To address the issues concerning the dynamic model errors and, in particular, measurement outliers, we propose an adaptive robust approach for two-antenna GPS/INS LC integration in which the IAE method is utilized to design a factor for the adaptive covariance. Several equations concerned with IAE drawbacks are derived to account for and overcome the numerical and feedback issues of two-antenna GPS/INS integration caused by measurement outliers and unknown state uncertainties, and an adaptive reconfiguration for measurement noise covariance according to measurement reliability is designed. The desirable properties of the proposed approach are summarized as follows:

- Adaptive modification of noise covariance can treat dynamic model errors and measurement disturbance to reduce their impact on state estimation, especially when the statistics of both measured and predicted noise have to be adapted. With filter updating, positive feedback and numerical issues can be reduced by quantifying statistical measurement noise on a more granular level based on the corresponding quantifications of measurement reliability in the case of measurement outliers.
- The proposed method can accurately quantify measurement reliability. It is an evidence-based regulation method with the benefit of attenuating the impact of innovation perturbation. In addition to the assured stability of filter updating, the performance of the augmented measurement equation in state error feedback for precious measurements is improved.

The paper is organized as follows. An overview of the two-antenna GPS/INS-integrated algorithm is provided in Section 2. In Section 3, the adaptive modification of noise covariance is discussed. The field experiment and results for different schemes are compared in Section 4 to verify the superiority of the proposed approach compared to existing methods. Finally, several conclusions of this work are drawn.

2. Two-Antenna GPS/INS

2.1. Inertial Dynamic Model

The inertial dynamic model is derived from the Psi-Angle error model based on INS error differential equations and summarized as [43]:

$$\begin{aligned}
\delta\dot{\boldsymbol{r}} &= -\boldsymbol{\omega}_{\mathrm{en}} \times \delta\boldsymbol{r} + \delta\boldsymbol{v} \\
\delta\dot{\boldsymbol{v}} &= \boldsymbol{f} \times \boldsymbol{\psi} - (2\boldsymbol{\omega}_{\mathrm{ie}} + \boldsymbol{\omega}_{\mathrm{en}}) \times \delta\boldsymbol{v} + \delta\boldsymbol{g} + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}\delta\boldsymbol{f}^{\mathrm{b}} \\
\dot{\boldsymbol{\psi}} &= -(\boldsymbol{\omega}_{\mathrm{ie}} + \boldsymbol{\omega}_{\mathrm{en}}) \times \boldsymbol{\psi} - \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}\delta\boldsymbol{\omega}^{\mathrm{b}}
\end{aligned} \tag{1}$$

where δ denotes the corresponding error or uncertainty of the vectors, $\dot{\bullet}$ denotes the first derivatives, $\times$ denotes the cross-product of two vectors, $\delta\boldsymbol{r}$, $\delta\boldsymbol{v}$, and $\boldsymbol{\psi}$ are the position, velocity, and attitude error state vectors, respectively, $\boldsymbol{f}$ is the specific force vector, $\boldsymbol{\omega}_{\mathrm{ie}}$ is the earth rotation vector, and $\boldsymbol{\omega}_{\mathrm{en}}$ is the craft-rate vector [44]. $\delta\boldsymbol{f}^{\mathrm{b}}$ and $\delta\boldsymbol{\omega}^{\mathrm{b}}$ are the accelerometer error and gyroscope error, respectively, and are written as:

$$\begin{aligned}
\delta\boldsymbol{f}^{\mathrm{b}} &= \boldsymbol{b}_{\mathrm{a}} + \mathrm{diag}(\boldsymbol{f}^{\mathrm{b}})\boldsymbol{s}_{\mathrm{a}} \\
\delta\boldsymbol{\omega}^{\mathrm{b}} &= \boldsymbol{b}_{\mathrm{g}} + \mathrm{diag}(\boldsymbol{\omega}^{\mathrm{b}})\boldsymbol{s}_{\mathrm{g}}
\end{aligned} \tag{2}$$

where $\mathrm{diag}(\bullet)$ denotes the diagonal form of the matrix, and $\boldsymbol{b}_{\mathrm{a}}$ and $\boldsymbol{s}_{\mathrm{a}}$ are the accelerometer bias and accelerometer scale factor vectors, respectively. $\boldsymbol{b}_{\mathrm{g}}$ and $\boldsymbol{s}_{\mathrm{g}}$ are the gyroscope bias and gyroscope scale factor vectors, respectively. The inertial measurement unit (IMU) sensor error terms $\boldsymbol{\varepsilon}$, such as bias and scale factors, are modeled as first-order Gauss–Markov (GM) processes:

$$\dot{\boldsymbol{\varepsilon}} = -T^{-1}\boldsymbol{\varepsilon} + \boldsymbol{w}_{\varepsilon} \tag{3}$$

where T is the correlation time, $\boldsymbol{w}_{\varepsilon}$ is the corresponding process noise vector, and $\delta\boldsymbol{g}$ is the gravity uncertainty error vector, projected as:

$$\delta\boldsymbol{g} = \mathrm{diag}\Big(-\omega_{\mathrm{s}}^2 - \omega_{\mathrm{s}}^2\ 2\omega_{\mathrm{s}}^2\Big)\delta\boldsymbol{r} \tag{4}$$

where ω_s denotes the Schuler frequency [43–47]. Subscript b indicates the body frame (b-frame) with vehicle axes, i.e., forward–transversal–down. Subscript n indicates the navigation frame (n-frame) is a local geodetic frame with the x-axis towards geodetic north, the z-axis towards an orthogonal to the reference ellipsoid pointing down, and the y-axis completing a right-handed orthogonal frame, i.e., north–east–down (NED). $\boldsymbol{C}_\mathrm{b}^\mathrm{n}$ is the direction cosine matrix (DCM) from the body frame to the n-frame.

In this research, an INS error with 21 states was developed according to the equations mentioned above. Nine navigation parameters expressed in the n-frame, and 12 inertial sensor error parameters expressed in the b-frame, are involved. The complete error state sequence is expressed as:

$$\delta\boldsymbol{x} = (\delta\boldsymbol{r}\ \delta\boldsymbol{v}\ \boldsymbol{\psi}\ \boldsymbol{b}_\mathrm{g}\ \boldsymbol{b}_\mathrm{a}\ \boldsymbol{s}_\mathrm{g}\ \boldsymbol{s}_\mathrm{a})^\mathrm{T} \tag{5}$$

2.2. Measurement Model

Figure 1 illustrates the physical location relationship between the IMU and GNSS rover with two antennas (Ant 1 and Ant 2) in the b-frame.

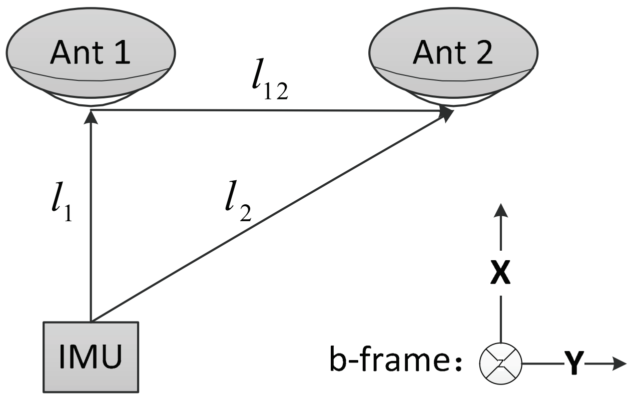

Figure 1. Physical-location relationship between the inertial measurement unit (IMU) and two Global Navigation Satellite System (GNSS) receiver antennas in the body frame.

Where $\boldsymbol{l}_i$ is the level arm vector between the IMU and i-th antenna, while $\boldsymbol{l}_{12}$ is the level arm vector between the two antennas, and can be expressed as:

$$\boldsymbol{l}_{12} = \boldsymbol{l}_2 - \boldsymbol{l}_1 \tag{6}$$

The linearized position error measurement equation is [43]:

$$\Delta\boldsymbol{Z}_r = \boldsymbol{r}_\mathrm{IMU} - \boldsymbol{r}_\mathrm{GPS1} + \boldsymbol{C}_\mathrm{b}^\mathrm{n}\boldsymbol{l}_1 = \delta\boldsymbol{r} + (\boldsymbol{C}_\mathrm{b}^\mathrm{n}\boldsymbol{l}_1\times)\boldsymbol{\psi} - \boldsymbol{e}_{\tilde{r}_1} \tag{7}$$

where Δ denotes the difference between measurements and predictions. The linearized velocity error measurement equation is [43]:

$$\begin{aligned}\Delta\boldsymbol{Z}_v &= (\boldsymbol{v}_\mathrm{IMU} + (\boldsymbol{C}_\mathrm{b}^\mathrm{n}(\boldsymbol{\omega}^\mathrm{b}\times) - (\boldsymbol{\omega}_\mathrm{in}\times))\boldsymbol{C}_\mathrm{b}^\mathrm{n}\boldsymbol{l}_1 - \boldsymbol{v}_\mathrm{GPS1}) \\ &= \delta\boldsymbol{v} - ((\boldsymbol{\omega}_\mathrm{in}\times)\boldsymbol{C}_\mathrm{b}^\mathrm{n}(\boldsymbol{l}_1\times) + \boldsymbol{C}_\mathrm{b}^\mathrm{n}((\boldsymbol{l}_1\times\boldsymbol{\omega}^\mathrm{b})\times))\boldsymbol{\psi} + \boldsymbol{C}_\mathrm{b}^\mathrm{n}(\boldsymbol{l}_1\times)\boldsymbol{b}_g + \boldsymbol{C}_\mathrm{b}^\mathrm{n}(\boldsymbol{l}_1\times)\mathrm{diag}(\boldsymbol{\omega}^\mathrm{b})\boldsymbol{s}_g - \boldsymbol{e}_{\tilde{v}_1}\end{aligned} \tag{8}$$

where $\bullet_\mathrm{IMU}$ and $\bullet_\mathrm{GPS}$ indicate that the corresponding vectors are obtained by the INS mechanization algorithm and GPS-RTK algorithm, respectively. $(\bullet\times)$ denotes the skew symmetric matrix form of a vector. $\boldsymbol{e}_{\tilde{r}_1}$ and $\boldsymbol{e}_{\tilde{v}_1}$ are the corresponding measurement white-noise vectors. $\boldsymbol{\omega}_\mathrm{in}$ is the rotation rate vector, and is expressed as:

$$\boldsymbol{\omega}_\mathrm{in} = \boldsymbol{\omega}_\mathrm{ie} + \boldsymbol{\omega}_\mathrm{en} \tag{9}$$

According to Equation (7), another position error measurement equation related to Rover 2 can be constructed as follows:

$$\Delta \boldsymbol{Z}_r = \boldsymbol{r}_{\mathrm{IMU}} - \boldsymbol{r}_{\mathrm{GPS2}} + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_2 = \delta \boldsymbol{r} + (\boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_2 \times) \psi - \boldsymbol{e}_{\tilde{r}_2} \tag{10}$$

where $\boldsymbol{e}_{\tilde{r}_2}$ is the corresponding measurement white noise vector.

$\boldsymbol{l}_{12}$ has already been accurately calibrated in the b-frame and is considered as a constraint that can be used for fast AR fixing, even for a low-cost single-frequency receiver. Assume that the two-antenna baseline vector $\boldsymbol{p}_{12}^{\mathrm{e}}$ in the earth frame (e-frame) can be accurately estimated by moving-reference-receiver GPS-RTK processing that a method used to determine relative position vector between the antennas mounted to a single platform. Then, the attitude error measurement equation can be constructed using the difference between Equations (7) and (10):

$$\begin{aligned} \Delta \boldsymbol{Z}_{\psi} &= \boldsymbol{r}_{\mathrm{IMU}} + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_1 - \boldsymbol{r}_{\mathrm{GPS1}} - (\boldsymbol{r}_{\mathrm{IMU}} + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_2 - \boldsymbol{r}_{\mathrm{GPS2}}) \\ &= \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_{12} - \boldsymbol{C}_{\mathrm{e}}^{\mathrm{n}} \boldsymbol{p}_{12}^{\mathrm{e}} \\ &= (\boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{l}_{12} \times) \boldsymbol{\psi} - \boldsymbol{e}_{\tilde{r}_{12}} \end{aligned} \tag{11}$$

where $\boldsymbol{e}_{\tilde{r}_{12}}$ is the differential measurement white-noise sequence, and $\boldsymbol{C}_{\mathrm{e}}^{\mathrm{n}}$ is the DCM from the e-frame to the n-frame.

Consequently, the ninth-order measurement error model of the proposed two-antenna GPS/INS integration is given by:

$$\Delta \boldsymbol{Z} = \boldsymbol{H} \delta \boldsymbol{x} + \boldsymbol{e} \tag{12}$$

where $\Delta \boldsymbol{Z}$ is the measurement vector, $\boldsymbol{H}$ is the measurement matrix calculated based on Equations (7), (8), and (11), $\boldsymbol{e}$ is the measurement white-noise sequence and, determined by GPS/RTKs, results in error variance that reflects measurement uncertainty.

$$\begin{aligned} \Delta \boldsymbol{Z} &= \left[\Delta \boldsymbol{Z}_r \ \Delta \boldsymbol{Z}_v \ \Delta \boldsymbol{Z}_{\psi}\right]^{\mathrm{T}} \\ \boldsymbol{e} &= \left[\boldsymbol{e}_{\tilde{r}_1} \ \boldsymbol{e}_{\tilde{v}_1} \ \boldsymbol{e}_{\tilde{r}_{12}}\right]^{\mathrm{T}} \\ \boldsymbol{H} &= \begin{bmatrix} \boldsymbol{I}_3 & 0 & \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}(\boldsymbol{l}_1 \times) & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{I}_3 & (\boldsymbol{\omega}_{\mathrm{in}} \times) \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}(\boldsymbol{l}_1 \times) + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}((\boldsymbol{l}_1 \times \boldsymbol{\omega}^{\mathrm{b}}) \times) & \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}(\boldsymbol{l}_1 \times) & 0 & \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}(\boldsymbol{l}_1 \times) \mathrm{diag}(\boldsymbol{\omega}^{\mathrm{b}}) & 0 \\ 0 & 0 & \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}(\boldsymbol{l}_{12} \times) & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{13}$$

where $\boldsymbol{I}_3$ is the third-order unit matrix, 0 is third-order zero matrix.

3. Adaptive Noise Covariance

As noted in the introduction, KFs have been widely used in data fusion, and their performance relies on the accuracy of the dynamic and measurement models, and the statistical accuracy of the noise covariance ($\boldsymbol{Q}$ and $\boldsymbol{R}$). Fortunately, $\boldsymbol{Q}$ and $\boldsymbol{R}$ can still be adjusted to reflect the actual uncertainties of state estimation in the long term.

The initial state error covariance matrix reflects the initial state-filtering accuracy, with little effect on the subsequent filter updating. The weight between $\boldsymbol{Q}$ and $\boldsymbol{R}$ determines the Kalman gain, which directly determines the impact that $\boldsymbol{Q}$ and $\boldsymbol{R}$ have on the state estimation. Therefore, the focus of the noise covariance reconstruction is partially shifted toward $\boldsymbol{Q}$ and $\boldsymbol{R}$.

3.1. Adaptive Process Noise Covariance

Process noise covariance $\boldsymbol{Q}$ should be adjusted by the filter algorithm as it cannot be easily controlled directly unless adaptive factor α is used to tune the predicted state covariance. $\boldsymbol{P}^{-}$ can be expressed as [26]:

$$\boldsymbol{P}^{-} = \alpha(\boldsymbol{\Phi} \hat{\boldsymbol{P}} \boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{Q} t) \tag{14}$$

where $\boldsymbol{\Phi}$ denotes the state-transition matrix computed from the INS dynamic system, t is the discretization time, and $\hat{\boldsymbol{P}}$ is the previous measurement updated state covariance. $\hat{\bullet}$ and $\bullet^-$ denotes the measurement updated parameter and predicted parameter, respectively. α is a scalar value, updated based on the weight between the covariance matrix of the predicted residual vector, denoted as $\hat{\boldsymbol{C}}_\xi$, and the theoretical covariance matrix of the predicted residual vector, denoted as $\boldsymbol{C}_\xi$. $\hat{\boldsymbol{C}}_\xi$ and $\boldsymbol{C}_\xi$ are expressed as [26]:

$$\begin{aligned} \boldsymbol{C}_\xi &= \boldsymbol{H}\boldsymbol{P}^-\boldsymbol{H}^\mathrm{T} + \boldsymbol{R} \\ \hat{\boldsymbol{C}}_\xi &= \boldsymbol{\xi}^\mathrm{T}\boldsymbol{\xi} \end{aligned} \tag{15}$$

where $\boldsymbol{\xi}$ is the innovation sequence and expressed as [25]:

$$\boldsymbol{\xi} = \boldsymbol{Z} - \boldsymbol{H}\delta\boldsymbol{x}^- \tag{16}$$

with predicted state sequence $\delta\boldsymbol{x}^-$.

Based on the Kalman filter principle, $\boldsymbol{C}_\xi$ reflects predicted measurement error and theoretically equals to $\hat{\boldsymbol{C}}_\xi$ in an ideal case (accurate dynamic model and measurement model, and their noise statistics) [26]. Assume measurements are measured without outliers and their noise probability density functions are accurate.

Let $\gamma = \mathrm{tr}(\hat{\boldsymbol{C}}_\xi)/\mathrm{tr}(\boldsymbol{C}_\xi)$, $\mathrm{tr}(\bullet)$ denotes the trace of a matrix. The adaptive function to determine the adaptive factors is expressed as [25,26]:

$$\alpha = \begin{cases} 1.0, \ \gamma \le \mathrm{c}_0 \\ \frac{1}{\mathrm{c}_0}\gamma, \ \mathrm{c}_0 \le \gamma \end{cases} \tag{17}$$

where c_0 is the corresponding empirical constant, which is typically equal to 1.5–2.0 according to Reference [25].

Under the condition of accurate measurements or accurate noise statistics, the KF time updating tends to be unstable when $\alpha > 1.0$. $\boldsymbol{P}^-$ is perturbed due to the dynamic model errors and should be tuned larger using α to ensure that $\boldsymbol{P}^-$ closely reflects the actual situation.

3.2. Adaptive Measurement Noise Covariance

Adaptive modification of the noise covariance is a tradeoff between the convergence rate and filter stability. The filter error propagation reflects the accuracy of the state estimations to some degree. In an ideal KF application, tuning the noise models to yield consistent estimation errors and uncertainties can also produce stable state estimates that track their true counterparts [46]. α makes $\boldsymbol{P}^-$ larger, and thus causes the filter gain value to increase, which increases the contribution of themeasurement outliers. Measurement outlier $\delta\boldsymbol{\xi}$ can be considered as the unknown or unidentified uncertainty contained in the innovation:

$$\tilde{\boldsymbol{\xi}} = \boldsymbol{\xi} + \delta\boldsymbol{\xi} \tag{18}$$

According to Equation (17), predicted state error propagation $\boldsymbol{P}^-$ can be increased β times without $\delta\boldsymbol{\xi}$, or inaccurately increased by α times with $\delta\boldsymbol{\xi}$ and

$$\begin{aligned} \beta &= \frac{(\boldsymbol{\xi}+\delta\boldsymbol{\xi})^\mathrm{T}(\boldsymbol{\xi}+\delta\boldsymbol{\xi})}{\mathrm{tr}(\boldsymbol{C}_\xi)} / \frac{\boldsymbol{\xi}^\mathrm{T}\boldsymbol{\xi}}{\mathrm{tr}(\boldsymbol{C}_\xi)} \\ &= 1 + (2\delta\boldsymbol{\xi}^\mathrm{T}\boldsymbol{\xi} + \delta\boldsymbol{\xi}^\mathrm{T}\delta\boldsymbol{\xi})/\boldsymbol{\xi}^\mathrm{T}\boldsymbol{\xi} \end{aligned} \tag{19}$$

In the measurement updating procedure, Kalman gain $\boldsymbol{K}$ is calculated by [47,48]:

$$\boldsymbol{K} = \boldsymbol{P}^-\boldsymbol{H}^\mathrm{T}(\boldsymbol{H}\boldsymbol{P}^-\boldsymbol{H}^\mathrm{T} + \boldsymbol{R})^{-1} \tag{20}$$

The measurement updating of the state estimate $\delta\hat{x}$ is formulated as follows:

$$\delta\hat{x} = \delta x^{-} + K\widetilde{\xi} = \delta x^{-} + K(\xi + \delta\xi) \tag{21}$$

A large proportion of the perturbations in the measurement is fed back to the state estimates because the corresponding gain value can be close to 1.0, subject to the large error magnitude of P^{-}.

If extreme measurement outliers occur, $\delta\xi$ tends to be extremely large. Then,

$$|\frac{\delta\xi^{\mathrm{T}}\delta\xi}{\mathrm{tr}\left(HP^{-}H^{\mathrm{T}} + R\right)} \to \infty \tag{22}$$

Compared to the uncorrected R, ξ contains large-magnitude errors. Therefore, α tends to be extremely large. For a more intuitive analysis of the disadvantage of large measurement errors in $\widetilde{\xi}$, we assume that α can be considered to tend to infinity. Based on L'Hôpital's rule [49], the corresponding Kalman gain can be derived as:

$$\begin{aligned} \lim_{\delta v \to \varepsilon} K &= \lim_{\alpha \to \infty} K \\ &= \lim_{\alpha \to \infty} \alpha P^{-}H^{\mathrm{T}}\left(H\alpha P^{-}H^{\mathrm{T}} + R\right)^{-1} \\ &= \left[\partial\left(\alpha P^{-}H^{\mathrm{T}}\right)/\partial\alpha\right]\left[\partial\left(H\alpha P^{-}H^{\mathrm{T}} + R\right)/\partial\alpha\right]^{-1} \\ &= P^{-}H^{\mathrm{T}}\left(HP^{-}H^{\mathrm{T}}\right)^{-1} \end{aligned} \tag{23}$$

According to Equation (13), every block of the $HP^{-}H^{\mathrm{T}}$ matrix consists of the linear transformation of skew-symmetric matrices, the determination of which equals to 0. Therefore, filter crashes can be produced due to singular-matrix inversion in the portion of the Kalman gain. This case rarely exists in general because the determination of R doesn't equals to 0 and numerical issues cannot be caused when processing matrix inversion in Equation (23). However, when the measurement outlier magnitude is much larger than the magnitude of R, the contribution of R in gain calculation can be degraded, even ignored, and positive feedback, filter divergence, even the numerical problem still exist. Small errors in P^{-} are relatively harmless; however, Equation (13) demonstrates that large P^{-}-matrix errors distort the Kalman gain matrix. R is often tuned by assigning state uncertainties that are substantially larger to an extent that nearly equivalent to $\xi\xi^{\mathrm{T}}$, and the modified measurement noise covariance that contains unknown uncertainties $\overline{R}$ can be expressed as:

$$\overline{R} = (\varsigma + \rho)(\varsigma + \rho)^{\mathrm{T}} \tag{24}$$

where sequence ς indicates the inaccurate statistical uncertainty, and is empirically smaller than the actual measurement uncertainties. The sequence ρ represents the unknown measurement uncertainties, which are assumed to approximately correspond with the actual outliers. Thus, the adaptive factor is:

$$\alpha = \frac{\gamma}{c_0} = \frac{1}{c_0}\frac{\xi^{\mathrm{T}}\xi + 2\delta\xi^{\mathrm{T}}\xi + \delta\xi^{\mathrm{T}}\delta\xi}{\mathrm{tr}\left(HP^{-}H^{\mathrm{T}}\right) + \varsigma^{\mathrm{T}}\varsigma + 2\varsigma^{\mathrm{T}}\rho + \rho^{\mathrm{T}}\rho} \tag{25}$$

If the hypothesis of Equation (22) is established, L'Hôpital's rule can be used to determine α, and it turns out that:

$$\alpha = \frac{\gamma}{c_0} = \frac{1}{c_0}\frac{\partial\left(\xi^{\mathrm{T}}\xi + 2\delta\xi^{\mathrm{T}}\xi + \delta\xi^{\mathrm{T}}\delta\xi\right)}{\partial\left(\mathrm{tr}\left(HP^{-}H^{\mathrm{T}}\right) + \varsigma^{\mathrm{T}}\varsigma + 2\varsigma^{\mathrm{T}}\rho + \rho^{\mathrm{T}}\rho\right)} = \frac{1}{c_0}\frac{||\xi||_2 + |\delta\xi||_2}{||\varsigma||_2 + ||\rho||_2} \tag{26}$$

where $||\bullet||_2$ indicates two norms of a vector.

Due to the introduction of ρ, the contribution of measurement outliers is almost reduced with respect to α and P^{-} afterwards. P^{-} reflects the relatively precise uncertainty of the predicted state estimates and cannot be extremely large. Therefore, L'Hôpital's rule cannot be used in the implementation of Equation (23), numerical issues concerning the inversion of $(\alpha \boldsymbol{H}\boldsymbol{P}^{-}\boldsymbol{H}^{\mathrm{T}} + \overline{\boldsymbol{R}})^{-1}$ are resolved, and the impact of positive feedback is attenuated because of the consideration of unknown measurement uncertainties in the Kalman gain determination. However, $\boldsymbol{\rho}$ is hard to be separated out; the most popular robust method in IAE is to tune α further by using an adaptive robust function that is expressed as [26]:

$$\alpha = \begin{cases} 1.0 & , \quad \gamma \leq c_0 \\ \frac{\gamma}{c_0} \times (\frac{c_1 - c_0}{c_1 - \gamma}), & c_0 \leq \gamma \leq c_1 \\ \frac{c_1}{\gamma} & , \quad c_1 \leq \gamma \end{cases} \tag{27}$$

where c_1 is the corresponding empirical constant, which is typically equal to 4.5–8.5 [50].

Unfortunately, some drawbacks still exist for the following reasons:

- When tuning a Kalman filter, unknown uncertainties cannot be easily separated from the measurement noise. Although filter stability is ensured by assigning substantially larger state uncertainties, subjective assumption is introduced.
- The performance of measurement error equations in INS calibration is weakened when the measurement is in the steady state and cannot continually provide high-accuracy positioning and velocity results. Due to dynamic model errors and self-drawbacks of INS mechanization, INS cannot provide accurate state prediction. Due to low-accuracy a priori solutions or GPS outages, γ may be also extremely large even without a measurement outlier because Equation (27) cannot figure out the source of larger innovation deviation. Therefore, α can be less than 1.0, and the impact of $\boldsymbol{P}^{-}$ matrix error is increased. According to Equations (21) and (23), a small $\boldsymbol{P}^{-}$ matrix error produces unresponsive state estimates, while $\boldsymbol{P}^{-}$ of a too-large error magnitude produces unstable, oscillatory state estimates [51].

The contributions of $\boldsymbol{P}^{-}$ and $\boldsymbol{R}$ determine the impact of dynamic and measurement models on the state estimation, respectively. Following Equations (17) and (27), the adaptive modification of $\boldsymbol{P}^{-}$ relies on $\boldsymbol{R}$, which demonstrates that focus should be placed on the measurement outlier detection and feedback. Hence, Figure 2 illustrates the proposed algorithm used to determine the $\boldsymbol{\rho}$ in Equation (24). In the two-antenna GPS/INS-integrated navigation application, the original $\boldsymbol{R}$ is modified based on filter measurement-updated covariance-solution processing by GPS-RTK, and reflects the filter accuracy of the GPS-RTK solution in detail ($\delta\widetilde{\boldsymbol{r}}_{\mathrm{GPS}} \in \Re^{3\times1}$, $\delta\widetilde{\boldsymbol{v}}_{\mathrm{GPS}} \in \Re^{3\times1}$, and $\delta\widetilde{\boldsymbol{p}}_{\mathrm{GPS}} \in \Re^{3\times1}$). $\boldsymbol{\rho} \in \Re^{3\times1}$ is designed based on the position dilution of precision (PDOP) value, number of valuable satellites ($Nsat$), AR ratio and length bias (dl) of $\widetilde{\boldsymbol{p}}_{12}$ which can be obtained by GPS-RTK processing. These parameters can objectively reflect the quality of RTK solutions, which also improves for the modification of measurement noise covariance matrix and the determination of adaptive factors. To tune the measurement noise covariance matrix such that it closely aligns with the actual error magnitudes, the modified $\overline{\boldsymbol{R}}$ is given by:

$$\begin{gathered} \overline{\boldsymbol{R}} = E[\boldsymbol{e}\boldsymbol{e}^{\mathrm{T}}] \\ \boldsymbol{e} = \mathrm{diag}[(\delta\widetilde{\boldsymbol{r}}_{\mathrm{GPS}} + \boldsymbol{\rho}_r)\ (\delta\widetilde{\boldsymbol{v}}_{\mathrm{GPS}} + \boldsymbol{\rho}_v)\ (\delta\widetilde{\boldsymbol{p}}_{\mathrm{GPS}} + \boldsymbol{\rho}_{\widetilde{\boldsymbol{p}}})]^{\mathrm{T}} \end{gathered} \tag{28}$$

where $\boldsymbol{\rho}_r = [\rho_r\ \rho_r\ \rho_r]^{\mathrm{T}}$, $\boldsymbol{\rho}_v = [\rho_v\ \rho_v\ \rho_v]^{\mathrm{T}}$ and $\boldsymbol{\rho}_{\widetilde{\boldsymbol{p}}} = [\rho_{\widetilde{\boldsymbol{p}}}\ \rho_{\widetilde{\boldsymbol{p}}}\ \rho_{\widetilde{\boldsymbol{p}}}]^{\mathrm{T}}$.

The bounded $dl (< \Delta)$ is given by:

$$dl = ||\widetilde{\boldsymbol{p}}_{12}||_2 - ||\boldsymbol{l}_{12}||_2 \tag{29}$$

Figure 2 shows the algorithm to adaptively modify the measurement noise covariance matrix. The factors ($\kappa, \mu, \gamma, \eta$) presented in the figure can quantify GPS-RTK reliability in detail to appropriately tune the measurement noise covariance matrix.

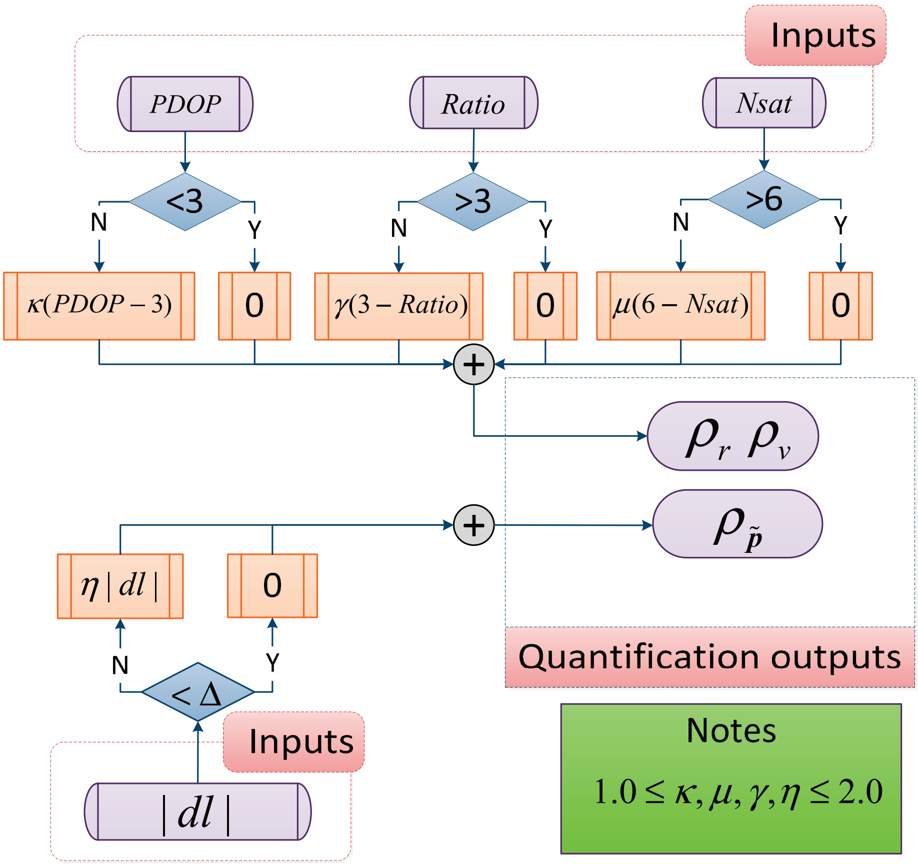

Figure 2. A quantification algorithm of Global Positioning System (GPS)-RTK reliability used in Equation (28) based on PDOP, number of valuable satellites, ratio, and length bias.

According to Figure 2, the adaptive measurement noise covariance matrix reconfiguration algorithm is formulated as the following equation. The algorithm is used to determine the driving state or measurement reliability based on the quantification results (ρ_r, ρ_v and $\rho_{\tilde{\boldsymbol{p}}}$), which can be equal or close to 0 when the GPS-RTK solution is reliable. Otherwise, state uncertainties larger than 1δ can occur in the instance when the measurement is less reliable. The factors mentioned in Figure 2, like PDOP and ratio, are directly related to positioning performance [52–56]; hence, the proposed measurement noise covariance matrix remodification method is more feasible and flexible compared with the adaptive robust method in Equation (27).

$$\begin{aligned} \rho_r, \rho_v &= \kappa(PDOP - 3) + \gamma(3 - Ratio) + \mu(6 - Nsat) \\ \rho_{\tilde{\boldsymbol{p}}} &= \eta|dl| \end{aligned} \tag{30}$$

In summary, the overall flowchart of the two-antenna GPS/INS integration algorithm is shown in Figure 3. The IMU outputs are processed by the INS mechanization algorithm into navigation parameters, including position, velocity, attitude vectors, and the derived baseline vector $\boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}\boldsymbol{l}_{12}$ between the two antennas. Two GPS-RTK processing schemes were adopted to estimate the position and velocity vectors of Rover 1 and the two-antenna baseline vector $\boldsymbol{p}_{12}^{\mathrm{n}}$. In this research, the implementation involves a filter based on the proposed approach.