INFORMATION SHEET 2021 ELEC4401 Part II

Electromagnetic Field Equations

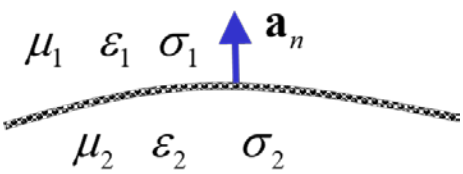
Differential Form	Integral Form	
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{S}$	Faraday's Law
$\nabla \bullet \mathbf{D} = \rho$	$\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho dV$	Gauss's Law
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	$\oint_C \mathbf{H} \bullet d\mathbf{l} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right) \bullet d\mathbf{S}$	Ampere's Law
$\nabla \bullet \mathbf{B} = 0$	$\oint_S \mathbf{B} \bullet d\mathbf{S} = 0$	
$\mathbf{B} = \nabla \times \mathbf{A} \quad \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi$	Potential Functions	
$\mathbf{A}(\mathbf{r}_2, t) = \oint_1 \frac{\mu_o I_1(t') d\mathbf{l}_1}{4\pi \lvert \mathbf{r}_2 - \mathbf{r}_1 \rvert}$ $t' = t - \frac{\lvert \mathbf{r}_2 - \mathbf{r}_1 \rvert}{c}$	Vector potential due a current element of length $d\mathbf{l}_1$	
$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} \quad \mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M})$	Fields in isotropic materials	
$\rho_D = -\nabla \bullet \mathbf{P}(\mathbf{r}) \quad \rho_S = -\mathbf{P}(\mathbf{r}) \bullet \mathbf{n}$	Volume and surface bound charge densities- isotropic materials	
$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$	Linear isotropic dielectric	
$\mathbf{E}_C(\mathbf{r}) = \frac{q(\mathbf{r} - \mathbf{r}_o)}{4\pi\varepsilon_o \lvert \mathbf{r} - \mathbf{r}_o \rvert^3}$	Point Charge Electric Field	
$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$	Magnetic Flux	
$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Lorentz Force	
$\nabla \bullet \mathbf{A} + \mu\varepsilon \frac{\partial \varphi}{\partial t} = 0$	Lorenz Gauge	
$\boldsymbol{\mathcal{P}} = \mathbf{E} \times \mathbf{H}$ (watts/m^2)	Poynting's Vector	
$w_E = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} \quad w_M = \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$ $-\frac{d}{dt} \int_V \left(\frac{1}{2} \mathbf{D} \cdot \mathbf{E} + \frac{1}{2} \mathbf{B} \cdot \mathbf{H} \right) d\mathrm{v} = \int_V \mathbf{J} \cdot \mathbf{E} \, d\mathrm{v} + \int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, d\mathrm{v}$		Energy Density

INFORMATION SHEET 2021 ELEC4401 Part II

Plane waves and Wave Impedance

$\gamma = \sqrt{-\left(\omega^2 \varepsilon\mu - j\omega\sigma\mu\right)} = \alpha + j\beta$	$u = \dfrac{\omega}{\beta}$ $\qquad \tan\vartheta = \dfrac{\sigma}{\omega\varepsilon}$
$\alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$	$\beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$
$\eta = \sqrt{\dfrac{\omega\mu}{\omega\varepsilon - j\sigma}} = \lvert\eta\rvert e^{j\vartheta_\eta}$	

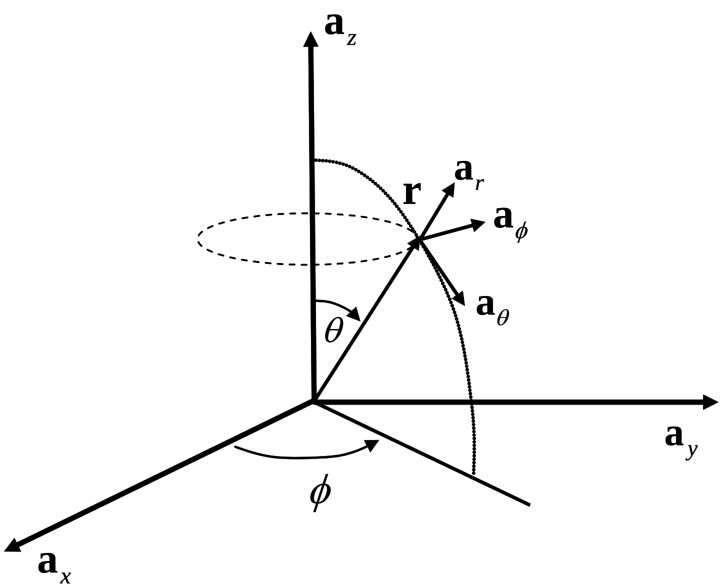
Boundary Conditions



$$\mathbf{a}_n \times \left(\mathbf{E}_1 - \mathbf{E}_2\right) = 0 \qquad \mathbf{a}_n \times \left(\mathbf{H}_1 - \mathbf{H}_2\right) = \mathbf{J}_s$$

$$\mathbf{a}_n \cdot \left(\mathbf{D}_1 - \mathbf{D}_2\right) = \rho_s \qquad \mathbf{a}_n \cdot \left(\mathbf{B}_1 - \mathbf{B}_2\right) = 0$$

Right Handed Co-ordinate System Unit Vectors



Vector Field Components in Cartesian Coordinates

$$A_x = \mathbf{A} \bullet \mathbf{a}_x \qquad A_y = \mathbf{A} \bullet \mathbf{a}_y \qquad A_z = \mathbf{A} \bullet \mathbf{a}_z$$

Vector Operations

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} = \left(A_y B_z - A_z B_y\right)\mathbf{a}_x + \left(A_z B_x - A_x B_z\right)\mathbf{a}_y + \left(A_x B_y - A_y B_x\right)\mathbf{a}_z$$

INFORMATION SHEET 2021 ELEC4401 Part II

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z \quad \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x \quad \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

$$\mathbf{a}_\rho \times \mathbf{a}_\phi = \mathbf{a}_z \quad \mathbf{a}_\phi \times \mathbf{a}_z = \mathbf{a}_\rho \quad \mathbf{a}_z \times \mathbf{a}_\rho = \mathbf{a}_\phi$$

$$\mathbf{a}_r \times \mathbf{a}_\theta = \mathbf{a}_\phi \quad \mathbf{a}_\theta \times \mathbf{a}_\phi = \mathbf{a}_r \quad \mathbf{a}_\phi \times \mathbf{a}_r = \mathbf{a}_\theta$$

$$\mathbf{A} \bullet (\mathbf{A} \times \mathbf{C}) = -\mathbf{A} \bullet (\mathbf{A} \times \mathbf{C}) = 0$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$$

$$\mathbf{A} = \mathbf{n} \times (\mathbf{A} \times \mathbf{n}) + (\mathbf{n} \bullet \mathbf{A})\mathbf{n} \qquad \mathbf{n} \bullet \mathbf{n} = 1$$

$$\nabla \equiv \mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z}$$

$$\nabla \times \nabla V = 0$$

$$\nabla \bullet (\nabla \times \mathbf{A}) = 0$$

$$\nabla \bullet (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \bullet (\nabla \times \mathbf{A}) - \mathbf{A} \bullet (\nabla \times \mathbf{B})$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \bullet \mathbf{A}) - \nabla^2 \mathbf{A}$$

$\nabla(\phi\chi) = \phi\nabla\chi + \chi\nabla\phi$, where ϕ and χ are any scalar fields.

$$\nabla\left(e^{-j\mathbf{k}\bullet\mathbf{r}}\right) = -j\mathbf{k}e^{-j\mathbf{k}\bullet\mathbf{r}}$$

$$\nabla^2\left(\mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}\right) = -(\mathbf{k} \bullet \mathbf{k})\mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}$$

$$\nabla \bullet \left(\mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}\right) = -j\mathbf{k} \bullet \mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}.$$

$$\nabla \times \left(\mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}\right) = -j\mathbf{k} \times \mathbf{A}e^{-j\mathbf{k}\bullet\mathbf{r}}$$

Divergence Theorem (Gauss Theorem) for a surface S enclosing a volume V

$$\oint_S \mathbf{A} \bullet d\mathbf{S} = \int_V \nabla \bullet \mathbf{A}\, dV$$

Stokes' Theorem for an open surface S with closed boundary L

$$\oint_L \mathbf{A} \bullet d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \bullet d\mathbf{S}$$

Helmholtz Transport Theorem

$$\frac{d}{dt}\int_{S(t)} \mathbf{F} \bullet d\mathbf{S} = \int_{S(t)} \left(\frac{\partial \mathbf{F}}{\partial t} + \mathbf{v}\nabla \bullet \mathbf{F} - \nabla \times (\mathbf{v} \times \mathbf{F})\right) \bullet d\mathbf{S}$$

INFORMATION SHEET 2021 ELEC4401 Part II

Green's Theorem: $\oint_S (\chi \mathbf{b}) \bullet d\mathbf{S} = \int_V (\chi \nabla \bullet \mathbf{b} + \mathbf{b} \bullet \nabla \chi) dV$

Physical constants

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Hm}^{-1} \quad \varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{Fm}^{-1} \quad e = 1.6 \times 10^{-19}\,\mathrm{C}$$

$$c = 3 \times 10^8\,\mathrm{m}s^{-1} \quad e/m = 1.76 \times 10^{11}\,\mathrm{Ckg}^{-1}$$

Some vector differential operators.

Gradient

$$\nabla \varphi = \frac{\partial \varphi}{\partial x}\mathbf{a}_x + \frac{\partial \varphi}{\partial y}\mathbf{a}_y + \frac{\partial \varphi}{\partial z}\mathbf{a}_z \qquad \nabla \varphi = \frac{\partial \varphi}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial \varphi}{\partial \phi}\mathbf{a}_\phi + \frac{\partial \varphi}{\partial z}\mathbf{a}_z$$

Laplacian

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \qquad \nabla^2 \varphi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \varphi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \varphi}{\partial \phi^2} + \frac{\partial^2 \varphi}{\partial z^2}$$

Curl of a vector field **F** in spherical co-ordinates:

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(F_\phi \sin\theta) - \frac{\partial F_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right]\mathbf{a}_\phi$$

DIFFERENTIATION

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x \qquad \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\mathrm{arc}\sec x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\sinh x) = \cosh x$$

INTEGRATION

INFORMATION SHEET 2021 ELEC4401 Part II

$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$

$\int \frac{1}{x} dx = \ln|x| + c$

$\int a^x dx = \frac{1}{\ln a} a^x + c$

$\int \sin x dx = -\cos x + c$

$\int \cos x dx = \sin x + c$

$\int \csc^2 x dx = -\cot x + c$

$\int \frac{dx}{1+x^2} = \arctan x + c$

$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$

$$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{1}{\sqrt{b^2-4ac}} \ln\left(\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right) & , b^2-4ac \geq 0 \\ \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) & , b^2-4ac < 0 \end{cases}$$

LAPLACE TRANSFORMS

F(s)	f (t) , t > 0
$Y(s) = \int_0^\infty \exp(-st) y(t)\, dt$	y(t)
Y(s)	$y(t) = \frac{1}{j2\pi} \int_{c-j\infty}^{c+j\infty} \exp(st) Y(s) ds$
$s^n Y(s) - s^{n-1}[y(0)] - s^{n-2}[y'(0)] - \dots - s\left[y^{(n-2)}(0)\right] - [y^{(n-1)}(0)]$	$y^{(n)}(t)$
(1/s) F(s)	$\int_0^t Y(\tau) d\tau$
F(s)G(s)	$\int_0^t f(t-\tau) g(\tau) d\tau$
$\frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$	f (αt)
F(s - α)	exp(-αt) f(t)
1	δ(t)

INFORMATION SHEET 2021 ELEC4401 Part II

$\exp(-\alpha s),\quad \alpha \geq 0$	$\delta(t-\alpha)$
$1/s$	$u(t)$
$\frac{1}{s}\exp(-\alpha s)$	$u(t-\alpha)$
$\frac{1}{s^n},\quad n = 1, 2, 3, \ldots$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{(s+\alpha)^n},\quad n = 1, 2, 3, \ldots$	$\left[\frac{t^{n-1}}{(n-1)!}\right]\exp(-\alpha t)$
$\frac{\alpha}{s(s+\alpha)}$	$1 - \exp(-\alpha t)$
$\frac{1}{(s+\alpha)(s+\beta)},\quad \beta \neq \alpha$	$\frac{1}{(\beta-\alpha)}[\exp(-\alpha t) - \exp(-\beta t)]$
$\frac{s}{(s+\alpha)(s+\beta)},\quad \beta \neq \alpha$	$\frac{1}{(\alpha-\beta)}[\alpha \exp(-\alpha t) - \beta \exp(-\beta t)]$
$\frac{\alpha}{s^2+\alpha^2}$	$\sin(\alpha t)$
$\frac{s}{s^2+\alpha^2}$	$\cos(\alpha t)$
$\frac{s^2-\alpha^2}{\left[s^2+\alpha^2\right]^2}$	$t\cos(\alpha t)$
$\frac{\alpha}{s^2(s+\alpha)}$	$t - \frac{1}{\alpha}[1 - \exp(-\alpha t)]$
$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\exp(-\alpha t)\sin(\beta t)$
$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\exp(-\alpha t)\sin(\beta t)$
$\frac{s+\lambda}{(s+\alpha)^2+\beta^2}$	$\exp(-\alpha t)\left\{\cos(\beta t) + \left[\frac{\lambda-\alpha}{\beta}\right]\sin(\beta t)\right\}$
$\frac{s+\alpha}{s^2+\beta^2}$	$\frac{\sqrt{\alpha^2+\beta^2}}{\beta}\sin(\beta t+\phi),\quad \phi = \arctan\left(\frac{\beta}{\alpha}\right)$