

# **Electrical, Electronic & Computer Engineering**

# **SEMESTER 1, 2021 EXAMINATIONS**

# ENSC3014 ELECTRONIC MATERIALS AND DEVICES -ATTACHMENT

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CONSTANTS	
Charge on an electron, $q$ , or $e$	$1.60 \times 10^{-19} \text{ C}$
Boltzmann's constant, $k$	$1.38 \times 10^{-23} \text{ JK}^{-1}$
Speed of light in free space, $c$	$3.00 \times 10^8 \text{ ms}^{-1}$
Plank's constant, h	$6.62 \times 10^{-34} \text{ Js}$
Reduced Plank's constant, $\hbar$	$\frac{h}{2\pi}$
Mass of an electron	$9.11 \times 10^{-31} \text{ kg}$
Mass of a proton or a neutron	$1.67 \times 10^{-27} \text{ kg}$
Permittivity of free space	$8.85 \times 10^{-14} \text{ Fcm}^{-1}$

PROPERTIES			
Material	Si	GaAs	$\mathrm{SiO}_2$
Intrinsic carrier concentration, $n_i$	$1.45 \times 10^{10} \text{ cm}^{-3}$	$1.79 \times 10^6 \ \mathrm{cm^{-3}}$	
Electron mobility, $\mu_e$	$1350 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$	$8500 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$	
Hole mobility, $\mu_h$	$480 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$	$400 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$	
Relative Permittivity, $\varepsilon_r$	11.7	13.1	3.9
Energy band gap, $E_G$	1.12 eV	1.42 eV	8.9 eV
Effective density of states in conduction band, $N_C$	$4.3 \times 10^{19} \text{ cm}^{-3}$	$4.7 \times 10^{17} \ \mathrm{cm^{-3}}$	
Effective density of states in valence band, $N_V$	$2.5 \times 10^{19} \text{ cm}^{-3}$	$7 \times 10^{18} \text{ cm}^{-3}$	

QUANTUM MECHANICS	
Shroedinger's wave equations in one dimension	$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \text{ (full)}$ $\Psi = \psi(x)\phi(t) \text{ (separable solutions)}$ $E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \text{ (time-independent)}$ $\phi(t) = e^{-j\frac{E}{\hbar}t} \text{ (time-dependent part)}$
De-Broglie's wave-particle duality	$p = \frac{h}{\lambda}$
Energy of a photon	$E = h\nu$ , where $\nu$ is the frequency of light

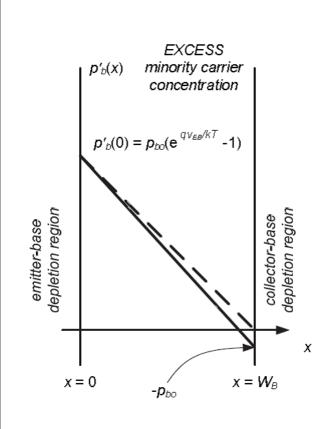
Effective mass	$m^* = \hbar^2 / \left( \frac{d^2 E}{dk^2} \right)$
The Fermi Level in a semiconductor	$E_F - E_V = \frac{E_G}{2} + \frac{3}{4}kT \ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{1}{2}kT \ln\left(\frac{n}{p}\right)$
Maxwell-Boltzmann approximation of carrier concentrations	$n = N_C \exp\left(-\frac{E_C - E_F}{kT}\right)$ $p = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$ where, $N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}}$ $N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}}$
Alternative Maxwell-Boltzmann expressions of carrier concentrations	$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$ $p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$
Thermal equilibrium condition	$p(x)n(x) = n_i^2$

Conductivity, resistivity and resistance	$\sigma = nq\mu_n + pq\mu_p$ $\rho = \frac{1}{\sigma}$ $R = \frac{\rho l}{A}$
Current Density	$J_n = nq\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$ $J_p = pq\mu_p \mathcal{E} - qD_p \frac{dp}{dx}$
Continuity equations	$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_n + G_n - R_n$ $\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot J_p + G_p - R_p$
Net rate of recombination	$U = R - G_{th}$
Trap assisted (Shockley-Read-Hall) net recombination rate	$U = \sigma v_{th} N_t \frac{pn - n_i^2}{p + n + 2n_i \cosh\left(\frac{E_F - E_i}{kT}\right)}$ where $\sigma$ is the capture cross-section and $v_{th}$ is the average thermal energy of carriers.
Approximate net rate of recombination in neutral regions	$U = R - G_{th}$ $= \frac{p_n - p_{n0}}{\tau_p} \text{ n-type material}$ $= \frac{n_p - n_{p0}}{\tau_n} \text{ p-type material}$

Poisson's Equation	$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_r \varepsilon_0}$
Relationship between electric field and electrostatic potential	${\cal E} = -rac{d\psi}{dx}$
Relationship between electrostatic potential and band structure	$\psi = -\frac{E_i}{q}$

Built in potential of pn junction	$V_{bi} = \frac{1}{q} \left( E_{Fn} - E_{Fp} \right)$ $\approx \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$
Depletion region widths (abrupt junctions)	$x_n = \sqrt{\frac{2\varepsilon_r \varepsilon_0}{qN_D} \left(\frac{N_A}{N_A + N_D}\right) (V_{bi} - V_J)}$ $x_p = \sqrt{\frac{2\varepsilon_r \varepsilon_0}{qN_A} \left(\frac{N_D}{N_A + N_D}\right) (V_{bi} - V_J)}$
	$W = x_n + x_p = \sqrt{\frac{2\varepsilon_r \varepsilon_0}{q} \left(\frac{N_A + N_D}{N_A N_D}\right) (V_{bi} - V_J)}$
Diode depletion region differential capacitance per unit area	$C = \frac{dQ_J}{dV_J}$ $= \frac{\varepsilon_T \varepsilon_0}{W}$ $= \frac{\varepsilon_T \varepsilon_0}{\sqrt{\frac{2\varepsilon_T \varepsilon_0}{q} \left(\frac{N_A + N_D}{N_A N_D}\right) (V_{bi} - V_J)}}$
Approximate max net rate of generation in an depletion region (works for both forward bias and reverse bias)	$U_{MAX} = \frac{n_i}{2\tau_0} \left( \exp\left(\frac{qV_J}{2kT}\right) - 1 \right)$ where, $\tau_0 = \frac{1}{\sigma v_{th} N_t}, \text{ is the minority carrier lifetime}$
pn junction diffusion currents (works for both forward and reverse bias). Replace $L_p$ or $L_n$ by $W$ for a short-base diode, where $W$ is the length of the neutral region between the depletion region to the contact.	$I_{diff,p} = qD_p \frac{p_{n0}}{L_p} A_J \left( \exp\left(\frac{qV_J}{kT}\right) - 1 \right)$ $I_{diff,n} = qD_n \frac{n_{p0}}{L_n} A_J \left( \exp\left(\frac{qV_J}{kT}\right) - 1 \right)$ where, $L_p = \sqrt{D_p \tau_p} \text{ (in neutral n-region)}$ $L_n = \sqrt{D_n \tau_n} \text{ (in neutral p-region)}$
pn junction generation/recombination currents in depletion region (works for both forward and reverse bias.)	$I_{rec/gen} = qA_J W_{\frac{n_i}{2\tau_0}} \left( \exp\left(\frac{qV_J}{2kT}\right) - 1 \right)$
Total pn junction currents	$I = I_{rec/gen} + I_{diff,p} + I_{diff,n}$
Quasi-static approximation for carrier concentrations at the edge of a depletion region in a pn junction	$p_n(x = 0) = p_{n0} \exp\left(\frac{qV_J}{kT}\right)$ $n_p(x = -W) = n_{p0} \exp\left(\frac{qV_J}{kT}\right)$

Excess minority carrier concentration in the base in forward active.



Excess minority carrier charge in the base in forward active.

$$q_F = Q_{F0} \left( \exp\left(\frac{qV_{EB}}{kT}\right) - 1 \right)$$

where,

$$Q_{F0} = \frac{qA_J W_B p_{b0}}{2}$$

Static collector current in forward active.

$$I_C \approx q A_J D_b \frac{p_b'(0)}{W_B}$$

$$= I_1 \left( \exp\left(\frac{qV_{EB}}{kT}\right) - 1 \right)$$

$$= \frac{q_F}{\tau_F}$$

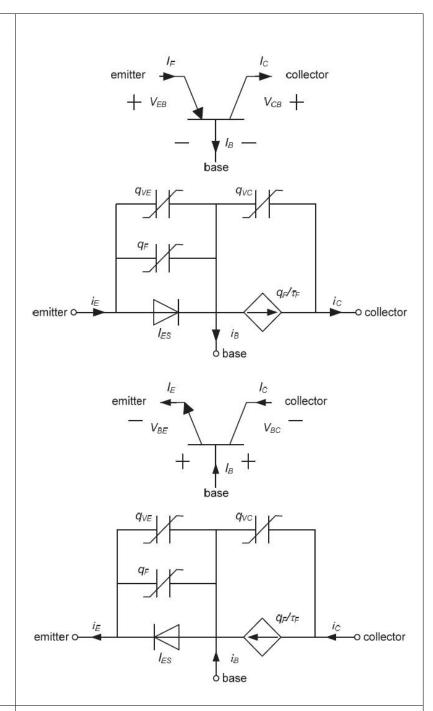
where,

$$I_1 = \frac{qA_JD_bp_{b0}}{W_B}$$

$$\tau_F = \frac{(W_B)^2}{2D_b}$$

Static base current in forward active	$I_B = I_{BA} + I_{BB}$
	$= \frac{qA_J W_B}{2\tau_b} p_b'(0) + \frac{qA_J D_e}{W_E} \left(\frac{n_{e0}}{p_{b0}}\right) p_b'(0)$
	$=rac{q_F}{ au_{BF}}$
	where,
	$ au_{BF} = rac{1}{rac{1}{ au_b} + rac{2D_e n_{e0}}{W_B W_E p_{b0}}}$
Static base current as a fraction of the static collector current in forward active.	
	where,
	$\delta = \frac{\frac{W_B}{2\tau_B} + \frac{D_e}{W_E} \left(\frac{n_{e0}}{p_{b0}}\right)}{D_b/W_b} $ (called the base defect)
Static emitter current in forward active	$I_E = I_C + I_B = (1 + \delta)I_C$
Charge control equations in forward active	$i_C = rac{q_F}{ au_F} - rac{dq_{VC}}{dt}$
	$i_B = \frac{dq_F}{dt} + \frac{dq_{VE}}{dt} + \frac{dq_{VC}}{dt} + \frac{q_F}{\tau_{BF}}$
	$i_E = \frac{dq_F}{dt} + \frac{dq_{VE}}{dt} + \frac{q_F}{\tau_{BF}} + \frac{q_F}{\tau_F}$

Charge control model in forward active



Complete charge control model equations

$$\begin{split} i_C &= \frac{q_F}{\tau_F} - \frac{dq_R}{dt} - q_R \left(\frac{1}{\tau_{BR}} + \frac{1}{\tau_R}\right) - \frac{dq_{VC}}{dt} \\ i_B &= \frac{dq_F}{dt} + \frac{q_F}{\tau_{BF}} + \frac{dq_R}{dt} + \frac{q_R}{\tau_{BF}} + \frac{dq_{VE}}{dt} + \frac{dq_{VC}}{dt} + \frac{q_F}{\tau_{BF}} \\ i_E &= -\frac{q_R}{\tau_R} + \frac{dq_F}{dt} + q_F \left(\frac{1}{\tau_{BF}} + \frac{1}{\tau_F}\right) + \frac{dq_{VE}}{dt} \\ \end{split}$$
 where, 
$$q_R = Q_{R0} \left(e^{\frac{q_{VCB}}{kT}} - 1\right)$$

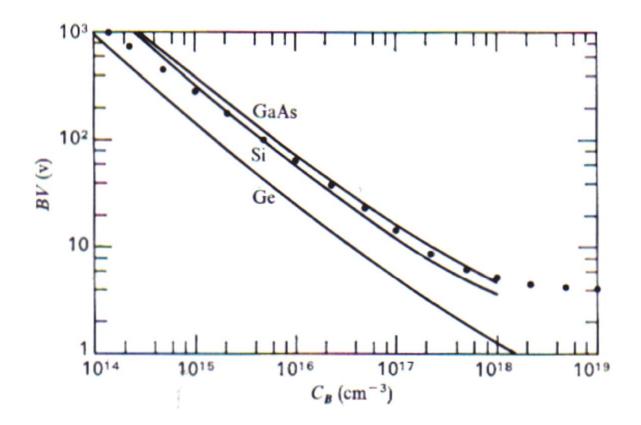
Complete charge-control model (valid in all regions of operation)  $I_C$ collector emitter base  $I_{CS}$ QR/TR emitter oo collector  $I_{ES}$  $i_B$ base  $i_C$ emitter collector base qR/TR emitter o--o collector IES  $i_B$ o base

Ideal n-channel MOSFET drain current	Triode: $I_D = \frac{Z}{L} \mu_n C_{ox} \left[ (V_G - V_T) V_D - \frac{1}{2} V_D^2 \right]$ Saturation: $I_{Dsat} = \frac{1}{2} \frac{Z}{L} \mu_n C_{ox} \left( V_G - V_T \right)^2$
Fermi Potential	p-type silicon: $\phi_p = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right)$ n-type silicon: $\phi_n = \frac{kT}{q} \ln \left( \frac{N_D}{n_i} \right)$
MOS Depletion width	p-type silicon: $W_d=\left(\frac{2\varepsilon_S\phi_s}{qN_A}\right)^{\frac{1}{2}}$ n-type silicon: $W_d=\left(\frac{2\varepsilon_S\phi_s}{qN_D}\right)^{\frac{1}{2}}$
MOS silicon charge	$V_G - V_{FB} = -\frac{Q_s}{C_{ox}} + \phi_s$
MOS voltage relationship	$V_G = \phi_{MS} + V_{ox} + \phi_s$
MOS inversion layer charge (p-type)	$Q_n = -\left(V_G - V_T\right)C_{ox}$
Ideal MOS saturation voltage	$V_{Dsat} = V_G - V_T$

Kinetic Energy	$E_{KE} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Relationship between frequency and wavelength of light	$\nu = \frac{c}{\lambda}$
Lorentz Force	$F = q\left(v \times B\right)$
Hall coefficient	$R_H = \frac{\mathcal{E}_H}{JB} = \frac{\mu}{\sigma} = \frac{1}{ne} = \frac{1}{pq}$

12 2 4 4 00026	neon 10	Ne	20,180	argon 18	Ā	39,948	krypton 36	Ż	83.80	xenon Z	Xe	131.29	1300 mg/s	Rh	[222]			
	fluorine 9	щ	18,998	chlorine 17	ರ	35.453	bromine 35	Ŗ	79.904	odine 53	_	126.90	astaline 85	At	[210]			
	oxygen 8	0	15.999	suffur 16	S	32.065	selenium 34	Se	78.96	tellurlur 52	Te	127.60	polonium 84	Ро	[209]			
4	nitrogen 7	z	14.007	phosphorus 15	۵	30.974	arsenic 33	As	74.922	antimony 51	Sb	121.76	bismuth 83	Bi	208.98			
	carbon 6	ပ	12,011	sticon 14	S	28,086	germanium 32	Ge	72.61	u 20	Sn	118.71	lead 82	Pb	207.2	ununquadum 114	Uuq	[289]
4	poron 5	Ω	10.811	atuminium 13	Ā	26,982	galllum 31	Ga	69.723	hdlum 49		114.82	thallium 81	F	204.38			_
1							zinc 30	Zu	66,39	cadmium 48	င္ပ	112.41	mercury 80	P	200.59	unurbium 112	Uub	1277
							copper 29	D C	63.546	silver 47	Ag	107.87	pop 79	Au	196.97	unununum 111	Dnn	[272]
							nickel 28	Z	58.693	palladium 46	Pd	106.42	platinum 78	Pt	195.08	110	Unn	271
							coball 27	ပိ	58.933	modium 45	뫈	102.91	iridium 77	_	192.22	109	Ĭ	[208]
							1ron 26	Fe	55.845	nuthentum 44	Ru	101.07	osmium 76	Os	190.23	nassium 108	H	[269]
							nanganese 25	Mn	54.938	technetium 43	ည	1861	rhenlum 75	Re	186.21	107	Bh	[564]
							chromium 24	င်	51,996	nolybdenum 42	è	95.94	tungsten 74	>	183,84	106	Sd	1500
							vanadlum 23	>	50.942	niobium 41	g	95.906	tantalum 73	Ta	180.95	dubnium 105	Op	[592]
							Wanium 22	F	47.867	zirconium 40	Zr	91.224	hafnium 72	Ŧ	178.49	104	R	[361]
							scandium 21	သွင	44.966	yttrium 39	>	88.906	lubetium 71	Ľ	174.97	103	۲	[262]
													57-70	*		89-102	*	
	beryllium 4	Be	9.0122	magnestum 12	Mg	24.305	caldium 20	Ca	40.078	strontlum 38	Š	87.62	barium 56	Ba	137,33	magirum 88	Ra	526
hydrogen 1,0079	Mhium 3	=	6,941	sodium 11	Na	22.990	mulsseton 19	¥	39.098	nubidium 37	Rb	85,468	caesium 55	Cs	132.91	francium 87	Ļ	822

opido opioo	fanthanum 57	Serlum 58	praseodymium 59	neodymium 60	promethium 61	samarlum 62	europlum 63	gadolinium 64	terbium 65	dysprosium 66	holmlum 67	erbium 68	mnijinu 69	ytterblum 70
000	Гa	Ce	Pr	N	Pm	Sm	En	gq	P	Ò	운	ш	Щ	Хp
	138,91		140.91	144.24	1145	150.36	151.96	157.25	158.93	162.50	164.93	167.26	168.93	173.04
	actinium		protactinam	uranium	neptunium	plufonium	americium	curium	berkeiturn	caffornium	einsteinium	fermium	mendelevium	nobelium
nide series	89	06	91	92	93	94	98	96	97	86	66	100	101	102
	Ac	H	Pa	⊃	QN N	Pu	Am	Cm	BK	ŭ	Es	Fm	Ma	8
	[227]	232.04	231.04	238.03	[237]	[244]	[243]	[247]	[247]	[251]	[252]	[257]	[258]	[259]



## Identities and Formulas

## Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
  $\cot \theta = \frac{\cos \theta}{\sin \theta}$ 

## Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$
  $\csc \theta = \frac{1}{\sin \theta}$ 
 $\cos \theta = \frac{1}{\sec \theta}$   $\sec \theta = \frac{1}{\cos \theta}$ 
 $\tan \theta = \frac{1}{\cot \theta}$   $\cot \theta = \frac{1}{\tan \theta}$ 

## Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

#### Even and Odd Formulas

$$\sin(-\theta) = -\sin\theta$$
  $\csc(-\theta) = -\csc\theta$   
 $\cos(-\theta) = \cos\theta$   $\sec(-\theta) = \sec\theta$   
 $\tan(-\theta) = -\tan\theta$   $\cot(-\theta) = -\cot\theta$ 

#### Periodic Formulas

If n is an integer

$$\sin(\theta + 2\pi n) = \sin \theta$$
  $\csc(\theta + 2\pi n) = \csc \theta$   
 $\cos(\theta + 2\pi n) = \cos \theta$   $\sec(\theta + 2\pi n) = \sec \theta$   
 $\tan(\theta + \pi n) = \tan \theta$   $\cot(\theta + \pi n) = \cot \theta$ 

#### **Double Angle Formulas**

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$
$$= 2\cos^2 \theta - 1$$
$$= 1 - 2\sin^2 \theta$$
$$\tan(2\theta) = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

 $\sin(2\theta) = 2\sin\theta\cos\theta$ 

## Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then:

$$\frac{\pi}{180^{\circ}} = \frac{t}{x} \qquad \Rightarrow \qquad t = \frac{\pi x}{180^{\circ}} \quad \text{and} \quad x = \frac{180^{\circ} t}{\pi} \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

### Half Angle Formulas

$$\sin \theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{2}}$$
$$\cos \theta = \pm \sqrt{\frac{1 + \cos(2\theta)}{2}}$$
$$\tan \theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}}$$

#### Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

#### Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

#### Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

#### Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$