Derivatives and Structured Products

The Shield Option

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i. The Shield Option – Introduction

a. The Product

When companies get their credit rating downgraded, they face an increase in their cost of capital which greatly affects their businesses.

The Shield Option is a derivative product that pays variable coupons linked to the average credit rating of a whole industry. As the credit rating of the industry gets worse, the coupons of this derivative product increase. The aim of this financial product is to provide agents with some positive and increasing payoffs whenever the credit risk of a specific industry increases. It is therefore an attractive product for all the agents which may be affected by an increase in the credit risk of a whole industry.

b. The underlying – The "Industry Rating" and its yield spread

The underlying of our derivative product is the credit rating of the whole industry with a payoff structure that has been created exclusively for the Shield Option. It follows these two steps. First, we create what we will call from now on the "Industry Rating". The Industry Rating is defined as the average rating of a basket of companies from the same industry. To compute this average rating, we take the ratings from Standard and Poor's and convert them from qualitative variables to numerical variables according to a basic scale. The conversion is reported in Table 1 in the Appendix.

Second, we compute the difference between the corporate bond yields of different credit ratings (Table 3). The coupons of the Shield Option are proportional to these yield spreads. For simplicity, we assume that these yield spreads for any given credit rating are constant through time. Figure 2 below shows the relation between rating values (x-axis) and bond yields (y-axis):

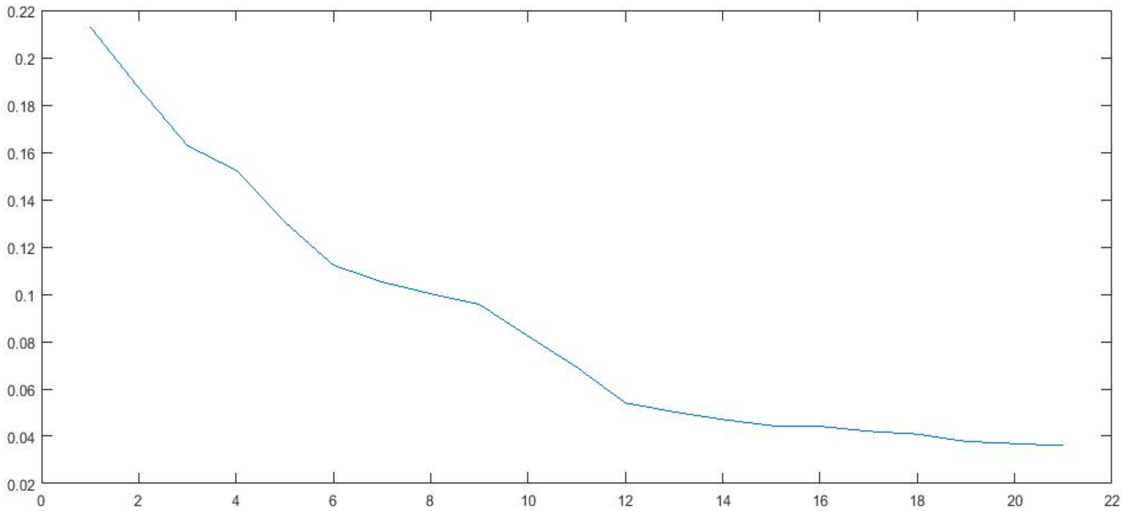

Figure 2: Bond yields as a function of credit ratings

c. Payoff Structure

We build the Shield option with a maturity of 5 years. The probability of having the same credit rating one year from now is very high, particularly for higher ratings so no agents would be interested in buying a derivative that protects against a decrease in the credit rating of the industry if there is a high chance that the credit rating stays the same. On the other hand, there is more uncertainty for the credit ratings 5 years from now.

The payoffs are linked to the differences between the average (historical) corporate bond yield of the credit rating at the beginning of the period and the average (historical) corporate bond yield of the credit rating at the end of the period. Therefore, the buyer of the derivative does not get any payoff if there is a momentary decrease in the credit rating within the 5 years of the lifetime of the derivative but reverts back to its original level at the end of the period. Here, the agent can capture the maximum payoff of the derivative product which is a coupon proportional to the difference between an AAA corporate bond yield and an CC or C corporate bond yield, assuming that the industry rating is AAA at the beginning of the period.

To make the option cheaper, we also set up different barriers for the option. For an AAA rated industry at the beginning of the period, we set up barriers at A, BBB, BB and B, meaning that if, for example, the industry rating decreases to a level below A at the end of the period, there is no payoff, the same goes for BBB, BB and B. We set up similar barriers for an A rated industry with barriers at BBB, BB, B and CCC, for a BBB rated industry with barriers at BB, B and CCC, and for a BB rated industry with barriers at B and CCC.

Therefore, the payoff of this option compensates the agents for an **expected** increase in the cost of capital and risk of one company based on the credit rating of the industry of that company.

ii. Target Clients

This derivative product can be sold to two main groups of participants in the market of financial derivatives products: hedgers and speculators.

Hedgers: In this group, we include all the companies and individuals that are directly and indirectly affected by a change in the industry credit rating. First, the companies that are part of the industry will be negatively affected by a decrease in the industry rating because this will be perceived by the market as an increase of risk at the industry level. Therefore, their borrowing costs will rise. Second, other stakeholders that are connected with one or more companies of this industry may also be indirectly affected by a decrease in the industry credit rating because the firms in the industry can change the prices of their products or try to bargain for better contracts with their suppliers to overcome the rise in their borrowing costs. This negotiation or increase in prices will have a negative effect on the stakeholders (for example, suppliers or consumers). Furthermore, an increase in the perceived default risk of a whole industry can affect the default risk or other industries that are interconnected.

This group of target clients will be interested in holding our derivative product to hedge against this rise in borrowing costs, in other words, when the industry rating decreases.

Speculators: This group is composed of investors that want to generate profits by betting on the rating changes of the industry in which the derivative product is made, or if they believe that the actual rating does not reflect the real risk that the industry is facing at that moment. We include investment banks, hedge funds, mutual funds, HNWI, wealth managers and asset managers. Their interest in our derivative product is the opposite of the interest that the hedgers have.

iii. Pricing The Shield Option

a. Data

We use credit rating data of 60 companies in the industrials sector from Bloomberg between 1994 and 2015, these 60 companies are representative of the whole sector. Within these 60 companies, we take 45 investment grade companies and 15 high yield companies. For the corporate bond yields, we use data from Ycharts and Bloomberg for all the ratings between the dates 1994 and 2015. We also take the historical average of the 3 months Treasury Bills between 1994 and 2015 from the Federal Reserve Bank of St. Louis. Finally, we take the CDS spread for each credit rating from Bloomberg.

b. Process description

To describe the industry credit rating converted to a numerical scale we use an Ornstein-Uhlenbeck process that has a modified doubly bounded Brownian motion, with an upper bound and a lower bound being the maximum and minimum values of the credit rating. The process is the following:

$$dR_t = \alpha(\bar{R} - R_t)dt + \frac{(a - R_t)(R_t - b)}{\sigma} dB_t$$

where $\bar{R}$ is the long-term mean of the industry rating, α is the speed of mean reversion, B_t is a standard Brownian motion, a and b are the upper and lower bounds, and σ is the volatility of the industry credit rating.

We chose this model to describe the process as it captures the mean reversion characteristic of credit ratings and it bounds the process with a maximum (AAA rating) and a minimum (D rating).

This process under Q becomes:

$$dR_t = \alpha(\theta - R_t)dt + \frac{(a - R_t)(R_t - b)}{\sigma} dB_t^Q$$

where $\theta = \bar{R} - \frac{(a-R_t)(R_t-b)}{\sigma} * \frac{\delta}{\alpha}$

where δ is the market price of risk

c. Estimation of The Parameters

First, we need to estimate the speed of mean reversion α of our process.

Following Smith (2010), we use an ordinary least squares regression in which the dependent variable is the difference of the industry rating between two subsequent periods (discretizing the process), and the independent variable is the value in the previous period. The estimated α will be equal to the negative sign of the coefficient found in the regression.

$$R_t - R_{t-1} = \beta R_{t-1} + \varepsilon_t$$

$$\hat{\alpha} = -\beta/dt$$

Second, the estimate of the long-term value of the industry rating $\bar{R}$ is the historical average of the industry rating for the period 1994-2015. Similarly, the volatility is estimated by computing the historical volatility of the industry rating between 1994 and 2015. The upper bound a corresponds to the maximum credit rating possible (AAA) which is 21 in our case and b corresponds to the minimum credit rating attainable (D) which is 1.

Finally, we need to estimate the market price of risk (δ). The process of the industry credit rating under Q poses an important issue: the market price of risk (δ) to go from the real probability measure P to the martingale measure Q is unknown since credit ratings are not traded. To solve this problem, we estimate θ by pricing a bond that pays \$100 at maturity if there is no default using our process dR_t and match the yield of such bond to the CDS yield of the same credit rating.

The price of the bond is as follows:

$$P_t = f(\theta) = E^{\mathrm{Q}}\left[e^{(-rt)} 100 \mathbf{I}_{R_t > \bar{K}}\right]$$

where $\mathbf{I}_{R_t > \bar{K}}$ is an indicator function equal to one if the bond does not default during the whole period.

Assuming that R_t is constant through time and equal to R_0 which corresponds to the initial credit rating, we find a closed form solution for this price which is a function of θ:

$$f(\theta) = e^{(-rt)} 100 \emptyset \left[\frac{\sigma(T-t)\left[\ln\left(\frac{R_0}{\bar{K}}\right) + \frac{1}{R_0}\alpha(\theta - R_0) - \frac{1}{2R_0^2}\frac{(a-R_0)^2(R_0-b)^2}{\sigma^2}\right]}{(a-R_0)(R_0-b)\sqrt{T-t}} \right]$$

where $\emptyset$ is a standard normal cdf.

We set R_0 equal to 21 (AA+), 17(A), 14(BBB), 11(BB), 8(B) and 5(CCC) and look at the yields of the bonds for each credit rating.

The yield is equal to:

$$f(\theta) * \exp(\text{yield} * \mathrm{T}) = 100 \Rightarrow \text{yield} = \frac{\ln\left(\frac{100}{f(\theta)}\right)}{T}$$

To get a price for the bond and subsequently its yield, we need to replace θ by a numerical value: we only use values of θ between 0 and 14 (long term mean of the credit rating) because if θ is superior than 14, it would mean that the market price of risk is negative (definition of θ). We want to test different numerical values of θ to minimize the sum of the squared differences between each yield found and the CDS risk premium with the same credit rating. If we minimize these differences for each credit rating individually, we will have a different θ and thus market price of risk for each credit rating but we want a constant market price of risk for all credit ratings so we minimize the sum of the differences as previously described using different constant θ for all credit ratings and we take the θ that gives us the lowest sum of the squared differences. Ultimately, we find that with $\theta = 10$, we have yields closest to the CDS risk premiums of the same rating.

Now, we know all the parameters of our process to simulate different paths for our industry rating.

d. Pricing

The price of the option is just the discounted value of the expected payoffs under the risk-neutral probability-measure Q.

First, we rely on Monte Carlo simulations to simulate paths for our industry rating, using the dynamic of the credit rating under Q and the Euler discretization method as it is more efficient for a smaller number of time steps and also easier to implement. Each of the simulations is made with 10000 number of simulations and 60 number of time steps (number of months within 5 years). Here, we assume that the credit ratings are updated every month to get more variation in the simulations. By discretizing the process of the credit rating, we have:

$$R_{t+h} = R_t + k(\theta - R_t)h + \frac{(22 - R_t)(R_t - 1)}{\sigma}\sqrt{h}Z$$

where h is the time increment and Z is a standard Brownian motion

Figure 8 below shows 100 simulations of credit rating over a 60 months period starting with a rating of 20 (AA):

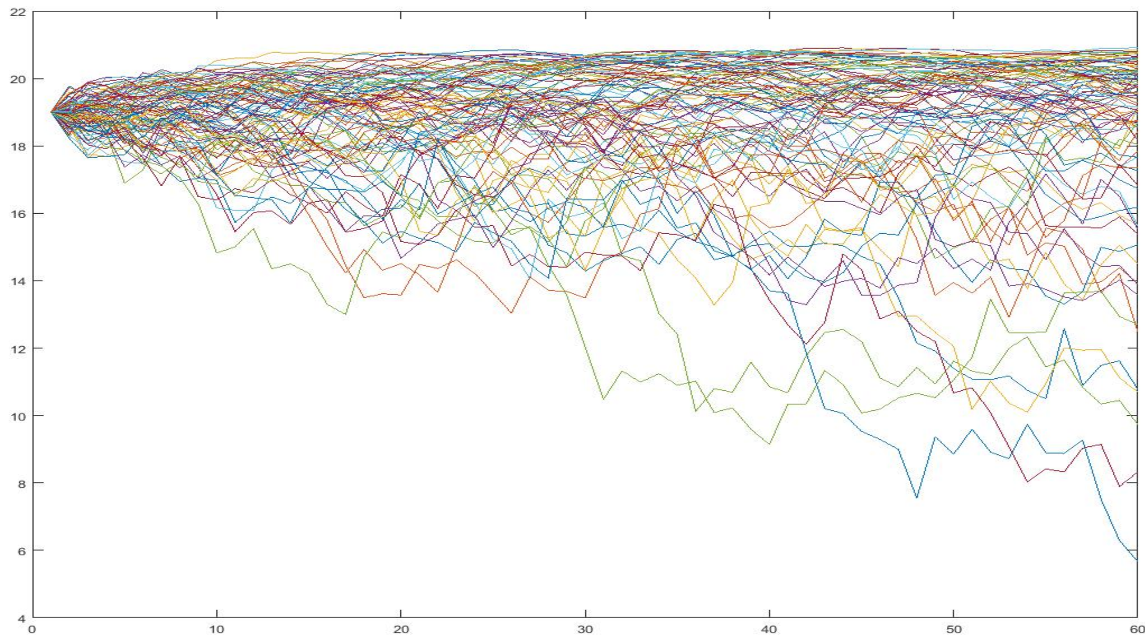

Figure 3: 100 Monte Carlo simulations of the industry credit rating's evolution starting at 20 (AA rating)

The simulation models the mean reversion characteristic of credit ratings, there are much more ratings around 20 (AA) at the end of the period.

We then round up each of the 10000 ratings at the end of the 5 years to get actual credit ratings according to table 1. We take the corresponding corporate bond yields for each of the 10000 credit ratings and we put equal to zero the yields corresponding to ratings above or equal to the initial credit rating. By taking the difference between the bond yield corresponding to the initial credit rating and each of the bond yields corresponding to the credit ratings below the initial rating at the end of the period, we have our payoffs, we then take the average of these payoffs and discount it to the risk free rate assumed to be constant to get our price.

Obviously, there will be different industry ratings at the end of the period depending on the industry rating at the beginning of the period. Consequently, we follow the same procedure to find the prices for each initial credit rating. We observe that the payoffs and thus the prices of the derivative where the initial credit ratings are close to each other are similar. For example, the price of the derivative where the initial rating is AA+ is very close to its price when the initial rating is AA, this is because of the close proximity between the bond yields of neighboring credit ratings. We thus create 6 categories in which the prices are very close to each other, the categories and the average price for each category are reported in table 4 below:

Maturity 5 years	Price (no barriers)
Category 1 (AAA to AA-)	0.5466
Category 2 (A+ to A-)	2.7156
Category 3 (BBB+ to BBB-)	3.6073
Category 4 (BB+ to BB-)	4.1393
Category 5 (B+ to B-)	4.6261
Category 6 (CCC+ to CCC-)	3.1670

Table 4: Prices of the derivative for different categories with no barriers

We observe that the price increases from category 1 (AAA to AA-) to category 5 (B+ to B-) and then decreases from category 5 to category 6 (CCC+ to CCC-): there are two effects affecting the prices here. First, as the credit rating gets worse, the chance of default increases so the probability of having a payoff also increases which will be reflected in a higher price. Second, as the initial credit rating goes lower, the set of payoffs decreases which will be reflected in a lower price since it is similar as setting a lower and lower up-and-out barrier as the credit rating gets worse (the credit ratings at the end of the period above or equal to the initial credit rating will give 0 payoff). Consequently, from category 1 to 5, the first effect overcomes the second effect explaining the observed increase in the price. However, from category 5 to 6, the second effect overcomes the first one leading to a decrease in the price of the derivative since the average yield spread (payoff) between the categories is actually very low. If we look at the price for each initial rating (table 9 of the appendix), the effects are less clear but we can still see a general trend of increasing prices up to the rating B- and then a decrease in the prices starting from CCC+.

Next, we offer different levels of protection given the preferences of the agents. Most agents will not be willing to buy a derivative that hedges against a downgrade of the industry going from AAA to CCC since the probability of this happening is very low. They are much more likely to buy a cheaper derivative with a down-and-out barrier. Consequently, we set up different barrier levels for different initial credit ratings: we do so for initial ratings AAA, A, BBB, BB and it makes sense that as the credit ratings goes lower, the possibilities of barriers become limited so for an initial rating of BB, we only offer 2 possibilities of barriers whereas for AAA we offer three barriers.

The results are reported from Table 5 to Table 8 below:

Maturity 5 years	Price with AAA-rated industry
barrier at A	0.0263
barrier at BBB	0.0451
barrier at BB	0 .1209

Table 5: Prices of the derivative for different barriers starting with a rating of AAA

Maturity 5 years	Price with A-rated industry
barrier at BBB	0.1630
barrier at BB	0.4259
barrier at B	0.9221

Table 6: Prices of the derivative for different barriers starting with a rating of A

Maturity 5 years	Price with BBB-rated industry
barrier at BB	0.2123
barrier at B	0.8021
barrier at CCC	1.5516

Table 7: Prices of the derivative for different barriers starting with a rating of BBB

Maturity 5 years	Price with BB-rated industry
barrier at B	0.2943
barrier at CCC	1.2657

Table 8: Prices of the derivative for different barriers starting with a rating of BB

We observe that the prices are much lower than in the case with no down-and-out barriers and they are increasing as the barriers get lower since the set of payoffs increases in this case. We also observe that on average, as the initial credit rating decreases, the prices increase for all level of barriers reflecting the fact that the chance of being downgraded increases as we go lower in the ratings and thus the chance of having a payoff increases.

Our derivative product naturally sets an up-and-out barrier and with a down-and-out barrier, we get much lower prices, making the derivative more attractive for certain agents.

Here, the prices obviously change when new simulations are made but the changes stay within a reasonable range of the prices reported from Table 4 to Table 8.

iv. Hedging Strategy

To delta hedge our derivative, we need to know the sensitivity of our derivative with respect to changes in the underlying (the industry rating). Table 9 of the appendix shows the deltas of the derivative (i.e. the percentage change of the prices when the rating changes by one level): these deltas are static so only a static hedging strategy is possible. Moreover, they differ depending on the initial value of the credit rating so different strategies are needed depending on the rating at the beginning of the period.

However, we face two important issues to delta hedge our derivative. The first and most obvious one is that our underlying ('the industry rating') is not a traded instrument so it is impossible to delta hedge the derivative but it is possible to use a traded instrument as a proxy for the credit rating and then delta hedge. We can think of a portfolio of bonds or CDSs of companies in the industry of interest as proxies for the credit rating and then take an offsetting position with this portfolio depending on the delta to get rid of our exposure to changes in the price of the derivative.

The second problem comes here: bonds and CDSs do not behave exactly according to changes in the credit ratings because they contain additional information and since our derivative covers a whole industry, we will also have to buy a considerable amount of CDSs or bonds that are representative of the industry in question to hedge our derivative which is very expensive.

Consequently, the most plausible and practical way to hedge our derivative is to take offsetting positions in the derivative. This is possible because our derivative fulfills a similar role as credit default swaps: the high speculation and liquidity characterizing CDS markets will also characterize our derivative. There is a demand on both sides of the market with hedgers and speculators and we are able to clear the market.

v. Conclusions

The Shield Option is a derivative product protecting agents from a downgrade of the rating of a whole industry and thus an expected increased cost of capital for each company. The main purpose for choosing this underlying (industry rating) is to simplify the model. We assume that, on average, a change in the industry rating will have an impact in the same direction on each company in that industry. The payoff is based on yield spreads so that the company is compensated for exactly the increase of the interest rate of the corporate bond when the credit rating gets worse. Obviously, each company will have to estimate their correlation to the overall industry in order to know if they should buy the derivative or not.

The derivative has a different price depending on the initial industry rating and they are affected by the chance of default (chance of having a higher payoff) and the set of payoffs, we see that the chance of default affects more the prices than the set of payoffs from the initial rating AAA until B- and the set of payoffs effect overcomes the first effect from CCC+.

In this report, we used the industrials sector but the derivative can be applied to any sector. Furthermore, in our case, because of the lack of data, we used the overall corporate bond yields for each rating and not the bond yields of companies from the industrials sector. The main limit of our derivative comes from the underlying, the industry rating, since a downgrade in the industry rating does not necessarily lead to a change in the credit rating every company in that industry, the industry rating will just be an average change in the credit rating of all the companies. Second, the credit ratings are not traded so it becomes very difficult to hedge this derivative.

vi. Appendix

Table 1: Conversion ratings

S&P Ratings	Our Ratings
AAA	22
AA+	21
AA	20
AA-	19
A+	18
A	17
A-	16
BBB+	15
BBB	14
BBB-	13
BB+	12
BB	11
BB-	10
B+	9
B	8
B-	7
CCC+	6
CCC	5
CCC-	4
CC	3
C	2
D	1

Table 2: Yields associated to each credit rating

S&P Ratings	Yield (%)
AAA	3.6
AA+	3.69
AA	3.78
AA-	4.09
A+	4.42
A	4.45
A-	4.71
BBB+	5.03
BBB	5.21
BBB-	5.41
BB+	6.92
BB	8.23
BB-	9.57
B+	10.02
B	10.52
B-	11.23
CCC+	13.05
CCC	15.24
CCC-	16.28
CC	18.72
C	21.3

Table 3: Yield Spreads

	AAA	AA+	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC+	CCC	CCC-	CC	C	D
AAA	0																				
AA+	0.09	0																			
AA-	0.18	0.09	0																		
A+	0.49	0.4	0.31	0																	
A	0.82	0.73	0.64	0.33	0																
A-	0.85	0.76	0.67	0.36	0.03	0															
BBB+	1.11	1.02	0.93	0.62	0.29	0.26	0														
BBB	1.43	1.34	1.25	0.94	0.61	0.58	0.32	0													
BBB-	1.61	1.52	1.43	1.12	0.79	0.76	0.5	0.18	0												
BB+	1.81	1.72	1.63	1.32	0.99	0.96	0.7	0.38	0.2	0											
BB	3.32	3.23	3.14	2.83	2.5	2.47	2.21	1.89	1.71	1.51	0										
BB-	4.63	4.54	4.45	4.14	3.81	3.78	3.52	3.2	3.02	2.82	1.31	0									
B+	5.97	5.88	5.79	5.48	5.15	5.12	4.86	4.54	4.36	4.16	2.65	1.34	0								
B	6.42	6.33	6.24	5.93	5.6	5.57	5.31	4.99	4.81	4.61	3.1	1.79	0.45	0							
B-	6.92	6.83	6.74	6.43	6.1	6.07	5.81	5.49	5.31	5.11	3.6	2.29	0.95	0.5	0						
CCC+	7.63	7.54	7.45	7.14	6.81	6.78	6.52	6.2	6.02	5.82	4.31	3	1.66	1.21	0.71	0					
CCC	9.45	9.36	9.27	8.96	8.63	8.6	8.34	8.02	7.84	7.64	6.13	4.82	3.48	3.03	2.53	1.82	0				
CCC-	11.6	11.6	11.5	11.2	10.8	10.8	10.5	10.2	10	9.83	8.32	7.01	5.67	5.22	4.72	4.01	2.19	0			
CC	12.7	12.6	12.5	12.2	11.9	11.8	11.6	11.3	11.1	10.9	9.36	8.05	6.71	6.26	5.76	5.05	3.23	1.04	0		
C	15.1	15	14.9	14.6	14.3	14.3	14	13.7	13.5	13.3	11.8	10.5	9.15	8.7	8.2	7.49	5.67	3.48	2.44	0	
D	17.7	17.6	17.5	17.2	16.9	16.9	16.6	16.3	16.1	15.9	14.4	13.1	11.7	11.3	10.8	10.1	8.25	6.06	5.02	2.58	0

Table 9: Prices and deltas for each individual rating

Initial rating	Prices	Deltas
AAA	0.0508	1.30
AA+	0.1169	4.89
AA	0.6894	0.93
AA-	1.3293	0.71
A+	2.2681	0.24
A	2.8270	0.079
A-	3.0517	0.078
BBB+	3.2914	0.16
BBB	3.8188	-0.28
BBB-	3.7119	0.06
BB+	3.9338	0.22
BB	4.5470	-0.13
BB-	3.9370	0.05
B+	4.1270	0.17
B	4.8301	0.02
B-	4.9213	-0.03
CCC+	4.7732	-0.37
CCC	3.0114	-0.43
CCC-	1.7165	

References

- **Smith, W.** (2010). *On the Simulation and Estimation of the Mean-Reverting Ornstein-Uhlenbeck Process.* [Accessed 27 May 2016]. Available at: https://commoditymodels.files.wordpress.com/2010/02/estimating-the-parameters-of-a-mean-reverting-ornstein-uhlenbeck-process1.pdf