Formula sheet

1. Complex power and power factor

$$\boldsymbol{S} = |V||I|(\cos(\theta-\gamma) \ + j\sin(\theta-\gamma)) = P + jQ = \boldsymbol{VI}^*,$$

where $\boldsymbol{V} = |V|\angle\theta, \boldsymbol{I} = |I|\angle\gamma$.

$$PF = \cos(\theta-\gamma) = \cos\phi$$

2. Balance three-phase voltage and current relations

For Y –connected components,

$$V_{1\phi} = V_{L-N} = \frac{1}{\sqrt{3}}V_{L-L}, I_{1\phi} = I_{L-L}.$$

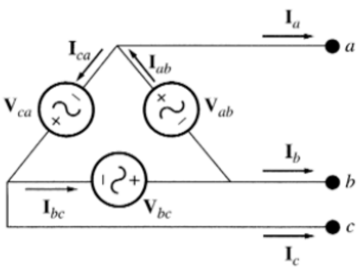
For Δ –connected components,

$$V_{1\phi} = V_{L-L}, I_{1\phi} = \frac{1}{\sqrt{3}}I_{L-L}.$$

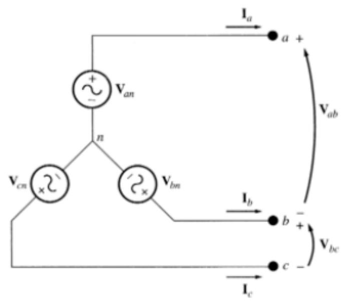
Three-phase power,

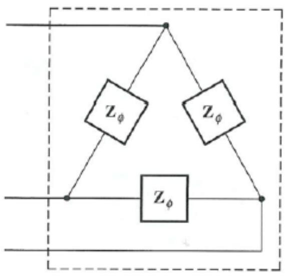
$$S_{3\phi} = 3|V_{1\phi}||I_{1\phi}| = \sqrt{3}|V_{L-L}||I_{L-L}| (\text{apparent power})$$
$$P_{3\phi} = 3|V_{1\phi}||I_{1\phi}|\cos\theta = \sqrt{3}|V_{L-L}||I_{L-L}|\cos\theta \ \ (\text{active power})$$
$$Q_{3\phi} = 3|V_{1\phi}||I_{1\phi}|\sin\theta = \sqrt{3}|V_{L-L}||I_{L-L}|\sin\theta \ \ (\text{reactive power})$$

$\Delta - Y$ transformation for balanced three phase source and load

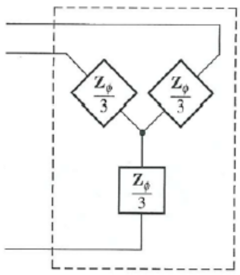


$$\boldsymbol{V_Y} = \frac{\boldsymbol{V_\Delta}}{\sqrt{3}\angle 30^o}$$





$$\boldsymbol{Z_Y} = \frac{\boldsymbol{Z_\Delta}}{3}$$



3. Transmission lines two-port network model

$$\begin{bmatrix} \boldsymbol{V_S} \\ \boldsymbol{I_S} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \boldsymbol{V_R} \\ \boldsymbol{I_R} \end{bmatrix}$$

Short:

$$A = 1,\ B = \boldsymbol{Z},$$
$$C = 0,\ D = 1,$$
$$\boldsymbol{Z} = \boldsymbol{z}l.$$

Medium-length:

$$A = D = 1 + \frac{\boldsymbol{YZ}}{2}; B = \boldsymbol{Z}; C = Y\left(1 + \frac{\boldsymbol{YZ}}{4}\right);$$
$$\boldsymbol{Y} = \boldsymbol{y}l; \boldsymbol{Z} = \boldsymbol{z}l.$$

Long:

$$A = \frac{\boldsymbol{Z'Y'}}{2} + 1 \qquad B = \boldsymbol{Z'}$$

$$C = Y'\left(\frac{\boldsymbol{Y'Z'}}{4} + 1\right) \qquad D = \frac{\boldsymbol{Z'Y'}}{2} + 1$$

$$\boldsymbol{Z'} = \boldsymbol{Z}\frac{\sinh \gamma l}{\gamma l}, \boldsymbol{Y'} = \boldsymbol{Y}\frac{\tanh(\gamma l/2)}{\gamma l/2}, \gamma = \sqrt{\boldsymbol{yz}}, \boldsymbol{Z} = \boldsymbol{z}l, \boldsymbol{Y} = \boldsymbol{y}l.$$

Hyperbolic relations for complex number calculations:

$$\cosh(x + jy) = \cosh(x)\cos(y) + j\sinh(x)\sin(y),$$
$$\sinh(x + jy) = \sinh(x)\cos(y) + j\cosh(x)\sin(y),$$

$$\tanh(x + jy) = \frac{\sinh(x + jy)}{\cosh(x + jy)}.$$

4. Per unit transformation

$$I_{1\phi,base} = \frac{S_{1\phi,base}}{V_{1\phi,\ base}}, Z_{\phi,base} = \frac{V_{1\phi,base}}{I_{1\phi,base}}, Z_{1\phi,\ base} = \frac{\left(V_{1\phi,base}\right)^2}{S_{1\phi,base}},$$

$$I_{1\phi,base} = \frac{S_{3\phi,base}}{3V_{1\phi,\ base}}, Z_{1\phi,base} = \frac{3\left(V_{1\phi,base}\right)^2}{S_{3\phi,base}}.$$

$$\boldsymbol{Z_{base2}} = \boldsymbol{Z_{base1}} \cdot \left(\frac{\boldsymbol{V_{base2}}}{\boldsymbol{V_{base1}}}\right)^2 \cdot \frac{\boldsymbol{S_{base1}}}{\boldsymbol{S_{base2}}}$$

5. Admittance matrix

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix},$$

$$\text{where } Y_{ik} = \begin{cases} -y_{ik}, & if\ i \neq k, \\ \displaystyle\sum_{k=0,\ k\neq i}^{n} y_{ik}, & \text{otherwise} \end{cases}$$

($k = 0$ for shunt admittance of a particular bus)

6. **Active power and reactive power balance equations**

$$S_i = S_{Gi} - S_{Di} = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

and

$$P_i = P_{Gi} - P_{Di} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}),$$
$$Q_i = Q_{Gi} - Q_{Di} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}).$$

Note:

- P_i and Q_i are the net active and reactive power injected into busbar i. '+' indicates power is being injected into the grid, and '-' indicates power is being absorbed from the grid.
- P_{Gi} and P_{Di}, both positive values, are the active power generation and demand.
- Q_{Gi} and Q_{Di}, both positive values, are the reactive power generation and demand.

7. **Gauss-Seidel method general form**

$$x = f(x).$$

Do

$$x^{(v+1)} = f\left(x^{(v)}\right).$$

until

$$\left|x^{(v+1)} - x^{(v)}\right| < \varepsilon$$

8. **Power balance equation in Gauss Seidel method format**

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right),$$
$$Q_i^{(v)} = -Im\left[V_i^{(v)*}\sum_{k=1}^{n} Y_{ik}V_k^{(v)}\right].$$

Note: please see note under item 5 above for sign convention.

9. **Newton Raphson method general form**

$$f(x) = 0.$$

Do

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)},$$
$$\Delta x^{(v)} = J^{-1}\left(x^{(v)}\right)\left(0 - f\left(x^{(v)}\right)\right),$$

until

$$\left|0 - f\left(x^{(v)}\right)\right| < \varepsilon$$

Jacobian matrix:

$$J(x) = \frac{\partial f(x)}{\partial x}$$

10. The inverse of a 3-by-3 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

then

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix},$$

where $|\cdot|$ indicates the determinant of a matrix, and

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}).$$

11. RL circuit transience

Component	Instantaneous current (A)	RMS current (A)
Symmetrical (AC)	$i_{ac}(t) = \frac{\sqrt{2}V}{Z} sin\,(\omega t + \alpha - \theta)$	$I_{ac} = \frac{V}{Z}$
DC offset	$i_{dc}(t) = -\frac{\sqrt{2}V}{Z} sin\,(\alpha - \theta) e^{\frac{-t}{T}}$	
Asymmetrical (total)	$i(t) = i_{ac}(t) + i_{dc}(t)$	$I_{rms}(t) = \sqrt{I_{ac}^2 + i_{dc}^2(t)}$
Asymmetrical total current with maximum DC offset		$I_{rms}(t) = I_{ac}\sqrt{1 + 2e^{\frac{-2t}{T}}}$

where time constant

$$T = \frac{L}{R} = \frac{X}{2\pi f R},$$

Angle α is the phase angle of the AC power source, and angle θ is the angle of the impedance, which is obtained by

$$\theta = \arctan\frac{\omega L}{R} = \arctan\frac{X}{R}.$$

12. Three-phase short circuit fault for synchronous generators

Component	Instantaneous current (A)	RMS current (A)
Symmetrical (AC)	$i_{ac}(t) = \sqrt{2}E_g\left[\left(\frac{1}{X_d''}-\frac{1}{X_d'}\right)e^{-t/T_d''}+\left(\frac{1}{X_d'}-\frac{1}{X_d}\right)e^{-t/T_d'} + \frac{1}{X_d}\right]\cdot \sin(\omega t+\alpha-\frac{\pi}{2})$	$I_{ac} = E_g\left[\left(\frac{1}{X_d''}-\frac{1}{X_d'}\right)e^{-t/T_d''} + \left(\frac{1}{X_d'}-\frac{1}{X_d}\right)e^{-t/T_d'}+\frac{1}{X_d}\right]$
Sub-transient		$I'' = \frac{E_g}{X_d''}$
Transient		$I' = \frac{E_g}{X_d'}$
Steady-state		$I = \frac{E_g}{X_d}$
Maximum DC offset	$i_{dc}(t) = \sqrt{2}I''e^{-\frac{t}{T_A}}$	
Asymmetrical (total)	$i(t) = i_{ac}(t) + i_{dc}(t)$	$I_{rms}(t) = \sqrt{I_{ac}^2 + i_{dc}^2(t)}$
Asymmetrical total current with maximum DC offset		$I_{rms}(t) = \sqrt{I_{ac}^2 + \left(\sqrt{2}I''e^{\frac{-t}{T_A}}\right)^2}$

T_d' and T_d'' are respectively the transient and sub-transient time constants, and T_A is the armature time constant.

13. Three-phase to ground fault for an N-bus system

Bus impedance matrix

$$Z_{bus} = Y_{bus}^{-1},$$

and

$$\begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \\ \vdots \\ V_i^{(1)} \\ \vdots \\ V_n^{(1)} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1,n-1} & Z_{1,n} \\ Z_{21} & Z_{22} & \cdots & Z_{2,n-1} & Z_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Z_{i,1} & Z_{i,2} & \cdots & Z_{i,n-1} & Z_{i,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Z_{n,1} & Z_{n,1} & \cdots & Z_{n,n-1} & Z_{n,n} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_F'' \\ \vdots \\ 0 \end{bmatrix}$$

With fault happening at node i,

$$V_i^{(1)} = -V_F = -Z_{ii}I_F''.$$

For $k = 1,2,\cdots n,\ \ k \neq i$

$$V_k^{(1)} = Z_{ki}\cdot(-I_F'') = -\frac{Z_{ki}}{Z_{ii}}\cdot V_F,$$

By applying superposition, we can obtain the busbar voltage as

$$V_{k,fault} = V_k^{(1)} + V_k^{(2)} = \left(1-\frac{Z_{ki}}{Z_{ii}}\right)V_F$$

where $V_k^{(2)}$ is the pre-fault voltage.

14. Phase-sequence transformation (only showing voltage, and current has the same relation)

$$\boldsymbol{V_S} = [V_a^0, V_a^+, V_a^-]^T, \boldsymbol{V_P} = [V_a, V_b, V_c]^T$$

$$\boldsymbol{V_P} = \boldsymbol{A}\boldsymbol{V_S} \qquad \boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$\boldsymbol{V_S} = \boldsymbol{A}^{-1}\boldsymbol{V_P} \qquad \boldsymbol{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \qquad \alpha = 1\angle 120^0$$

15. Impedance phase-sequence transformation:

$$\boldsymbol{Z_s} = \boldsymbol{A}^{-1}\boldsymbol{Z_p}\boldsymbol{A}$$

Balanced $\boldsymbol{Y}$ impedance load. The impedance of each phase is designated Z_Y, and a neutral impedance Z_n is connected between the load neutral and ground.

$$\boldsymbol{Z_S} = \boldsymbol{A}^{-1}\boldsymbol{Z_p}\boldsymbol{A} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}.$$

$$\boldsymbol{V_S} = \begin{bmatrix} V^0 \\ V^+ \\ V^- \end{bmatrix} = \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}\begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix}$$

Series impedance loads:

$$\boldsymbol{Z_p} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix}.$$

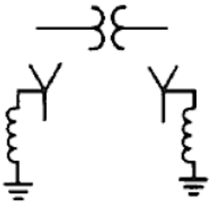
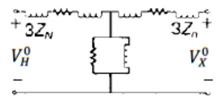
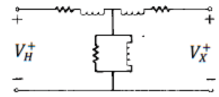
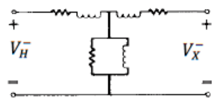
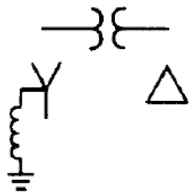
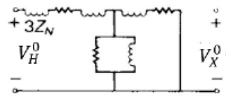
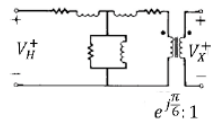
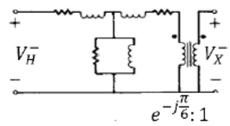
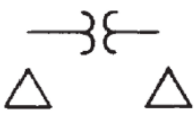
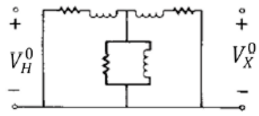
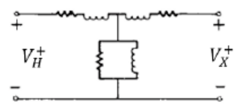
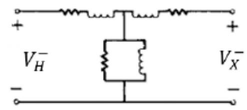
Assuming $Z_{aa} = Z_{bb} = Z_{cc}$, $Z_{ab} = Z_{ac} = Z_{bc}$, and the neutral is grounded (bolted), the sequence impedance is

$$\boldsymbol{Z_s} = \boldsymbol{A}^{-1}\boldsymbol{Z_p}\boldsymbol{A} = \begin{bmatrix} Z_{aa} + 2Z_{ab} & 0 & 0 \\ 0 & Z_{aa} - Z_{ab} & 0 \\ 0 & 0 & Z_{aa} - Z_{ab} \end{bmatrix}.$$

$$\boldsymbol{V_s} - \boldsymbol{V_s'} = \boldsymbol{Z_s}\boldsymbol{I_s}$$

$$V^0 - V^{0'} = Z^0 I^0$$
$$V^+ - V^{+'} = Z^+ I^+$$
$$V^- - V^{-'} = Z^- I^-$$

16. Sequence networks for transformers

Topology	Single-line diagram	Sequence network	
$Y - Y$		Zero sequence	
		Positive sequence	
		Negative sequence	
$Y - \Delta$		Zero sequence	
		Positive sequence	
		Negative sequence	
$\Delta - \Delta$		Zero sequence	
		Positive sequence	
		Negative sequence	

17. Steps for analysing faults:

(1) Draw sequence networks for the pre-fault balanced, uncoupled power system.
(2) Find the Thevenin equivalent impedance view from the fault terminal.
(3) Implement fault and calculate desired quantities.

18. General form for Lagrangian function

Minimize $f(x)$,
Subject to $g(x) = 0$.

The Lagrangian function is defined as

$$L(x, \lambda) = f(x) + \lambda^T g(x),$$

then a necessary condition for a minimum is

$$\nabla L_x(x, \lambda) = 0 \text{ and } \nabla L_\lambda(x, \lambda) = 0.$$

19. Single machine infinite bus (SMIB) output power, assuming infinite bus voltage angle is zero.

$$P_e = \frac{|E_a| \cdot |V_{bus}|}{|X_{eq}|} \sin \delta$$

20. Generator swing equation

$$P_m - P_e(\delta) = M \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt},$$

$$M = \frac{H}{\pi f_s}.$$

Left and right equilibrium points for angle δ_{EP1} and $\delta_{EP2}(= 180^o - \delta_{EP1})$ are sought by solving $P_m = P_e(\delta)$.

21. Equal area criterion for single machine infinite bus (SMIB) system

The accelerating area equals the decelerating area, i.e.,

$$\int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta = \int_{\delta_1}^{\delta_2} (P_e - P_m) d\delta,$$

where δ_0 is the left equilibrium point, δ_1 is the fault clearing angle, and δ_2 is the maximum rotor angle.

If $\delta_2 > \delta_{EP2}$, the SMIB system is considered unstable.

The fault clearing angle δ_1 that makes $\delta_2 = \delta_{EP2}$ is called the critical fault clearing angle, or δ_{cr}.

END OF ATTACHMENT