Number Theory Algorithms

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Abstract

This paper is the documentation for the Euclidean Algorithm module in Number Theory Algorithms mobile application.

Extended Euclidean Algorithm

The Extended Euclidean Algorithm is used to compute integers x, y for $ax + by = GCD(a, b)$ where $a, b \in \mathbb{N}$ and $x, y \in \mathbb{Z}$. The implementation of this algorithm is based on ([1] pg. 16).

Algorithm 1: Extended Euclidean Algorithm

Input: $a, b \in \mathbb{N}$

Output: $x, y \in \mathbb{Z}$ for $ax + by = GCD(a, b)$

$r_{n-2} := a$
$r_{n-1} := b$
$q_{n-1} :=$ quotient of r_{n-2}/r_{n-1}
$r_n :=$ remainder of r_{n-2}/r_{n-1}
$x_{n-2} := 1$, $x_{n-1} := 0$, $x_{temp} := x_{n-1}$, $x_{n-1} := x_{n-2} - x_{n-1} \cdot q_{n-1}$, $x_{n-2} := x_{temp}$
$y_{n-2} := 0$, $y_{n-1} := 1$, $y_{temp} := y_{n-1}$, $y_{n-1} := y_{n-2} - y_{n-1} \cdot q_{n-1}$, $y_{n-2} := y_{temp}$

while $r_n > 0$ **do**

$r_{n-2} := r_{n-1}$
$r_{n-1} := r_n$
$q_{n-1} :=$ quotient of r_{n-2}/r_{n-1}
$r_n :=$ remainder of r_{n-2}/r_{n-1}
$x_{temp} := x_{n-1}$
$x_{n-1} := x_{n-2} - x_{n-1} \cdot q_{n-1}$
$x_{n-2} := x_{temp}$
$y_{temp} := y_{n-1}$
$y_{n-1} := y_{n-2} - y_{n-1} \cdot q_{n-1}$
$y_{n-2} := y_{temp}$

end

return $x = x_{n-2}$, $y = y_{n-2}$

References

[1] Cohen, Henri. *A course in computational algebraic number theory.* Springer-Verlag, 1996.