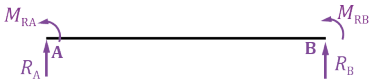
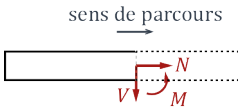
Formulaire résistance des matériaux – Calcul des poutres

Hypothèses et conventions

- Les sollicitations internes sont définies comme agissant de la partie à droite de la coupure sur la partie à gauche ;
- Les conventions de signe pour les réactions sont:
  - forces verticales positives orientées vers le haut ;
  - moments positifs dans le sens trigonométrique.



- Le repère local pour la position des sections et le calcul de la flèche est indiqué sur chaque figure ;
- Les conventions de signe pour les sollicitations internes sont :
  - effort normal positif dans le sens x local positif ;
  - effort tranchant positif dans le sens y local positif ;
  - moment fléchissant positif dans le sens trigonométrique.



- Les conventions de signe pour les diagrammes sont indiquées sur chaque figure.

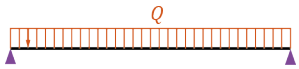
Configurations traitées

- Poutres isostatiques à une travée sur deux appuis (série Axx) ;
- Poutres isostatiques à deux travées sur deux appuis (série ACxx) ;
- Poutres hyperstatiques bi-encastrées (série Bxx) ;
- Poutres isostatiques en console (série Cxx) ;
- Poutres hyperstatiques à une travée encastrées-appuyées (série Dxx) ;
- Poutres hyperstatiques à deux travées, encastrées-appuyées (série DCxx).

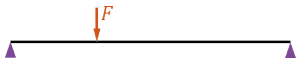
Pour chaque configuration, le formulaire donne généralement :

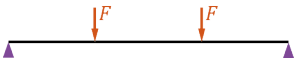
- Les réactions aux appuis ;
- L'effort tranchant et le moment fléchissant le long de la poutre ;
- La position et la valeur du moment maximal ;
- La déformée en flexion ;
- La flèche verticale en diverses sections, y compris la flèche maximale ;
- Les rotations au droit des sections singulières, le cas échéant ;
- L'énergie de déformation en flexion.

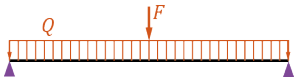
POUTRES SUR 2 APPUIS

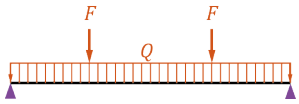
A01


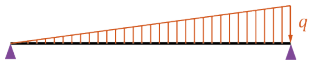
A02

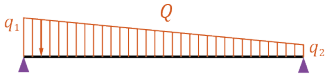

A03


A04


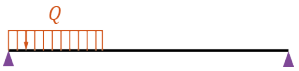
A05


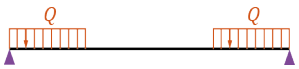
A06


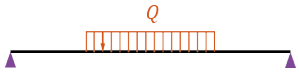
A07


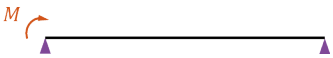
A08


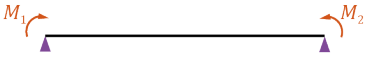
A09

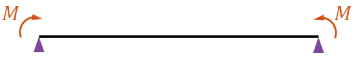

A10


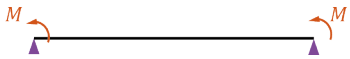
A11


A12


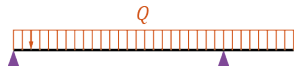
A13


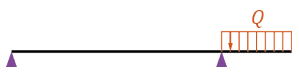
A14


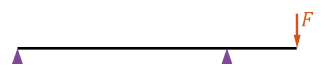
A15


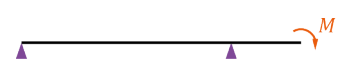
A16


A17


AC01


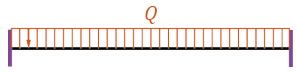
AC02


AC03


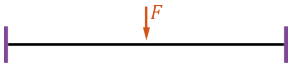
AC04


POUTRES BI-ENCASTRÉES

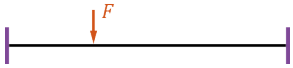
B01



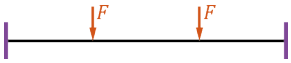
B02



B03

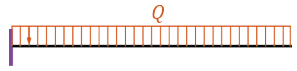


B04

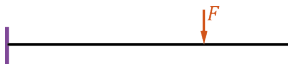


POUTRES CONSOLES

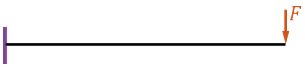
C01



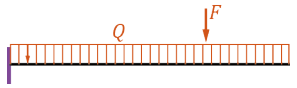
C02



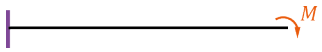
C03



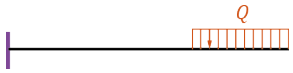
C04



C05

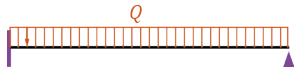


C06

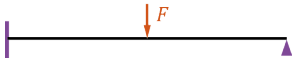


POUTRES ENCASTRÉES-APPUYÉES

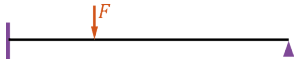
D01



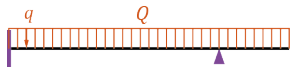
D02



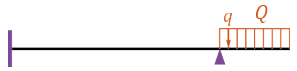
D03



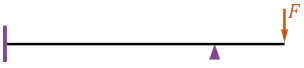
DC01



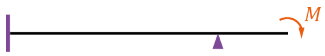
DC02



DC03



DC04

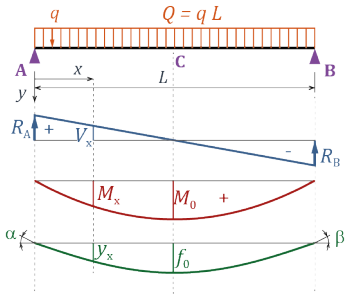


POUTRES SUR DEUX APPUIS SIMPLES

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

A01



$$Q = q\,L$$

$$R_A = R_B = \frac{Q}{2}$$

$$V(x) = \frac{Q}{2\,L}(L - 2\,x)$$

$$M(x) = \frac{Q}{2\,L}\,x\,(L - x)$$

$$M_0 = \frac{Q\,L}{8} \text{ pour } x_0 = \frac{L}{2}$$

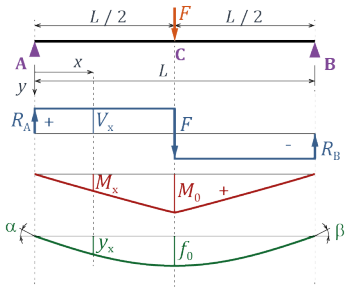
$$y(x) = \frac{Q}{24\,EI}\frac{x}{L}(L - x)(L^2 + L\,x - x^2)$$

$$f_0 = f_C = \frac{5}{384}\frac{Q\,L^3}{EI} \text{ pour } x_0 = \frac{L}{2}$$

$$\tan\alpha = -\tan\beta = \frac{Q\,L^2}{24\,EI}$$

$$W_d = \frac{Q^2\,L^3}{240\,EI} = \frac{8}{25}\,f_C\,Q$$

A02



$$R_A = R_B = \frac{F}{2}$$

$$V_{AC}(x) = \frac{F}{2} \qquad V_{CB}(x) = -\frac{F}{2}$$

$$M_{AC}(x) = \frac{F}{2}\,x \qquad M_{CB}(x) = \frac{F}{2}\,(L - x)$$

$$M_0 = \frac{F\,L}{4} \text{ pour } x_0 = \frac{L}{2}$$

$$y_{AC}(x) = \frac{F\,x}{48\,EI}(3\,L^2 - 4\,x^2)$$

$$y_{CB}(x) = \frac{F}{48\,EI}(L - x)(-L^2 - 4\,x^2 + 8\,L\,x)$$

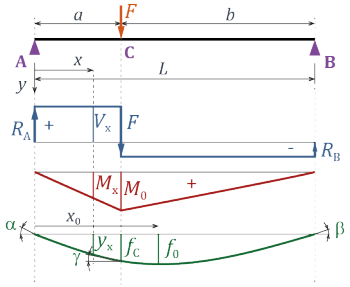
$$f_0 = f_C = \frac{F\,L^3}{48\,EI} \text{ pour } x_0 = \frac{L}{2}$$

$$\tan\alpha = -\tan\beta = \frac{F\,L^2}{16\,EI}$$

$$W_d = \frac{F^2\,L^3}{96\,EI} = \frac{1}{2}\,f_C\,F$$

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions aux appuis	f	Flèche
V	Effort tranchant	f_0	Flèche maximale
M	Moment fléchissant	α, β	Déformations angulaires
M_0	Moment fléchissant maximal	W_d	Energie de déformation élastique en flexion

A03



$$R_A = \frac{F\,b}{L} \qquad R_B = \frac{F\,a}{L}$$

$$V_{AC}(x) = \frac{F\,b}{L} \qquad V_{CB}(x) = -\frac{F\,a}{L}$$

$$W_d = \frac{F^2\,a^2 b^2}{6\,EI\,L} = \frac{1}{2}\,f_C\,F$$

$$y_{AC}(x) = \frac{F\,b\,x}{6\,EI\,L}\,(a\,(L+b) - x^2)$$

$$y_{CB}(x) = \frac{F\,a\,(L-x)}{6\,EI\,L}\,(b\,(L+a) - (L-x)^2)$$

$$f_C = \frac{F\,a^2 b^2}{3\,EI\,L}$$

$$M_{AC}(x) = \frac{F\,b\,x}{L} \qquad M_{CB}(x) = \frac{F\,a}{L}\,(L - x)$$

$$M_0 = M_C = \frac{F\,a\,b}{L} \quad \text{pour } x_0 = a$$

$$f_0 = \frac{F\,c}{27\,EI\,L}\sqrt{3\,(L^2 - c^2)^3}$$

avec $c = \min\{a; b\}$

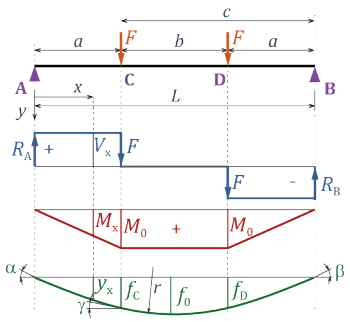
$$x_0 = \frac{L}{2}\left(1 \pm \left(1 - \sqrt{\frac{4}{3}\left(1 - \frac{c^2}{L^2}\right)}\right)\right)$$

$si\ a < b$: signe +

$si\ a \geq b$: signe −

$$\tan\alpha = \frac{F\,a\,b}{6\,EI\,L}(L+b) \qquad \tan\beta = -\frac{F\,a\,b}{6\,EI\,L}(L+a) \qquad \tan\gamma = \frac{F\,a\,b}{3\,EI\,L}(b-a)$$

A04



$$c = a + b = L - a$$

$$R_A = R_B = F$$

$$V_{AC}(x) = F \qquad M_{AC}(x) = F\,x$$

$$V_{CD}(x) = 0 \qquad M_{CD}(x) = F\,a$$

$$V_{DB}(x) = -F \qquad M_{DB}(x) = F\,(L - x)$$

$$M_0 = M_C = M_D = F\,a \quad \text{constant sur C-D}$$

$$W_d = \frac{F^2\,a^2\,(L + 2\,b)}{6\,EI}$$

$$\tan\alpha = \frac{F\,a\,c}{2\,EI} \qquad \tan\gamma = \frac{F\,a\,b}{2\,EI}$$

$$\tan\beta = -\tan\alpha$$

$$y_{AC}(x) = \frac{F\,x}{6\,EI}(3\,a\,(L-a) - x^2)$$

$$y_{CD}(x) = \frac{F\,a}{6\,EI}(3\,x\,(L-x) - a^2)$$

$$y_{DB}(x) = y_{AC}(L - x)$$

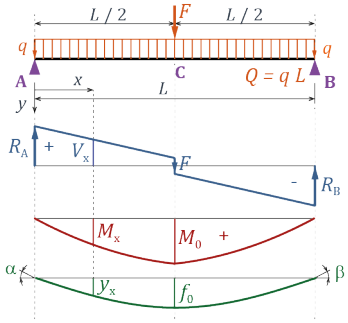
$$f_C = f_D = \frac{F\,a^2}{6\,EI}\,(3\,L - 4\,a)$$

$$f_0 = \frac{F\,a}{24\,EI}(3\,L^2 - 4\,a^2) \quad \text{pour } x_0 = L/2$$

courbure constante en CD: $\frac{1}{r} = \frac{F\,a}{EI}$

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal
$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

A05



$$Q = q\,L$$

$$R_A = R_B = \frac{Q+F}{2}$$

$$V_{AC}(x) = \frac{Q}{2\,L}(L-2\,x) + \frac{F}{2}$$

$$V_{CB}(x) = \frac{Q}{2\,L}(L-2\,x) - \frac{F}{2}$$

$$y_{AC}(\xi) = \frac{L^3}{8\,EI}\left[\frac{Q}{3}\xi^4 - \frac{2}{3}(Q+F)\,\xi^3 + \left(\frac{Q}{3}+\frac{F}{2}\right)\xi\right] \quad \text{où } \xi = \frac{x}{L}$$

$$y_{CD}(\xi) = \frac{L^3}{8\,EI}\left[\frac{Q}{3}(1-\xi)^4 - \frac{2}{3}(Q+F)\,(1-\xi)^3 + \left(\frac{Q}{3}+\frac{F}{2}\right)(1-\xi)\right]$$

$$f_0 = f_C = \frac{L^3}{48\,EI}\left[\frac{5}{8}Q + F\right] \quad \text{pour } x_0 = \frac{L}{2}$$

$$\tan\alpha = -\tan\beta = \frac{L^2}{8\,EI}\left[\frac{Q}{3}+\frac{F}{2}\right]$$

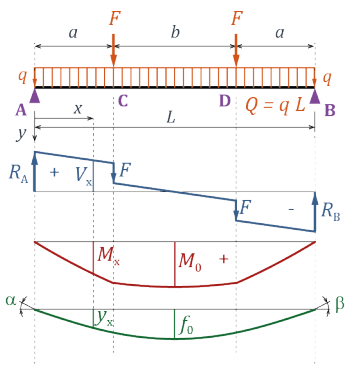
$$M_{AC}(x) = \frac{Q\,x}{2\,L}(L-x) + \frac{F}{2}\,x$$

$$M_{CB}(x) = \left[\frac{Q\,x}{2\,L} + \frac{F}{2}\right](L-x)$$

$$M_0 = \left[\frac{Q}{2}+F\right]\frac{L}{4} \text{ pour } x_0 = \frac{L}{2}$$

$$W_d = \frac{L^3}{48\,EI}\left[\frac{Q^2}{5} + \frac{F^2}{2} + \frac{5\,F\,Q}{8}\right]$$

A06



$$Q = q\,L$$

$$R_A = R_B = F + \frac{Q}{2}$$

$$V_{AC}(x) = F + \frac{Q}{2}\left(1 - 2\,\frac{x}{L}\right)$$

$$V_{CD}(x) = \frac{Q}{2}\left(1 - 2\,\frac{x}{L}\right)$$

$$V_{DB}(x) = -F + \frac{Q}{2}\left(1 - 2\,\frac{x}{L}\right)$$

$$y(x) = y_{A01}(x) + y_{A04}(x)$$

$$f_0 = \frac{F\,a}{24\,EI}(3\,L^2 - 4\,a^2) + \frac{5\,Q\,L^3}{384\,EI} \quad \text{pour } x_0 = \frac{L}{2}$$

$$f_C = f_D = \frac{F\,a^2}{6\,EI}(3\,L - 4\,a) + \frac{Q\,a}{24\,EI\,L}(L^3 - 2\,a^2 L + a^3)$$

$$M_{AC}(x) = F\,x + \frac{Q\,x}{2\,L}(L-x)$$

$$M_{CD}(x) = F\,a + \frac{Q\,x}{2\,L}(L-x)$$

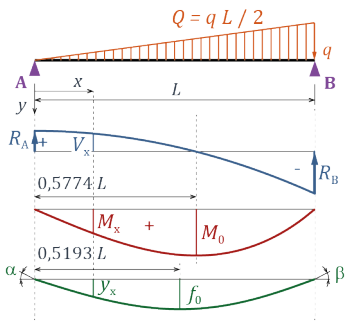
$$M_{DB}(x) = F\,(L-x) + \frac{Q\,x}{2\,L}(L-x)$$

$$M_0 = F\,a + \frac{Q\,L}{8} \quad \text{pour } x_0 = \frac{L}{2}$$

$$\tan\alpha = -\tan\beta = \frac{F\,a\,(L-a)}{2\,EI} + \frac{Q\,L^2}{24\,EI}$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal
$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

A07



$$Q = q\,L/2$$

$$R_A = \frac{Q}{3} \qquad R_B = \frac{2\,Q}{3}$$

$$V(x) = \frac{Q}{3\,L^2}(L^2 - 3\,x^2)$$

$$M(x) = \frac{Q\,x}{3\,L^2}\,(L^2 - x^2)$$

$$M_0 = \frac{2\,Q\,L}{9\,\sqrt{3}} \text{ pour } x_0 = \frac{L}{\sqrt{3}} = 0{,}5774\,L$$

$$W_d = \frac{4}{945}\,\frac{Q^2\,L^3}{EI}$$

$$\xi = \frac{x}{L}$$

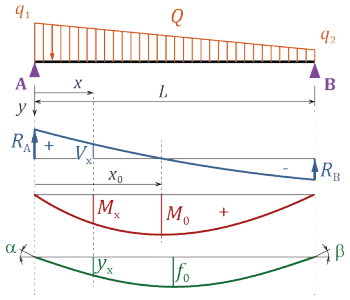
$$y(\xi) = \frac{Q\,L^3}{180\,EI}\,\xi\,(3\,\xi^4 - 10\,\xi^2 + 7)$$

$$f_0 = k_f\,\frac{Q\,L^3}{EI} \text{ pour } \frac{x_0}{L} = \sqrt{1 - \frac{4}{\sqrt{30}}} = 0{,}5193$$

$$k_f = \frac{10 + \sqrt{30}}{3375}\sqrt{30 - 4\,\sqrt{30}} = 0{,}01304$$

$$\tan\alpha = \frac{7}{180}\,\frac{Q\,L^2}{EI} \qquad \tan\beta = -\frac{2}{45}\,\frac{Q\,L^2}{EI}$$

A08



Pour q_1 et q_2 de même signe (*)

$$\xi = \frac{x}{L} \qquad \varepsilon = \sqrt{1 + \frac{\psi^2}{12}}$$

$$Q = \frac{q_1 + q_2}{2}\,L \qquad \psi = 2\,\frac{q_2 - q_1}{q_2 + q_1}$$

$$R_A = \frac{Q}{2}\left(1 - \frac{\psi}{6}\right) \qquad R_B = \frac{Q}{2}\left(1 + \frac{\psi}{6}\right)$$

$$V(\xi) = \frac{Q}{2}\,(1 - 2\,\xi) - \frac{\psi\,Q}{2}\left(\frac{1}{6} - \xi + \xi^2\right)$$

$$M(\xi) = \frac{Q\,L}{2}(\xi - \xi^2) - \frac{\psi\,Q\,L}{12}(\xi - 3\,\xi^2 + 2\,\xi^3)$$

$$y(\xi) = \frac{Q\,L^3\,\xi}{24\,EI}\left[\xi^3 - 2\,\xi^2 + 1 + \frac{\psi}{30}(6\,\xi^4 - 15\,\xi^3 + 10\,\xi^2 - 1)\right]$$

$$M_0 = \frac{Q\,x_0}{72\,\psi}\,(18\,\psi - \psi^2 + 6\,(\varepsilon - 1)(\psi - 2))$$

$$x_0 = \frac{L}{2}\left[1 + \frac{2}{\psi}(\varepsilon - 1)\right]$$

$$W_d = \frac{Q^2\,L^3}{240\,EI}\left(1 + \frac{\psi^2}{252}\right)$$

$$\tan\alpha = \frac{Q\,L^2}{24\,EI}\left(1 - \frac{\psi}{30}\right)$$

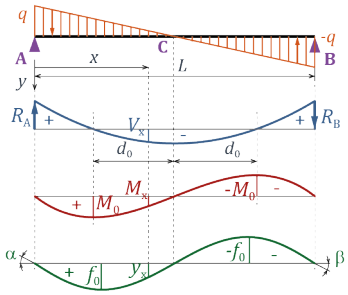
$$\tan\beta = -\frac{Q\,L^2}{24\,EI}\left(1 + \frac{\psi}{30}\right)$$

$$f_0 \approx \frac{5}{384}\,\frac{Q\,L^3}{EI}\left(1 + \frac{\psi}{30}\right)$$

(*) : quand q_1 et q_2 sont de signe opposé, le moment peut changer de signe en fonction des valeurs de q_1 et q_2. Les formules présentées ici ne donnent que la position et la valeur du moment maxi positif.

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions aux appuis	f	Flèche
V	Effort tranchant	f_0	Flèche maximale
M	Moment fléchissant	α, β	Déformations angulaires
M_0	Moment fléchissant maximal	W_d	Energie de déformation élastique en flexion

A09



$$R_A = \frac{q\,L}{6} \qquad R_B = -\frac{q\,L}{6}$$

$$V(\xi) = q\,L\left(\xi^2 - \xi + \frac{1}{6}\right)$$

$$M(\xi) = \frac{q\,L^2}{6}\,(2\,\xi^3 - 3\,\xi^2 + \xi)$$

$$M_0 = \frac{\sqrt{3}}{108} q L^2 \quad \text{en } x_0 = \frac{L}{2} - d_0$$

$$\text{avec } d_0 = \frac{L}{2\sqrt{3}}$$

$$y(\xi) = -\frac{q\,L^4}{360\,EI}\,\xi\,(6\,\xi^4 - 15\,\xi^3 + 10\,\xi^2 - 1)$$

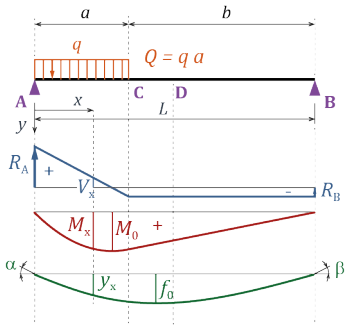
$$y_C = 0$$

$$f_0 \approx \frac{q\,L^4}{2457\,EI} \quad \text{pour } x_0 \approx \frac{L}{2} \pm 0{,}260\,L$$

$$\tan\alpha = \tan\beta = \frac{q\,L^3}{360\,EI}$$

$$W_d = \frac{q^2\,L^5}{15120\,EI}$$

A10



$$Q = q\,a \qquad d = a\left(2 - \frac{a}{L}\right)$$

$$R_A = \frac{Q\,d}{2\,a} \qquad R_B = \frac{Q\,a}{2\,L}$$

$$V_{AC}(x) = \frac{Q}{2\,a}\,(d - 2\,x)$$

$$V_{CB}(x) = -\frac{Q\,a}{2\,L}$$

$$M_{AC}(x) = \frac{Q\,x}{2\,a}\,(d - x)$$

$$M_{CB}(x) = \frac{Q\,a}{2\,L}\,(L - x)$$

$$M_C = \frac{Q\,a\,b}{2\,L}$$

$$M_0 = \frac{Q\,d^2}{8\,a} \quad \text{en } x_0 = \frac{d}{2}$$

$$y_{AC}(x) = \frac{Q\,x}{24\,EI\,a}\,(x^3 - 2\,d\,x^2 + d^2 L)$$

$$y_{CB}(x) = \frac{Q\,a}{24\,EI\,L}\,(L - x)(4\,L\,x - a^2 - 2\,x^2)$$

$$y_C = \frac{Q\,a^2 b}{24\,EI\,L}\,(4\,L - 3\,a)$$

$$y_D = \frac{Q\,L^2}{384\,EI\,a}\,(L^2 - 4\,d\,L + 8\,d^2) \quad \text{pour } a \geq \frac{L}{2}$$

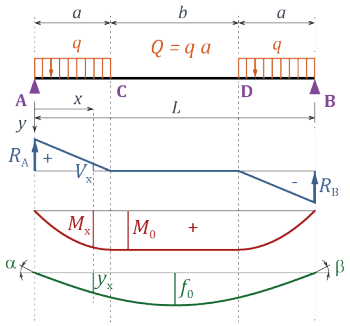
$$\tan\alpha = \frac{Q\,d^2\,L}{24\,EI\,a}$$

$$\tan\beta = -\frac{Q\,a}{24\,EI\,L}\,(2L^2 - a^2)$$

$$W_d = \frac{Q^2\,a^2}{240\,EI\,L}\,(10\,L^2 - 14\,L\,a + 5\,a^2)$$

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions aux appuis	f	Flèche
V	Effort tranchant	f_0	Flèche maximale
M	Moment fléchissant	α, β	Déformations angulaires
M_0	Moment fléchissant maximal	W_d	Energie de déformation élastique en flexion

A11



$Q = q\,a$

$R_A = Q$ $\qquad R_B = Q$

$V_{AC}(x) = Q\left(1 - \dfrac{x}{a}\right)$ $\qquad V_{CD}(x) = 0$

$V_{DB}(x) = -Q\left(1 - \dfrac{L-x}{a}\right)$

$M_{AC}(x) = \dfrac{Q\,x}{2\,a}(2\,a - x)$ $\qquad M_{CD}(x) = \dfrac{Q\,a}{2}$

$M_{CB}(x) = \dfrac{Q\,(L-x)}{2\,a}(2\,a - L + x)$

$M_C = M_D = M_0 = \dfrac{Q\,a}{2}$

$y_{AC}(x) = \dfrac{Q\,x}{24\,EI\,a}(x^3 - 4\,a\,x^2 + 2\,d\,a^2)$

$d = 3L - 2a$

$y_{CD}(x) = \dfrac{Q\,a}{24\,EI}(6\,L\,x - a^2 - 6\,x^2)$

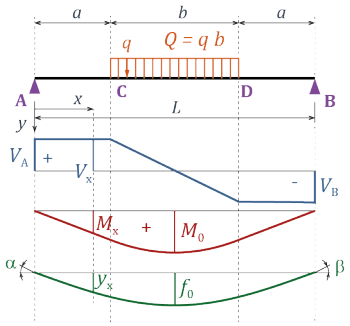
$y_C = y_D = \dfrac{Q\,a^2}{24\,EI}(6\,L - 7\,a)$

$f_0 = \dfrac{Q\,a}{48\,EI}(3\,L^2 - 2\,a^2)$ pour $x_0 = \dfrac{L}{2}$

$\tan\alpha = -\tan\beta = \dfrac{Q\,a}{12\,EI}(3L - 2a)$

$W_d = \dfrac{Q^2\,a^2}{120\,EI}(16\,a + 15\,b)$

A12



$Q = q\,b$

$R_A = R_B = \dfrac{Q}{2}$

$V_{AC}(x) = \dfrac{Q}{2}$ $\qquad V_{DB}(x) = -\dfrac{Q}{2}$

$V_{CD}(x) = \dfrac{Q}{2\,b}(L - 2\,x)$

$M_{AC}(x) = \dfrac{Q\,x}{2}$ $\qquad M_{DB}(x) = \dfrac{Q\,(L-x)}{2}$

$M_{CD}(x) = \dfrac{Q}{2\,b}(L\,x - x^2 - a^2)$

$M_C = M_D = \dfrac{Q\,a}{2}$

$M_0 = \dfrac{Q}{8}(L + 2\,a)$ $\quad$ en $x_0 = \dfrac{L}{2}$

$y_{DB}(x) = y_{AC}(L - x)$

$y_{AC}(x) = \dfrac{Q\,x}{48\,EI}(3\,L^2 - b^2 - 4\,x^2)$

$y_{CB}(x) = \dfrac{Q}{24\,EI\,b}(x^4 - 2\,L\,x^3 + 6\,a^2\,x^2 + L\,(L^2 - 6\,a^2)\,x + a^4)$

$y_C = y_D = \dfrac{Q\,a}{24\,EI}(L^2 + 2\,a\,b)$

$\tan\alpha = \dfrac{Q}{48\,EI}(3\,L^2 - b^2) = -\tan\beta$

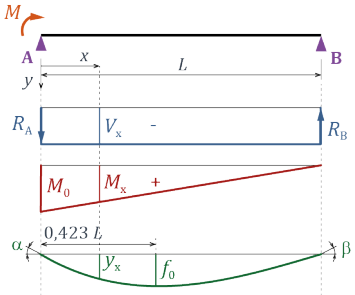
$f_0 = \dfrac{Q}{384\,EI\,b}(5\,L^4 + 16\,a^4 - 24\,a^2\,L^2)$

$W_d = \dfrac{Q^2}{240\,EI}(b^3 + 10\,ab^2 + 30\,a^2 b + 20\,a^3)$

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

A13



$$R_A = -R_B = -\frac{M}{L}$$

$$V(x) = -\frac{M}{L}$$

$$M(x) = M\left(1 - \frac{x}{L}\right)$$

$$M_0 = M \quad \text{pour } x_0 = 0$$

$$W_d = \frac{M^2 L}{6\,EI}$$

$$\xi = x/L$$

$$y(\xi) = \frac{M L^2}{6\,EI}\,\xi(\xi - 1)(\xi - 2)$$

$$f_0 = \frac{\sqrt{3}}{27}\frac{M L^2}{EI} \quad \text{pour } x_0 = k_0 L$$

$$\text{avec } k_0 = 1 - \frac{\sqrt{3}}{3} = 0{,}423$$

$$f_C = \frac{M L^2}{16\,EI} \quad \text{pour } x_C = 0{,}5\,L$$

$$\tan\alpha = \frac{M L}{3\,EI} \qquad \tan\beta = -\frac{M L}{6\,EI}$$

A14

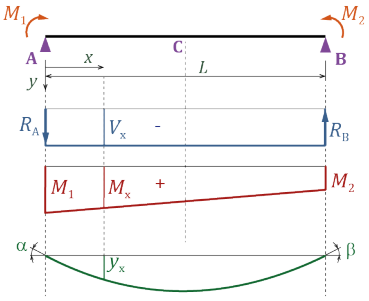


schéma pour $0 \le M_2 \le M_1$

$$M_m = \frac{M_1 + M_2}{2} \qquad \Delta M = M_2 - M_1$$

$$R_A = -R_B = \frac{\Delta M}{L}$$

$$V(x) = \frac{\Delta M}{L}$$

$$M(x) = M_2\frac{x}{L} + M_1\left(1 - \frac{x}{L}\right)$$

$$M_0 = \max\{M_1; M_2\}$$

$$W_d = \frac{L}{24\,EI}\left(12\,M_m^2 + \Delta M^2\right)$$

$$\xi = \frac{x}{L} \qquad \varepsilon = \frac{\Delta M}{M_m}$$

$$y(\xi) = \frac{M_m L^2}{12\,EI}\,\xi(1 - \xi)(6 - \varepsilon + 2\,\varepsilon\,\xi)$$

$$f_C = \frac{M_m L^2}{8\,EI} \quad \text{pour } x_C = \frac{L}{2}$$

$$f_0 \approx f_C \quad \text{pour } |\Delta M| \le 2\,|M_m|$$

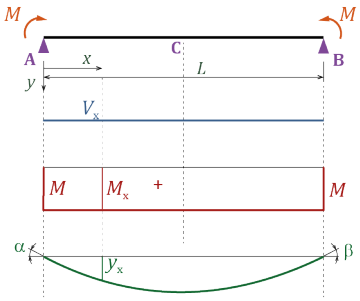
$$\tan\alpha = \frac{L}{6\,EI}\,(2\,M_1 + M_2)$$

$$\tan\beta = -\frac{L}{6\,EI}\,(M_1 + 2\,M_2)$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

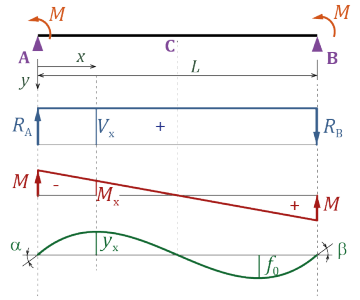
A15



$$R_A = -R_B = 0$$
$$V(x) = 0$$
$$M(x) = M$$
$$W_d = \frac{M^2\,L}{2\,EI}$$

$$y(x) = \frac{M}{2\,EI} x\,(L - x)$$
$$f_0 = f_C = \frac{M\,L^2}{8\,EI} \quad \text{pour } x_0 = x_C = \frac{L}{2}$$
$$\tan\alpha = -\tan\beta = \frac{M\,L}{2\,EI}$$

A16



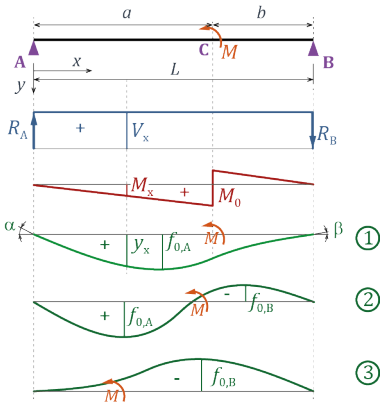
$$R_A = -R_B = 2\,\frac{M}{L}$$
$$V(x) = 2\,\frac{M}{L}$$
$$M(x) = M\left(2\,\frac{x}{L} - 1\right)$$
$$W_d = \frac{M^2 L}{6\,EI}$$

$$y(x) = \frac{M\,x}{6\,EI\,L}(-2\,x^2 + 3x\,L - L^2)$$
$$f_0 = \pm\frac{M\,L^2}{36\,\sqrt{3}\,EI} \quad \text{pour } x_0 = \frac{L}{2} \pm \frac{L}{2\sqrt{3}}$$
$$\tan\alpha = \tan\beta = -\frac{M\,L}{6\,EI}$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

A17



L'allure de la déformée dépend de la position de M :

① : $a > \frac{L}{\sqrt{3}}$

② : $L - \frac{L}{\sqrt{3}} < a < \frac{L}{\sqrt{3}}$

③ : $L - \frac{L}{\sqrt{3}} > a$

$$R_A = -R_B = \frac{M}{L}$$

$$V(x) = \frac{M}{L}$$

$$M_{AC}(x) = M\frac{x}{L} \qquad M_{CB}(x) = M\left(\frac{x}{L} - 1\right)$$

$$M_0 = M\frac{a}{L} \text{ pour } a \geq \frac{L}{2}$$

$$M_0 = -M\frac{b}{L} \text{ pour } a \leq \frac{L}{2}$$

$$W_d = \frac{M^2}{6\,EI\,L^2}\,(a^3 + b^3)$$

$$\tan\alpha = \frac{M}{6\,EI\,L}(L^2 - 3\,b^2)$$

$$\tan\beta = \frac{M}{6\,EI\,L}(L^2 - 3\,a^2)$$

$$\tan\gamma = \frac{M}{6\,EI\,L}(L - 3\,a^2 - 3\,b^2)$$

$$y_{AC}(x) = \frac{M\,x}{6\,EI\,L}(L^2 - 3\,b^2 - x^2)$$

$$y_{CB}(x) = \frac{M\,(L-x)}{6\,EI\,L}(x^2 + 3\,a^2 - 2\,L\,x)$$

si $b < \frac{L}{\sqrt{3}}$, flèche positive maxi pour $x_0 = d_b$:

$$f_{0,A} = \frac{M\,d_b^3}{3\,EI\,L}$$

si $a < \frac{L}{\sqrt{3}}$, flèche négative maxi pour $x_0 = d_a$:

$$f_{0,B} = -\frac{M\,d_a^3}{3\,EI\,L}$$

$$d_a = L\left(\frac{1}{3} - \frac{a^2}{L^2}\right)^{0,5} \qquad d_b = L\left(\frac{1}{3} - \frac{b^2}{L^2}\right)^{0,5}$$

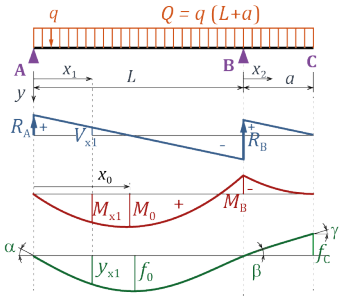
$$f_C = \frac{M}{3\,EI}\,\frac{a\,b}{L}\,(a - b)$$

POUTRES SUR 2 APPUIS ET PARTIE EN CONSOLE

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

AC01



$$R_A = \frac{Q\,(L-a)}{2\,L} \qquad R_B = \frac{Q\,(L+a)}{2\,L}$$

$$V_{AB}(x_1) = \frac{Q}{2}\left(\frac{L-a}{L} - \frac{2\,x_1}{L+a}\right)$$

$$V_{BC}(x) = Q\,\frac{a-x_2}{L+a}$$

$$\tan\alpha = \frac{Q\,L\,(L^2-2\,a^2)}{24\,EI\,(L+a)}$$

$$\tan\beta = \frac{Q\,L\,(4\,a^2-L^2)}{24\,EI\,(L+a)}$$

$$\tan\gamma = \frac{Q}{24\,EI\,(L+a)}\,[4\,a^2(L+a) - L^3]$$

$$Q = q\,(L+a) \qquad x_2 = x_1 - L$$

$$M_{AB}(x_1) = \frac{Q\,x_1}{2}\left(\frac{L-a}{L} - \frac{x_1}{L+a}\right)$$

$$M_{BC}(x_2) = -\frac{Q}{2\,(L+a)}\,(a-x_2)^2$$

$$M_B = -\frac{Q\,a^2}{2\,(L+a)}$$

$$M_0 = \frac{Q}{8\,L^2}\,(L+a)(L-a)^2$$

$$\text{en } x_0 = \frac{L^2-a^2}{2\,L} \qquad (\text{quand } L > a)$$

$$y_{AB}(x_1) = \frac{Q\,x_1}{24\,EI\,L\,(L+a)}\,[L\,x_1^3 - 2\,(L^2-a^2)\,x_1^2 + L^2\,(L^2-2\,a^2)]$$

$$y_{BC}(x_2) = \frac{Q\,x_2}{24\,EI\,(L+a)}\,[x_2^3 - 4\,a\,x_2^2 + 6\,a^2\,x_2 + L\,(4\,a^2-L^2)]$$

$$f_C = \frac{Q\,a}{24\,EI\,(L+a)}\,[3\,a^3 - L\,(L^2-4\,a^2)]$$

$$f_0 \approx \frac{k_0\,Q\,L^4}{24\,EI\,(L+a)} \quad \text{sur } [AB]$$

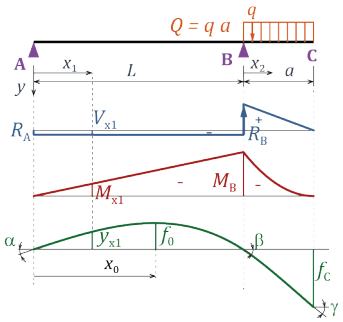
pour $a \le 0{,}6\,L$: $k_0 = -0{,}6632\,(a/L)^2 - 0{,}0312\,(a/L) + 0{,}3138$

pour $a > 0{,}6\,L$: $k_0 = -0{,}8787\,(a/L)^2 + 0{,}2704\,(a/L) + 0{,}1364$

$$W_d = \frac{Q^2}{240\,EI\,(L+a)^2}\,(L^5 + 10\,L\,a^4 - 5\,L^3a^2 + 6\,a^5)$$

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions (efforts verticaux) aux appuis	f	Flèche
M_{RA}, M_{RB}	Réactions (moments) aux appuis		
V	Effort tranchant	f_0	Flèche maximale
M	Moment fléchissant	α, β	Déformations angulaires
M_0	Moment fléchissant maximal	W_d	Energie de déformation élastique en flexion

AC02



$$R_A = -\frac{Q\,a}{2\,L} \qquad R_B = \frac{Q\,(2\,L + a)}{2\,L}$$

$$V_{AB}(x_1) = -\frac{Q\,a}{2\,L}$$

$$V_{BC}(x_2) = Q\left(1 - \frac{x_2}{a}\right)$$

$$W_d = \frac{Q^2\,a^2}{120\,EI}\,(5\,L + 3\,a)$$

$$y_{AB}(x_1) = \frac{Q\,a\,x_1}{12\,EI\,L}\,(x_1^2 - L^2)$$

$$y_{BC}(x_2) = \frac{Q\,x_2}{24\,EI\,a}\,[x_2^3 - 4\,a\,x_2^2 + 6\,a^2\,x_2 + 4\,a^2 L]$$

$$f_C = \frac{Q\,a^2}{24\,EI}\,(4\,L + 3\,a)$$

$$f_0 = -\frac{Q\,a\,L^2}{18\sqrt{3}\,EI} \quad \text{en } x_0 = \frac{L}{\sqrt{3}}$$

$$Q = q\,a \qquad x_2 = x_1 - L$$

$$M_{AB}(x_1) = -\frac{Q\,a\,x_1}{2\,L}$$

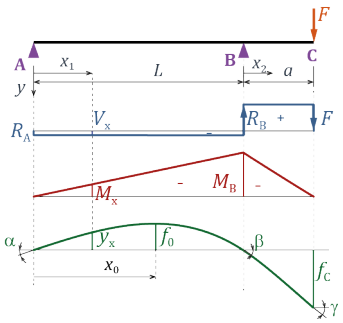
$$M_{BC}(x_2) = -\frac{Q}{2\,a}\,(a - x_2)^2$$

$$M_B = -\frac{Q\,a}{2}$$

$$\tan\alpha = -\frac{Q\,a\,L}{12\,EI} \qquad \tan\beta = \frac{Q\,a\,L}{6\,EI}$$

$$\tan\gamma = \frac{Q\,a\,(L + a)}{6\,EI}$$

AC03



$$x_2 = x_1 - L$$

$$R_A = -F\,\frac{a}{L} \qquad R_B = F\,\frac{L + a}{L}$$

$$V_{AB}(x_1) = -F\,\frac{a}{L} \qquad V_{BC}(x_2) = F$$

$$M_{AB}(x_1) = -F\,\frac{a\,x_1}{L} \qquad M_{BC}(x_2) = F\,(x_2 - a)$$

$$M_B = -F\,a$$

$$W_d = \frac{F^2\,a^2}{6\,EI}\,(L + a)$$

$$y_{AB}(x_1) = \frac{F\,a\,x_1}{6\,EI\,L}\,(x_1^2 - L^2)$$

$$y_{BC}(x_2) = \frac{F\,x_2}{6\,EI}\,(2\,a\,L + 3\,a\,x_2 - x_2^2)$$

$$f_0 = -\frac{F\,a\,L^2}{9\sqrt{3}\,EI} \text{ pour } x_0 = \frac{L}{\sqrt{3}} = 0{,}577\,L$$

$$f_C = \frac{F a^2}{3\,EI}\,(L + a)$$

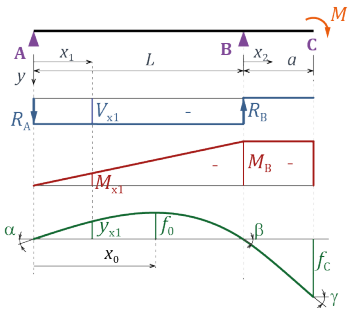
$$\tan\alpha = -\frac{F\,a\,L}{6\,EI} \qquad \tan\beta = \frac{F\,a\,L}{3\,EI}$$

$$\tan\gamma = \frac{F\,a}{6\,EI}\,(2\,L + 3\,a)$$

$\xi = x/l$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

AC04



$$x_2 = x_1 - L$$

$$R_A = -\frac{M}{L} \qquad R_B = \frac{M}{L}$$

$$V_{AB}(x_1) = -\frac{M}{L} \qquad V_{BC}(x_2) = 0$$

$$M_{AB}(x_1) = -M\frac{x_1}{L} \qquad M_{BC}(x_2) = -M$$

$$M_B = -M$$

$$W_d = \frac{M^2}{6\,EI}(L + 3\,a)$$

$$y_{AB}(x_1) = \frac{M\,x_1}{6\,EI\,L}(x_1^2 - L^2)$$

$$y_{BC}(x_2) = \frac{M\,x_2}{6\,EI}(3\,x_2 + 2\,L)$$

$$f_0 = -\frac{M\,L^2}{9\sqrt{3}\,EI} \text{ pour } x_0 = \frac{L}{\sqrt{3}} = 0{,}577\,L$$

$$f_C = \frac{M\,a}{6\,EI}(2\,L + 3\,a)$$

$$\tan\alpha = -\frac{M\,L}{6\,EI} \qquad \tan\beta = \frac{M\,L}{3\,EI}$$

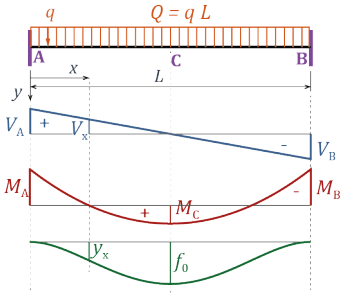
$$\tan\gamma = \frac{M}{3\,EI}(L + 3\,a)$$

POUTRES BI-ENCASTRÉES

$\xi = x/l$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

B01



$$Q = q\,L$$

$$R_A = R_B = \frac{Q}{2}$$

$$M_{RA} = -M_{RB} = \frac{Q\,L}{12}$$

$$V(x) = \frac{Q}{2\,L}(L - 2\,x)$$

$$y(x) = \frac{Q}{24\,EI}\frac{x^2}{L}\,(x^2 - 2\,L\,x + L^2)$$

$$f_0 = f_C = \frac{Q\,L^3}{384\,EI} \quad \text{pour} \quad x_0 = \frac{L}{2}$$

$$M(x) = \frac{Q}{12\,L}(6\,x\,(L - x) - L^2)$$

$$M_A = M_B = -\frac{Q\,L}{12}$$

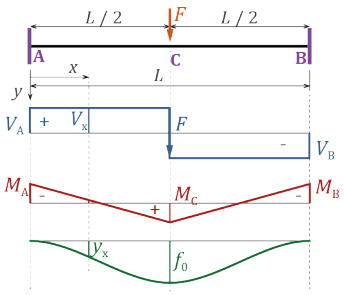
$$M_C = +\frac{Q\,L}{24} \text{ pour } x_0 = \frac{L}{2}$$

$$M_A = -2\,M_C$$

$$M = 0 \text{ pour } x = \frac{L}{2}\left(1 \pm \frac{1}{\sqrt{3}}\right)$$

$$W_d = \frac{Q^2\,L^3}{1440\,EI} = \frac{4}{15}\,f_C\,Q$$

B02



$$R_A = R_B = \frac{F}{2} \qquad M_{RA} = -M_{RB} = \frac{F\,L}{8}$$

$$V_{AC}(x) = \frac{F}{2} \qquad V_{CB}(x) = -\frac{F}{2}$$

$$y_{AC}(x) = \frac{F}{48\,EI}x^2\,(3\,L - 4\,x)$$

$$y_{CD}(x) = \frac{F}{48\,EI}(L - x)^2\,(4\,x - L)$$

$$f_0 = f_C = \frac{F\,L^3}{192\,EI} \quad \text{pour} \quad x_0 = \frac{L}{2}$$

$$M_{AC}(x) = \frac{F}{8}(4\,x - L)$$

$$M_{CB}(x) = \frac{F}{8}\,(3\,L - 4\,x)$$

$$M_A = M_B = -\frac{F\,L}{8}$$

$$M_C = \frac{F\,L}{8} \text{ pour } x_0 = \frac{L}{2}$$

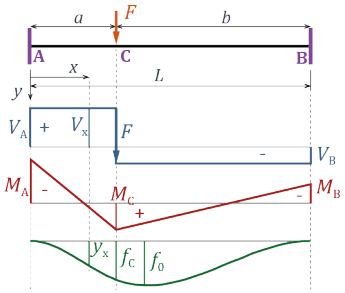
$$M_A = -M_C$$

$$M = 0 \text{ pour } x = \frac{L}{2}\left(1 \pm \frac{1}{2}\right)$$

$$W_d = \frac{F^2\,L^3}{384\,EI} = \frac{1}{2}\,f_C\,F$$

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions (efforts verticaux) aux appuis	f	Flèche
M_{RA}, M_{RB}	Réactions (moments) aux appuis		
V	Effort tranchant	f_0	Flèche maximale
M	Moment fléchissant	α, β	Déformations angulaires
M_0	Moment fléchissant maximal	W_d	Energie de déformation élastique en flexion

B03



$$d_A = 3\,a + b \qquad d_B = 3\,b + a$$

$$R_A = \frac{F\,b^2}{L^3}\,d_A \qquad R_B = \frac{F\,a^2}{L^3}\,d_B$$

$$M_{RA} = \frac{F\,a\,b^2}{L^2} \qquad M_{RB} = -\frac{F\,a^2\,b}{L^2}$$

$$V_{AC}(x) = R_A \qquad V_{CB}(x) = -R_B$$

$$W_d = \frac{F^2 a^3 b^3}{6\,EI\,L^3}$$

$$M_{AC}(x) = -\frac{F\,a\,b^2}{L^2} + \frac{F\,b^2}{L^3}\,d_A\,x$$

$$M_{CB}(x) = \frac{F\,a^2}{L^2}(a+2b) - \frac{F\,a^2}{L^3}\,d_B\,x$$

$$M_A = -F\,\frac{a\,b^2}{L^2} \qquad M_B = -F\,\frac{a^2\,b}{L^2}$$

$$M_C = +2\,F\,\frac{a^2\,b^2}{L^3}$$

$$y_{AC}(x) = \frac{F}{6\,EI}\frac{b^2}{L^3}\,(3\,a\,L\,x^2 - d_A\,x^3)$$

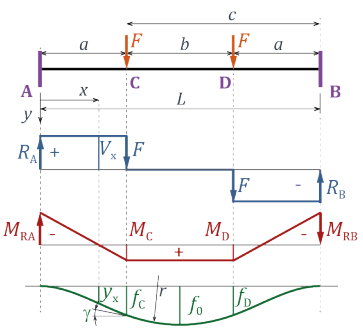
$$y_{CB}(x) = \frac{F}{6\,EI}\frac{a^2}{L^3}\,(3\,b\,L\,(L-x)^2 - d_B\,(L-x)^3)$$

$$f_C = \frac{F\,a^3\,b^3}{3\,EI\,L^3}$$

$$f_0 = \frac{2}{3}\frac{F\,a^2\,b^2}{EI}\frac{c}{(L+2\,c)^2} \quad \text{pour } x_0 = \frac{2\,L\,c}{L+2\,c}$$

$$c = \max\{a; b\}$$

B04



$$c = a + b = L - a \qquad d = \frac{a\,c}{L}$$

$$R_A = R_B = F$$

$$M_{RA} = -Fd \qquad M_{RB} = Fd$$

$$V_{AC}(x) = F \qquad V_{CB}(x) = 0 \qquad V_{DB}(x) = -F$$

$$M_{AC}(x) = F\,(x-d)$$

$$M_{CD}(x) = \frac{F\,a^2}{L}$$

$$M_{DB}(x) = F\,(L-x-d)$$

$$M_A = M_B = -F\,d \qquad M_C = +F\,\frac{a^2}{L}$$

$$\frac{M_A}{M_C} = -\frac{c}{a}$$

$$y_{AC}(x) = \frac{F}{6\,EI}x\,(3\,d\,x - x^2)$$

$$y_{CD}(x) = \frac{F}{6\,EI}\frac{a^2}{L}(3\,(L-x)\,x - a\,L)$$

$$f_C = f_D = \frac{F\,a^3}{6\,EI\,L}(a+2\,b)$$

$$f_0 = \frac{F\,a^2}{24\,EI}(2\,a+3\,b) \qquad \text{pour } x_0 = \frac{L}{2}$$

$$\tan\gamma = \frac{F\,a^2\,b}{2\,EI\,L}$$

$$W_{db} = \frac{F^2 a^4 b}{2\,EI\,L^2}$$

$$W_{da} = \frac{F^2 a^3}{6\,EI\,L^2}\,(a^2 + b^2 + a\,b)$$

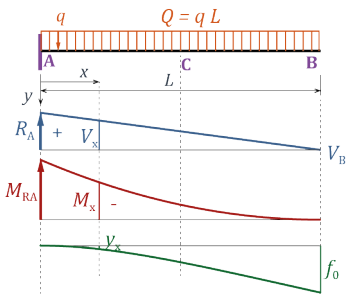
$$W_d = 2\,W_{da} + W_{db}$$

POUTRES CONSOLES

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

C01



$$Q = q\,L$$

$$R_A = Q \qquad M_{RA} = \frac{Q\,L}{2}$$

$$V(x) = Q\left(1 - \frac{x}{L}\right)$$

$$W_d = \frac{Q^2\,L^3}{40\,EI} = \frac{1}{5}\,f_B\,Q$$

$$M(x) = -\frac{Q}{2\,L}(L - x)^2$$

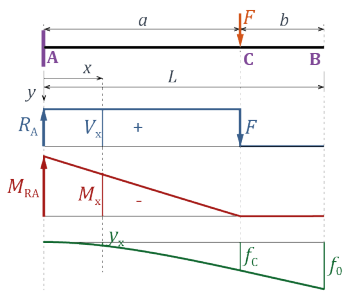
$$M_A = -\frac{Q\,L}{2}$$

$$y(x) = \frac{Q}{24\,EI}\frac{x^2}{L}\,(x^2 - 4\,L\,x + 6\,L^2)$$

$$f_0 = f_B = \frac{Q\,L^3}{8\,EI} \text{ pour } x_0 = L$$

$$\tan\beta = \frac{Q\,L^2}{6\,EI}$$

C02



$$R_A = F \qquad M_{RA} = F\,a$$

$$V_{AC}(x) = F \qquad V_{CB}(x) = 0$$

$$M_{AC}(x) = -F\,(a - x) \qquad M_{CB}(x) = 0$$

$$M_A = -F\,a$$

$$W_d = \frac{F^2\,a^3}{6\,EI} = \frac{1}{2}\,f_C\,F$$

$$y_{AC}(x) = \frac{F}{6\,EI}x^2\,(3\,a - x)$$

$$y_{CB}(x) = \frac{F}{6\,EI}a^2\,(3\,x - a)$$

$$f_0 = f_B = \frac{F\,a^2}{6\,EI}(3\,L - a) \text{ pour } x_0 = L$$

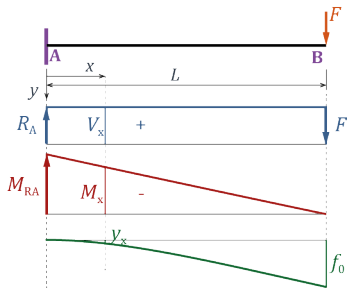
$$f_C = \frac{F\,a^3}{3\,EI}$$

$$\tan\beta = \frac{F\,a^2}{2\,EI}$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

C03



$$R_A = F \qquad M_{RA} = F\,L$$

$$V(x) = F$$

$$M(x) = -F\,(L - x)$$

$$M_A = -F\,L$$

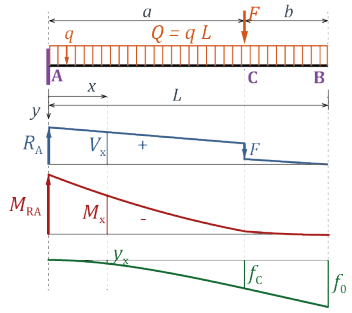
$$y(x) = \frac{F}{6\,EI} x^2\,(3\,L - x)$$

$$f_0 = \frac{F\,L^3}{3\,EI}$$

$$\tan\beta = \frac{F\,L^2}{2\,EI}$$

$$W_d = \frac{F^2\,L^3}{6\,EI} = \frac{1}{2}\,f_0\,F$$

C04



$$R_A = F + Q \qquad M_{RA} = Fa + \frac{Q}{2}L$$

$$V_{AC}(x) = F + Q\left(1 - \frac{x}{L}\right)$$

$$V_{CB}(x) = Q\left(1 - \frac{x}{L}\right)$$

$$M_{AC}(x) = -F\,(a - x) - \frac{Q}{2\,L}(L - x)^2$$

$$M_{CB}(x) = -\frac{Q}{2\,L}(L - x)^2$$

$$M_A = -F\,a - Q\,L/2$$

$$W_d = \frac{Q^2\,L^3}{40\,EI} + \frac{F^2\,a^3}{6\,EI} + \frac{QF\,a^2}{24\,EI\,L}\,(6\,L^2 + a^2 - 4La)$$

$$Q = q\,L$$

$$y(x) = y_{CAS\,CO1}(x) + y_{CAS\,CO2}(x)$$

$$f_0 = f_B = \frac{F\,a^2}{6\,EI}(3\,L - a) + \frac{Q\,L^3}{8\,EI}$$

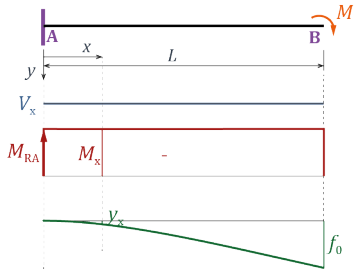
$$f_C = \frac{F\,a^3}{3\,EI} + \frac{Q\,a^2}{24\,EI\,L}\,(a^2 + 6\,L^2 - 4\,L\,a)$$

$$\tan\beta = \frac{3\,F\,a^2 + Q\,L^2}{6\,EI}$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

C05



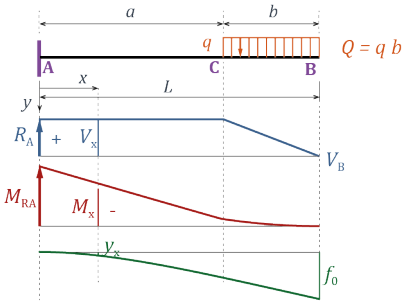
$$R_A = 0 \qquad M_{RA} = M$$

$$V(x) = 0$$

$$M(x) = -M$$
$$M_A = -M$$

$$y(x) = \frac{M}{2\,EI} x^2$$
$$f_0 = \frac{M\,L^2}{2\,EI}$$
$$\tan\beta = \frac{M\,L}{EI}$$
$$W_d = \frac{M^2\,L}{2\,EI}$$

C06



$$Q = q\,b \qquad d = a + \frac{b}{2}$$
$$R_A = Q \qquad M_{RA} = Q\,d$$

$$V_{AC}(x) = Q \qquad V_{CB}(x) = \frac{Q}{b}(L - x)$$
$$M_{AC}(x) = Q\,(x - d)$$
$$M_{CB}(x) = -\frac{Q}{2\,b}(L - x)^2$$
$$M_A = -Q\,d \qquad M_C = -Q\frac{b}{2}$$
$$W_d = \frac{Q^2}{120\,EI}(3\,b^3 + 20\,a^3 + 30\,a\,b\,d)$$

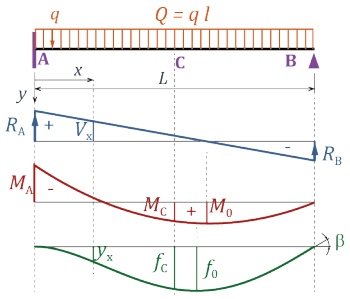
$$y_{AC}(x) = \frac{Q}{6\,EI} x^2\,(3\,d - x)$$
$$y_{CB}(x) = \frac{Q}{24\,EI\,b}(x^4 - 4\,L\,x^3 + 6\,L^2 x^2 - 4\,a^3 x + a^4)$$

$$f_C = \frac{Q\,a^2}{12\,EI}(4\,a + 3\,b)$$
$$f_0 = f_B = \frac{Q}{24\,EI\,b}(3\,L^4 - 3\,a^4 - 4\,a^3 b)$$
$$\tan\gamma = \frac{Q\,a\,L}{2\,EI} = \frac{q\,a\,b\,L}{2\,EI}$$
$$\tan\beta = \frac{Q}{6\,EI}\frac{(L^3 - a^3)}{b}$$

POUTRES ENCASTRÉES-APPUYÉES

$\xi = x/L$	Position relative	$y(x)$	Déformée élastique en flexion
R_A, R_B	Réactions (efforts verticaux) aux appuis	f	Flèche
M_{RA}, M_{RB}	Réactions (moments) aux appuis	f_0	Flèche maximale
V	Effort tranchant	α, β	Déformations angulaires
M	Moment fléchissant	W_d	Energie de déformation élastique en flexion
M_0	Moment fléchissant maximal		

D01



$$Q = q\,L$$

$$R_A = \frac{5}{8}\,Q \qquad R_B = \frac{3}{8}\,Q$$

$$M_{RA} = \frac{Q\,L}{8}$$

$$V(x) = \frac{Q}{8\,L}(5\,L - 8\,x)$$

$$V_A = \frac{5}{8}Q \qquad V_B = -\frac{3}{8}\,Q$$

$$y(x) = \frac{Q}{48\,EI}\frac{x^2}{L}\,(x - L)(2\,x - 3\,L)$$

$$f_0 \approx \frac{Q\,L^3}{185\,EI}$$

$$\text{pour } x_0 = \frac{15}{16}L\left(1 - \frac{\sqrt{33}}{15}\right) = 0{,}5784\,L$$

$$M(x) = \frac{Q}{8\,L}(L - x)(4\,x - L)$$

$$M_A = -\frac{Q\,L}{8} = -\frac{16}{9}\,M_0$$

$$M_C = +\frac{Q\,L}{16} \text{ pour } x_C = \frac{L}{2}$$

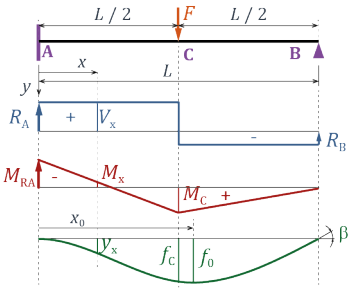
$$M_0 = \frac{9}{128}Q\,L \text{ pour } x_0 = \frac{5}{8}\,L$$

$$f_C = \frac{Q\,L^3}{192\,EI} \text{ pour } x_C = \frac{L}{2}$$

$$\tan\beta = -\frac{Q\,L^2}{48\,EI}$$

$$W_d = \frac{Q^2\,L^3}{640\,EI}$$

D02



$$R_A = \frac{11}{16}\,F \qquad R_B = \frac{5}{16}\,F$$

$$M_{RA} = \frac{3}{16}FL$$

$$V_{AC}(x) = \frac{11}{16}\,F \qquad V_{CB}(x) = -\frac{5}{16}\,F$$

$$M_{AC}(x) = \frac{F}{16}(11\,x - 3\,L)$$

$$M_{CB}(x) = \frac{5}{16}\,F\,(L - x)$$

$$M_A = -\frac{3}{16}\,FL = -\frac{6}{5}\,M_C$$

$$M_C = \frac{5}{32}F\,L \text{ pour } x_0 = \frac{L}{2}$$

$$y_{AC}(x) = \frac{F}{96\,EI}x^2\,(9\,L - 11\,x)$$

$$y_{CD}(x) = \frac{F}{96\,EI}\,(x - L)(5\,x^2 - 10\,L\,x + 2\,L^2)$$

$$f_0 = \frac{\sqrt{5}}{240}\frac{F\,L^3}{EI}$$

$$\text{pour } x_0 = L\left(1 - \frac{1}{\sqrt{5}}\right) = 0{,}552\,L$$

$$f_C = \frac{7\,F\,L^3}{768\,EI} \text{ pour } x_C = \frac{L}{2}$$

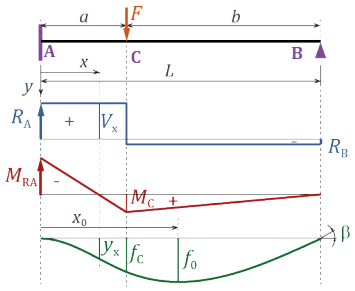
$$\tan\beta = -\frac{F\,L^2}{32\,EI}$$

$$W_d = \frac{7}{1536}\frac{F^2\,L^3}{EI} = \frac{1}{2}\,f_C\,F$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

D03



$$d_1 = \sqrt{L^2 - b^2} \qquad d_2 = \sqrt{3\,L^2 - b^2} \qquad d_3 = 3\,L - a$$

$$R_A = \frac{F\,b}{2\,L^3}\,d_2^2 \qquad R_B = \frac{F\,a^2}{2\,L^3}\,(2\,L + b)$$

$$M_{RA} = \frac{F\,b}{2\,L^2}\,d_1^2$$

$$M_{AC}(x) = \frac{F\,b}{2\,L^3}\,[d_2^2\,x - d_1^2\,L]$$

$$M_{CB}(x) = \frac{F\,d_3\,a^2}{2\,L^3}\,(L - x)$$

$$M_A = -M_{RA}$$

$$M_C = +\frac{F\,d_3\,a^2\,b}{2\,L^3}$$

$$V_{AC}(x) = R_A \qquad V_{CB}(x) = -R_B$$

$$W_d = \frac{F^2\,a^3 b^2}{24\,EI\,L^3}(3\,L + b) = \frac{1}{2}\,F\,f_C$$

$$y_{AC}(x) = \frac{F\,b}{12\,EI}\frac{x^2}{L^2}\left[3\,d_1^2 - d_2^2\frac{x}{L}\right]$$

$$y_{CB}(x) = \frac{F\,a^2}{12\,EI\,L^3}\,[(2\,L + b)(x - 3\,L)\,x^2 + 2\,L^3\,(3\,x - a)]$$

$$k_L = \frac{\sqrt{2}}{\sqrt{2}+1} = 0{,}586$$

$$k_F = \sqrt{\frac{b}{2\,L + b}}$$

si $a \geq k_L\,L$

$$f_0 = \frac{F\,b}{3\,EI}\frac{d_1^6}{d_2^4}$$

pour $x_0 = 2\,L\left(\frac{d_1}{d_2}\right)^2$

si $a < k_L\,L$

$$f_0 = k_F\frac{F\,a^2\,b}{6\,EI}$$

pour $x_0 = L\,(1 - k_F)$

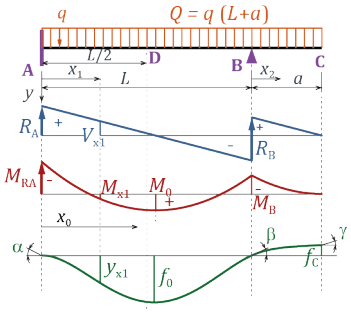
$$\tan\beta = -\frac{F\,a^2\,b}{4\,EI\,L}$$

$$f_C = \frac{F\,a^3\,b^2}{12\,EI\,L^3}\,(3\,L + b) \text{ pour } x_C = a$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

DC01



$$Q = q\,(L+a) = Q_L + Q_a$$
$$Q_L = q\,L \qquad Q_a = q\,a$$
$$R_A = \frac{d}{L} Q_L \qquad R_B = Q_a + Q_L\left(1 - \frac{d}{L}\right)$$
$$M_{RA} = \frac{Q_L}{12}(4\,d - L)$$
$$V_{AB}(x_1) = \frac{Q_L}{L}(d - x_1)$$
$$V_A = R_A \qquad V_{B1} = -\,Q_L\left(1 - \frac{d}{L}\right)$$
$$V_{BC}(x_2) = Q_a\left(1 - \frac{x_2}{a}\right)$$
$$V_C = 0 \qquad V_{B2} = +\,Q_a$$

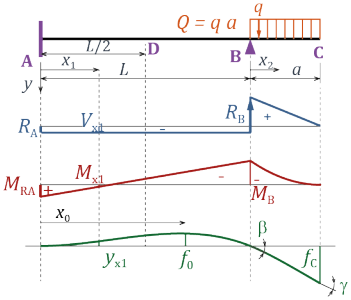
$$d = L\left(\frac{5}{8} - \frac{3}{4}\frac{a^2}{L^2}\right)$$
$$M_{AB}(x_1) = -\frac{Q_L}{8\,L}(4\,x_1^2 - 8\,d\,x_1 + L^2 - 2\,a^2)$$
$$M_{BC}(x_2) = -\frac{Q_a}{2\,a}(a - x_2)^2$$
$$M_A = M_{RA} \qquad M_B = -Q_a\frac{a}{2}$$
$$M_0 = \frac{Q_L}{8\,L}(4\,d^2 - L^2 + 2\,a^2) \quad \text{pour } x_0 = d$$
$$W_d = \frac{Q_L^2\,L^3}{1920\,EI}\left(3 + 60\frac{a^4}{L^4} - 20\frac{a^2}{L^2}\right) + \frac{Q_a^2\,a^3}{40\,EI}$$

$$y_{AC}(x_1) = \frac{Q_L}{48\,EI\,L}x_1^2\,(2\,x_1^2 - 8\,d\,x_1 + 3\,L^2 - 6a^2)$$
$$y_{BC}(x_2) = \frac{Q\,x_2}{48\,EI\,(L+a)}(2\,x_2^3 - 8\,a\,x_2^2 + 12\,a^2\,x_2 + 6\,a^2L - L^3)$$

$$f_C = \frac{Q_a}{48\,EI}(6\,a^3 + 6\,a^2\,L - L^3)$$
$$f_D = \frac{Q_L}{192\,EI}\,L\,(L^2 - 3\,a^2)$$

$$\tan\beta = \frac{Q_L}{48\,EI}(6\,a^2 - L^2)$$
$$\tan\gamma = \frac{Q}{48\,EI}\,\frac{8\,a^3 + 6\,a^2L - L^3}{L+a}$$

DC02



$$Q = Q_a = q\,a$$
$$R_A = -\frac{3}{4}\frac{a}{L}Q \qquad R_B = Q\left(1 + \frac{3}{4}\frac{a}{L}\right)$$
$$M_{RA} = -\frac{Q\,a}{4}$$
$$V_{AB}(x_1) = -\frac{3}{4}\frac{a}{L}Q \qquad V_A = V_{B1} = -\frac{3}{4}\frac{a}{L}Q$$
$$V_{BC}(x_2) = Q\left(1 - \frac{x_2}{a}\right) \qquad V_{B2} = Q$$

$$M_{AB}(x_1) = \frac{Q\,a}{4}\left(1 - 3\frac{x_1}{L}\right)$$
$$M_{BC}(x_2) = -\frac{Q}{2\,a}(a - x_2)^2$$
$$M_A = -M_{RA} = \frac{Q\,a}{4} \qquad M_B = -\frac{Q\,a}{2}$$
$$M_D = -\frac{Q\,a}{8}$$
$$W_d = \frac{Q^2\,a^2}{160\,EI}(5\,L + 4\,a)$$

$$y_{AB}(x_1) = \frac{Q\,a}{8\,EI\,L}x_1^2\,(x_1 - L) \qquad y_{BC}(x_2) = \frac{Q\,x_2}{24\,EI\,a}[x_2^3 - 4\,a\,x_2^2 + 6\,a^2\,x_2 + 3\,a^2L]$$

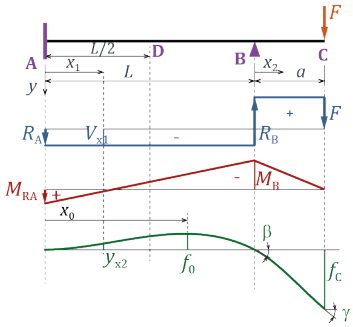
$$f_C = \frac{Q\,a^2}{8\,EI}(L + a) \qquad f_D = \frac{Q\,a\,L^2}{64\,EI}$$
$$f_0 = \frac{Q\,a\,L^2}{54\,EI} \qquad \text{pour } x_{1,0} = \frac{2}{3}L$$

$$\frac{f_c}{f_0} = \frac{27}{4}\frac{a}{L}\left(1 + \frac{a}{L}\right)$$
$$\tan\beta = \frac{Q\,a\,L}{8\,EI}$$
$$\tan\gamma = \frac{Q\,a}{24\,EI}(3\,L + 4\,a)$$

$\xi = x/L$	Position relative
R_A, R_B	Réactions (efforts verticaux) aux appuis
M_{RA}, M_{RB}	Réactions (moments) aux appuis
V	Effort tranchant
M	Moment fléchissant
M_0	Moment fléchissant maximal

$y(x)$	Déformée élastique en flexion
f	Flèche
f_0	Flèche maximale
α, β	Déformations angulaires
W_d	Energie de déformation élastique en flexion

DC03



$$R_A = -\frac{3}{2}\frac{F\,a}{L} \qquad R_B = \frac{F}{2\,L}(2\,L + 3\,a)$$

$$M_{RA} = -\frac{F\,a}{2}$$

$$V_{AC}(x) = -\frac{3}{2}\frac{F\,a}{L} \qquad V_{CB}(x) = F$$

$$M_{AB}(x_1) = \frac{F\,a}{2\,L}(L - 3\,x_1)$$

$$M_{BC}(x_2) = F\,(x_2 - a)$$

$$M_A = -M_{RA} = \frac{F\,a}{2} \qquad M_B = -F\,a$$

$$W_{dL} = \frac{F^2\,a^2\,L}{8\,EI} \qquad W_{da} = \frac{F^2\,a^3}{6\,EI}$$

$$W_d = \frac{F^2\,a^2}{24\,EI}(3\,L + 4\,a)$$

$$y_{AB}(x_1) = \frac{F\,a}{4\,EI\,L}x_1^2\,(x_1 - L)$$

$$y_{BC}(x_2) = \frac{F\,x_2}{12\,EI}(3\,a\,L + 6\,a\,x_2 - 2\,x_2^2)$$

$$f_C = \frac{F\,a^2}{12\,EI}(3\,L + 4\,a) \qquad f_D = \frac{Q\,a\,L^2}{32\,EI}$$

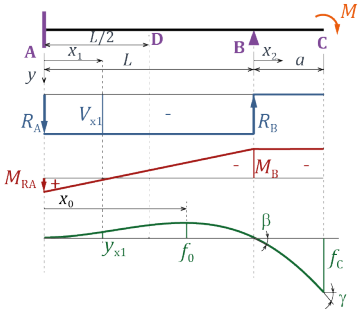
$$f_0 = \frac{F\,a\,L^2}{27\,EI} \quad \text{pour } x_0 = \frac{2}{3}\,L$$

$$\frac{f_c}{f_0} = \frac{27}{4}\frac{a}{L}\left(1 + 2\frac{a}{L}\right)$$

$$\tan\beta = \frac{F\,a\,L}{4\,EI}$$

$$\tan\gamma = \frac{F\,a}{4\,EI}(L + 2\,a)$$

DC04



$$R_A = -R_B = -\frac{3}{2}\frac{M}{L}$$

$$M_{RA} = -\frac{M}{2}$$

$$V_{AB}(x_1) = -\frac{3}{2}\frac{M}{L} \qquad V_{BC}(x_2) = 0$$

$$M_{AB}(x_1) = \frac{M}{2\,L}(L - 3\,x_1) \qquad M_{BC}(x_2) = -M$$

$$M_A = -M_{RA} \qquad M_B = -M$$

$$W_{dL} = \frac{M^2\,L}{8\,EI} \qquad W_{da} = \frac{M^2\,a}{2\,EI}$$

$$W_d = \frac{M^2}{8\,EI}(L + 4\,a)$$

$$y_{AB}(x_1) = \frac{M}{4\,EI\,L}x_1^2\,(x_1 - L)$$

$$y_{BC}(x_2) = \frac{M\,x_2}{4\,EI}(2\,x_2 + L)$$

$$f_C = \frac{M\,a}{4\,EI}(L + 2\,a)$$

$$f_D = \frac{M\,L^2}{32\,EI}$$

$$f_0 = \frac{M\,L^2}{27\,EI} \quad \text{pour } x_0 = \frac{2}{3}\,L$$

$$\tan\beta = \frac{M\,L}{4\,EI}$$

$$\tan\gamma = \frac{M}{4\,EI}(L + 4\,a)$$