The Laplace Transform

$$F(s) = \mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}dt$$

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{j2\pi}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st}ds$$

Properties of the Laplace Transform (one sided)

Linearity: $\mathcal{L}[af_1(t) + bf_2(t)] = a\mathcal{L}[f_1(t)] + b\mathcal{L}[f_2(t)]$

Time Differentiation:

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1}f(0) - s^{n-2}\left.\frac{df}{dt}\right|_{t=0} - \cdots - \left.\frac{d^{n-1}f}{dt^{n-1}}\right|_{t=0}$$

Time Integration: $\mathcal{L}\left[\int_{0^-}^{t} f(t')dt'\right] = F(s)/s$

Scaling: $\mathcal{L}[f(at)] = \frac{1}{|a|}F\left(\frac{s}{a}\right)$

Time Shift: $\mathcal{L}[f(t-\tau)] = F(s)e^{-\tau s}$

Frequency Shift: $\mathcal{L}\left[e^{-at}f(t)\right] = F(s+a)$

Initial Value: $\lim_{s\to\infty} sF(s) = f(0)$

Final Value: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

R L C V-I Relations in the Laplace domain

$$i = C\frac{dv}{dt} \qquad v = L\frac{di}{dt} \qquad v = iR$$

$$V(s) = \left(\frac{1}{Cs}\right)I(s) + \frac{v(0^-)}{s} \qquad V(s) = LsI(s) - Li(0^-) \qquad V(s) = RI(s)$$

Solution to first order Differential Equation

$$\tau\frac{dx}{dt} + x(t) = B \qquad x(t) = Ae^{\left(-\frac{t}{\tau}\right)} + B$$

$$\tau = RC \ \text{ or } \ L/R$$

$$B = x(\infty), \quad A = x(0) - x(\infty)$$

Roots of a Quadratic equation

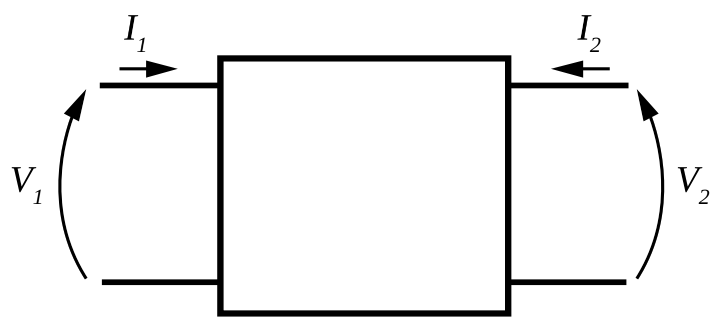
$ax^2 + bx + c = 0$ has roots $\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laplace Transform Pairs

$f(t)$	$F(s)$	$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$	$\left(\frac{1}{\omega}\sin\omega t\right)u(t)$	$\frac{1}{s^2+\omega^2}$
$t.u(t)$	$\frac{1}{s^2}$	$\cos\omega t.u(t)$	$\frac{s}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$(1-\cos\omega t)u(t)$	$\frac{\omega^2}{s(s^2+\omega^2)}$
$e^{at}.u(t)$	$\frac{1}{s-a}$	$\sin(\omega t+\theta).u(t)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$t.e^{at}.u(t)$	$\frac{1}{(s-a)^2}$	$\cos(\omega t+\theta).u(t)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{at}u(t)$	$\frac{1}{(s-a)^n}$	$e^{-\alpha t}\sin\omega t.u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\frac{1}{(a-b)}(e^{at}-e^{bt})u(t)$	$\frac{1}{(s-a)(s-b)}$	$e^{-\alpha t}\cos\omega t.u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
$\left[\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(a-b)(c-b)}+\frac{e^{-ct}}{(a-c)(b-c)}\right]u(t)$	$\frac{1}{(s+a)(s+b)(s+c)}$	$\sinh\alpha t.u(t)$	$\frac{\alpha}{s^2-\alpha^2}$
$(1-e^{at})u(t)$	$\frac{-a}{s(s-a)}$	$\cosh\alpha t.u(t)$	$\frac{s}{s^2-\alpha^2}$

Two Port Parameters

Name	Function		Equation
	Express	In terms of	
Impedance (z)	V_1, V_2	I_1, I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$
Admittance (y)	I_1, I_2	V_1, V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$
Transmission (T)	V_1, I_1	V_2, $-I_2$	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse transmission (t)	V_2, I_2	V_1, $-I_1$	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
Hybrid (h)	V_1, I_2	V_2, I_1	$V_1 = \mathrm{h}_{11}I_1 + \mathrm{h}_{12}V_2$ $I_2 = \mathrm{h}_{21}I_1 + \mathrm{h}_{22}V_2$
Inverse hybrid (g)	V_2, I_1	V_1, I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$



RLC circuits

Resonant frequency: $\omega_0 = \dfrac{1}{\sqrt{LC}}$,

Series circuit: $\alpha = \dfrac{R}{2L}$, Parallel circuit: $\alpha = \dfrac{1}{2RC}$

Response to a DC forcing function V_f: $v(t) = v_f(t) + v_n(t) = V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Where: $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0{}^2}$

Miller's Theorem:

$Z_1 = \dfrac{Z}{1-K}$, $Z_2 = \dfrac{KZ}{K-1}$, $K = \dfrac{V_2}{V_1}$

Impedance and Frequency Scaling:

$R^* = bR$, $L^* = \dfrac{b}{a} L$, $C^* = \dfrac{1}{ab} C$

Differentiation

$$\frac{d\left(g(h(x))\right)}{dx}=\frac{d\left(g(h(x))\right)}{dh}\frac{d\left(h(x)\right)}{dx}$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right)=f(x)\cdot g'(x)+g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}e^x=e^x$$

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}\left(\ln x\right)=\frac{1}{x}$$

$$\frac{d}{dx}\left(a^x\right)=\ln a\cdot a^x$$

$$\frac{d}{dx}\left(\sin x\right)=\cos x$$

$$\frac{d}{dx}\left(\cos x\right)=-\sin x$$

$$\frac{d}{dx}\left(\tan x\right)=\sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right)=-\csc^2 x$$

$$\frac{d}{dx}\left(\sec x\right)=\sec x\tan x$$

$$\frac{d}{dx}\left(\arcsin x\right)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\arctan x\right)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\operatorname{arcsec} x\right)=\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\sinh x\right)=\cosh x$$

Integration

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\quad n\neq -1$$

$$\int \frac{1}{x}dx=\ln|x|+c$$

$$\int a^x dx=\frac{1}{\ln a}a^x+c$$

$$\int \sin x dx=-\cos x+c$$

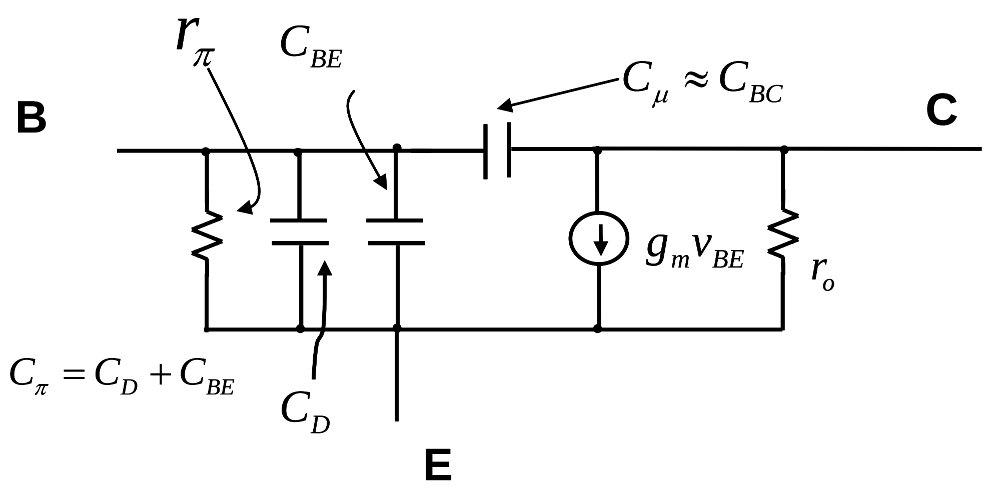
$$\int \cos x dx=\sin x+c$$

$$\int \csc^2 x dx=-\cot x+c$$

$$\int \frac{dx}{1+x^2}=\arctan x+c$$

$$\int \frac{dx}{\sqrt{1-x^2}}=\arcsin x+c$$

BJT Forward Active Region Hybrid π Model



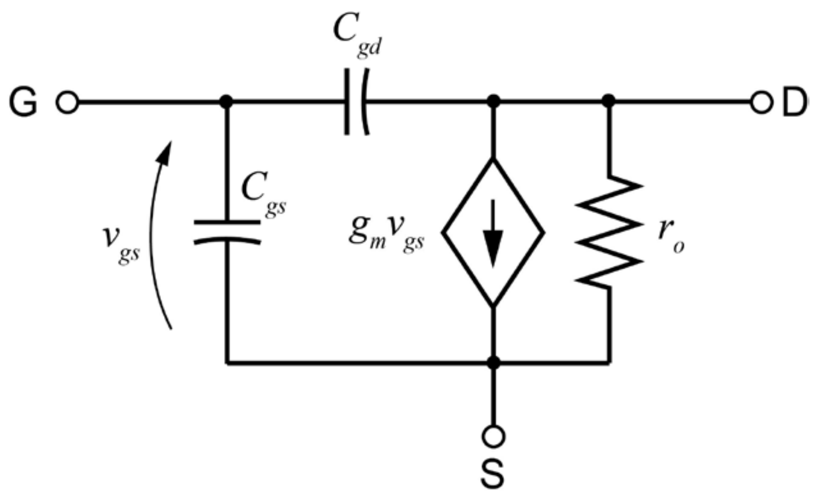
$I_E = (\beta+1)I_B$	$I_C = \beta I_B$ $I_E = (\beta+1)I_B$	$g_m = \frac{I_c}{V_T}$	$V_T = \frac{kT}{q}$
$r_\pi = \frac{\beta}{g_m}$	$\frac{1}{r_o} \approx \frac{I_C}{V_A}$	$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$	V_A Early Voltage

Enhancement FET Model

NMOS Transistor	PMOS Transistor
$K_n{}' = \mu_n C_{ox} = \mu_n \frac{\varepsilon_{ox}}{t_{ox}}$	$K_p{}' = \mu_p C_{ox} = \mu_p \frac{\varepsilon_{ox}}{t_{ox}}$
The cutoff region ($V_{GS}<V_t$): $I_{DS}=0$	The cutoff region ($V_{GS}>V_t$): $I_{DS}=0$
The linear region ($0<V_{DS}<V_{GS}-V_t$) $I_{DS} = K_n{}' \frac{W}{L}[(V_{GS}-V_t)V_{DS} - \frac{V_{DS}{}^2}{2}]$	The linear region ($0>V_{DS}>V_{GS}-V_t$) $I_{DS} = -K_p{}' \frac{W}{L}[(V_{GS}-V_t)V_{DS} - \frac{V_{DS}{}^2}{2}]$
The saturation region ($0< V_{GS}-V_t<V_{DS}$) $I_{DS} = \frac{K_n{}'}{2}\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$	The saturation region ($0> V_{GS}-V_t>V_{DS}$) $I_{DS} = -\frac{K_p{}'}{2}\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$

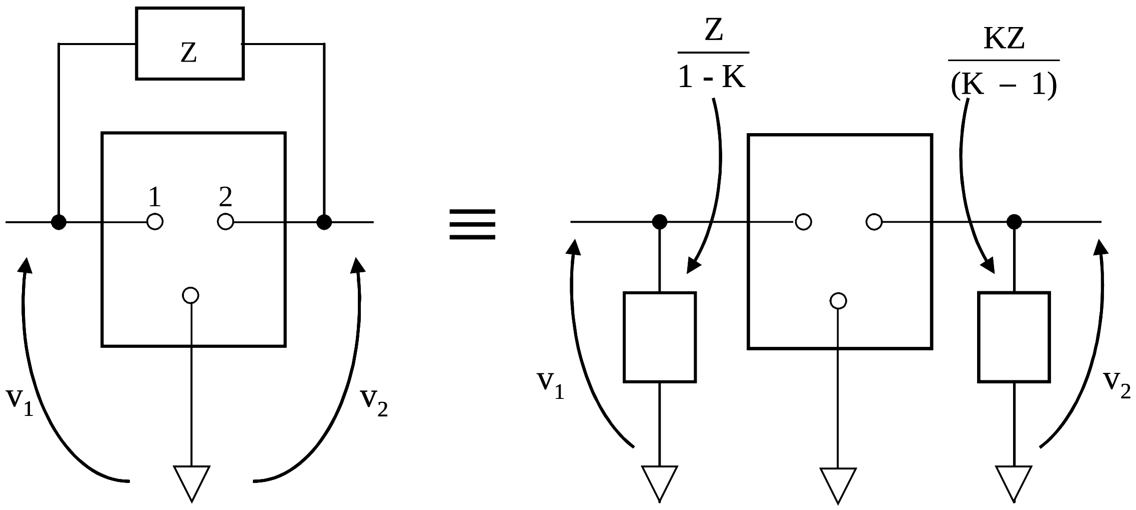
Enhancement N Channel FET Saturated Region Small Signal Model

(Source connected to substrate)



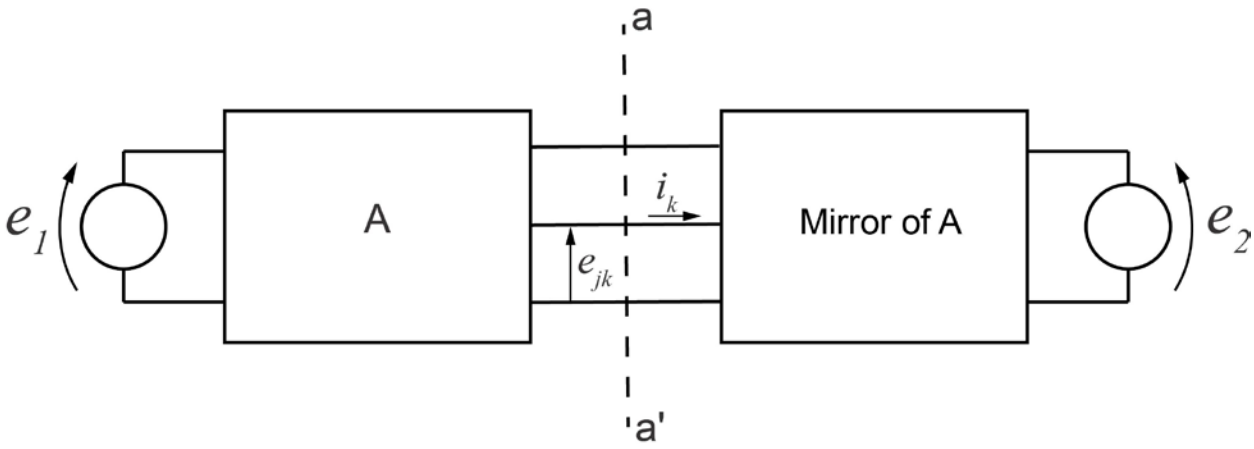
$g_m = \frac{2I_D}{V_{OV}}$	$r_O = \frac{1}{\lambda I_D}$
$V_{OV} \triangleq V_{GS} - V_T$ (*overdrive voltage*)	

Miller's Theorem

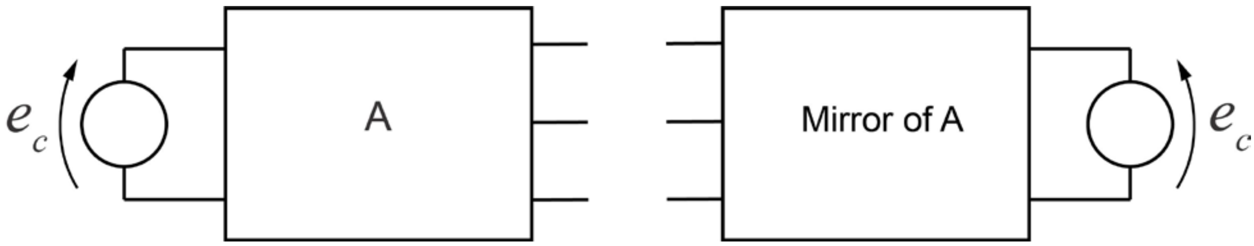


$$K \triangleq \frac{v_2}{v_1}$$

Bisection Theorem



(a) Bisection Theorem Common Mode Equivalence



(b) Bisection Theorem Differential Mode Equivalence