THE UNIVERSITY OF WESTERN AUSTRALIA

Electrical, Electronic & Computer Engineering

SEMESTER 1, 2020 EXAMINATIONS

ELEC5506
PROCESS INSTRUMENTATION AND CONTROL -ATTACHMENT

FAMILY NAME:__________________________ GIVEN NAMES:______________________

STUDENT ID: | | | | | | | | | SIGNATURE: ________________________

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1 Laplace transforms

Table 1: Table of Laplace transforms

$F(s)$	$f(t),\ t \geq 0$
1	$\delta(t)$
$1/s$	$1(t)$
$1/s^2$	t
$2!/s^3$	t^2
$3!/s^4$	t^3
$m!/s^{m+1}$	t^m
$\frac{1}{s+a}$	$\exp(-at)$
$\frac{1}{(s+a)^2}$	$t\exp(-at)$
$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}\exp(-at)$
$\frac{a}{s(s+a)}$	$1-\exp(-at)$
$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+\exp(-at))$
$\frac{b-a}{(s+a)(s+b)}$	$\exp(-at)-\exp(-bt)$
$\frac{s}{(s+a)^2}$	$(1-at)\exp(-at)$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\exp(-at)$
$\frac{(b-a)s}{(s+a)(s+b)}$	$b\exp(-bt)-a\exp(-at)$
$\frac{a}{s^2+a^2}$	$\sin(at)$
$\frac{s}{s^2+a^2}$	$\cos(at)$
$\frac{s+a}{(s+a)^2+b^2}$	$\exp(-at)\cos(bt)$
$\frac{b}{(s+a)^2+b^2}$	$\exp(-at)\sin(bt)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\exp(-at)[\cos(bt)+\frac{a}{b}\sin(bt)]$

Table 2: Properties of Laplace transforms

Laplace Transform	Time Function	Comment
$F(s)$	$f(t)$	Transform pair
$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
$F(s)\exp(-\lambda s)$	$f(t-\lambda)$	Time delay ($\lambda \geq 0$)
$\frac{1}{\lvert a\rvert}F(\frac{s}{a})$	f(at)	Time scaling
$F(s+a)$	$f(t)\exp(-at)$	Shift in frequency
$s^m F(s) - s^{m-1}f(0)$ $-s^{m-2}f'(0) - \ldots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differention
$\frac{F(s)}{s}$	$\int_0^t f(\eta)d\eta$	Integration
$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial value theorem
$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final value theorem
$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
$\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)d\omega$	$\int_0^\infty y(t)u(t)dt$	Parseval's theorem
$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

2 First-order systems

The settling-time is the time to reach within a specified percentage of the final value:

- 2 time-constants is $\exp(-2) \approx 0.14$ which means that after 2 time-constants the response has reached 86% of the final value;
- 3 time-constants is $\exp(-3) \approx 0.05$ which means that after 3 time-constants the response has reached 95% of the final value;
- 4 time constants is $\exp(-4) \approx 0.02$ which means that after 4 time-constants the response has reached 98% of the final value;
- 5 time constants is $\exp(-5) \approx 0.01$ which means that after 5 time-constants the response has reached 99% of the final value.

$$
\begin{aligned}
T_s\,(14\%) &= 2\tau && (1)\\
T_s\,(5\%) &= 3\tau && (2)\\
T_s\,(2\%) &= 4\tau && (3)\\
T_s\,(1\%) &= 5\tau && (4)
\end{aligned}
$$

The rise-time is the time it takes for the response to go from 10% to 90% of the final value:

$$
\begin{aligned}
10\%: &\quad 1 - \exp(-t_{10\%}/\tau) = 0.1 && (5)\\
90\%: &\quad 1 - \exp(-t_{90\%}/\tau) = 0.9 && (6)\\
\implies &\quad \exp([t_{90\%} - t_{10\%}]/\tau) = 9 && (7)\\
\implies &\quad T_r = t_{90\%} - t_{10\%} = \tau \ln(9) \approx 2.2\tau && (8)
\end{aligned}
$$

3 Second-order or Oscillatory systems

Consider the general function

$$G(s) = \frac{c}{s^2 + bs + c} \tag{9}$$

When $b = 0$ the system is undamped, $s = \pm j\sqrt{c}$, and hence $\omega_n = \sqrt{c}$ or $c = \omega^2$. For the underdamped system $b \neq 0$, $s = -\sigma \pm j\omega_n$ and $\sigma = -b/2$.

Define the damping factor:

$$\zeta = \frac{|\sigma|}{\omega_n} = \frac{b/2}{\omega_n} \tag{10}$$

which implies $b = 2\zeta\omega_n$.

In general then

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{11}$$

where K is the gain, $\omega = \omega_n$ is the natural frequency, and ζ is the damping factor. The poles occur at $s_{1.2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\sigma_d \pm j\omega_d$ where

$$
\begin{aligned}
\sigma_d &= \zeta\omega_n && (12)\\
\omega_d &= \omega_n\sqrt{1 - \zeta^2} && (13)
\end{aligned}
$$

The step response for a second-order system is

$$\begin{aligned}
y(t) &= A\Big[1-\exp(-\zeta\omega_n t)\cos(\omega_n\sqrt{1-\zeta^2}\,t) \\
&\quad -\frac{\zeta}{\sqrt{1-\zeta^2}}\exp(-\zeta\omega_n t)\sin(\omega_n\sqrt{1-\zeta^2}\,t)\Big] \qquad (14)\\
y(t) &= A\left[1-\frac{1}{\sqrt{1-\zeta^2}}\exp(-\zeta\omega_n t)\left[\sqrt{1-\zeta^2}\cos(\omega_n\sqrt{1-\zeta^2}\,t)\right.\right. \\
&\quad \left.\left.+\zeta\sin(\omega_n\sqrt{1-\zeta^2}\,t)\right]\right] \qquad (15)\\
& \qquad (16)
\end{aligned}$$

Let $\sqrt{1-\zeta^2}=\cos\phi$ and $\zeta=\sin\phi$, then

$$\begin{aligned}
y(t) &= A\left[1-\frac{1}{\sqrt{1-\zeta^2}}\exp(-\zeta\omega_n t)\left[\cos\phi\cos(\omega_n\sqrt{1-\zeta^2}\,t)+\sin\phi\sin(\omega_n\sqrt{1-\zeta^2}\,t)\right]\right] \qquad (17)\\
&= A[1-\frac{1}{\sqrt{1-\zeta^2}}\exp(-\zeta\omega_n t)\cos(\omega_n\sqrt{1-\zeta^2}\,t-\phi)] \qquad (18)\\
&= A[1-\frac{1}{\sqrt{1-\zeta^2}}\exp(-\sigma_d t)\cos(\omega_d t-\phi)] \qquad (19)
\end{aligned}$$

$$y(t)=A[1-\frac{1}{\sqrt{1-\zeta^2}}\exp(-\zeta\omega_n t)\sin(\frac{\omega_n}{\sqrt{1-\zeta^2}}+\arccos\zeta)] \qquad (20)$$

3.1 % overshoot

For a second order system with two poles and no zeros, the percentage overshoot is approximately given as

$$\%\ \text{overshoot} \approx (1-\zeta/0.6)100 \qquad (21)$$

3.2 Settling time

Define T_s to be the settling time to within 2% of the final value.

$$T_s \approx \frac{4}{\zeta\omega_n}=\frac{4}{\sigma_d}=4\tau \qquad (22)$$

$$(23)$$

If we are interested in the time it takes to settle within 5% of the final value then a reasonable approximation is:

$$T_{s,0.05} \approx \frac{3}{\zeta\omega_n}=\frac{3}{\sigma_d}=3\tau \qquad (24)$$

$$(25)$$

3.3 Peak time

The peak time T_p is the time it takes to reach the maximum value. This is where the slope is zero

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} \tag{26}$$

$$\tag{27}$$

The maximum overshoot will then be the value at $t = T_p$ and hence

$$\%OS = M_p \times 100 = \exp(-\sigma_d T_p) \times 100 = \exp(-\zeta\pi/\sqrt{1-\zeta^2}) \times 100\% \tag{28}$$

For $0 \le \zeta \le 0.6$

$$M_p \approx 1 - \frac{\zeta}{0.6} \tag{29}$$

3.4 Rise time

The rise time for an under-damped system is the time for the step response to reach the final value for the first time.

$$T_r = \frac{1}{\omega_d}\tan^{-1}\left(-\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{30}$$

$$= \frac{1}{\omega_d}(\frac{\pi}{2} + \sin^{-1}\zeta) \tag{31}$$

$$= \frac{\pi/2 + \sin^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \tag{32}$$

$$\approx \frac{\pi/2 + \zeta}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi + 2\zeta}{2\omega_n\sqrt{1-\zeta^2}} \approx \frac{\pi}{2\omega_n} \approx \frac{1.6}{\omega_n} \quad \text{for small } \zeta \tag{33}$$

This result indicates that in order to reduce the rise time we need to increase ω_n.

Taking $\zeta = 0.5$ to be a mid-range value, the time it takes to go from 0.1 to 0.9 of the final value is

$$t_r \approx \frac{1.8}{\omega_n} \tag{34}$$

4 Zeigler-Nichols

	k_p/k_c	T_i/T_c	T_d/T_c	T_p/T_c
P	0.5			1.0
PI	0.4	0.8		1.4
PID	0.6	0.5	0.125	0.85

Table 3: Controller parameters for the Ziegler-Nichols frequency response. method.

	ak_p	T_i/T_{del}	T_d/T_{del}	T_p/T_{del}
P	1			4
PI	0.9	3		5.7
PID	1.2	2	$T_{del}/2$	3.4

Table 4: Controller parameters for the alternative Ziegler-Nichols frequency response. method.