THE UNIVERSITY OF WESTERN AUSTRALIA

DESK No. | | | |

FAMILY NAME: ______________________

GIVEN NAMES: ______________________

SIGNATURE: ______________________

STUDENT NUMBER: | | | | | | | | |

SEMESTER 2, 2021 EXAMINATIONS - ATTACHMENTS

ENSC3015

Engineering

Electrical, Electronic and Computer Engineering

Signals and Systems

.

This paper contains: **12** Pages **(including title page)**

Time Allowed: **2:00** hours

INSTRUCTIONS:

Please use the included tables of pairs and properties and useful formulas as required.

THIS IS A CLOSED BOOK EXAMINATION (SEE ALLOWABLE ITEMS)

SUPPLIED STATIONERY	**ALLOWABLE ITEMS**
1 x Answer Booklet 18 Pages	**UWA Approved Calculator with Sticker** **Open Book with Student Notes**

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Zero-input Response

Real Roots (continuous-time systems)

Assume r of the N roots are real but identical, that is $\lambda_1 = \lambda_2 = \cdots = \lambda_r = \lambda$ with the remaining $(N-r)$ roots being distinct, that is $\lambda_{r+1}, \lambda_{r+2}, \dots, \lambda_N$:

$$y_0(t) = (c_1 + c_2 t + \cdots + c_r t^{r-1})e^{\lambda t} + c_{r+1}e^{\lambda_{r+1}t} + c_{r+2}e^{\lambda_{r+2}t} + \cdots + c_N e^{\lambda_N t}$$

Real Roots (discrete-time systems)

Assume r of the N roots are real but identical ($= \lambda$), with the remaining $(N-r)$ roots being distinct:

$$y_0[n] = (c_1 + c_2 n + \cdots + c_r n^{r-1})\lambda^n + \sum_{k=r+1}^{N} c_k \lambda_k^n$$

Convolution

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau$$

$$x_1[n] * x_2[n] = \sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m]$$

Partial Fraction Expansion

Simple, distinct poles, $d_i \neq d_j$ for $X(s)$ with $M < N$

$$X(s) = \sum_{k=1}^{N} \frac{A_k}{(s-d_k)}$$

where:

$$A_k = (s-d_k)X(s)|_{s=d_k} \quad \text{and} \quad \frac{A_k}{(s-d_k)} \leftrightarrow A_k e^{d_k t}u(t)$$

Simple, distinct poles, $d_i \neq d_j$ for $X(z)$ with $M < N$

$$X(z) = \sum_{k=1}^{N} \frac{A_k}{(1-d_k z^{-1})}$$

where:

$$A_k = (1-d_k z^{-1})X(z)|_{z=d_k} \quad \text{and} \quad \frac{A_k}{(1-d_k z^{-1})} \leftrightarrow A_k (d_k)^n u[n]$$

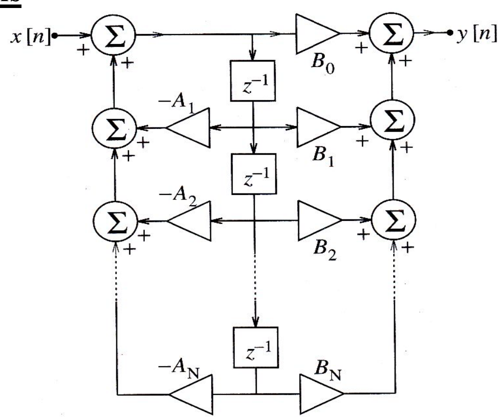
Causal and Stable Systems

For a continuous-time system with $M \leq N$ **all the poles of $H(s)$ must lie in the left half of the *s*-plane**, or left hand plane (LHP), that is $\text{Re}(d_k) < 0$, since the ROC is to the right of all poles (causal) and the $j\omega$-axis is included (stable). **If $M > N$ the system is deemed unstable**.

For a discrete-time system **all the poles of $H(z)$ must lie inside the unit circle,** that is $|d_k| < 1$, since the ROC is to the exterior of all poles (causal) and the unit circle is included (stable).

Direct Form II Realisation of Discrete-Time Systems

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} B_k z^{-k}}{1 + \sum_{k=1}^{N} A_k z^{-k}}$$



D.1 *Basic Laplace Transforms*

Signal $x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$	*Transform* $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$	ROC
$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
$tu(t)$	$\frac{1}{s^2}$	$\text{Re}\{s\} > 0$
$\delta(t-\tau), \quad \tau \ge 0$	$e^{-s\tau}$	for all s
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} > -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\text{Re}\{s\} > -a$
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	$\text{Re}\{s\} > 0$
$[\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	$\text{Re}\{s\} > 0$
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	$\text{Re}\{s\} > -a$
$[e^{-at}\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	$\text{Re}\{s\} > -a$

Signal	*Bilateral Transform*	*ROC*
$\delta(t-\tau), \tau < 0$	$e^{-s\tau}$	for all s
$-u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
$-tu(-t)$	$\frac{1}{s^2}$	$\text{Re}\{s\} < 0$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} < -a$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\text{Re}\{s\} < -a$

D.2 *Laplace Transform Properties*

Signal	Unilateral Transform $x(t) \xleftrightarrow{\mathcal{L}_u} X(s)$, $y(t) \xleftrightarrow{\mathcal{L}_u} Y(s)$	Bilateral Transform $x(t) \xleftrightarrow{\mathcal{L}} X(s)$, $y(t) \xleftrightarrow{\mathcal{L}} Y(s)$	ROC $s \in R_x$, $s \in R_y$
$ax(t) + by(t)$	$aX(s) + bY(s)$	$aX(s) + bY(s)$	At least $R_x \cap R_y$
$x(t-\tau)$	$e^{-s\tau}X(s)$ if $x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_o t}x(t)$	$X(s-s_o)$	$X(s-s_o)$	$R_x + \mathrm{Re}\{s_o\}$
$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right)$, $a > 0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$ if $x(t) = y(t) = 0$ for $t < 0$	$X(s)Y(s)$	At least $R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$	$sX(s)$	At least R_x
$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{s}\int_{-\infty}^{0^-} x(\tau)\, d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	At least $R_x \cap \{\mathrm{Re}\{s\} > 0\}$

Unilateral Transform: $\frac{d^n}{dt^n}x(t) \leftrightarrow s^n X(s) - \sum_{k=1}^{n} s^{n-k}x^{(k-1)}(0^-)$ where $x^{(r)}(0^-)$ is $\left.\frac{d^r x}{dt^r}\right|_{t=0^-}$

E.1 *Basic z-Transforms*

Signal	Transform	ROC
$x[n] = \frac{1}{2\pi j}\oint X(z)\, z^{n-1}\, dz$	$X[z] = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$	
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$[\cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}\cos\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[\sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}\sin\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}r\cos\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2 z^{-2}}$	$\lvert z\rvert > r$
$[r^n\sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}r\sin\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2 z^{-2}}$	$\lvert z\rvert > r$

Signal	*Bilateral Transform*	*ROC*
$u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$

E.2 *z-Transform Properties*

Signal	*Unilateral Transform* $x[n] \xleftrightarrow{z_u} X(z)$ $y[n] \xleftrightarrow{z_u} Y(z)$	*Bilateral Transform* $x[n] \xleftrightarrow{z} X(z)$ $y[n] \xleftrightarrow{z} Y(z)$	*ROC* $z \in R_x$ $z \in R_y$
$ax[n]+by[n]$	$aX(z)+bY(z)$	$aX(z)+bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	See below	$z^{-k}X(z)$	R_x, except possibly $\lvert z\rvert = 0, \infty$
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$	$X\left(\frac{z}{\alpha}\right)$	$\lvert\alpha\rvert R_x$
$x[-n]$	—	$X\left(\frac{1}{z}\right)$	$\frac{1}{R_x}$
$x[n]*y[n]$	$X(z)Y(z)$ if $x[n]=y[n]=0$ for $n<0$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x, except possibly addition or deletion of $z=0$

E.2.1 UNILATERAL Z-TRANSFORM TIME-SHIFT PROPERTY

$$x[n-k] \xleftrightarrow{z_u} x[-k] + x[-k+1]z^{-1} + \cdots + x[-1]z^{-k+1} + z^{-k}X(z) \quad \text{for } k>0$$

$$x[n+k] \xleftrightarrow{z_u} -x[0]z^{k} - x[1]z^{k-1} - \cdots - x[k-1]z + z^{k}X(z) \quad \text{for } k>0$$

Euler's Relation

$$e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)} = 2\cos(\omega t+\phi)$$
$$e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)} = 2j\sin(\omega t+\phi)$$

Trigonometric Formulas

$$2\sin A\sin B = \cos(A-B) - \cos(A+B) \qquad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$
$$2\cos A\sin B = \sin(A+B) - \sin(A-B) \qquad 2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B \qquad \cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$$

Sifting Property

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)*\delta(t) \qquad x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = x[n]*\delta[n]$$

Definite Integrals

$$\int_a^b x^n\,dx = \frac{1}{n+1}x^{n+1}\Big|_a^b, \qquad n \neq -1$$

$$\int_a^b e^{cx}\,dx = \frac{1}{c}e^{cx}\Big|_a^b$$

$$\int_a^b xe^{cx}\,dx = \frac{1}{c^2}e^{cx}(cx-1)\Big|_a^b$$

$$\int_a^b \cos(cx)\,dx = \frac{1}{c}\sin(cx)\Big|_a^b$$

$$\int_a^b \sin(cx)\,dx = -\frac{1}{c}\cos(cx)\Big|_a^b$$

$$\int_a^b x\cos(cx)\,dx = \frac{1}{c^2}(\cos(cx) + cx\sin(cx))\Big|_a^b$$

$$\int_a^b x\sin(cx)\,dx = \frac{1}{c^2}(\sin(cx) - cx\cos(cx))\Big|_a^b$$

$$\int_a^b e^{gx}\cos(cx)\,dx = \frac{e^{gx}}{g^2+c^2}(g\cos(cx) + c\sin(cx))\Big|_a^b$$

$$\int_a^b e^{gx}\sin(cx)\,dx = \frac{e^{gx}}{g^2+c^2}(g\sin(cx) - c\cos(cx))\Big|_a^b$$

Geometric Series

$$\sum_{n=0}^{M-1}\beta^n = \begin{cases} \dfrac{1-\beta^M}{1-\beta}, & \beta \neq 1 \\ M, & \beta = 1 \end{cases}$$

$$\sum_{n=k}^{l}\beta^n = \begin{cases} \dfrac{\beta^k-\beta^{l+1}}{1-\beta}, & \beta \neq 1 \\ l-k+1, & \beta = 1 \end{cases}$$

$$\sum_{n=0}^{\infty}\beta^n = \frac{1}{1-\beta}, \quad |\beta| < 1$$

$$\sum_{n=k}^{\infty}\beta^n = \frac{\beta^k}{1-\beta}, \quad |\beta| < 1$$

$$\sum_{n=-k}^{-\infty}\beta^n = \beta^{-k}\left(\frac{\beta}{\beta-1}\right), \quad |\beta| > 1$$

$$\sum_{n=0}^{\infty}n\beta^n = \frac{\beta}{(1-\beta)^2}, \quad |\beta| < 1$$

Quadratic formula

The solution to:

$$ax^2 + bx + c = 0$$

is:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

In the following table you can use any convenient period rather than the default $\int_0^T$ for the Fourier Series and $\sum_{n=0}^{N-1}$ for the Discrete-Time Fourier Series

TABLE 3.2 ***The Four Fourier Representations.***

Time Domain	*Periodic* (t, n)	*Non periodic* (t, n)	
Continuous (t)	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t}$ $X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}\,dt$ $x(t)$ has period T $\omega_o = \frac{2\pi}{T}$	Fourier Transform $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$ $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$	*Nonperiodic* (k, ω)
Discrete (n)	Discrete-Time Fourier Series $x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_o n}$ $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega_o n}$ $x[n]$ and $X[k]$ have period N $\Omega_o = \frac{2\pi}{N}$	Discrete-Time Fourier Transform $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$ $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$ $X(e^{j\Omega})$ has period 2π	*Periodic* (k, Ω)
	Discrete (k)	*Continuous* (ω, Ω)	*Frequency Domain*

$$g(t) \xleftrightarrow{FS;\ \omega_o = 2\pi/T} G[k]$$

$$v[n] \xleftrightarrow{DTFT} V(e^{j\Omega})$$

$$w[n] \xleftrightarrow{DTFS;\ \Omega_o = 2\pi/N} W[k]$$

C.8.1 FT Representation for a Continuous-Time Periodic Signal

$$g(t) \xleftrightarrow{FT} G(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} G[k]\delta(\omega - k\omega_o)$$

C.8.2 DTFT Representation for a Discrete-Time Periodic-Signal

$$w[n] \xleftrightarrow{DTFT} W(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} W[k]\delta(\Omega - k\Omega_o)$$

C.8.3 FT Representation for a Discrete-Time Nonperiodic Signal

$$v_\delta(t) = \sum_{n=-\infty}^{\infty} v[n]\delta(t - nT_s) \xleftrightarrow{FT} V_\delta(j\omega) = V(e^{j\Omega})\Big|_{\Omega=\omega T_s}$$

C.8.4 FT Representation for a Discrete-Time Nonperiodic Signal

$$w_\delta(t) = \sum_{n=-\infty}^{\infty} w[n]\delta(t - nT_s) \xleftrightarrow{FT} W_\delta(j\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty} W[k]\delta\left(\omega - \frac{k\Omega_o}{T_s}\right)$$

Fourier Transform Pairs

Time Domain	*Frequency Domain*
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
$x(t) = \begin{cases} 1, & \lvert t\rvert \le T_o \\ 0, & \text{otherwise} \end{cases}$	$X(j\omega) = \frac{2\sin(\omega T_o)}{\omega}$
$x(t) = \frac{1}{\pi t}\sin(Wt)$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert \le W \\ 0, & \text{otherwise} \end{cases}$
$x(t) = \delta(t)$	$X(j\omega) = 1$
$x(t) = 1$	$X(j\omega) = 2\pi\delta(\omega)$
$x(t) = u(t)$	$X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$
$x(t) = e^{-at}u(t), \quad \text{Re}\{a\} > 0$	$X(j\omega) = \frac{1}{a + j\omega}$
$x(t) = te^{-at}u(t), \quad \text{Re}\{a\} > 0$	$X(j\omega) = \frac{1}{(a + j\omega)^2}$
$x(t) = e^{-a\lvert t\rvert}, \quad a > 0$	$X(j\omega) = \frac{2a}{a^2 + \omega^2}$
$x(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$	$X(j\omega) = e^{-\omega^2/2}$

Periodic Time-Domain Signal	*Fourier Transform*
$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t}$	$X(j\omega) = 2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_o)$
$x(t) = \cos(\omega_o t)$	$X(j\omega) = \pi\delta(\omega - \omega_o) + \pi\delta(\omega + \omega_o)$
$x(t) = \sin(\omega_o t)$	$X(j\omega) = \frac{\pi}{j}\delta(\omega - \omega_o) - \frac{\pi}{j}\delta(\omega + \omega_o)$
$x(t) = e^{j\omega_o t}$	$X(j\omega) = 2\pi\delta(\omega - \omega_o)$
$x(t) = \sum_{n=-\infty}^{\infty}\delta(t - nT_s)$	$X(j\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega - k\frac{2\pi}{T_s}\right)$
$x(t) = \begin{cases} 1, & \lvert t\rvert \le T_o \\ 0, & T_o < \lvert t\rvert < T/2 \end{cases}$ $x(t + T) = x(t)$	$X(j\omega) = \sum_{k=-\infty}^{\infty}\frac{2\sin(k\omega_o T_o)}{k}\delta(\omega - k\omega_o)$

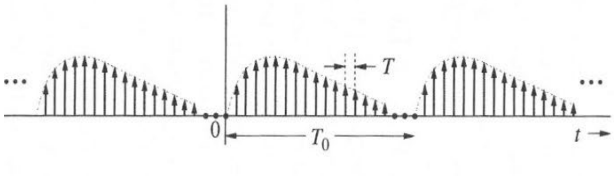
Fourier Transform Properties

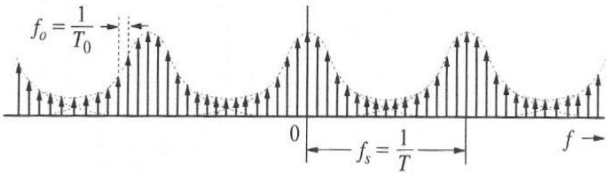
Property	Fourier Transform $x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$
Linearity	$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$
Time shift	$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$
Frequency shift	$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$
Scaling	$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Differentiation in time	$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$
Differentiation in frequency	$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$
Integration/ Summation	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega-\nu))\,d\nu$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$
Symmetry	$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \mathrm{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \mathrm{Re}\{X(j\omega)\} = 0$

Effect of Sampling

$$\sum_{n} x(nT)\delta(t-nT) \xleftrightarrow{FT} \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\omega - k\frac{2\pi}{T}\right)$$

Sampling in time and frequency (digital signal processing)





$$N_0 = \frac{T_0}{T} = \frac{f_S}{f_0},$$ where N_0 is the number of samples over T_0 or f_S

Discrete-Time Fourier Transform Pairs

Time Domain	*Frequency Domain*
$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$	$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
$x[n] = \begin{cases} 1, & \lvert n\rvert \le M \\ 0, & \text{otherwise} \end{cases}$	$X(e^{j\Omega}) = \dfrac{\sin\left[\Omega\left(\frac{2M+1}{2}\right)\right]}{\sin\left(\frac{\Omega}{2}\right)}$
$x[n] = \alpha^n u[n], \quad \lvert\alpha\rvert < 1$	$X(e^{j\Omega}) = \dfrac{1}{1-\alpha e^{-j\Omega}}$
$x[n] = \delta[n]$	$X(e^{j\Omega}) = 1$
$x[n] = u[n]$	$X(e^{j\Omega}) = \dfrac{1}{1-e^{-j\Omega}} + \pi\sum_{p=-\infty}^{\infty}\delta(\Omega - 2\pi p)$
$x[n] = \frac{1}{\pi n}\sin(Wn), \quad 0 < W \le \pi$	$X(e^{j\Omega}) = \begin{cases} 1, & \lvert\Omega\rvert \le W \\ 0, & W < \lvert\Omega\rvert \le \pi \end{cases}$ $X(e^{j\Omega})$ is 2π periodic
$x[n] = (n+1)\alpha^n u[n]$	$X(e^{j\Omega}) = \dfrac{1}{(1-\alpha e^{-j\Omega})^2}$

Periodic Time-Domain Signal	*Discrete-Time Fourier Transform*
$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_o n}$	$X(e^{j\Omega}) = 2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\Omega - k\Omega_o)$
$x[n] = \cos(\Omega_1 n)$	$X(e^{j\Omega}) = \pi\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_1 - k2\pi) + \delta(\Omega + \Omega_1 - k2\pi)$
$x[n] = \sin(\Omega_1 n)$	$X(e^{j\Omega}) = \frac{\pi}{j}\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_1 - k2\pi) - \delta(\Omega + \Omega_1 - k2\pi)$
$x[n] = e^{j\Omega_1 n}$	$X(e^{j\Omega}) = 2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_1 - k2\pi)$
$x[n] = \sum_{k=-\infty}^{\infty}\delta(n - kN)$	$X(e^{j\Omega}) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\Omega - \frac{k2\pi}{N}\right)$

Discrete-Time Fourier Transform Properties

Property	*Discrete-Time FT* $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$
Linearity	$ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\Omega}) + bY(e^{j\Omega})$
Time shift	$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o}X(e^{j\Omega})$
Frequency shift	$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Gamma)})$
Scaling	$x_z[n] = 0, \quad n \neq 0, \pm p, \pm 2p, \pm 3p, \ldots$ $x_z[pn] \xleftrightarrow{DTFT} X_z(e^{j\Omega/p})$
Differentiation in time	—
Differentiation in frequency	$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega}X(e^{j\Omega})$
Integration/ Summation	$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}}$ $+ \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$
Convolution	$\sum_{l=-\infty}^{\infty} x[l]y[n - l] \xleftrightarrow{DTFT} X(e^{j\Omega})Y(e^{j\Omega})$
Multiplication	$x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma})Y(e^{j(\Omega-\Gamma)})\, d\Gamma$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\Omega})\rvert^2\, d\Omega$
Duality	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$
Symmetry	$x[n]$ real $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = X(e^{-j\Omega})$ $x[n]$ imaginary $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = -X(e^{-j\Omega})$ $x[n]$ real and even $\xleftrightarrow{DTFT} \mathrm{Im}\{X(e^{j\Omega})\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFT} \mathrm{Re}\{X(e^{j\Omega})\} = 0$