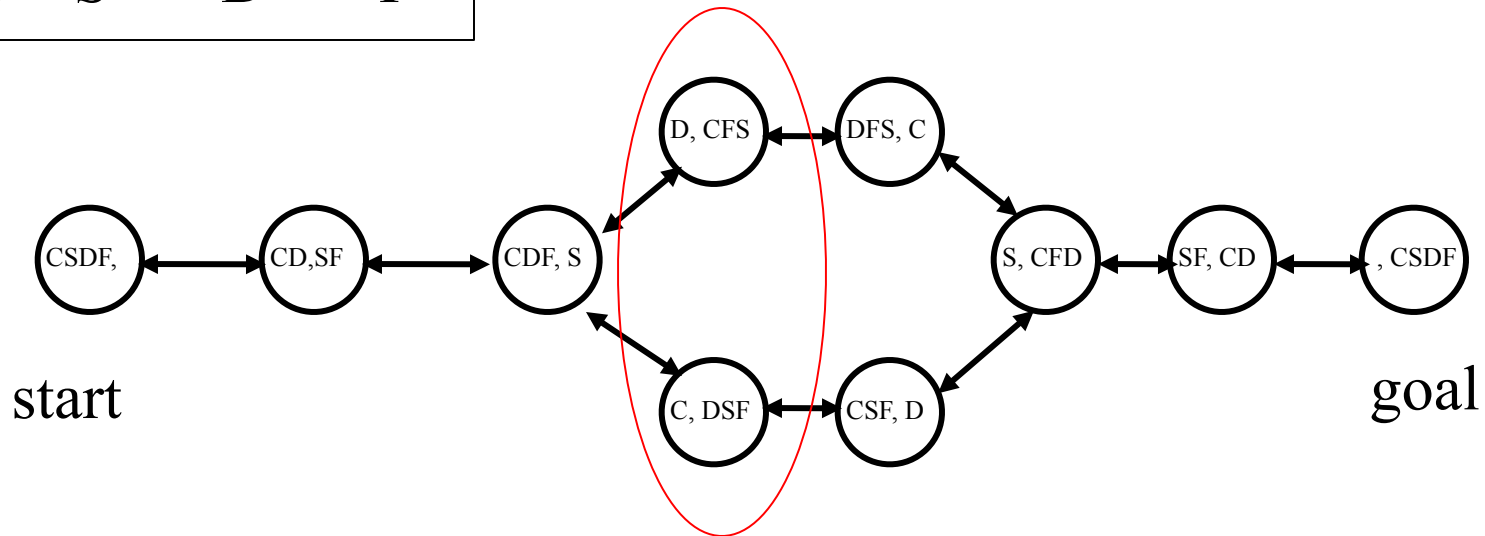
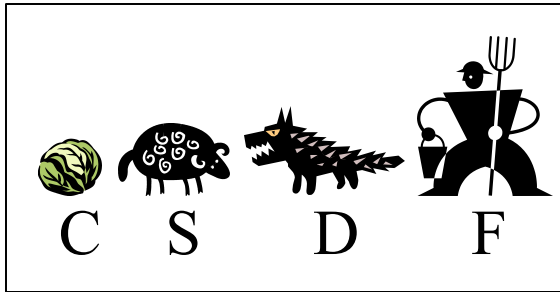
[Reverse directions](#)

- 1 Head east from W Dayton St - go 0.5 mi
- 2 Turn left at N Frances St - go 0.2 mi
- 3 Turn right at W Gilman St - go 0.3 mi
- 4 Turn right at N Henry St - go 0.1 mi
- 5 Turn right at W Gorham St - go 0.1 mi

These directions are for planning purposes only. You may find that construction projects, traffic, or other events may cause road conditions to differ from the map results.

Map data ©2005 NAVTEQ™, Tele Atlas

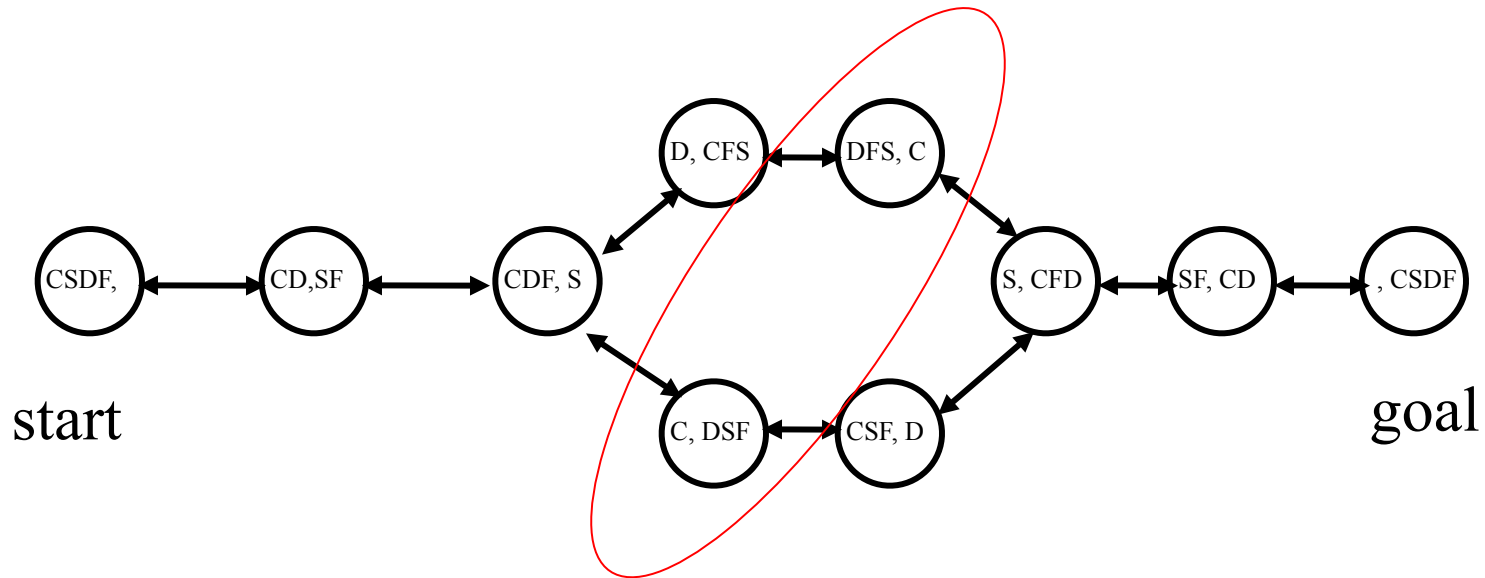
A directed graph in state space



- In general there will be many generated, but un-expanded states at any given time
- One has to choose which one to expand next

Different search strategies

- The generated, but not yet expanded states form the **fringe** (**OPEN**).
- The essential difference is **which one to expand first**.
- Deep or shallow?



Uninformed search on trees

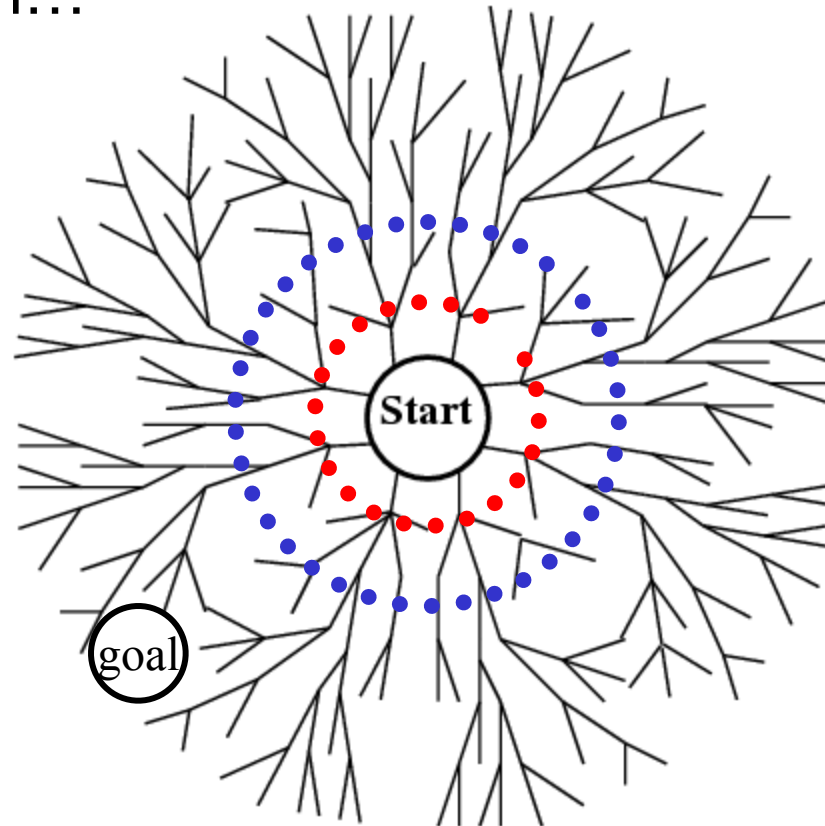
- **Uninformed** means we only know:
 - The goal test
 - The *succs()* function
- But **not** which non-goal states are better: that would be informed search (next topic).
- For now, we also assume *succs()* graph is **a tree**.
 - Won't encounter repeated states.
 - We will relax it later.
- Many search strategies:
 - We will see BFS, UCS, DFS, IDS
- Differ by what un-expanded nodes to expand

Breadth-first search (BFS)

Expand the shallowest node first

- Examine states **one** step away from the initial states
- Examine states **two** steps away from the initial states
- and so on...

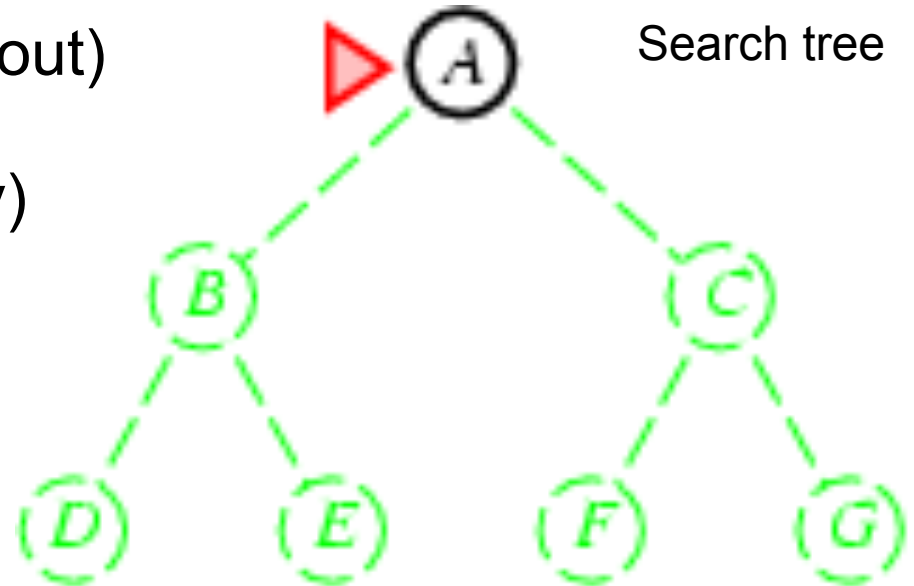
ripple



Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



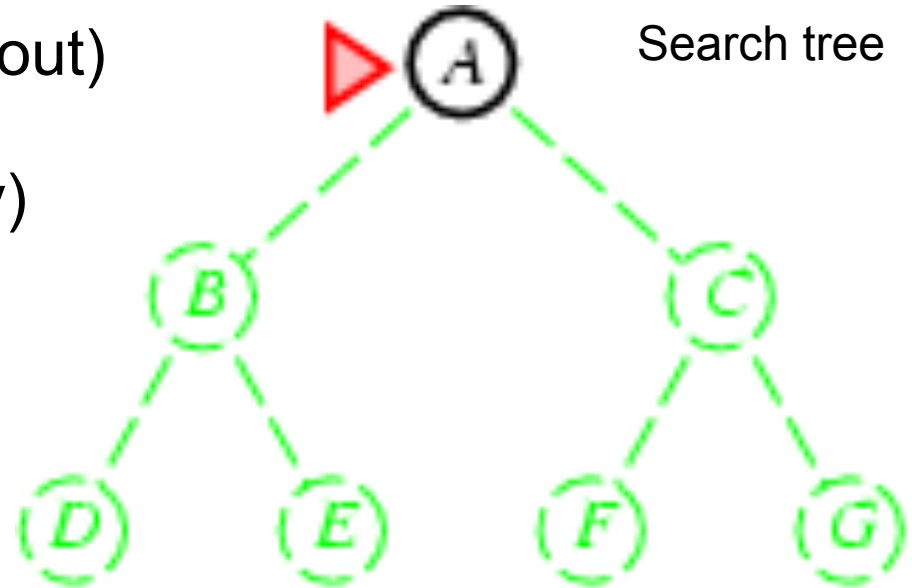
Initial state: **A**

Goal state: **G**

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue (**fringe**, **OPEN**)
→ [A] →

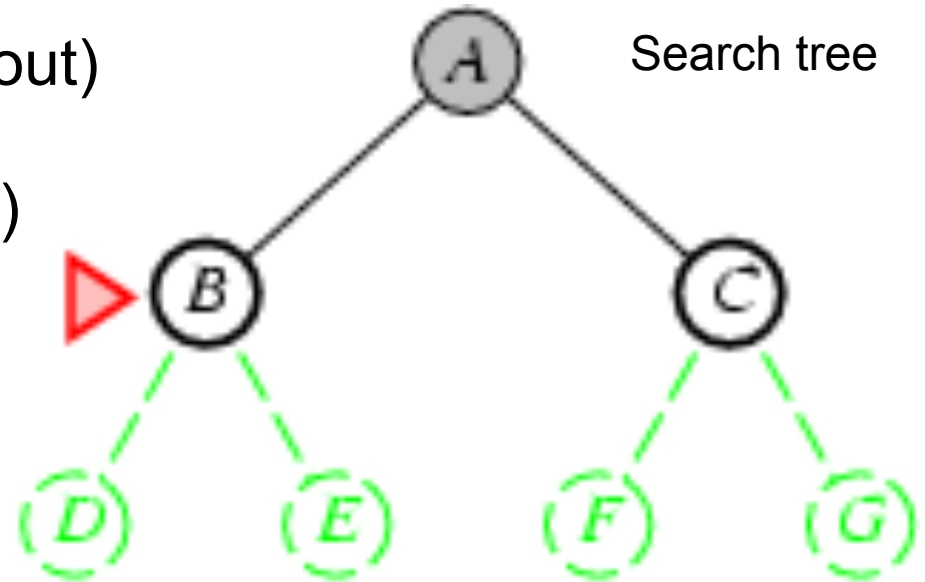
Initial state: **A**

Goal state: **G**

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue (**fringe**, **OPEN**)
→ [CB] → A

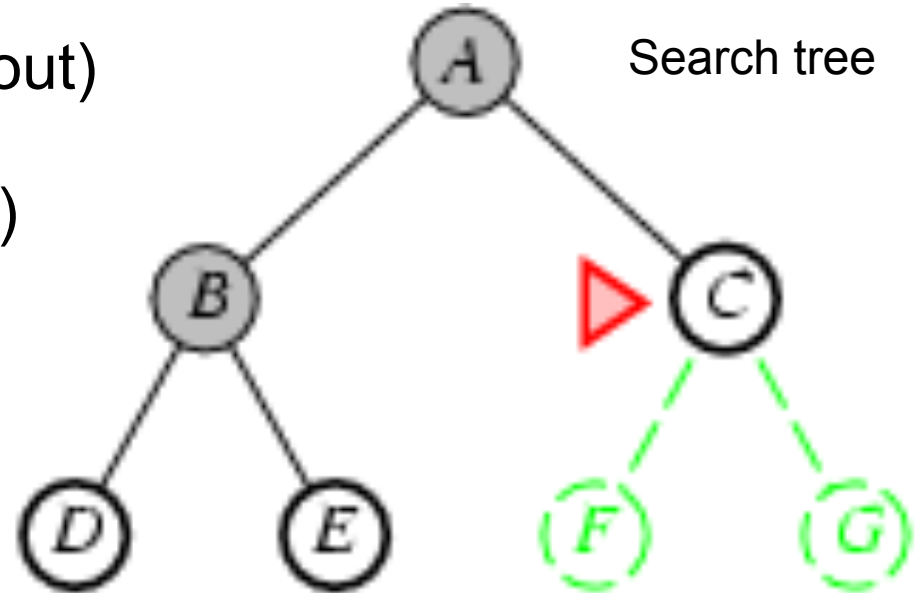
Initial state: **A**

Goal state: **G**

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue (**fringe**, **OPEN**)
→ [EDC] → B

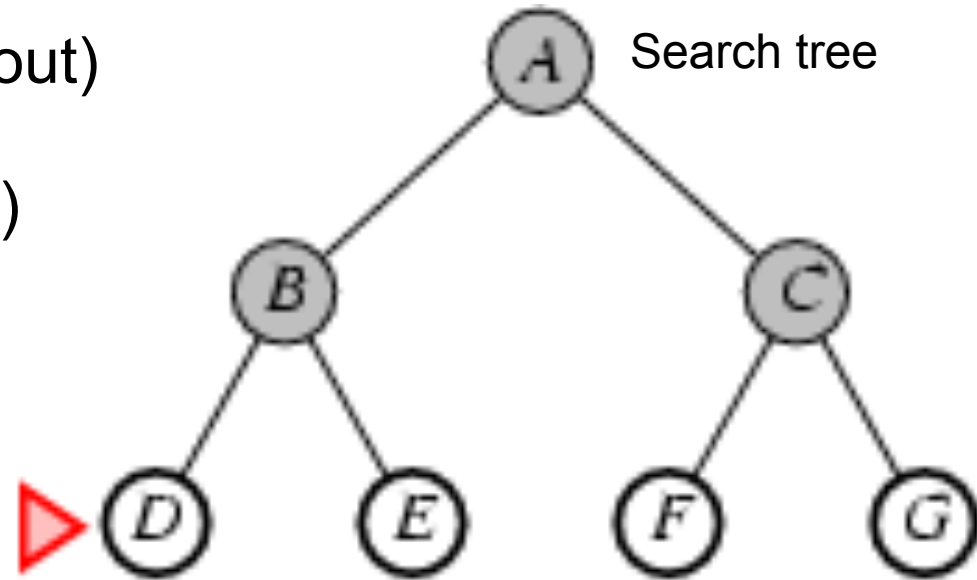
Initial state: **A**

Goal state: **G**

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue (**fringe, OPEN**)
→[GFED] → C

Initial state: **A**

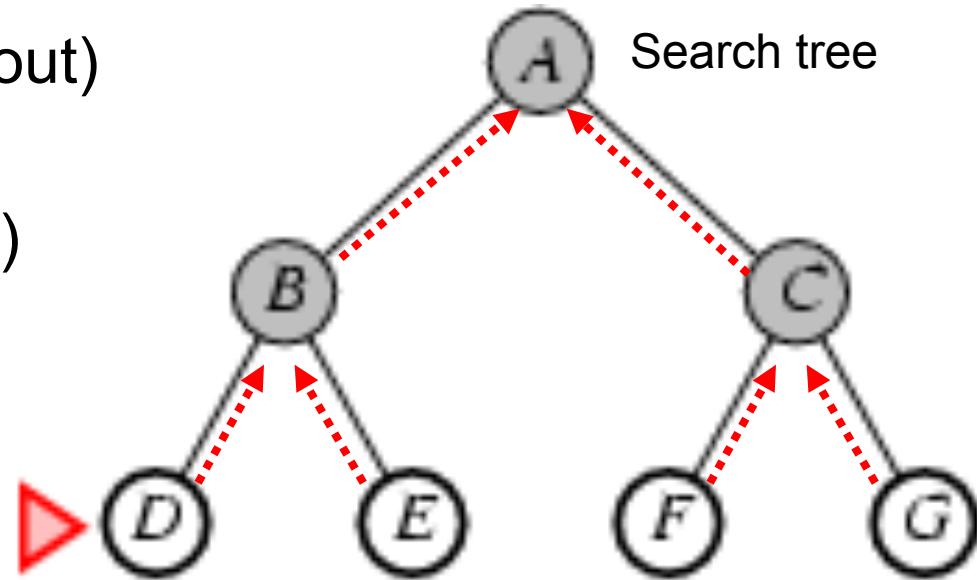
Goal state: **G**

If G is a goal, we've seen it, but we don't stop!

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue
→[] →G

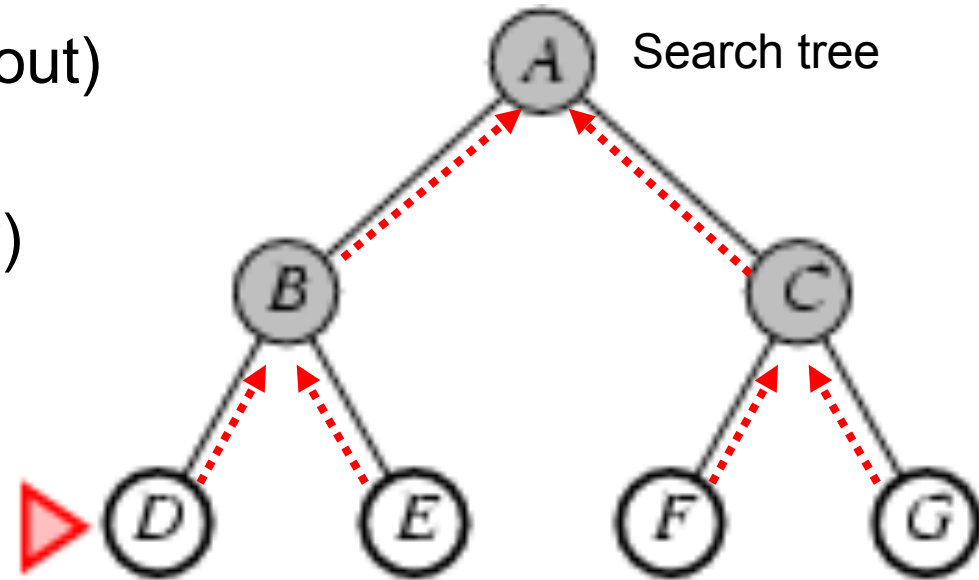
... until much later we pop G.

Looking foolish?
Indeed. But let's be
consistent...

Breadth-first search (BFS)

Use a **queue** (First-in First-out)

1. en_queue(Initial states)
2. While (queue not empty)
3. s = de_queue()
4. if (s==goal) success!
5. T = succs(s)
6. en_queue(T)
7. endwhile



queue
→ [] → G

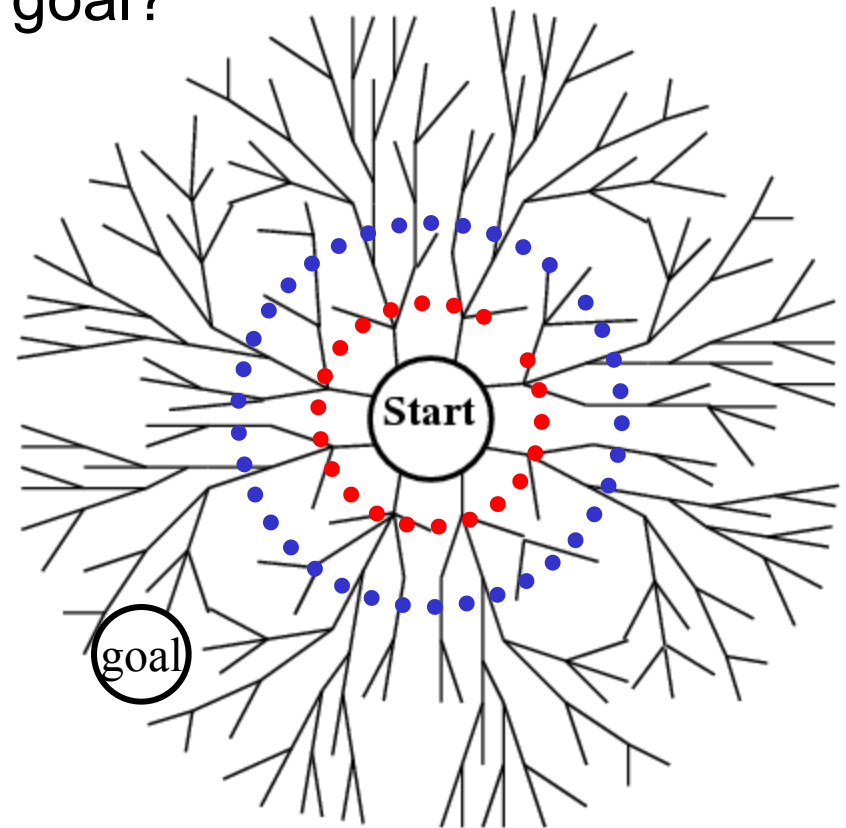
... until much later we pop G.

Looking foolish?
Indeed. But let's be
consistent...

We need **back pointers** to
recover the solution path.

Performance of BFS

- Assume:
 - the graph may be infinite.
 - Goal(s) exists and is only finite steps away.
- Will BFS find at least one goal?
- Will BFS find the least cost goal?
- Time complexity?
 - # states generated
 - Goal d edges away
 - Branching factor b
- Space complexity?
 - # states stored



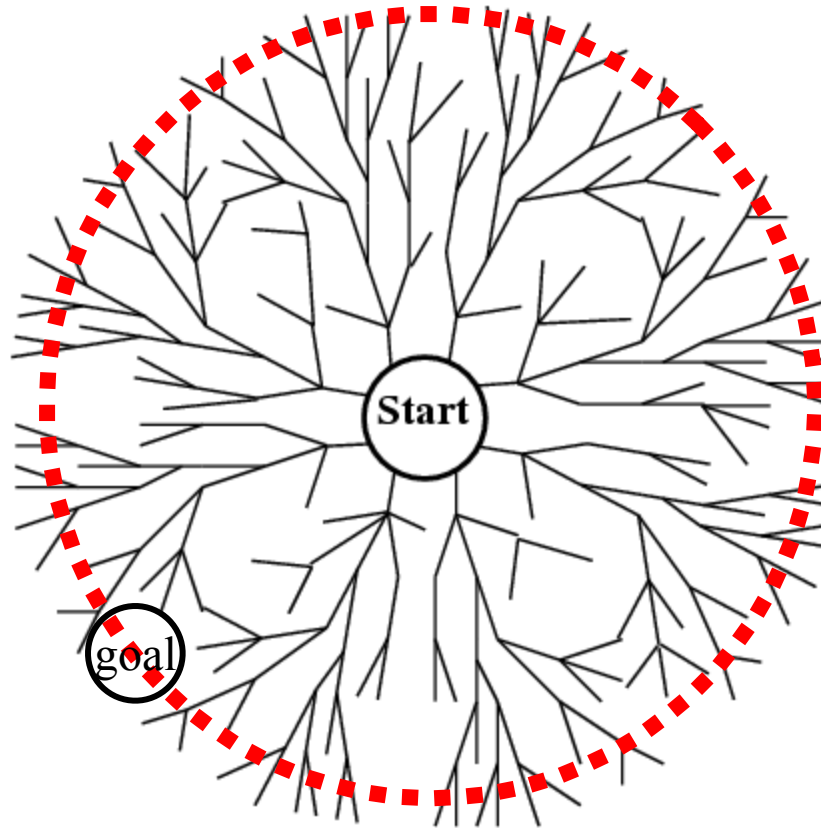
Performance of BFS

Four measures of search algorithms:

- **Completeness** (**not** finding all goals): yes, BFS will find a goal.
- **Optimality**: yes if edges cost 1 (more generally positive non-decreasing in depth), **no otherwise**.
- **Time** complexity (worst case): goal is the last node at radius d .
 - Have to generate all nodes at radius d .
 - $b + b^2 + \dots + b^d \sim O(b^d)$
- **Space** complexity (**bad**)
 - Back pointers for all generated nodes $O(b^d)$
 - The queue / fringe (smaller, but still $O(b^d)$)

What's in the fringe (queue) for BFS?

- Convince yourself this is $O(b^d)$



Performance of search algorithms on trees

b: branching factor (assume finite) d: goal depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if 1	$O(b^d)$	$O(b^d)$

1. Edge cost constant, or positive non-decreasing in depth

Q1-1: You are running BFS on a finite tree-structured state space graph that does not have a goal state. What is the behavior of BFS?

1. Visit all N nodes, then return one at random
2. Visit all N nodes, then stop and return “failure”
3. Visit all N nodes, then return the node farthest from the initial state
4. Get stuck in an infinite loop

Q1-1: You are running BFS on a finite tree-structured state space graph that does not have a goal state. What is the behavior of BFS?

1. Visit all N nodes, then return one at random
2. Visit all N nodes, then stop and return “failure”
3. Visit all N nodes, then return the node farthest from the initial state
4. Get stuck in an infinite loop



Performance of BFS

Four measures of search algorithm

Solution:
Uniform-cost
search

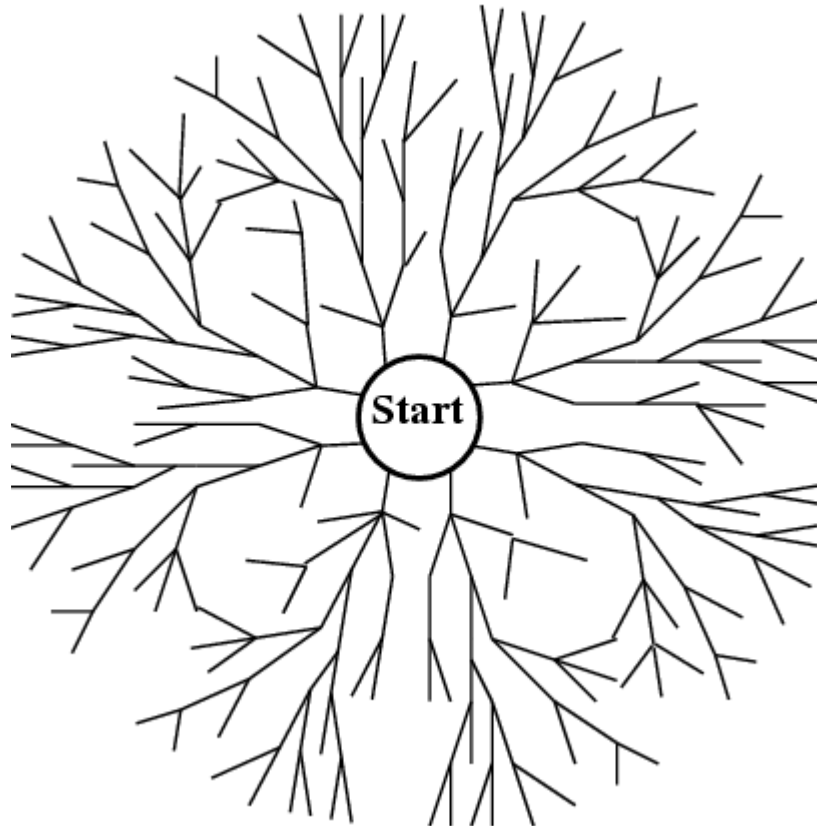
- **Completeness** (**not** finding all goals will find a goal).
- **Optimality**: yes if edges cost 1 (most generally positive non-decreasing in depth), **no otherwise**.
- **Time** complexity (worst case): goal is the last node at radius d .
 - Have to generate all nodes at radius d .
 - $b + b^2 + \dots + b^d \sim O(b^d)$
- **Space** complexity (**bad**)
 - Back pointers for all generated nodes $O(b^d)$
 - The queue / fringe (smaller, but still $O(b^d)$)

Uniform-cost search

- Find the least-cost goal
- Each node has a path cost from start (= sum of edge costs along the path).
- Expand the least cost node first.
- Use a **priority queue** instead of a normal queue
 - Always take out the least cost item

Uniform-cost search (UCS)

- Complete and optimal (if edge costs $\geq \epsilon > 0$)
- Time and space: can be much worse than BFS
 - Let C^* be the cost of the least-cost goal
 - $O(b^{C^*/\epsilon})$



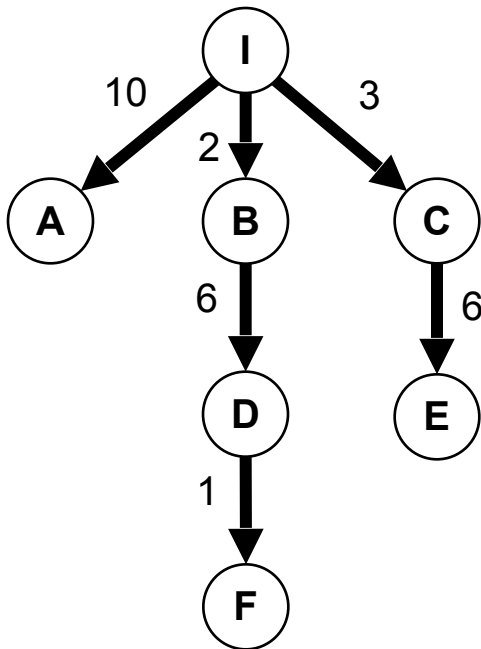
Performance of search algorithms on trees

b: branching factor (assume finite) d: goal depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if 1	$O(b^d)$	$O(b^d)$
Uniform-cost search ²	Y	Y	$O(b^{C^*/\epsilon})$	$O(b^{C^*/\epsilon})$

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \epsilon > 0$. C^* is the best goal path cost.

Q1-2: You are running UCS in the state space graph below. You just called the successor function on node D. What is the cost of node F?



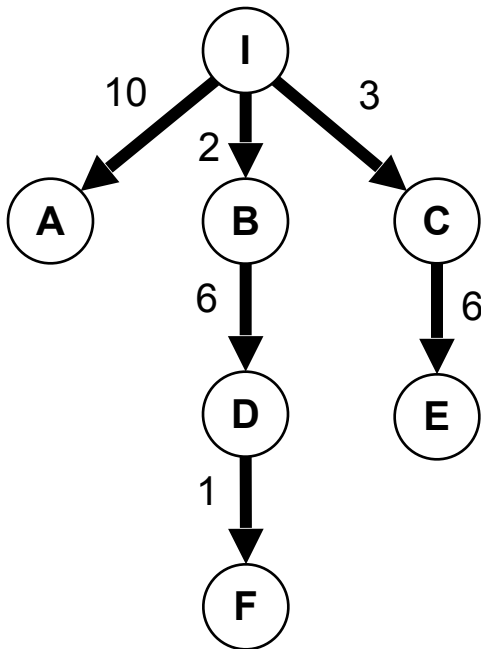
1. 2

2. 7

3. 8

4. 9

Q1-2: You are running UCS in the state space graph below. You just called the successor function on node D. What is the cost of node F?



1. 2

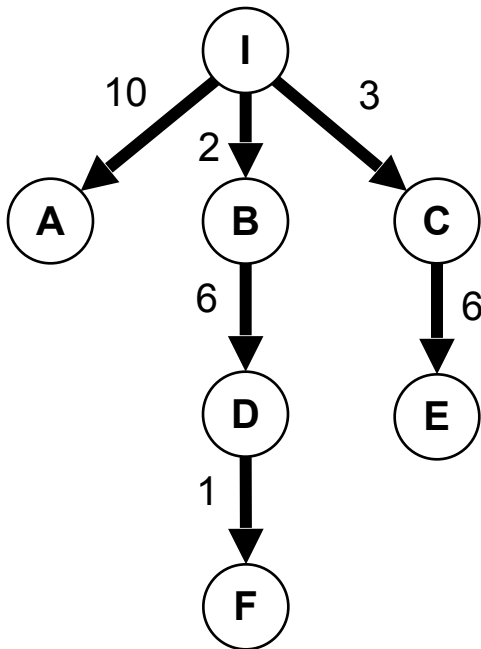
2. 7

3. 8

4. 9



Q1-3: You are running UCS in the state space graph below. You just expanded (visited) node C. What node will the search expand next?



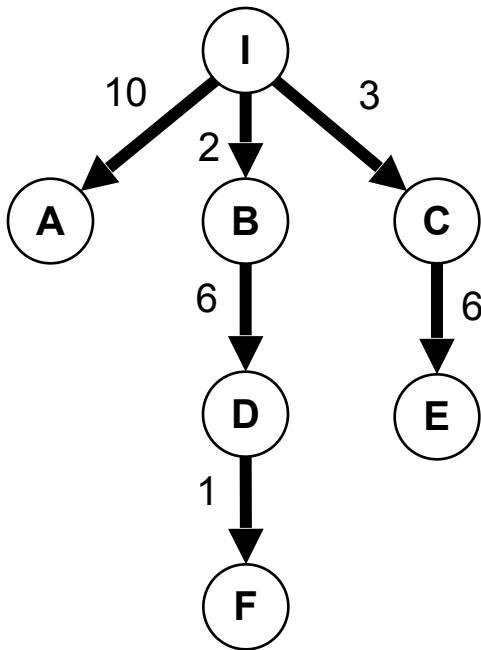
1. A

2. D

3. E

4. F

Q1-3: You are running UCS in the state space graph below. You just expanded (visited) node C. What node will the search expand next?



1. A
2. D
3. E
4. F
- ➔

General State-Space Search Algorithm

```
function general-search(problem, QUEUEING-FUNCTION)
  # problem describes the start state, operators, goal test, and
  #   operator costs
  # queueing-function is a comparator function that ranks two states
  # general-search returns either a goal node or "failure"

  nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
    if EMPTY(nodes) then return "failure"
    node = REMOVE-FRONT(nodes)
    if problem.GOAL-TEST(node.STATE) succeeds then return node
    nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
                                             problem.OPERATORS))
  # succ(s)=EXPAND(s, OPERATORS)
  # Note: The goal test is NOT done when nodes are generated
  # Note: This algorithm does not detect loops
end
```

Performance of BFS

Four measures of search algorithm

- **Completeness** (not finding all goals will find a goal.
- **Optimality**: yes if edges cost 1 (most generally positive non-decreasing in depth), no otherwise.
- **Time** complexity: goal is the last node at radius d .
 - Have to generate all nodes at radius d .
 - $b + b^2 + \dots + b^d \sim O(b^d)$
- **Space** complexity (bad)
 - Back pointers for all generated nodes $O(b^d)$
 - The queue / fringe (smaller, but still $O(b^d)$)

Solution:
Uniform-cost
search

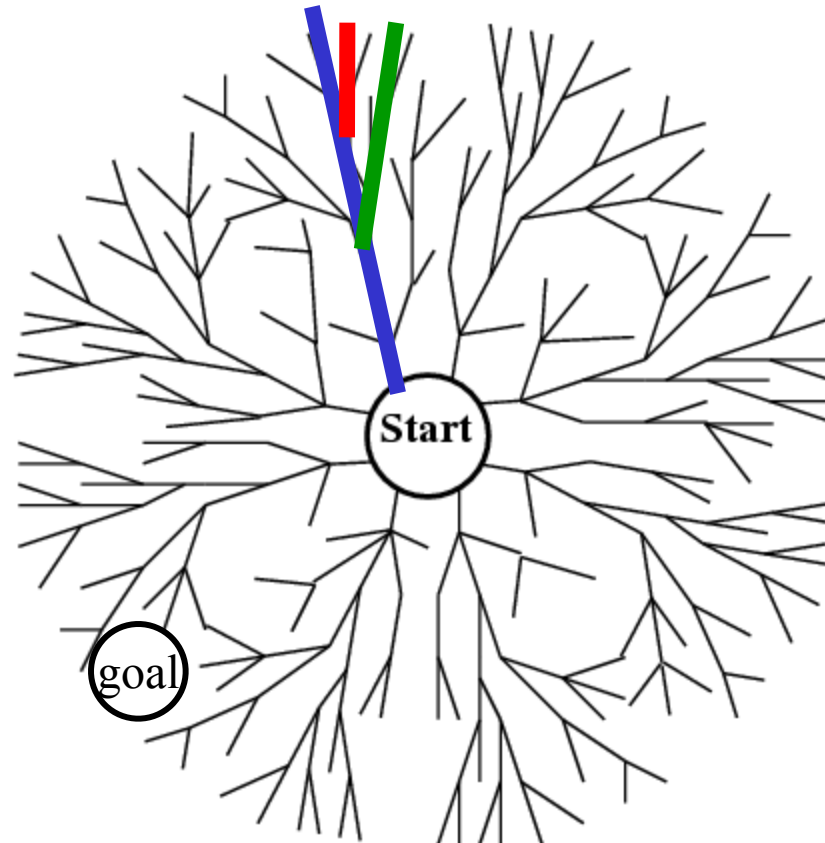
Solution:
Depth-first
search

Depth-first search

Expand the deepest node first

1. Select a direction, go deep to the end ————
2. Slightly change the end ————
3. Slightly change the end some more... ————

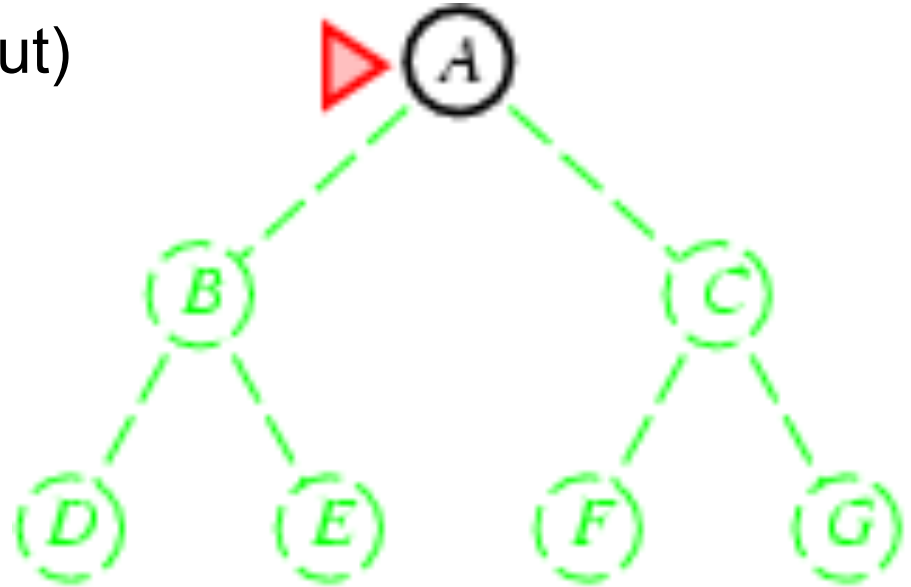
fan



Depth-first search (DFS)

Use a **stack** (First-in Last-out)

1. push(Initial states)
2. While (stack not empty)
3. s = pop()
4. if (s==goal) success!
5. T = succs(s)
6. push(T)
7. endwhile

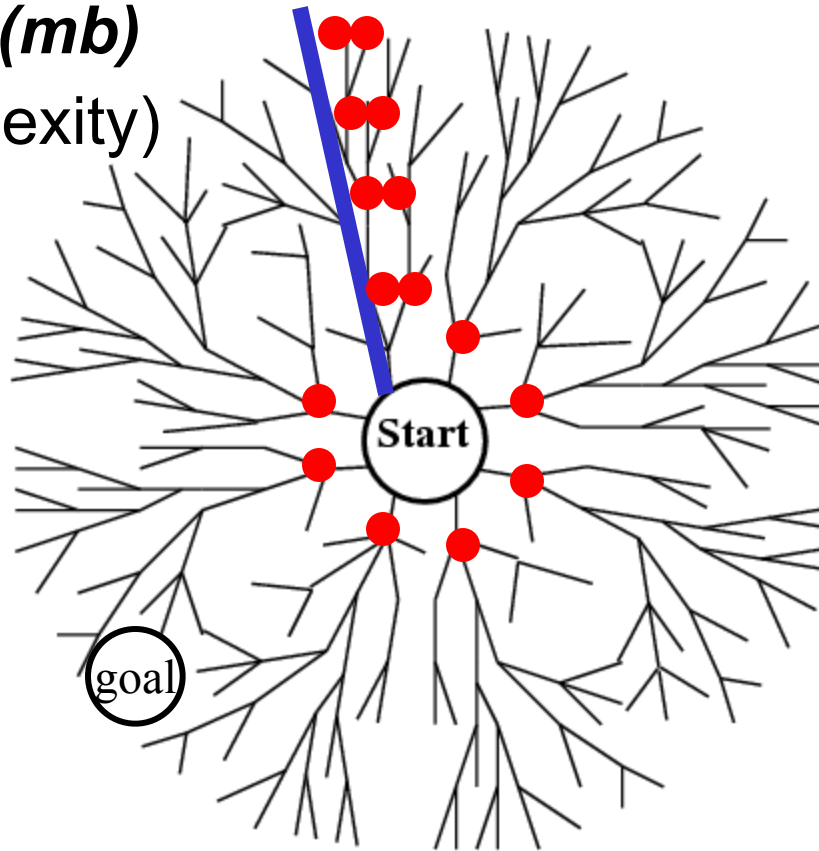


stack (**fringe**)

1. A, [B, C]
2. B, [D, E, C]
3. D, [E, C]
4. E, [C]
5. C, [F, G]
6. F, [G]
7. G

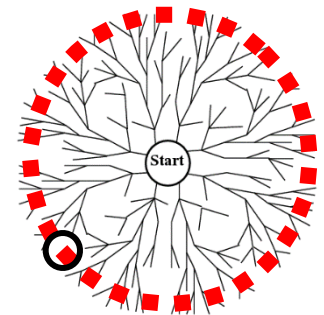
What's in the fringe for DFS?

- m = maximum depth of graph from start
- $m(b-1) \sim O(mb)$
(Space complexity)



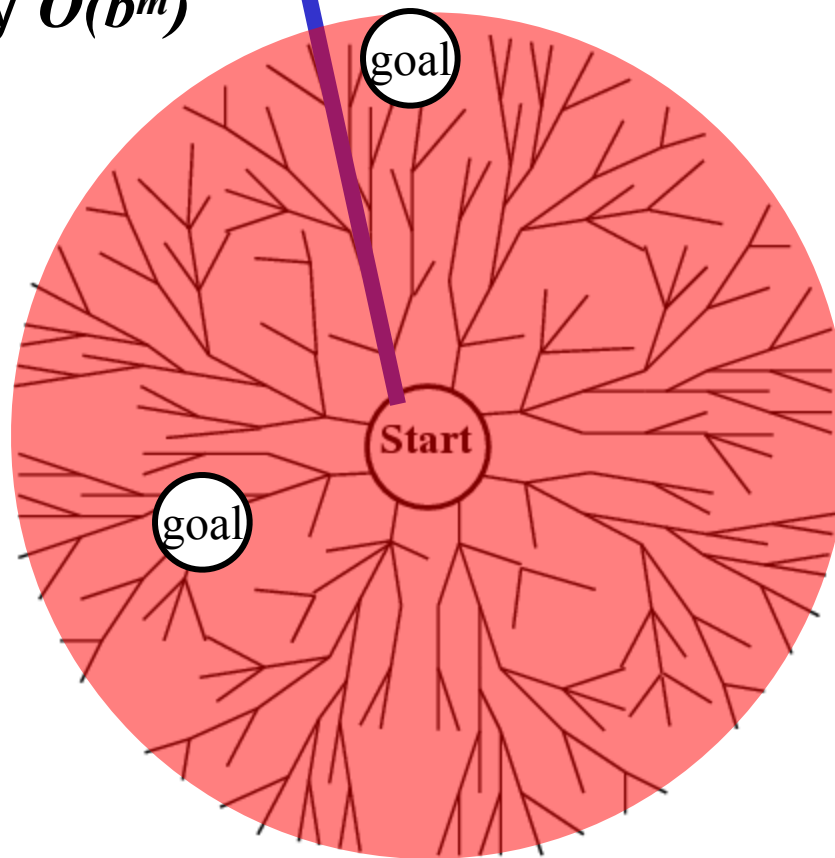
- “backtracking search” even less space
 - generate siblings (if applicable)

c.f. BFS $O(b^d)$

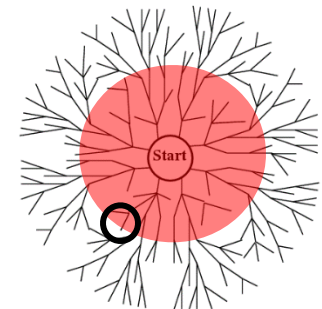


What's wrong with DFS?

- Infinite tree: may not find goal (incomplete)
- May not be optimal
- Finite tree: may visit almost all nodes, time complexity $O(b^m)$



c.f. BFS $O(b^d)$



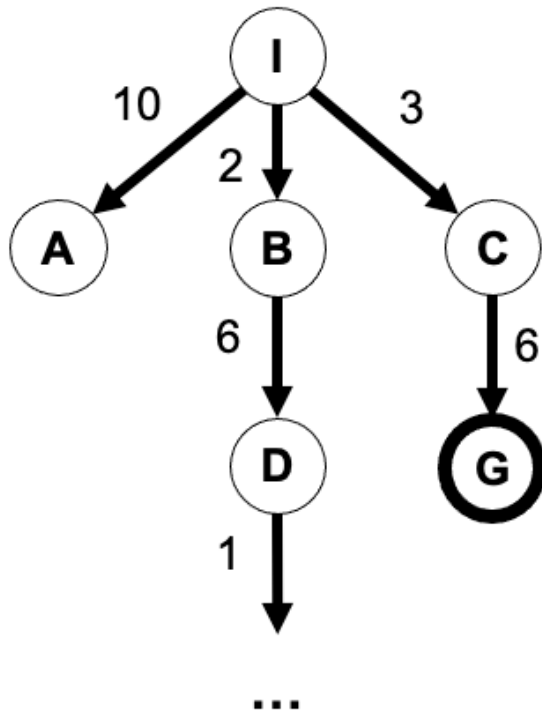
Performance of search algorithms on trees

b: branching factor (assume finite) d: goal depth m: graph depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if ¹	$O(b^d)$	$O(b^d)$
Uniform-cost search ²	Y	Y	$O(b^{C^*/\epsilon})$	$O(b^{C^*/\epsilon})$
Depth-first search	N	N	$O(b^m)$	$O(bm)$

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \epsilon > 0$. C^* is the best goal path cost.

Q2-1: You are running DFS in the state space graph below. DFS expands nodes left to right. G is the goal state. The state space graph is infinite (the path after D does not terminate). What is the behavior of DFS?



1. Get stuck in an infinite loop
2. Return A
3. Return G
4. Return "failure"

Q2-2: You need to search a randomly generated state space graph with one goal, uniform edges costs, $d=2$, and $m=100$. Considering worst case behavior, do you select BFS or DFS for your search?

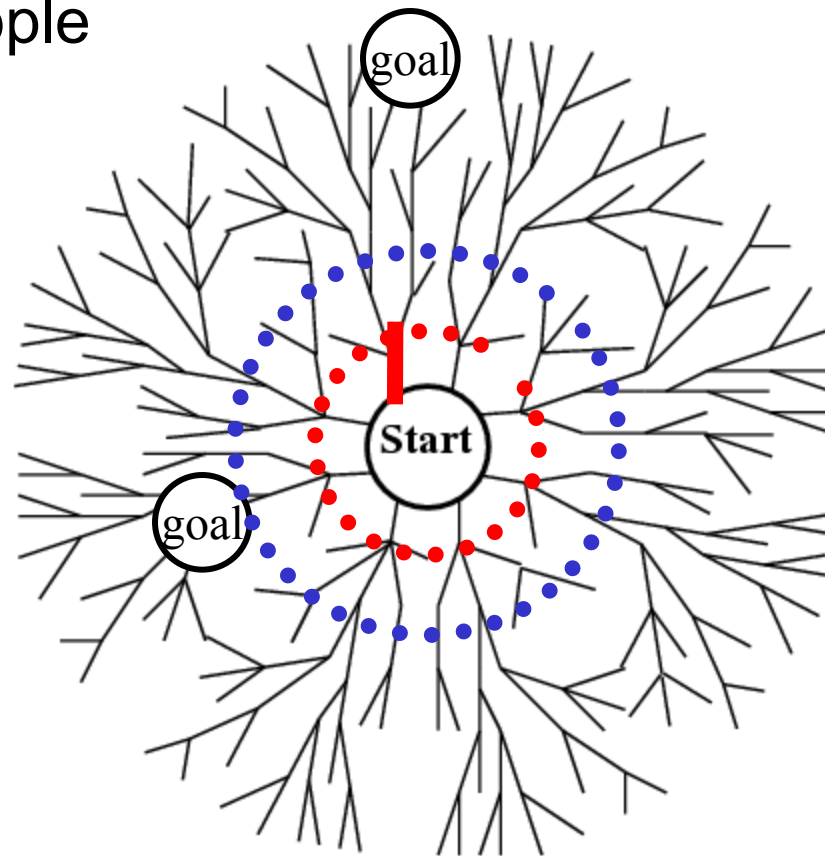
1. BFS 

2. DFS

How about this?

1. DFS, but stop if path length > 1 .
2. If goal not found, repeat DFS, stop if path length > 2 .
3. And so on...

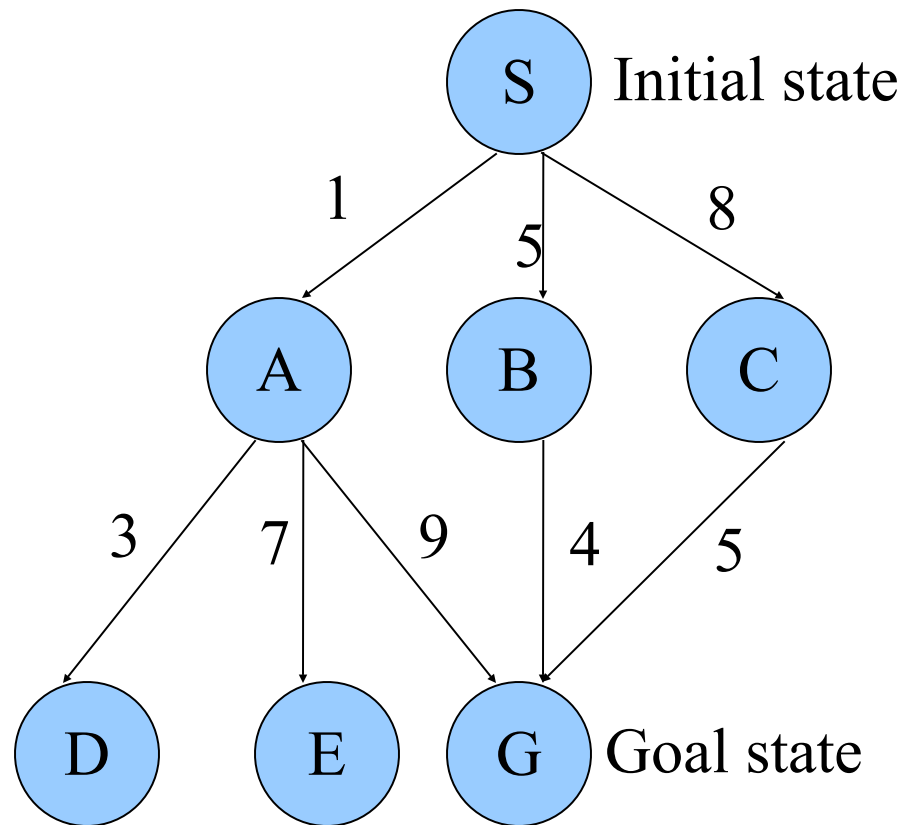
fan within ripple



Iterative deepening

- Search proceeds like BFS, but fringe is like DFS
 - Complete, optimal like BFS
 - Small space complexity like DFS
 - Time complexity like BFS
- Preferred uninformed search method

Example



(All edges are directed, pointing downwards)

Nodes expanded by:

- Breadth-First Search: S A B C D E G

Solution found: S A G

- Uniform-Cost Search: S A D B C E G

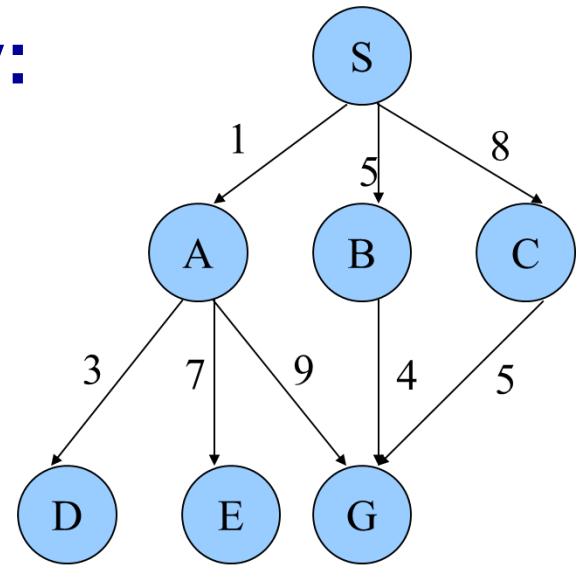
Solution found: S B G (This is the only uninformed search that worries about costs.)

- Depth-First Search: S A D E G

Solution found: S A G

- Iterative-Deepening Search: S A B C S A D E G

Solution found: S A G



Performance of search algorithms on trees

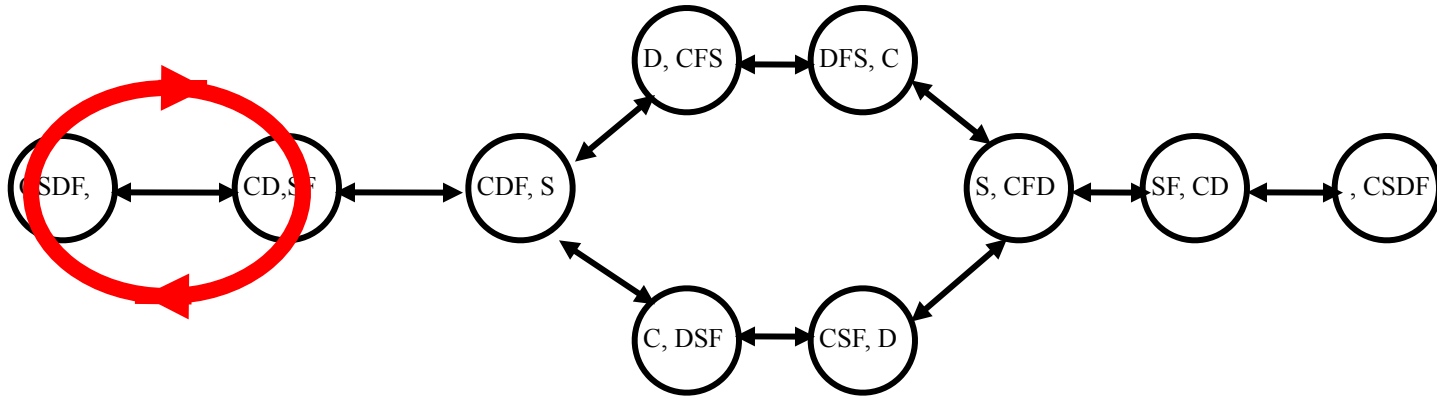
b: branching factor (assume finite) d: goal depth m: graph depth

	Complete	optimal	time	space
Breadth-first search	Y	Y, if ¹	$O(b^d)$	$O(b^d)$
Uniform-cost search ²	Y	Y	$O(b^{C^*/\epsilon})$	$O(b^{C^*/\epsilon})$
Depth-first search	N	N	$O(b^m)$	$O(bm)$
Iterative deepening	Y	Y, if ¹	$O(b^d)$	$O(bd)$

1. edge cost constant, or positive non-decreasing in depth
2. edge costs $\geq \epsilon > 0$. C^* is the best goal path cost.

If state space graph is not a tree

- The problem: repeated states

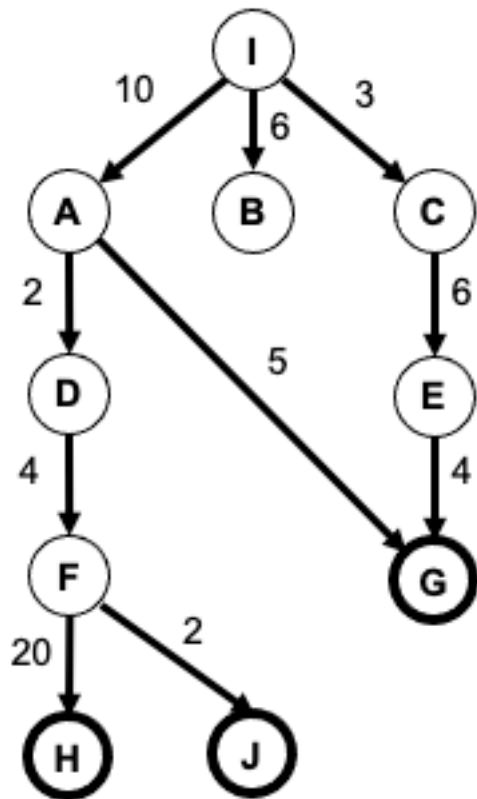


- Ignore the danger of repeated states: wasteful (BFS) or impossible (DFS). Can you see why?
- How to prevent it?

If state space graph is not a tree

- We have to remember already-expanded states (**CLOSED**).
- When we take out a state from the fringe (OPEN), check whether it is in CLOSED (already expanded).
 - If yes, throw it away.
 - If no, expand it (add successors to OPEN), and move it to CLOSED.

Q3-1: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by BFS?



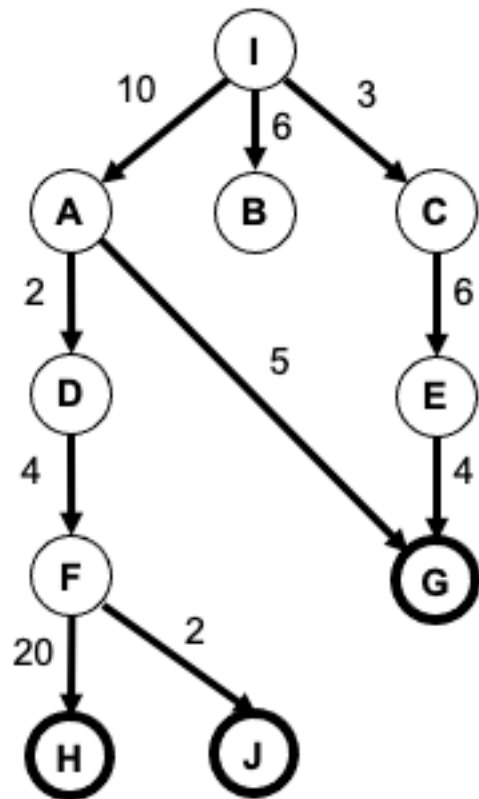
1. IADFH

2. IADFJ

3. IAG

4. ICEG

Q3-2: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by UCS?



1. IADFH

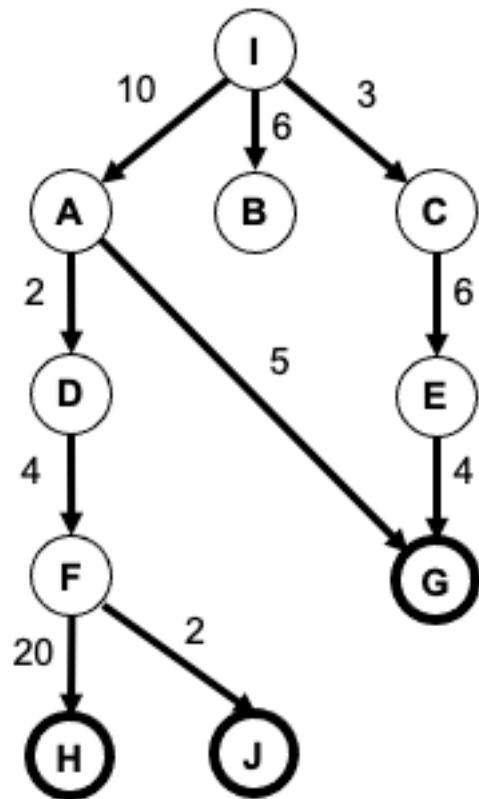
2. IADFJ

3. IAG



4. ICEG

Q3-3: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by DFS?



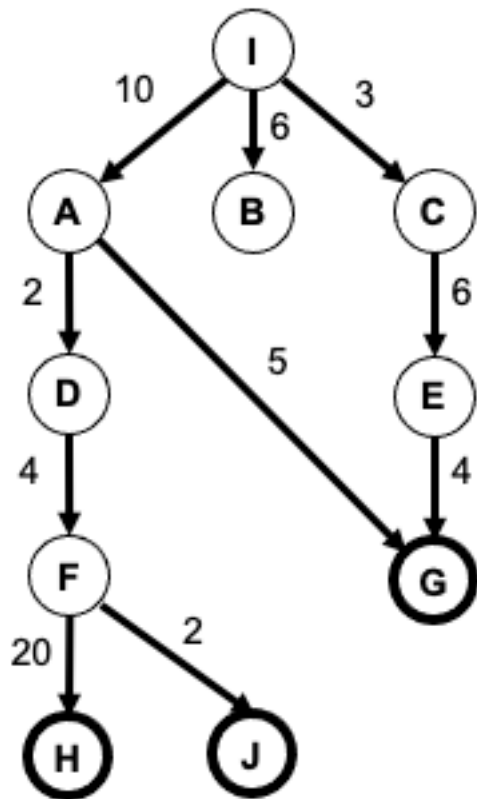
1. IADFH

2. IADFJ

3. IAG

4. ICEG

Q3-4: Consider the state space graph below. Goal states have bold borders. Nodes are expanded left to right when there are ties. What solution path is returned by IDS?



1. IADFH

2. IADFJ

3. IAG

4. ICEG

What you should know

- Problem solving as search: state, successors, goal test
- Uninformed search
 - Breadth-first search
 - Uniform-cost search
 - Depth-first search
 - Iterative deepening



- Can you unify them using the same algorithm, with different priority functions?
- Performance measures
 - Completeness, optimality, time complexity, space complexity