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Tableau Equations

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{S} \\ -\mathbf{S}^T & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_0\mathbf{p}+\mathbf{M}_1 & \mathbf{N}_0\mathbf{p}+\mathbf{N}_1 \end{bmatrix} \begin{bmatrix} \mathbf{e}(t) \\ \mathbf{v}(t) \\ \mathbf{i}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_\mathbf{s}(t) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{Q} \\ -\mathbf{Q}^T & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_0\mathbf{p}+\mathbf{M}_1 & \mathbf{N}_0\mathbf{p}+\mathbf{N}_1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_t(t) \\ \mathbf{v}(t) \\ \mathbf{i}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_\mathbf{s}(t) \end{bmatrix}$$

$$\begin{bmatrix} -\mathbf{B}^T & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_0\mathbf{p}+\mathbf{M}_1 & \mathbf{N}_0\mathbf{p}+\mathbf{N}_1 \end{bmatrix} \begin{bmatrix} \mathbf{i}_L(t) \\ \mathbf{v}(t) \\ \mathbf{i}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_\mathbf{s}(t) \end{bmatrix}$$

R L C V-I Relations in Laplace Domain

$$i = C\frac{dv}{dt} \qquad v = L\frac{di}{dt} \qquad v = iR$$

$$, V(s) = \left(\frac{1}{Cs}\right) I(s) + \frac{v(0^-)}{s}, \quad V(s) = LsI(s) - Li(0^-), \; V(s) = RI(s)$$

The Origin of Initial Conditions

$$\int_a^x \frac{df(t)}{dt}dt = f(x) - f(a) \qquad f(x) = \int_a^x \frac{df(t)}{dt}dt + f(a)$$

SOLUTION TO FIRST ORDER LINEAR DIFFERNTUIAL EQUATION

$$\tau\frac{dx}{dt} + x(t) = B \qquad x(t) = Ae^{\left(-\frac{t}{\tau}\right)} + B$$

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TRANSMISISON LINES

$v(x,t)=v_o+v_1\left(t-\frac{x}{u_p}\right)+v_2\left(t+\frac{x}{u_p}\right)$	forward backward wave decomposition
$i(x,t)=i_o+i_1\left(t-\frac{x}{u_p}\right)+i_2\left(t+\frac{x}{u_p}\right)$	forward backward wave decomposition
$i_1(x,t)-i_1(x,t_o)=\frac{1}{Z_0}(v_1(x,t)-v_1(x,t_o))$	
$i_2(x,t)-i_2(x,t_o)=-\frac{1}{Z_0}(v_2(x,t)-v_2(x,t_o))$	
Voltage Reflection Coefficient for a lossless line at an impedance R $\rho=\frac{R-Z_0}{R+Z_0}$	
Transfer of an impedance Z from $x=0$ to $x=l$ on a lossless transmission line in the frequency domain. $Z_l=Z_0\left(\frac{Z\cos(\beta l)+jZ_0\sin(\beta l)}{Z_0\cos(\beta l)+jZ\sin(\beta l)}\right)$	
Transfer of source with impedance Z_S at $x=0$ to $x=l$ on a lossless transmission line in the frequency domain. $V_{SE}=\frac{Z_0V_S}{Z_0\cos(\beta l)+jZ_S\sin(\beta l)}$	
$V_x=V_+e^{-jbx}+V_-e^{jbx}$	
$\beta=\omega\sqrt{LC}$ $\quad Z_0=\sqrt{\frac{L}{C}}$ $\quad u_p=\frac{1}{\sqrt{LC}}$	
$\rho(s)=\frac{Z(s)-Z_0(s)}{Z(s)+Z_0(s)}$	Reflection coefficient at an impedance Z(s)
Uniform Loaded Line Equations Approximations Load C_x at spacing l_x . Unloaded line capacitance C_0 F/m $a=\left(1+\frac{C_x}{l_xC_0}\right)=\left(1+\frac{Z_0C_x}{T_x^{Unloaded}}\right)$	

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$$\tau_d^{Loaded} = \tau_d^{Unloaded}\sqrt{a} \qquad Z_0^{Loaded} = \frac{Z_0^{Unloaded}}{\sqrt{a}}$$

Approximate Second Order Modelling of Loaded Lossless Line	
Backward or Source end damping	$\omega_n = \frac{1}{\sqrt{Z_0 C_L T_d}}, \ \xi = \frac{R_S\left(Z_0 C_L + T_d\right)}{2Z_0\sqrt{Z_0 C_L T_d}} \approx \frac{R_S}{2Z_0}\sqrt{\frac{Z_0 C_L}{T_d}}$
Forward or Destination end damping	$\omega_n = \frac{1}{\sqrt{Z_0 C_L T_d}}, \ \xi = \frac{Z_0}{2R_L}\sqrt{\frac{T_d}{Z_0 C_L}}$

Two Port Model of Transmission Line

$$\begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma(s)l) & -Z_0(s)\sinh(\gamma(s)l) \\ -\frac{1}{Z_0(s)}\sinh(\gamma(s)l) & \cosh(\gamma(s)l) \end{bmatrix} \begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix}$$

$$Z_0(s) = \sqrt{\frac{(R+sL)}{(G+sC)}} \qquad \gamma(s) \triangleq \sqrt{(R+sL)(G+sC)}$$

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INCIDENCE MATRIX CONVENTION

$$\begin{cases} +1 \text{ edge } k \text{ leaves vertex } i \\ -1 \text{ edge } k \text{ enters vertex } i \\ \ \ 0 \text{ edge } k \text{ does not touch vertex } i \end{cases}$$

1. Given a connected digraph **G**, a ***loop*** **L** is defined to be a connected *subgraph* of **G** in which precisely two edges (branches) are incident with each node.
2. Any loop formed by branches of a circuit is called a ***mesh*** iff the loop encloses no other branches, or wires in its interior.
3. Given a connected digraph **G**, a subset of branches **C** of **G** is called a ***cut set*** iff the following two conditions are satisfied:
   - the removal of all the branches of the cut set results in a digraph that is not a connected digraph, and
   - the removal of all but any one branch of **G** leaves the digraph connected.
4. A (spanning) ***tree*** T of a connected graph G is a subgraph which satisfies three fundamental properties:
   - It is connected.
   - It contains all connected vertices of G.
   - It has no loops (circuits).
5. Given a tree **T**, the edges of **G** can be partitioned into two disjoint sets:
   - Edges which belong to **T**, called tree branches or ***twigs*** for short
   - Edges which do not belong to **T**, called *links* or chords or cotrees branches.
6. Every twig of **T** together with some links defines a unique cut set, called the ***fundamental cut set*** associated with the twig.
7. Every link of **T** and the unique path on the tree between its two nodes constitutes a unique loop, called the ***fundamental loop*** associated with the link.
8. If $\mathbf{S}$ is the incidence for a given connected digraph G contained with one node used as the datum, a theorem due to Kirchoff states that the number of spanning trees is given by $\det(\mathbf{S}\mathbf{S}^T)$.

LAPLACE TRANSFORMS

F(s)	f (t) , t > 0
$Y(s) = \int_0^{\infty} \exp(-st)\, y(t)\, dt$	y(t)
Y(s)	$y(t) = \frac{1}{j2\pi} \int_{c-j\infty}^{c+j\infty} \exp(st)\, Y(s)\, ds$
$s^n\, Y(s) - s^{n-1}\,[y(0)]$ $-s^{n-2}\,[y'(0)] - \ldots - s\left[y^{(n-2)}(0)\right]$ $-[y^{(n-1)}(0)]$	$y^{(n)}(t)$
(1/s) F(s)	$\int_0^t Y(\tau)\, d\tau$
F(s)G(s)	$\int_0^t f(t-\tau) g(\tau)\, d\tau$
$\frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$	f (αt)
F(s - α)	exp(-αt) f(t)
1	δ(t)
$\exp(-\alpha s), \quad \alpha \ge 0$	δ(t-α)
1/s	u(t)
$\frac{1}{s} \exp(-\alpha s)$	u(t-α)
$\frac{1}{s^n}, \quad n = 1, 2, 3, \ldots$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{(s+\alpha)^n}, \quad n = 1, 2, 3, \ldots$	$\left[\frac{t^{n-1}}{(n-1)!}\right] \exp(-\alpha t)$
$\frac{\alpha}{s(s+\alpha)}$	1 - exp(-αt)
$\frac{1}{(s+\alpha)(s+\beta)}, \quad \beta \neq \alpha$	$\frac{1}{(\beta-\alpha)} [\exp(-\alpha t) - \exp(-\beta t)]$

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$\frac{s}{(s+\alpha)(s+\beta)}, \quad \beta \neq \alpha$	$\frac{1}{(\alpha-\beta)}\left[\alpha\exp(-\alpha t)-\beta\exp(-\beta t)\right]$
$\frac{\alpha}{s^2+\alpha^2}$	$\sin(\alpha t)$
$\frac{s}{s^2+\alpha^2}$	$\cos(\alpha t)$
$\frac{s^2-\alpha^2}{\left[s^2+\alpha^2\right]^2}$	$t\cos(\alpha t)$
$\frac{\alpha}{s^2(s+\alpha)}$	$t-\frac{1}{\alpha}\left[1-\exp(-\alpha t)\right]$
$\frac{s+\lambda}{(s+\alpha)^2+\beta^2}$	$\exp(-\alpha t)\left\{\cos(\beta t)+\left[\frac{\lambda-\alpha}{\beta}\right]\sin(\beta t)\right\}$
$\frac{s+\alpha}{s^2+\beta^2}$	$\frac{\sqrt{\alpha^2+\beta^2}}{\beta}\sin(\beta t+\phi), \quad \phi=\arctan\left(\frac{\beta}{\alpha}\right)$

$f(0^+)=\lim_{s\to\infty} sF(s)$ $\qquad$ $f(\infty)=\lim_{s\to 0} sF(s)$

DIFFERENTIATION

$$\frac{d\left(g(h(x))\right)}{dx}=\frac{d\left(g(h(x))\right)}{dh}\frac{d\left(h(x)\right)}{dx}$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right)=f(x)\cdot g'(x)+g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1} \qquad \frac{d}{dx}\left(\ln x\right)=\frac{1}{x}$$

$$\frac{d}{dx}\left(a^x\right)=\ln a\cdot a^x \qquad \frac{d}{dx}\left(\sin x\right)=\cos x$$

$$\frac{d}{dx}\left(\cos x\right)=-\sin x \qquad \frac{d}{dx}\left(\tan x\right)=\sec^2 x$$

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$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arc}\sec x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

INTEGRATION

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int a^x dx = \frac{1}{\ln a} a^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \csc^2 x dx = -\cot x + c$$

$$\int \frac{dx}{1+x^2} = \arctan x + c$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$$

$$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{1}{\sqrt{b^2-4ac}} \ln\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right) & , b^2-4ac \geq 0 \\ \dfrac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}} \right) & , b^2-4ac < 0 \end{cases}$$