

Noise Measurements

1. Noise Measurements

Adapted from Prof. Keith Ruddick's Write-Up

In this experiment, you will make two series of measurements in order to determine two of the fundamental constants of physics: k the Boltzmann constant, and e the charge on the electron. k will be determined by measurements of the thermal (Johnson) noise in a series of resistors, while e will be determined by measurement of the shot noise in a weakly illuminated photodiode.

This experiment consists of three steps:

- First, you calibrate the preamplifier and filters to obtain their gain vs. frequency response (see figure 1.1.) This is done by comparing the output signal from the preamplifier and filters to a known input signal from the function generator. Since the preamplifier has a very large gain, the corresponding input signal must be very small. Therefore, it is first sent through an attenuator circuit to avoid overloading the preamplifier.
- Second, use ordinary carbon resistors as noise "signal generators." Connect them to the preamplifier input. Since you have measured the preamplifier and filters response to a small input signal, you can now determine k from the measured output voltages.

It may seem a bit strange that a carbon resistor could act like a signal source because so far you have treated carbon resistors as passive devices to reduce voltages. For this experiment you should consider that in addition to the $V = IR$ relation, carbon resistors also produce a very small Johnson noise voltage V_j . Since this noise voltage is an AC source, you must rectify the signal, i.e., measure its RMS voltage.

- Third, you measure the small current fluctuations due to shot noise from a phototransistor illuminated by an LED and determine e . The shot noise voltage is measured by passing the current from the phototransistor through a resistor which in turn is connected through some coupling capacitors to the preamplifier and filters. Clearly, the resistor must be chosen carefully so that the observed noise voltage is not dominated by Johnson noise.

1.1. Theory: Determination of k from Johnson Noise

Spurious voltages or currents which interfere with electrical signals are termed *noise*. A typical example would be pick-up of 60 Hz signals from the ac mains. Such noise can arise from poor shielding or poor grounding of the apparatus, but some types of noise are intrinsic to electrical circuits. Johnson noise, sometimes called thermal noise, is one of these.

Thermal fluctuations of the electromagnetic field inside a resistor produce a corresponding random fluctuation in the voltage. You can read about Johnson's experiments and their explanation by Nyquist in two important papers in *Physical Review*, Volume 32, pp 97, 110 (1928). (These are highly recommended reading). H&H discuss noise from a more practical point of view in chapter 7.

Using classical physics arguments and statistics, Nyquist showed that the RMS noise voltage across any resistance R in a frequency range B is:

$$V_J = \sqrt{4kTRB} \quad (1.1.)$$

where k is the Boltzmann constant and T the absolute temperature. For example, the RMS noise voltage of a 10 k Ω resistor at room temperature measured over a bandwidth (frequency range) of 10kHz is 1.3 μ V, i.e. an amplifier with a gain of 10^6 and this bandwidth connected across a 10 k Ω will have an output of 1.3 V even when no signal is applied! The noise power is the same for equal frequency intervals; i.e., the frequency spectrum is flat, and Johnson noise is said to be "white noise", analogous with the uniform spectral distribution of white light.

In this experiment, you will actually measure a portion of the Johnson noise power spectrum. Since the noise voltages are small, these must be amplified using an *instrumentation amplifier*. Such amplifiers are used for measuring small differential voltages and have extremely large common-mode rejection ratios. The frequency bandwidth will be determined by a combination of the gain characteristics of the amplifier and by additional low-pass and high-pass filters.

If a sinusoidal input signal V_{in} of frequency f becomes V_{out} after amplification and filtering, then the effective gain $g(f)$ is defined by:

$$g(f) = (V_{out}/V_{in})_f \quad (1.2.)$$

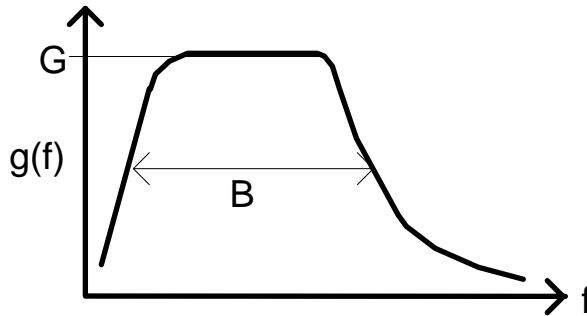
Thus, after amplification the contribution dV_J^2 to the Johnson noise in a differential frequency interval becomes:

$$dV_{Jout}^2 = g(f)^2 dV_J^2 \quad (1.3.)$$

and after integrating over the frequencies of the pass band we obtain:

$$V_{Jout}^2 = 4RkT \int_0^\infty g(f)^2 df = 4RkT(G^2 B) \quad (1.4.)$$

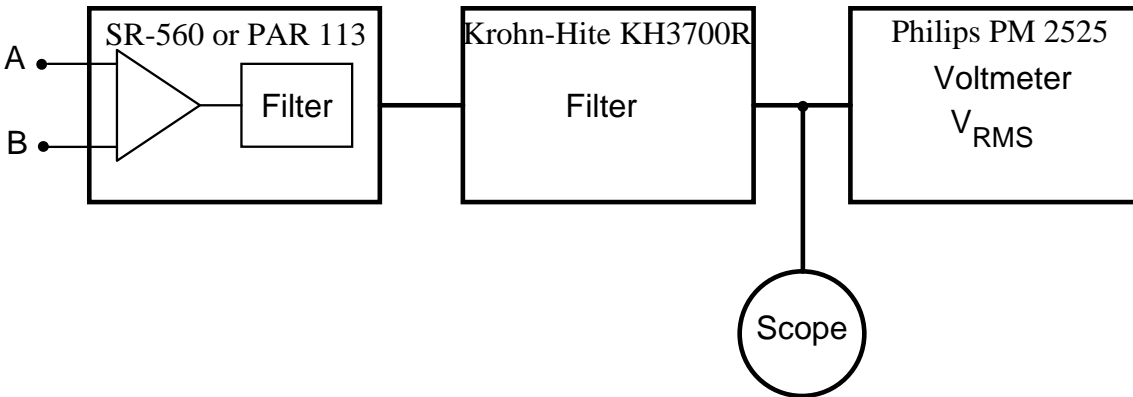
where the factor $G^2 B$ is essentially an effective bandwidth B multiplied by the square of some average gain, G .



• Figure 1.1. Gain as a function of frequency. (Note: do not be alarmed if your data has not as 'flat' a response as shown in the picture above.)

The first part of this experiment consists of a calibration of the amplifier and associated filters to determine the factor G^2B and, the second part, the measurement of V_{Jout} for a range of resistor values. This is a high precision experiment, in principle. With these data, and with a little care you should be able to get a precision of about 1% in your final result.

1.2. Apparatus



• Figure 1.2. Johnson noise measurement set-up.

Figure 1.2. shows the electronic apparatus. Set the system up and familiarize yourself with the low-noise instrumentation amplifier (either a Stanford Research SR-560 or PAR-113), the band-pass filter (Krohn-Hite KH-3700R), and the RMS voltmeter (Philips PM-2525 or HP34401A). The Stanford SR560 amplifier incorporates both low-pass and high pass filters for defining the measurement bandwidth, and the KH-3700R is used to provide even better definition of the bandpass region.

Some possible settings for the components, which should eliminate effectively all traces of the 60 Hz and its harmonics are:

SR 560 or PAR 113	Low-Pass:	30kHz
	High-Pass:	300Hz
	Coupling:	AC
	Source:	A – B
	Gain Mode:	Low Noise
	Gain:	10^4
	Power:	BATT (if possible)
KH 3700	LF:	300Hz
	HF:	20kHz
	Gain:	0 dB
	Line Opr and plugged in!	

• Table 1.1. Settings for the Stanford SR560 and the Krohn-Hite KH3700R low noise amplifier filter. Note that for the SR560, both the 300Hz and the 30kHz filter cutoff lights must be on; if this is not the case keep pressing the button located directly over the label "ROLLOFF".

During the experiment disconnect the power cord to the Stanford Research SR-560; it contains a rechargeable battery and will run a lot quieter on the battery than when it is connected to the 110VAC. Since the battery needs to recharge, *reconnect the power cord when you are done* with the experiment so that other people can use it. The OVERLOAD signal light indicates that the signal fed into the

amplifier is too large; reduce either the signal or the gain if this happens; you may have to turn the amplifier off temporarily. If it refuses to turn on, you need to further reduce the input signal.

Use short coaxial cables to connect to the A and B inputs of the preamp (each input switch in the AC position and the source set to "A-B").

Quick Equipment Test

With the setup as indicated in figure 1.2 and table 1.1, connect a short, one foot, coaxial cable directly from A input to B input. Your V_{RMS} output reading from this setup should be about 20 mVAC or less if the equipment is working properly while running on battery. (Note: that the V_{RMS} is roughly proportional to the cable length and, therefore, will yield a larger value for a longer cable.) If this is not the case see your TA.

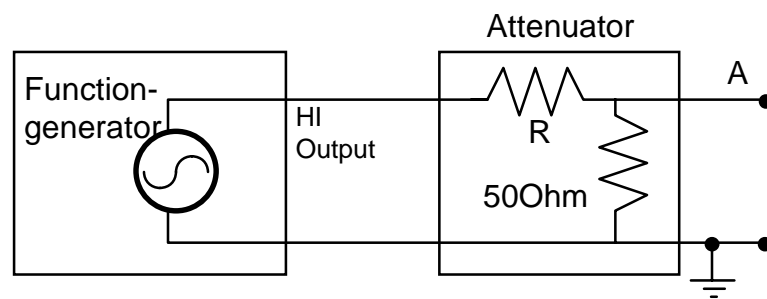
1.3. Calibration

a) Set-up

The calibration consists of generating a known sinusoidal voltage V_{in} at a frequency f and measuring the output voltage V_{out} to determine the actual frequency dependent gain of the system $g(f)$ over the range of the chosen bandpass interval. Use the settings as shown in table 1.1.

b) Using the Attenuators

The first problem is that of obtaining a small, measurable input voltage. The maximum output voltage of the system is around 5 V and wave distortions occur at higher output voltages. Since the amplifier is used with a gain setting of 10^4 , you would need input voltages of 10's of μV . The function generator can only provide pulses as small as a few mV. The solution to this dilemma is to place an **attenuator** or resistive voltage divider on the output of the function generator before the calibration signals are sent to the amplifier. You should find appropriate attenuators with the equipment; they provide approximately an attenuation factor of 1000. See figure 1.3.

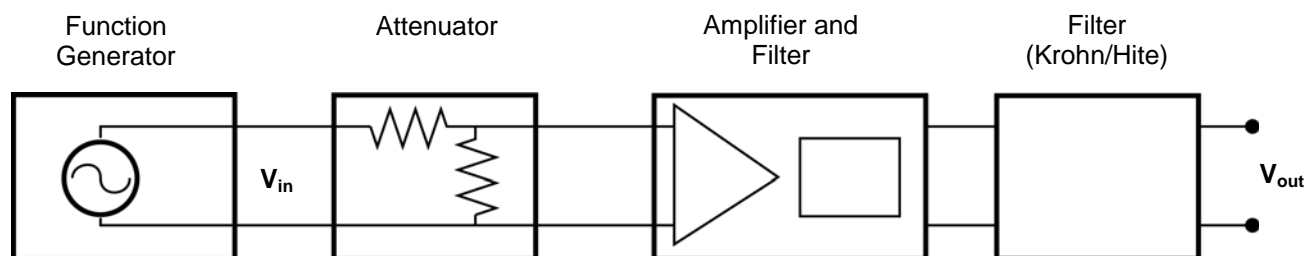


- Figure 1.3. Setup for using the attenuators to obtain a small input signal. (If you wish to make your own attenuator to provide more or less attenuation, ideally it should have an output impedance of $50\ \Omega$ so that it has the same output impedance as the function generator).

First calibrate the attenuator: You should be able to determine the attenuation factor by opening the box and directly measuring the values of the resistors with the digital multimeter.

c) Measuring Gain and Bandwidth

You are now ready to measure $g(f)$.



• Figure 1.4.

(i) Connect the attenuator between the function generator and the amplifier as shown in Figure 1.4. (On the PAR113 make sure that input B is grounded if you use input A, on the SR 560, select A.) Also connect one digital voltmeter directly to function generator output to measure V_{in} and another one to the Krohn-Hite filter output to measure V_{out} . See figure 1.4. Make sure both meters are set to measure RMS voltages! If you are using the HP34401A multimeter press “AC V,” on the Philips PM2525 press the V~ button. For the appropriate settings of the amplifier and filters see table 1.1.

(ii) Before you begin taking data you, check that the equipment is working properly and that you know how to compute $g(f)$. Set the output amplitude of the function generator, V_{in} , to 0.1 V RMS and adjust its frequency until you find the value for which V_{out} is a maximum. From these readings and taking the attenuator into consideration, calculate $g(f)$, i.e., the gain of the amplifier and filter system. Your result should be very close to the nominal gain setting of the amplifier. If you are off by more than 15% do not continue; instead see your TA.

(iii) Now systematically measure V_{in} and V_{out} as a function of frequency. Record sufficient data so that you will be able to accurately evaluate the integral $\int g(f)^2 df$ using numerical and graphical methods. Taking readings from 100Hz to 40 kHz at 1 to 2 kHz intervals should be sufficient. While doing so, keep an eye on V_{out} : if it exceeds 5 V RMS you are probably overloading the amplifier and your results are meaningless; if V_{out} drops below 5 mV RMS, your readings are too small to be accurate. In either case, adjust the amplitude (V_{in}) of the function generator. A good starting value for V_{in} is between 0.1V and 1V RMS.

(iv) Using an Excel spreadsheet, you should now be able to obtain a value for the quantity

$$\int_0^{\infty} g(f)^2 df = G^2 B \quad (1.5.)$$

One method to evaluate equation 1.5, though not the most accurate one, is to use the integration by summation. You sum the products of $g^2(f)$ and Δf where $g^2(f)$ is obtained from the individual V_{out} and V_{in} measurements in (iii) and Δf corresponds to the frequency interval of each measurement.

Before you continue and use a more sophisticated numerical integration method, you should check the result obtained by the simple integration by summation method with the right hand side of 1.5: use the maximum gain measured in (ii) for G and a good estimate of B , which is the bandwidth, can be

obtained by looking at a plot of $g(f)$ vs. f data. This rough estimate of G^2B should agree in its magnitude to the result obtained by the integration by summation method! Check this before proceeding!

If your rough estimate agrees, you should improve the accuracy of the integration by using a numerical method such as the Trapezoidal or Simpson's rule of integration. It is up to you which one you choose, nevertheless, make sure that you check the result from the more sophisticated method against the more simple one. (Keep this rule of programming in mind: while a simple numerical method may not produce a very accurate result, it is sometimes better than the result from a more accurate method which, due to its complexity, may contain bugs!)

Finally, estimate the uncertainty of your result and report the result and its uncertainty in your report. Also, keep the spreadsheet with the calculations because you will need it to adjust your result later for an additional correction factor. (See eq. 1.7.)

1.4. Johnson Noise

The Shielded Aluminum Box

The signal that you will measure from the Johnson noise is very small, on the order of a few mV / k Ω . Therefore, it is crucial that the resistors and all the cables connected to the resistor are shielded from external noise sources, especially from the ubiquitous 60 Hz. You will find an aluminum box with a lid next to the experiment and you will use it to shield the resistor during the measurements. Note that you are a fairly efficient antenna yourself and that your position relative to the resistor may provide significant noise unless you provide adequate shielding, by closing the lid. Tapping on the box, or any other vibrations produce noise signals (microphonics).

Measurement Setup

Before you start measuring V_R for a series of carbon resistors disconnect the attenuator box and function generator. They are no longer needed. Connect two very short coaxial BNC cables directly from the BNC jacks of the aluminum box to the SAR 560 amplifier inputs A and B. For the remaining measurements, set the amplifier in its A - B mode!

The carbon resistors are held between a pair of alligator clips inside the shielded aluminum box. Do not get distracted by all the other stuff in the box. As long as the switch $S1$ is in the "JOHNSON" position, as it should be for these measurements, then the BNC connectors are *only* connected across the resistor R and all the other stuff inside the box will be disconnected. (The disconnected circuit is not needed until the next part of this experiment, the shot noise measurements.) For a detailed circuit diagram see figure 1.5.

Measurements and Preliminary Analysis

To familiarize yourself with the equipment and to test that the setup is (still) working properly, take the preliminary measurements below and then perform a very rough analysis. (You will be able to use the data collected here later, so do not skip this important step!) For these measurements, the voltages are the RMS AC voltages observed as shown in Figure 1.2.

- To verify that the system is working properly, insert a 1 M Ω resistor. You should get a V_{RMS} reading of about 100 ± 20 mV $_{\text{RMS}}$. Discard this reading for now.

- Measure the resistance and V_R for 5 different carbon resistors between 500 Ω and about 15k Ω .
- Measure V_{zero} for the case when the resistance is zero, i.e. for a piece of wire connected between the alligator clips. Estimate the uncertainty of each measurement that you make.

All contributions to the measured RMS voltage should be statistically uncorrelated and must therefore add in quadrature. The desired mean square Johnson noise is then

$$V_{Jout}^2 = V_R^2 - V_{zero}^2 \quad (1.6.)$$

Next, perform a quick, preliminary analysis and plot: $V_{Jout}^2 / (4 T G^2 B)$ vs. R . For now, you may ignore errors and using Excel's Trendline utility obtain the slope of your data. (Click on a data point in your chart and from the main menu select: "Chart / Add Trendline;" select the linear fit and under the "Options" tab select "Display Equation on Chart." You may need to reformat the displayed equation to see more significant figures.) From the slope you should be able to calculate k . If you are off by more 30% of the accepted value, stop right now! Go back over the setup or see your TA because if you continue further your results will only get more meaningless.

- Only if the result of your preliminary analysis of k looks promising, continue taking 7 more V_R readings for carbon resistors between 15 k Ω to 1 M Ω . Keep measuring V_{zero} . You should take several of these readings during the course of your measurements since any additional interference may vary with time. The V_{zero} measurements are important in order to determine the equivalent noise input of the amplifier. Again estimate the uncertainty of each measurement that you make.

1.5. Data Analysis

If you add the newly collected data to the preliminary analysis from the previous section and again plot $V_{Jout}^2 / (4 T G^2 B)$ vs. R you will find the data no longer follows a straight line. Instead, with R above 100 k Ω , the slope keeps decreasing with increasing R .

The reason for this is an effect which has not yet been considered: stray capacitance of the input cables and the input capacitance of the preamplifier itself. This capacitance, which is effectively in parallel with the resistance R , forms a low pass filter, which shunts part of the AC Johnson noise signal. It can simply accounted for by including an additional factor in the expression for V_{Jout}^2 :

$$V_{Jout}^2 = 4RkT \int_0^{\infty} \frac{g(f)^2}{1 + (2\pi fRC)^2} df \quad (1.7.)$$

This additional factor clearly has a negligible effect for low frequencies and small resistances, such as the ones used in the previous preliminary analysis, but for larger resistances it must be included in your spreadsheet calculations to determine $G^2 B$.

In other words, $G^2 B$ can no longer be treated as a constant but instead depends on R and C . The previous calculations of $G^2 B$ in section 1.3 must be adjusted for each resistor to reflect this fact:

$$G^2 B(R, C) = \int_0^\infty \frac{g(f)^2}{1 + (2\pi f RC)^2} df \quad (1.7a.)$$

An additional complication arises since C is not known and is difficult to measure. We can estimate its magnitude since the value quoted by the manufacturer for RG58 coaxial cable is 29 pF/ft though such an estimate does not include the stray capacitance of the amplifier itself.

Although we do not know the exact value of C , we can obtain it indirectly from our data and equation 1.7. If we plot, or fit, $V_{Jout}^2 / (4 T G^2 B(R, C))$ vs. R , then the data should fall on a straight line for the “correct” or “best” value of C . An “incorrect” value of C would result for the data to fall on a curved line. Actually, the more C deviates from the “best” value, the more the data would deviate from falling on a straight line.

This observation allows us to find a “best” value for C by fitting our data to a straight line:

- First, we use a least squares fit of $V_{Jout}^2 / (4 T G^2 B(R, C))$ vs. R selecting a “reasonable” estimate of C .
- Second, since χ^2 is a measurement of how well data fits a theoretical prediction, we adjust C until we get the smallest value of χ^2 , at which point we have found the “best” value for C .
- Third, having found the “best” value for C , the results of the least squares fit should immediately reveal the value of k .

While this completes the theoretical discussion of the data analysis, what follows are some practical hints on how to set up the spreadsheet calculations. You may skip this section and you may choose your own method to calculate C and k and its uncertainties.

Note that such measurements of T (instead of k) can, and do, provide a means for direct temperature measurement. With special techniques, they can be used to measure temperatures down to the mK (milliKelvin!) scale.

Details and Hints

1: Calculating the Individual $G^2 B(R, C)$ For Each R

Use your $G^2 B$ calculations from the previous section and modify your spreadsheet to include the lowpass filter correction to reflect the “new” $G^2 B(R, C)$. (See equation 1.7a.) Adjust your spreadsheet in such a way that both C and R can easily be adjusted, i.e., set your spreadsheet up that all calculations involving C (or R) point to a single cell that contains the value of C (or R .) (Ask your TA if you need help with this.) Use a reasonable estimate of C based on the value quoted by the manufacturer for RG58 coaxial cable as 29 pF/ft.

Next make a copy of the entire spreadsheet and paste it back into your workbook. (From the main menu select: “Edit / Move or Copy Sheet;” select the “Create Copy” check box.) Repeat this for each value of R . Lay it out so that all sheets refer to one and only one cell on a separate worksheet that contains a value for C . You should now have all the individual $G^2 B(R, C)$ for each R .

2: LSQ fit and Iteration to find the best value of C

Use a LSQ fit for $V_{Jout}^2 / (4 T G^2 B(R,C))$ vs. R using the $G^2 B(R,C)$ calculated previously for each R . Either use Excel's "Solver" utility or adjust C until you get the smallest χ^2 .

Before you can fit the data, you must calculate the error in $V_{Jout}^2 / (4 T G^2 B(R,C))$ which depends on the uncertainties in V_{Jout}^2 , T and $G^2 B(R,C)$. Of all the error propagations involved, the most difficult is the one related to $G^2 B(R,C)$. (Note: this is by no means the most significant error in the $V_{Jout}^2 / (4 T G^2 B(R,C))$ term!) For simplicity, assume that $G^2 B(R,C)$'s only significant error contribution comes from

v_{out} when determining $g(f)$. If this is the case, you should be able to show that: $\frac{\sigma_{G^2 B(R,C)}}{G^2 B(R,C)} \approx \frac{2\sigma_{vout}}{v_{out}}$. Of

course, v_{out} and σ_{vout} are frequency dependent; nevertheless, you should be able to estimate a rough (percentage) value for it.

You will determine the error in C after you obtained its best value by minimizing χ^2 . To determine the error in C , see Appendix A, figure A.4. and the comments relating to it.

1.6. Determination of e by Shot Noise Measurements

In contrast to a normal steady current, a source of current in which the charge carriers are created or released by statistically independent events must deliver a "noisy" current, which fluctuates about some average value. An example of such a current is the current generated by light falling on a photodiode; electrons are excited into the conduction band and swept away by the local electric field in the depletion region. The fluctuating component in such currents was first called "shot noise" by Schottky who likened it to the acoustic noise generated by a hail of shot striking a target; a better analogy might be to the sound of raindrops falling on a tin roof. Shot noise is a white noise, like thermal noise.

Some features of shot noise can be obtained from statistical arguments: If, on average, N electrons are produced in a time interval τ then the resulting average current will be:

$$I_{ave} = \frac{Ne}{\tau} \quad (1.8.)$$

This current will have a fluctuating component I_{fluct} proportional to the fluctuations in the quantity N , given by Poisson statistics as \sqrt{N} , i.e. it then follows that

$$I_{fluct}^2 = \frac{I_{ave} e}{\tau} \quad (1.9.)$$

In this analysis, we must assume that τ is large enough so that the numbers of electrons produced in successive time intervals τ are statistically independent.

In general, a single electron injected into any circuit will produce a pulse of current $i(t)$ which will be a damped oscillation due to inductance in the circuit. τ will then be the effective duration of this oscillation. Obviously $\int i(t) dt$. Many individual electrons generally will be produced in the time τ and the result is a superposition of many wave forms $i(t)$ initiated at random times. A full analysis of the problem requires Fourier analysis of such random noise signals and yields

$$I_{fluct}^2 = 2I_{ave} eB \quad (1.10.)$$

where B is the frequency bandwidth of the measurement, as before. This implies that a current of $1\mu A$ will have a fluctuating component of $0.6 \times 10^{-4} \mu A$ when measured over a 10 kHz bandwidth, for example.

In this experiment, the current produced by a weakly illuminated photodiode is passed through a resistor R . (See figure 1.5.) The average voltage across this resistor is then $V_{ave} = I_{ave} R$. The small fluctuating voltage component $V_{fluct} = I_{fluct} R$ can be separated off by a blocking capacitor, then amplified and filtered, and finally measured by an RMS voltmeter. After amplification $g(f)$ at frequency f we have:

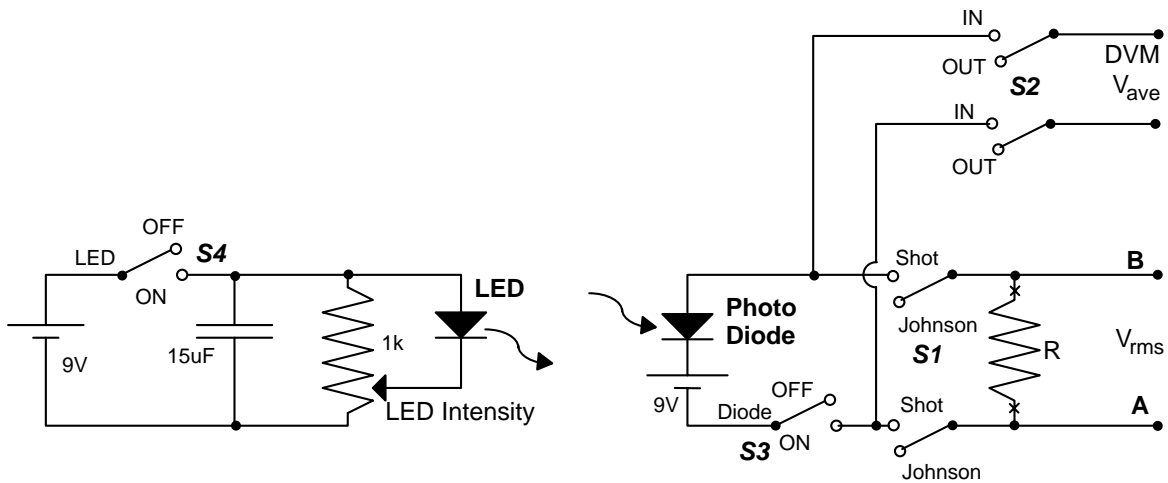
$$V_{rms} = g(f)V_{fluct} \quad (1.11.)$$

and then

$$V_{rms}^2 = 2eR^2I_{ave} \int g(f)^2 df = 2eRV_{ave}(G^2B) \quad (1.12.)$$

1.7. Shot Noise Setup

By setting the switch on the shielded box to the "SHOT" position, the photodiode (Motorola MRD 500) and its battery power supply are connected across the resistor and BNC output connectors:



• Figure 1.5. The "noise box" circuit. Be sure to turn on S3 and S4 for the shot noise measurements.

The source of illumination is an LED whose intensity may be varied by means of a potentiometer. Turn both the LED and Photodiode switch on; adjust the LED intensity and observe the range of V_{ave} , which is measured by a digital voltmeter as a **DC voltage** across R .

The settings that you used in the measurement chain for the Johnson noise measurements will probably prove adequate (and save you the necessity of recalibration) for the V_{rms} measurements; remember to keep the multimeter in its **RMS AC voltage** measurement mode, i.e., press the AC V (HP) or V~ button (Philips) respectively.

The currents that you will be measuring are very small, fractions of a μA typically. You must choose an appropriate value of R so that the shot noise signal is neither too small nor completely dominated by the Johnson and amplifier noise. On the other hand, adding the diode to the circuit must necessarily add significantly to the capacitance connected across the resistor: As discussed in the Johnson noise measurements, the effect of this stray capacitance is enhanced with large resistance values. Resistor values around $100k\Omega$ have produced good data. You should take measurements with two resistors to check that your result is independent of stray capacitance.

When taking data you will encounter a problem with the current setup: when the DVMs for the V_{rms} and V_{ave} are both connected at the same time then the OVERLOAD LED on the amplifier will turn on, indicating that the V_{rms} readings are meaningless. This is caused by the DVM measuring V_{ave} injecting excess noise into the circuit. Although the box is equipped with a switch that disconnects this DVM from the circuit, turning this switch off does not sufficiently prevent the injection of noise from the DVM back into the circuit. Therefore, you must physically disconnect the DVM measuring V_{ave} each time while measuring V_{rms} !

A possible data strategy is:

1. Connect the DVMs for measuring V_{ave} and V_{rms} . Since you already have a feel for the range of V_{ave} , adjust the intensity of the LED with the potentiometer on the front of the box to a value within that range.
2. Read V_{ave} quickly because the battery is slowly being drained and V_{ave} decreases while you are reading it. (This should be a DC voltage!)
3. Physically disconnect the V_{ave} DVM and wait till the OVERLOAD LED goes off. Now read V_{rms} . (This should be an AC voltage!)
4. Connect the V_{ave} DVM again and adjust the intensity of the LED again, i.e., return to step 2.

1.8. Shot Noise Analysis

Plot V_{rms}^2 as a function of $2RV_{ave} \int_0^\infty g^2(f)df$. Find the slope and intercept to determine the charge on the electron and the amplifier noise at the input.

Noise Measurements Experiment Grading Form

Name:.....

Total Pts:.....

Theory

	Max Pts.		Comments
		Overall	
		Johnson Noise	
		Shot Noise	

Questions/Analysis/Results

	Max Pts		Comments
		determine G^2B	
		C-correction for large R	
		Johnson noise k	
		Johnson noise σ_k	
		Johnson noise discussion	
		Johnson noise overall	
		shot noise e	
		shot noise σ_e	
		shot noise discussion	
		amplifier noise	

Presentation

	Max Pts		Comments
		Sections labeled, figures labeled	
		Figures referenced, units	
		Overall	

Comments: