CS 540 Introduction to Artificial Intelligence

Search II: Informed Search

University of Wisconsin-Madison

Fall 2023

Today’s Goals

- Finish and review of uninformed search strategies.
- Understand the difference between uninformed and informed search.
- Introduce A* Search
  - Heuristic properties, stopping rules, analysis
- Extensions: Beyond A*
  - Iterative deepening, beam search

Breadth-First Search

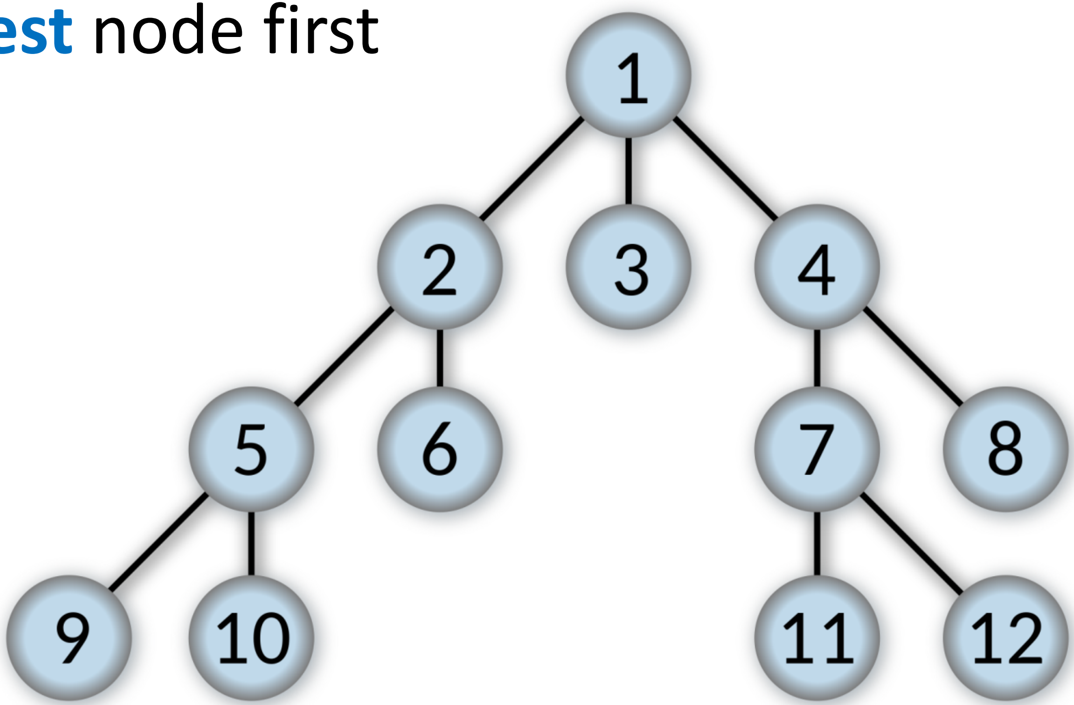
Recall: expand **shallowest** node first

- Data structure: queue
- **Properties**:
  - Complete
  - Optimal (if edge cost 1)
  - Time $O(b^d)$

Depth

Branching Factor

  - Space $O(b^d)$



Wiki

Uniform Cost Search

Like BFS, but keeps track of cost

- Expand least cost node
- Data structure: priority queue
- **Properties**:
  - Complete
  - Optimal (if weight lower bounded by ε)
  - Time O($b^{C^*/\varepsilon}$)
  - Space O($b^{C^*/\varepsilon}$)

    C* is optimal path cost to goal.

    ϵ is cost of edge with smallest cost.

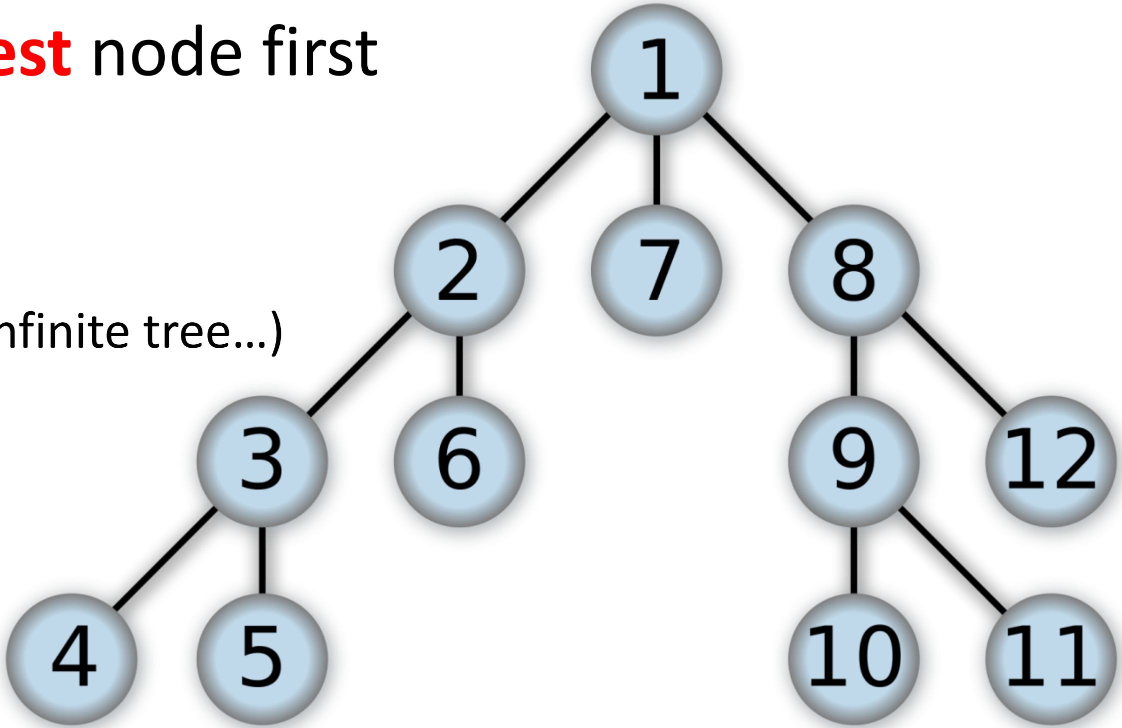
Depth-First Search

Recall: expand **deepest** node first

- Data structure: stack
- **Properties**:
  - Incomplete (stuck in infinite tree...)
  - Suboptimal
  - Time $O(b^m)$

Max Depth

  - Space $O(bm)$



Wiki

Iterative Deepening DFS

Repeated limited DFS

- Search like BFS, fringe like DFS
- **Properties**:
  - Complete
  - Optimal (if edge cost 1)
  - Time $O(b^d)$
  - Space $O(bd)$

A good option!

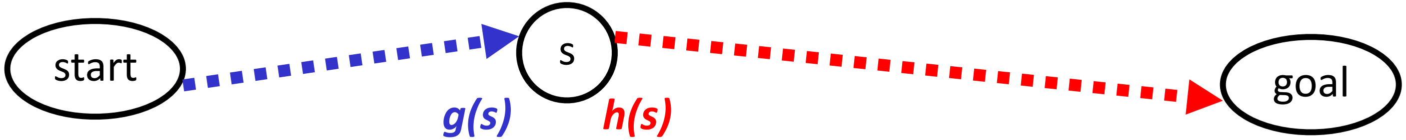
Uninformed vs Informed Search

Uninformed search (all of what we saw). Know:

- Path cost ***g(s)*** from start to state *s.*
- Successors.

Informed search. Know:

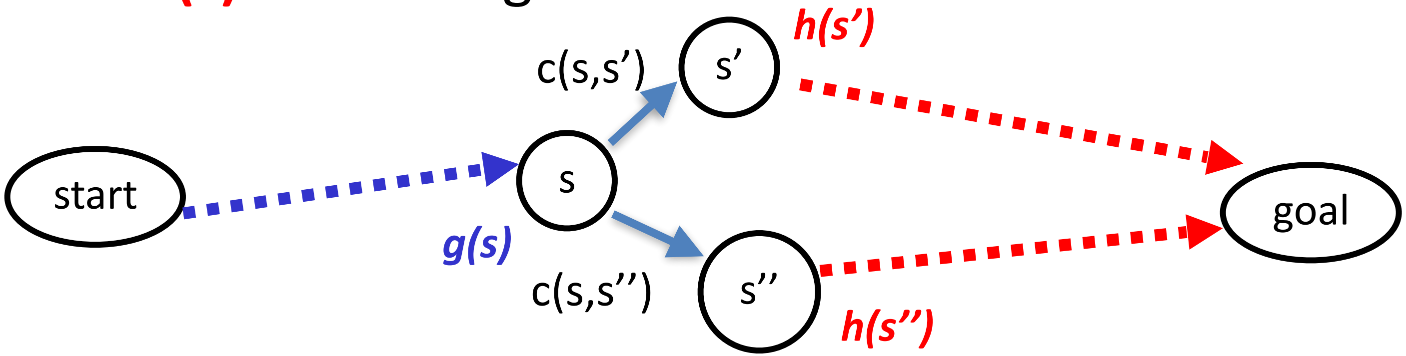
- All uninformed search properties, plus
- Heuristic **h(s)** from s to goal.



Informed Search

Informed search. Know:

- All uninformed search properties, plus
- Heuristic ***h(s)*** from *s* to goal.

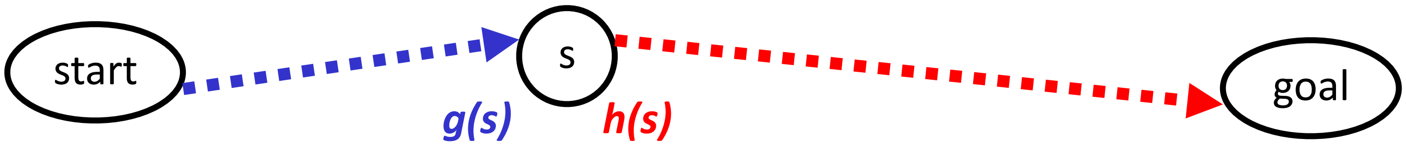


- Goal: **speed up search.**

Using the Heuristic

Recall uniform-cost search

- We store potential next states with a priority queue
- Expand the state with the smallest ***g*(*s*)**
  - ***g*(*s*)** "first-half-cost"

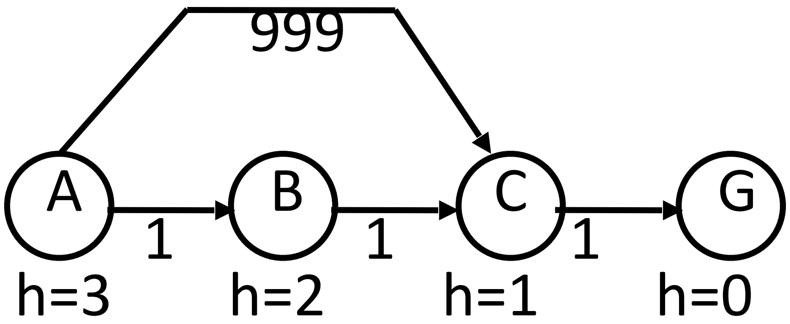


- Now let's use the heuristic ("second-half-cost")
  - Several possible approaches: let's see what works

Attempt 1: Best-First Greedy

One approach: just use ***h(s)*** alone

- Specifically, expand the state with smallest ***h(s)***
- This isn't a good idea. Why?

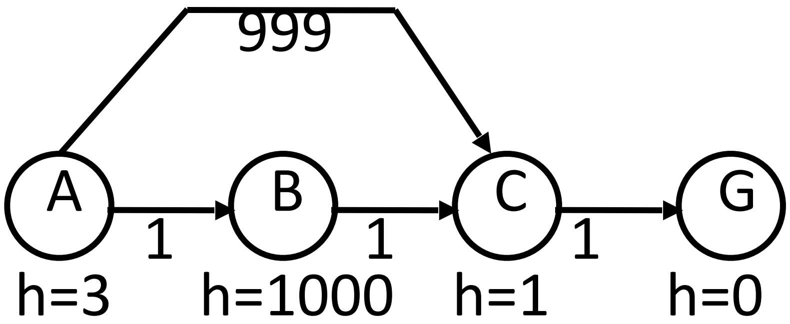


- Not optimal! **Get** A → C → G. **Want**: A →B → C → G

Attempt 2: **A Search**

Next approach: use both $g(s) + h(s)$

- Specifically, expand state with smallest $g(s) + h(s)$
- Again, use a priority queue
- Called **"A" search**



- **Still not optimal!** (Does work for former example).

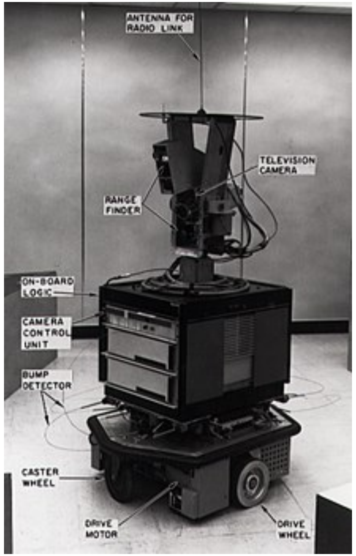
Attempt 3: **A* Search**

Same idea, use $g(s)$ + $h(s)$, with one **requirement**

- Demand that $h(s) \leq h^*(s)$ where $h^*(s)$ is true cost from s to goal.
- If heuristic has this property, it is called "admissible"
  - Optimistic! Never over-estimates
- Still need $h(s) \geq 0$
  - Negative heuristics can lead to strange behavior
- This is **A* search**

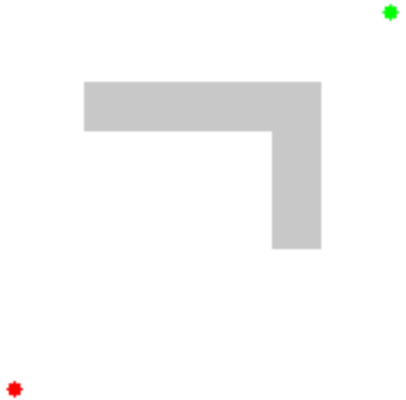
Attempt 3: **A* Search**

Origins: robots and planning



Shakey the Robot, 1960’s

Credit: Wiki



Animation: finding a path around obstacle

Credit: Wiki

Admissible Heuristic Functions

Have to be careful to ensure admissibility (**optimism!)**

- Example: **8 Game**

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- One useful approach: **relax constraints**
  - ***h*(*s*)** = number of tiles in wrong position
    - allows tiles to fly to destination in a single step

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

- A. An admissible heuristic
- B. Not an admissible heuristic

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

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- **B. Not an admissible heuristic**

Break & Quiz

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- A. An admissible heuristic No: riding your bike takes longer.
- **B. Not an admissible heuristic**

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

(i) $h(s) = h^*(s)$

(ii) $h(s) = \max(2, h^*(s))$

(iii) $h(s) = \min(2, h^*(s))$

(iv) $h(s) = h^*(s) - 2$

(v) $h(s) = \mathrm{sqrt}(h^*(s))$

- A. All of the above
- B. (i), (iii), (iv)
- C. (i), (iii)
- D. (i), (iii), (v)

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

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- **C. (i), (iii)**
- D. (i), (iii), (v)

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

(i) $h(s) = h^*(s)$

(ii) $h(s) = \max(2, h^*(s))$ No: $h(s)$ might be too big

(iii) $h(s) = \min(2, h^*(s))$

(iv) $h(s) = h^*(s)-2$ No: $h(s)$ might be negative

(v) $h(s) = \text{sqrt}(h^*(s))$ No: if $h^*(s) < 1$ then $h(s)$ *is bigger*

- A. All of the above
- B. (i), (iii), (iv)
- **C. (i), (iii)**
- D. (i), (iii), (v)

Heuristic Function Tradeoffs

Dominance: h_2 dominates h_1 if for all states s,

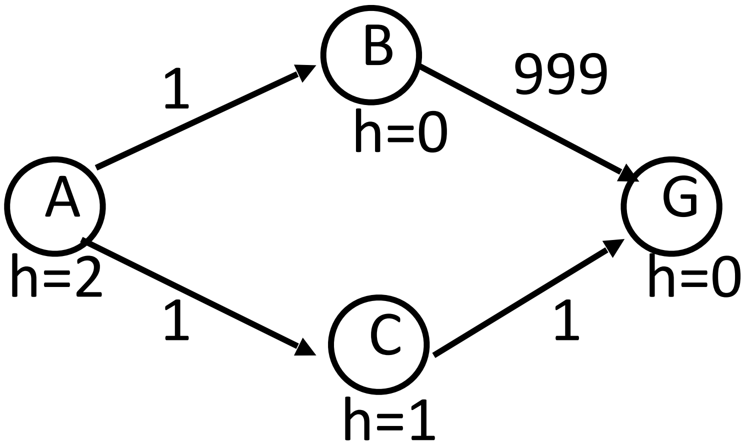
$$h_1(s) \leq h_2(s) \leq h^*(s)$$

- **Idea**: we want to be as close to h^* as possible
  - But not over! **Must under-estimate true cost.**
- **Tradeoff**: being very close might require a very complex heuristic, expensive computation
  - Might be better off with cheaper heuristic & expand more nodes.

A* Termination

When should A* **stop**?

- One idea: as soon as we reach goal state?

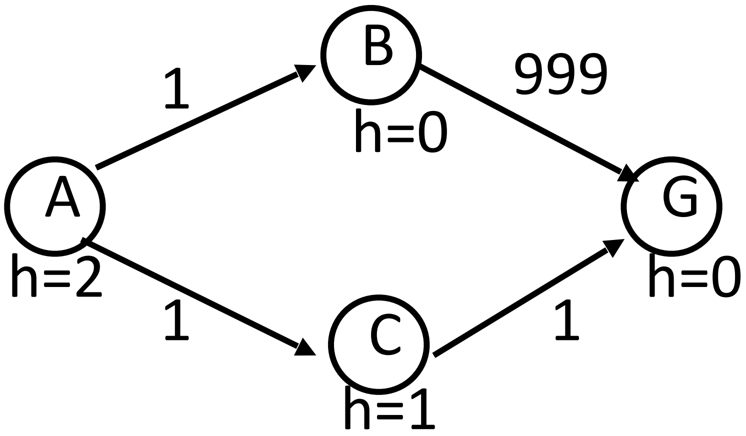


- ***h*** is admissible, but note that we get A →B → G (**cost 1000**)!

A* Termination

When should A* **stop**?

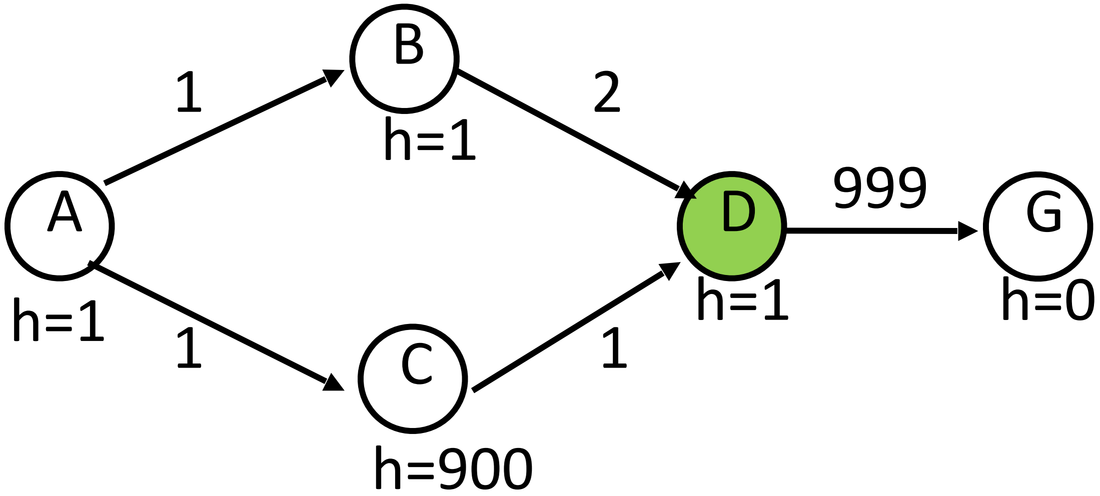
- **Rule**: terminate **when a goal is popped** from queue.



- Note: taking ***h*** =0 reduces to uniform cost search rule.

A* Revisiting Expanded States

Possible to revisit an expanded state, get a shorter path:



- Put D back into priority queue, smaller g+h.
- **Note:** uninformed search methods will not revisit expanded states.

A* Full Algorithm

1. Put the start state S on the priority queue. We call the priority queue OPEN
2. If OPEN is empty, exit with failure

States we have already expanded

3. Remove from OPEN and place on CLOSED a node n for which $f(n)$ is minimum (note that $f(n)=g(n)+h(n)$)
4. If n is a goal node, exit (recover path by tracing back pointers from n to S)
5. Expand n, generating all successors and attach to pointers back to n. For each successor n' of n
   1. If n' is not already on OPEN or CLOSED compute $h(n')$, $g(n')=g(n)+c(n,n')$, $f(n')=g(n')+h(n')$, and place it on OPEN.
   2. If n' is already on OPEN or CLOSED, then check if $g(n')$ is lower for the new version of n'. If so, then:
      1. Redirect pointers backward from n' along path yielding lower $g(n')$.
      2. If (n' is already on OPEN) then update n' on OPEN; else add n' to OPEN
   3. If $g(n')$ is not lower for the new version, do nothing.
6. Goto 2.

A* Analysis

Some properties:

- Terminates!
- A* can use **lots of memory**:
  - O(# states).
- Will run out on large problems.

- Next, we will consider some alternatives to deal with this.

Break & Quiz

Q 2.1: Consider two heuristics for the 8 puzzle problem. $\boldsymbol{h_1}$ is the number of tiles in wrong position. $\boldsymbol{h_2}$ is the l_1/Manhattan distance between the tiles and the goal location. How do $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$ relate?

- A. $\boldsymbol{h_2}$ dominates $\boldsymbol{h_1}$
- B. $\boldsymbol{h_1}$ dominates $\boldsymbol{h_2}$
- C. Neither dominates the other

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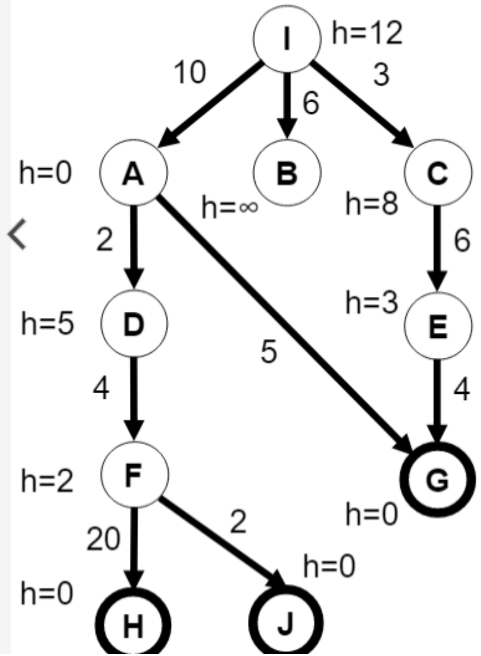
- **A. $\boldsymbol{h_2}$ dominates $\boldsymbol{h_1}$**
- B. $\boldsymbol{h_1}$ dominates $\boldsymbol{h_2}$ (No: $\boldsymbol{h_1}$ is a distance where each entry is at most 1, $\boldsymbol{h_2}$ can be greater)
- C. Neither dominates the other

Break & Quiz

Q 2.2: Consider the state space graph below. Goal states have bold borders. ***h*(*s*)** is show next to each node. What node will be expanded by A* after the initial state I?



- A. A
- B. B
- C. C

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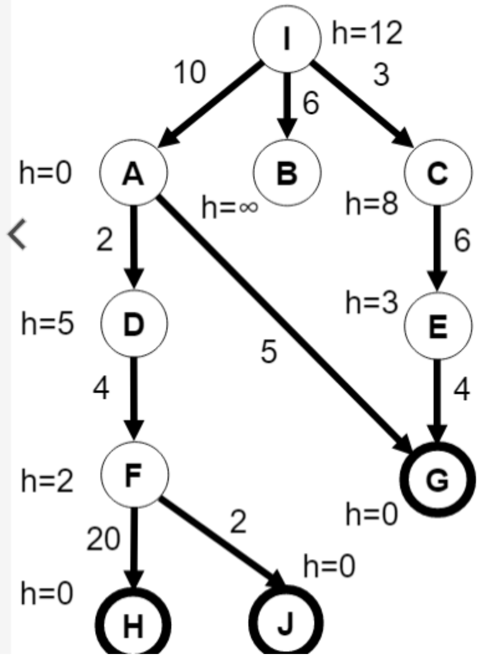


- **A. A**
- B. B
- C. C

IDA*: Iterative Deepening A*

Similar idea to our earlier iterative deepening.

- Bound the memory in search.
- At each phase, don't expand any node with $\boldsymbol{g(s)} + \boldsymbol{h(s)} > k$,
  - Assuming integer costs, do this for k=0, then k=1, then k=2, and so on
- Complete + optimal, might be costly time-wise
  - Revisit many nodes
- Lower memory use than A*

IDA*: Properties

How many restarts do we expect?

- With integer costs, optimal solution C^*, at most C^*

What about non-integer costs?

- Initial threshold k. Use the same rule for non-expansion
- Set new k to be the min $g(s) + h(s)$ for non-expanded nodes
- Worst case: restarted for each state

Beam Search

General approach (beyond A* too)

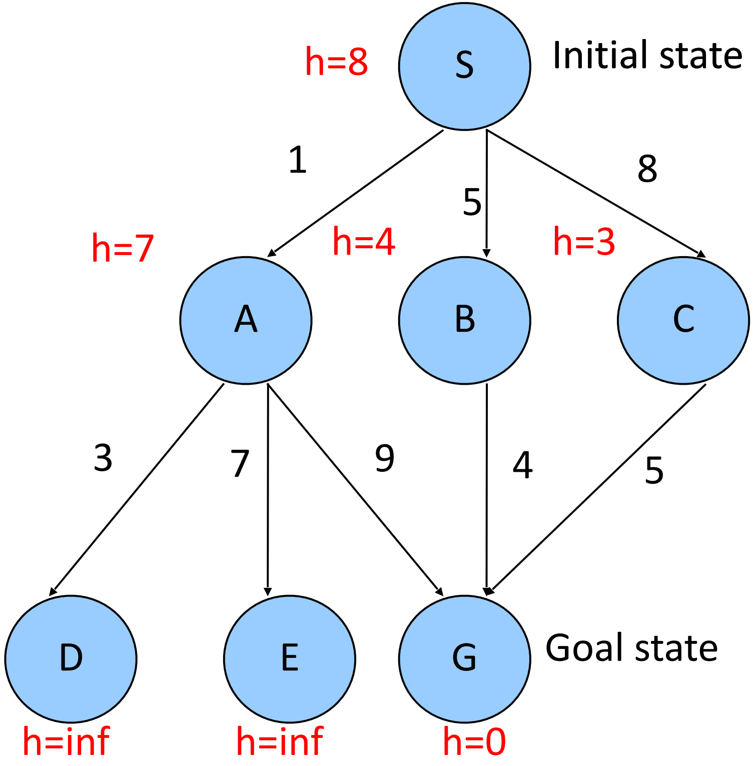
- Priority queue with fixed size *k*; beyond *k* nodes, **discard**!
- **Upside**: good memory efficiency
- **Downside**: not complete or optimal

Variation:

- Priority queue with nodes that **are at most ε worse** than best node.

Recap and Examples

Example for A*:

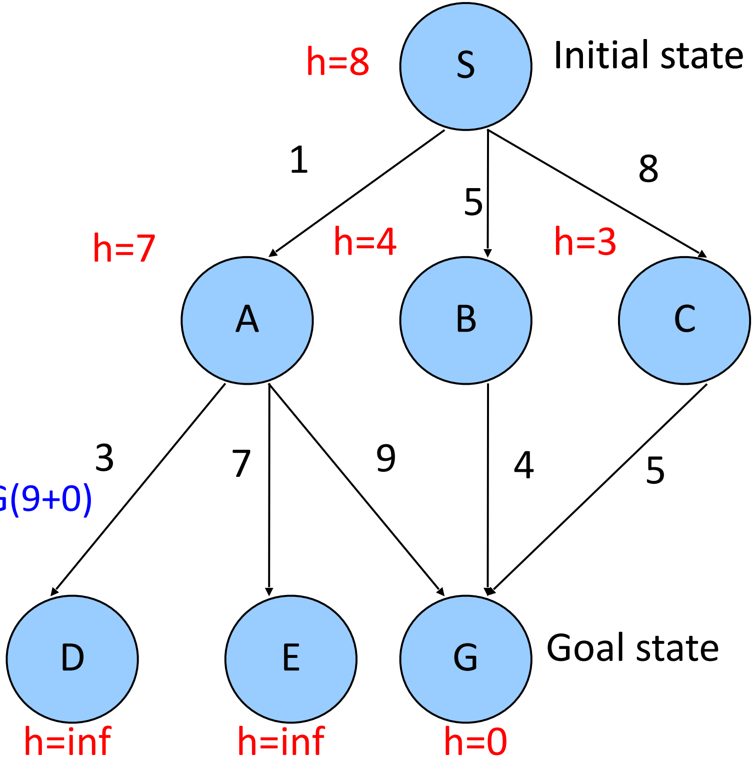


Recap and Examples

Example for A*:

OPEN	CLOSED
S(0+8)	-
A(1+7) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+7)
C(8+3) D(4+inf) E(8+inf) G(9+0)	S(0+8) A(1+7) B(5+4)
C(8+3) D(4+inf) E(8+inf)	S(0+8) A(1+7) B(5+4) G(9+0)

G → B → S

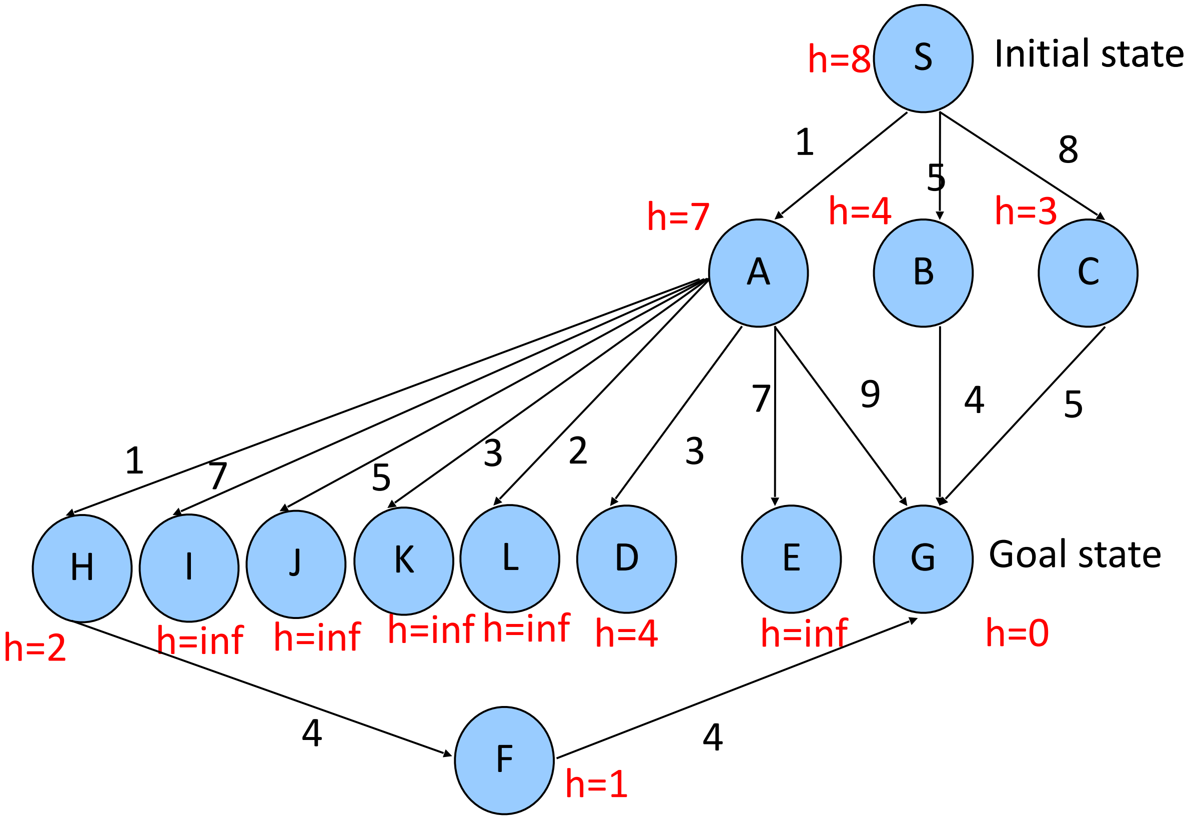


Recap and Examples

Example for IDA*:

Threshold = 8

PATH PREFIX	OPEN
-	S(0+8)
S	A(1+7)
S A	H(2+2) D(4+4)
S A H	D(4+4) F(6+1)
S A H F	D(4+4)
S A D	

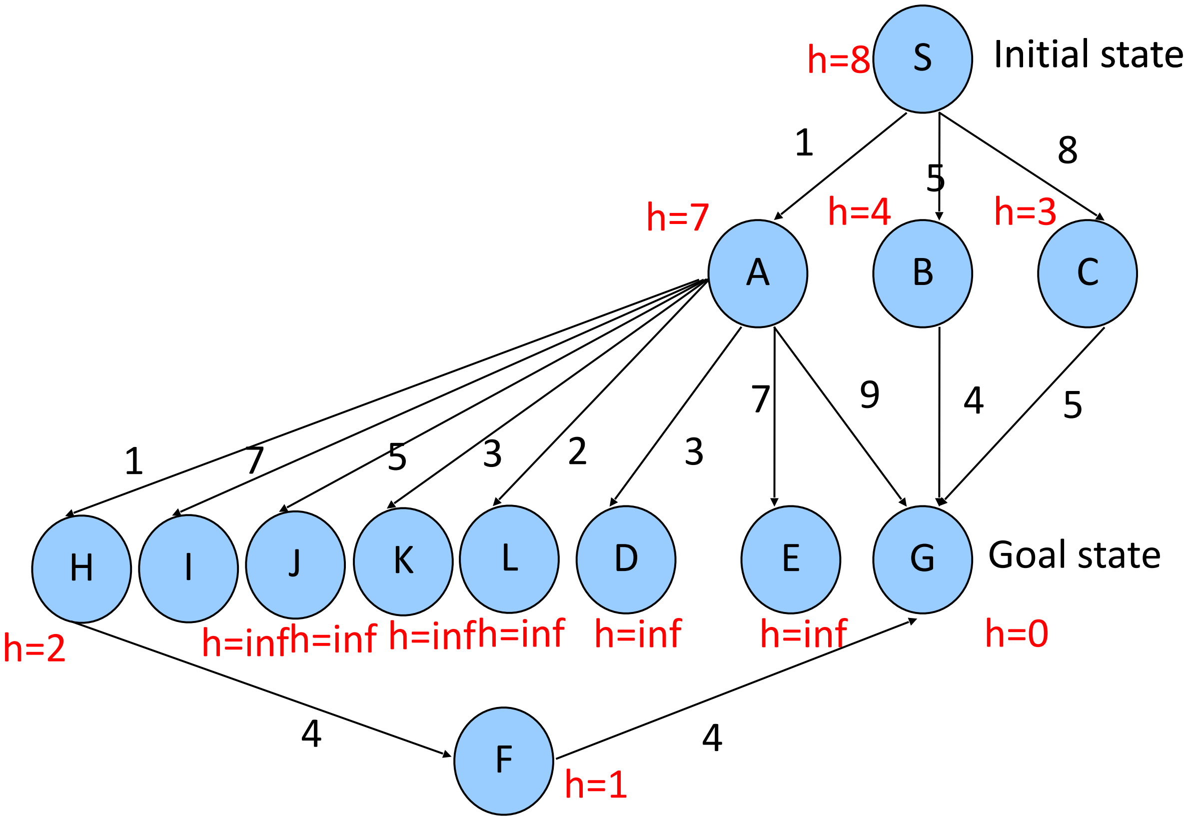


Recap and Examples

Example for IDA*:

Threshold = 9

PREFIX	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
S A	B(5+4) H(2+2) D(4+4)
S A H	B(5+4) D(4+4) F(6+1)
S A H F	B(5+4) D(4+4)
S A D	B(5+4)
S B	G(9+0)
S B G	



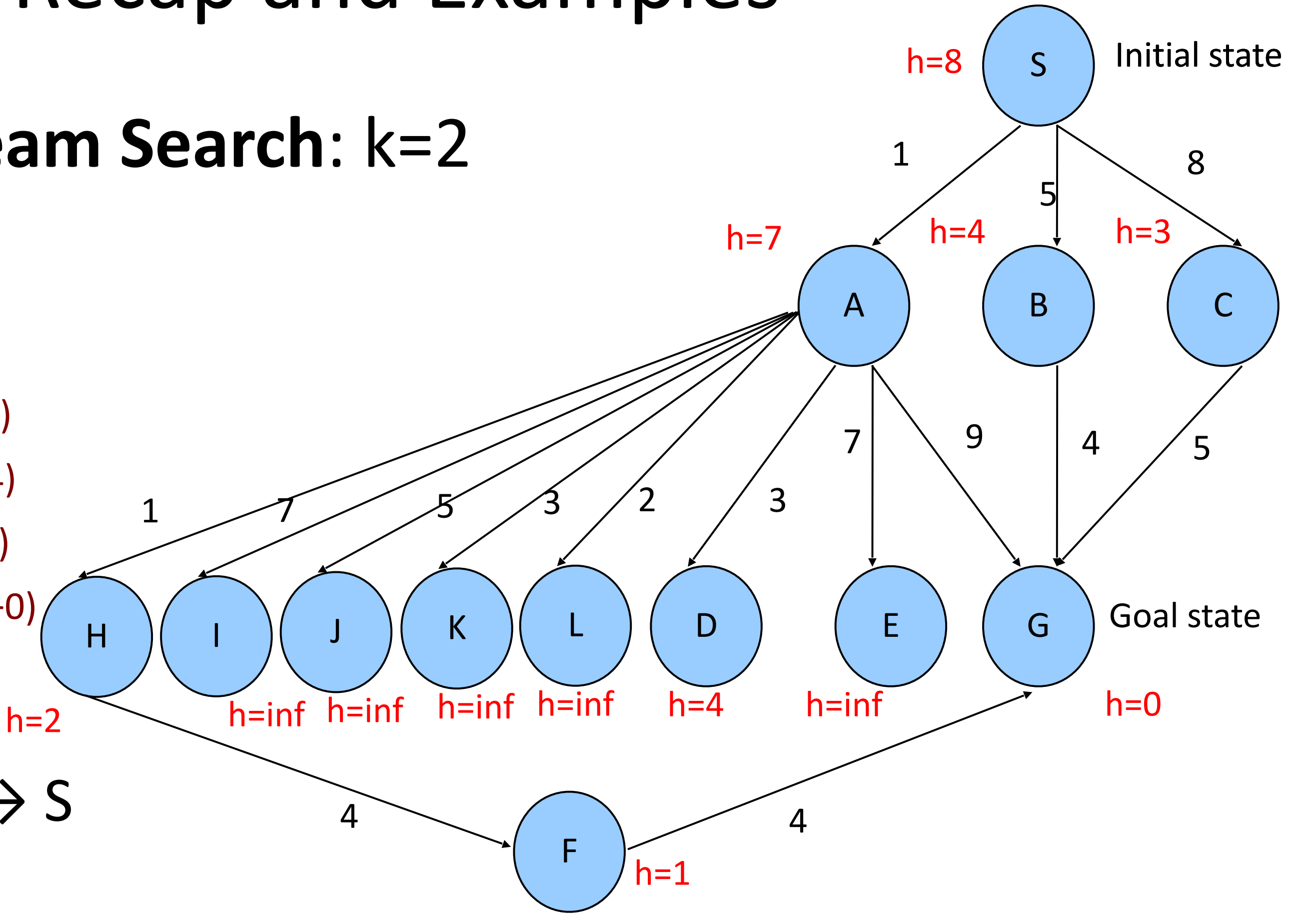
Recap and Examples

Example for **Beam Search**: k=2

CURRENT	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
A	H(2+2) D(4+4)
H	D(4+4) F(6+1)
F	D(4+4) G(10+0)
D	G(10+0)
G	

G → F → H → A → S

Not optimal!



Summary

- Informed search: introduce heuristics
  - Not all approaches work: best-first greedy is bad
- A* algorithm
  - Properties of A*, idea of admissible heuristics
- Beyond A*
  - IDA*, beam search. Ways to deal with space requirements.