MUSIC ANALYSIS WITH A BAYESIAN MODEL

Lilia Ben Baccar
MS-DS, ENSAE Paris
lilia.benbaccar@ensae.fr

Erwan Rahis
MS-DS, ENSAE Paris
erwan.rahis@ensae.fr

January 2021

Abstract

The goal of this project is to use Bayesian inference to analyze a music piece. We consider a music piece as a sequential data and segment the piece using a non-parametric Bayesian model. The music, to be represented as a discrete sequence of observations is processed and then modeled using a Hidden Markov Chain model. The segmentation is inferred by the model and compared to our music-theoretic analysis. Our method is mainly a combination of the papers by Lu Ren and al.**??** and Emily B. Fox and al. **??**.

1 Bayesian model

To solve this problem, the approach taken is a non-parametric Bayesian model based on the *Hidden Markov Model* and a prior generated by a *Hierarchical Dirichlet Process* (HDP-HMM). We want to segment a waveform into a set of time intervals with no *a priori* on the number of intervals. The sequence is modelled using the Hidden Markov Model with the states to categorise the interval. In other words, we want the piece to be divided into intervals and each interval will be labelled to a state that we will later on analyse. For example a music piece can be divided into parts that are : intro, chorus, pre-chorus, outro. Those parts can be easily identified by the human ear and brain (music-theoretic analysis). We will compare our analysis to the results of the model.
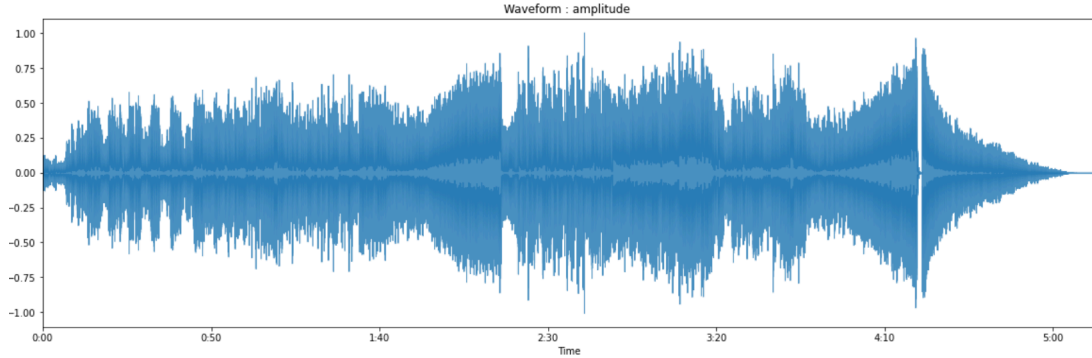

Figure 1: Audio waveform of the piece considered

1.1 Hidden Markov Model

The Hidden Markov Model used is described as follows : A discrete sequence of observed data $x = \{x_t\}_{t=1}^T$ with $x_t \in \{1, \ldots, M\}$ and a corresponding hidden state sequence $s = \{s_t\}_{t=1}^T$ with $s_t \in \{1, \ldots, I\}$. The states will be analysed as in Section 1 (intro, chorus, etc.). Each observation is associated to a state. The discrete HMM model is represented by its parameters $\theta = \{A, B, \pi\}$ such that :

- A is a set of state transition probabilities, the probability to transition from one state to another. For two states ρ and ξ, the transition probability from ρ to ξ is $a_{\rho\xi} = \mathbb{P}(s_{t+1} = \xi | s_t = \rho)$

- B is a set of emission probabilities, the probability that an observation associated to a state ρ is equal to m : $b_{\rho m} = \mathbb{P}(x_t = m | s_t = \rho)$

- π is the set of initial state distribution, the probability that the first observation in time $t = 1$ is equal to the state ρ : $\pi_\rho = \mathbb{P}(s_1 = \rho)$

Because we will use a Hierchical Dirichlet Process to model the parameters θ, we will divide the sequence into J subsequences. This means that we will model J Hidden Markov Models. By taking the notation at the beginning of this section, we will have $j \in \{1, \ldots, J\}$ and $x = \{x_j\}_{j=1}^J$ with $x_j = \{x_{jt}\}_{t=1}^T$. Note that the total length of the discrete sequence is $J \times T$ and every subsequence x_j has a length T. We allow the parameters to be different in each subsequence such that :

$$x_j \sim HMM(\theta_j)$$

The joint distribution given θ is :

$$p(x|\theta) = \prod_{j=1}^J \left\{ \sum_{s_j} \pi_{j,1} \prod_{t=1}^{T-1} a_{s_{j,t+1}} \prod_{t=1}^T b_{s_{j,t}x_{j,t}} \right\}$$

1.2 Hierchical Dirichlet Process

A Hierchical Dirichlet Process is defined as a collection of Dirichlet Processes. Our collection of DP processes is the J parameters θ_j linked to the HMM. This kind of setting allows for analysing how inter-related are the subsequences one to the others. The first process of the collection is denoted as :

$$G_0 \sim DP(\gamma, H)$$

with γ a concentration parameter for the Dirichlet distribution which is symmetrical and H a base probability measure on Θ. The formal definition of a Dirichlet process is $\forall (B_k)_{k=1,\ldots,K}$ finite partition of Θ :

$$(G_0(B_k))_{k=1,\ldots,K}|\gamma, H \sim Dir((\gamma H(B_k))_{k=1,\ldots,K})$$

The Dirichlet Process can be written wirrten such that :

$$G_0 = \sum_{k=1}^\infty \beta_k \delta_{\theta_k}$$

The first process G_0 is then used as the base measure for the other processes :

$$G_j \sim DP(\alpha, G_O)$$

2 Data processing

We consider an acoustic signal in a mp3 format loaded in Python using the librosa package. The sampling rate is 22.05kHz which means we have 22050 observations par second of audio signal. The acoustic signal is first loaded as a waveform or amplitude as shown in Figure 1. It represents the pressure change recorded by a recording instrument such as a microphone. We then divide the piece into 100ms contiguous frames. Each amplitude frame is processed by computing its Mel frequency sceptral coefficients (MFCCs). These coefficients are derived from the Melspectrogram that is a log-scale of pitches judged to be equally distanced to one another. The Mel spectrogram is the representation showing the mel frequencies according to time and intensity. The MFCCs are the most popular feature used in audio processing. We compute 40 columns of coefficients for each frame. To obtain a vector sequential data, we quantize each frame of coefficient. First, we *whiten* the columns so each one of them have a unit variance. Then we use the k-means algorithm with 16 codebooks and pass them to the vector quantization algorithm. The result is a 1D discrete sequential data representing the coefficients.

3 Computational method

4 Results

References

[1] Emily B. Fox, Erik B. Sudderth, Michael I. Jordan, and Alan S. Willsky. A sticky HDP-HMM with application to speaker diarization. *The Annals of Applied Statistics*, 5(2A):1020--1056, June 2011.

[2] Lu Ren, David B. Dunson, Scott Lindroth, and Lawrence Carin. Music analysis with a Bayesian dynamic model. *IEEE*, 2009.

[3] Lu Ren, David Dunson, Scott Lindroth, and Lawrence Carin. Dynamic Nonparametric Bayesian Models for Analysis of Music. *Journal of the American Statistical Association*, 105(490):458--472, June 2010.

[4] Satyanand Singh and E.G. Rajan. Vector Quantization Approach for Speaker Recognition using MFCC and Inverted MFCC. *International Journal of Computer Applications*, 17(1):1--7, March 2011.

[5] IEEE Signal Processing Society, editor. *Proceedings / 2009 IEEE International Conference on Acoustics, Speech, and Signal Processing: April 19 - 24, 2009, Taipei International Convention Center, Taipei, Taiwan*. IEEE, Piscataway, NJ, 2009. Meeting Name: IEEE International Conference on Acoustics, Speech, and Signal Processing.