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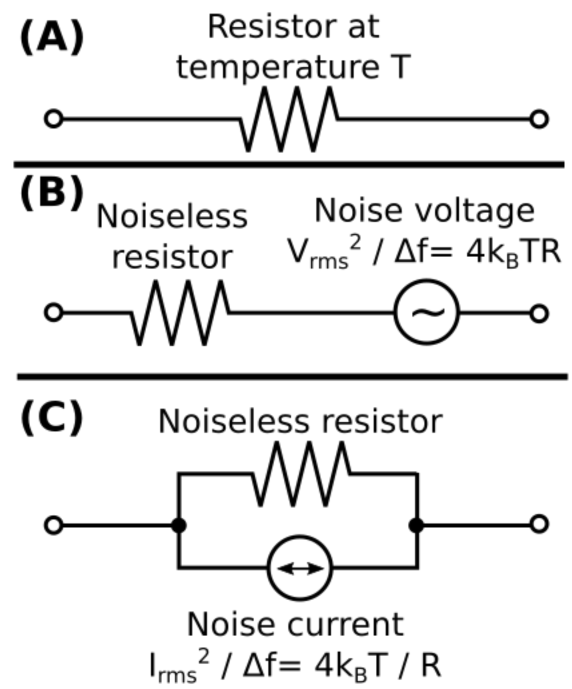
Johnson–Nyquist noise

Johnson–Nyquist noise (**thermal noise**, **Johnson noise**, or **Nyquist noise**) is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage. The generic, statistical physical derivation of this noise is called the fluctuation-dissipation theorem, where generalized impedance or generalized susceptibility is used to characterize the medium.



These three circuits are all equivalent: **(A)** A resistor at nonzero temperature, which has Johnson noise; **(B)** A noiseless resistor in series with a noise-creating voltage source (i.e. the Thévenin equivalent circuit); **(C)** A noiseless resistance in parallel with a noise-creating current source (i.e. the Norton equivalent circuit).

Thermal noise in an ideal resistor is approximately white, meaning that the power spectral density is nearly constant throughout the frequency spectrum (however see the section below on extremely high frequencies). When limited to a finite bandwidth, thermal noise has a nearly Gaussian amplitude distribution.[1]

Contents

History

This type of noise was discovered and first measured by John B. Johnson at Bell Labs in 1926.[2][3] He described his findings to Harry Nyquist, also at Bell Labs, who was able to explain the results.[4]

Noise voltage and power

Thermal noise is distinct from shot noise, which consists of additional current fluctuations that occur when a voltage is applied and a macroscopic current starts to flow. For the general case, the above definition applies to charge carriers in any type of conducting medium (e.g. ions in an electrolyte), not just resistors. It can be modeled by a voltage source representing the noise of the non-ideal resistor in series with an ideal noise free resistor.

The one-sided power spectral density, or voltage variance (mean square) per hertz of bandwidth, is given by

$$\overline{v_n^2} = 4k_{\mathrm{B}}TR$$

where k_{B} is Boltzmann's constant in joules per kelvin, T is the resistor's absolute temperature in kelvins, and R is the resistor value in ohms (Ω). Use this equation for quick calculation, at room temperature:

$$\sqrt{\overline{v_n^2}} = 0.13\sqrt{R}\,\mathrm{nV}/\sqrt{\mathrm{Hz}}.$$

For example, a 1 kΩ resistor at a temperature of 300 K has

$$\sqrt{\overline{v_n^2}} = \sqrt{4\cdot 1.38\cdot 10^{-23}\ \mathrm{J/K}\cdot 300\ \mathrm{K}\cdot 1\ \mathrm{k\Omega}} = 4.07\ \mathrm{nV}/\sqrt{\mathrm{Hz}}.$$

For a given bandwidth, the root mean square (RMS) of the voltage, v_n, is given by

$$v_n = \sqrt{\overline{v_n^2}}\sqrt{\Delta f} = \sqrt{4k_{\mathrm{B}}TR\Delta f}$$

where Δf is the bandwidth in hertz over which the noise is measured. For a 1 kΩ resistor at room temperature and a 10 kHz bandwidth, the RMS noise voltage is 400 nV.[5] A useful rule of thumb to remember is that 50 Ω at 1 Hz bandwidth correspond to 1 nV noise at room temperature.

A resistor in a short circuit dissipates a noise power of

$$P = v_n^2/R = 4k_{\mathrm{B}}\,T\Delta f.$$

The noise generated at the resistor can transfer to the remaining circuit; the maximum noise power transfer happens with impedance matching when the Thévenin equivalent resistance of the remaining circuit is equal to the noise generating resistance. In this case each one of the two participating resistors dissipates noise in both itself and in the other resistor. Since only half of the source voltage drops across any one of these resistors, the resulting noise power is given by

$$P = k_{\mathrm{B}}\,T\Delta f$$

where P is the thermal noise power in watts. Notice that this is independent of the noise generating resistance.

Noise current

The noise source can also be modeled by a current source in parallel with the resistor by taking the Norton equivalent that corresponds simply to dividing by R. This gives the root mean square value of the current source as:

$$i_n = \sqrt{\frac{4k_{\mathrm{B}}T\Delta f}{R}}.$$

Thermal noise is intrinsic to all resistors and is not a sign of poor design or manufacture, although resistors may also have excess noise.

Noise power in decibels

Signal power is often measured in dBm (decibels relative to 1 milliwatt). From the equation above, noise power in a resistor at room temperature, in dBm, is then:

$$P_{\mathrm{dBm}} = 10\ \log_{10}(k_{\mathrm{B}} T \Delta f \times 1000)$$

where the factor of 1000 is present because the power is given in milliwatts, rather than watts. This equation can be simplified by separating the constant parts from the bandwidth:

$$P_{\mathrm{dBm}} = 10\ \log_{10}(k_{\mathrm{B}} T \times 1000) + 10\ \log_{10}(\Delta f)$$

which is more commonly seen approximated for room temperature (T = 300 K) as:

$$P_{\mathrm{dBm}} = -174 + 10\ \log_{10}(\Delta f)$$

where Δf is given in Hz; e.g., for a noise bandwidth of 40 MHz, Δf is 40,000,000.

Using this equation, noise power for different bandwidths is simple to calculate:

Bandwidth (Δf)	Thermal noise power	Notes
1 Hz	−174 dBm	
10 Hz	−164 dBm	
100 Hz	−154 dBm	
1 kHz	−144 dBm	
10 kHz	−134 dBm	FM channel of 2-way radio
100 kHz	−124 dBm	
180 kHz	−121.45 dBm	One LTE resource block
200 kHz	−121 dBm	GSM channel
1 MHz	−114 dBm	Bluetooth channel
2 MHz	−111 dBm	Commercial GPS channel
3.84 MHz	−108 dBm	UMTS channel
6 MHz	−106 dBm	Analog television channel
20 MHz	−101 dBm	WLAN 802.11 channel
40 MHz	−98 dBm	WLAN 802.11n 40 MHz channel
80 MHz	−95 dBm	WLAN 802.11ac 80 MHz channel
160 MHz	−92 dBm	WLAN 802.11ac 160 MHz channel
1 GHz	−84 dBm	UWB channel

Thermal noise on capacitors

Thermal noise on capacitors is referred to as *kTC* noise. Thermal noise in an RC circuit has an unusually simple expression, as the value of the resistance (R) drops out of the equation. This is because higher R contributes to more filtering as well as to more noise. The noise bandwidth of the RC circuit is $1/(4RC)$,[6] which can substituted into the above formula to eliminate R. The mean-square and RMS noise voltage generated in such a filter are:[7]

$$\overline{v_n^2} = k_{\mathrm{B}}T/C$$

$$v_n = \sqrt{k_{\mathrm{B}}T/C}.$$

Thermal noise in the resistor accounts for 100% of *kTC* noise.

In the extreme case of the *reset noise* left on a capacitor by opening an ideal switch, the resistance is infinite, yet the formula still applies; however, now the RMS must be interpreted not as a time average, but as an average over many such reset events, since the voltage is constant when the bandwidth is zero. In this sense, the Johnson noise of an RC circuit can be seen to be inherent, an effect of the thermodynamic distribution of the number of electrons on the capacitor, even without the involvement of a resistor.

The noise is not caused by the capacitor itself, but by the thermodynamic fluctuations of the amount of charge on the capacitor. Once the capacitor is disconnected from a conducting circuit, the thermodynamic fluctuation is *frozen* at a random value with standard deviation as given above.

The reset noise of capacitive sensors is often a limiting noise source, for example in image sensors. As an alternative to the voltage noise, the reset noise on the capacitor can also be quantified as the electrical charge standard deviation, as

$$Q_n = \sqrt{k_{\mathrm{B}}TC}.$$

Since the charge variance is $k_{\mathrm{B}}TC$, this noise is often called *kTC noise*.

Any system in thermal equilibrium has state variables with a mean energy of $kT/2$ per degree of freedom. Using the formula for energy on a capacitor ($E = \frac{1}{2}CV^2$), mean noise energy on a capacitor can be seen to also be $\frac{1}{2}C(kT/C)$, or also $kT/2$. Thermal noise on a capacitor can be derived from this relationship, without consideration of resistance.

The *kTC* noise is the dominant noise source at small capacitors.

Noise of capacitors at 300 K

Capacitance	$\sqrt{k_{\mathrm{B}}T/C}$	**Electrons**
1 fF	2 mV	12.5 e⁻
10 fF	640 μV	40 e⁻
100 fF	200 μV	125 e⁻
1 pF	64 μV	400 e⁻
10 pF	20 μV	1250 e⁻
100 pF	6.4 μV	4000 e⁻
1 nF	2 μV	12500 e⁻

Generalized forms

The $4k_\text{B}TR$ voltage noise described above is a special case for a purely resistive component for low frequencies. In general, the thermal electrical noise continues to be related to resistive response in many more generalized electrical cases, as a consequence of the fluctuation-dissipation theorem. Below a variety of generalizations are noted. All of these generalizations share a common limitation, that they only apply in cases where the electrical component under consideration is purely passive and linear.

Reactive impedances

Nyquist's original paper also provided the generalized noise for components having partly reactive response, e.g., sources that contain capacitors or inductors.[4] Such a component can be described by a frequency-dependent complex electrical impedance $Z(f)$. The formula for the power spectral density of the series noise voltage is

$$S_{v_n v_n}(f) = 4 k_\text{B} T \eta(f) \operatorname{Re}[Z(f)].$$

The function $\eta(f)$ is simply equal to 1 except at very high frequencies, or near absolute zero (see below).

The real part of impedance, $\operatorname{Re}[Z(f)]$, is in general frequency dependent and so the Johnson–Nyquist noise is not white noise. The rms noise voltage over a span of frequencies f_1 to f_2 can be found by integration of the power spectral density:

$$\sqrt{\langle v_n^2 \rangle} = \sqrt{\int_{f_1}^{f_2} S_{v_n v_n}(f) df}.$$

Alternatively, a parallel noise current can be used to describe Johnson noise, its power spectral density being

$$S_{i_n i_n}(f) = 4 k_\text{B} T \eta(f) \operatorname{Re}[Y(f)].$$

where $Y(f) = 1/Z(f)$ is the electrical admittance; note that $\operatorname{Re}[Y(f)] = \operatorname{Re}[Z(f)]/|Z(f)|^2$

Quantum effects at high frequencies or low temperatures

Nyquist also pointed out that quantum effects occur for very high frequencies or very low temperatures near absolute zero.[4] The function $\eta(f)$ is in general given by

$$\eta(f) = \frac{hf/k_\text{B}T}{e^{hf/k_\text{B}T} - 1},$$

where h is Planck's constant.

At very high frequencies $f \gtrsim k_\text{B}T/h$, the function $\eta(f)$ starts to exponentially decrease to zero. At room temperature this transition occurs in the terahertz, far beyond the capabilities of conventional electronics, and so it is valid to set $\eta(f) = 1$ for conventional electronics work.

Relation to Planck's law

Nyquist's formula is essentially the same as that derived by Planck in 1901 for electromagnetic radiation of a blackbody in one dimension—i.e., it is the one-dimensional version of Planck's law of blackbody radiation.[8] In other words, a hot resistor will create electromagnetic waves on a transmission line just as a hot object will create electromagnetic waves in free space.

In 1946, Dicke elaborated on the relationship,[9] and further connected it to properties of antennas, particularly the fact that the average antenna aperture over all different directions cannot be larger than $\lambda^2/(4\pi)$, where λ is wavelength. This comes from the different frequency dependence of 3D versus 1D Planck's law.

Multiport electrical networks

Richard Q. Twiss extended Nyquist's formulas to multi-port passive electrical networks, including non-reciprocal devices such as circulators and isolators.[10] Thermal noise appears at every port, and can be described as random series voltage sources in series with each port. The random voltages at different ports may be correlated, and their amplitudes and correlations are fully described by a set of cross-spectral density functions relating the different noise voltages,

$$S_{v_m v_n}(f) = 2k_{\mathrm{B}}T\eta(f)(Z_{mn}(f) + Z_{nm}(f)^*)$$

where the Z_{mn} are the elements of the impedance matrix $\mathbf{Z}$. Again, an alternative description of the noise is instead in terms of parallel current sources applied at each port. Their cross-spectral density is given by

$$S_{i_m i_n}(f) = 2k_{\mathrm{B}}T\eta(f)(Y_{mn}(f) + Y_{nm}(f)^*)$$

where $\mathbf{Y} = \mathbf{Z}^{-1}$ is the admittance matrix.

Continuous electrodynamic media

The full generalization of Nyquist noise is found in fluctuation electrodynamics, which describes the noise current density inside continuous media with dissipative response in a continuous response function such as dielectric permittivity or magnetic permeability. The equations of fluctuation electrodynamics provide a common framework for describing both Johnson–Nyquist noise and free space blackbody radiation.[11]

See also

- Fluctuation-dissipation theorem
- Shot noise
- 1/f noise
- Langevin equation

References

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External links

- Amplifier noise in RF systems (http://www4.tpgi.com.au/users/ldbutler/AmpNoise.htm)
- Thermal noise (undergraduate) with detailed math (http://www.physics.utoronto.ca/~phy225h/experiments/thermal-noise/Thermal-Noise.pdf)
- Johnson–Nyquist noise or thermal noise calculator – volts and dB (http://www.sengpielaudio.com/calculator-noise.htm)
- Derivation of the Nyquist relation using a random electric field, H. Sonoda (via archive.org) (https://web.archive.org/web/20140714152536/http://www.phys.sci.kobe-u.ac.jp/~sonoda/notes/nyquist_random.ps)
- Applet of the thermal noise. (https://web.archive.org/web/20140202091900/http://xformulas.net/applets/thermal_noise.html)