

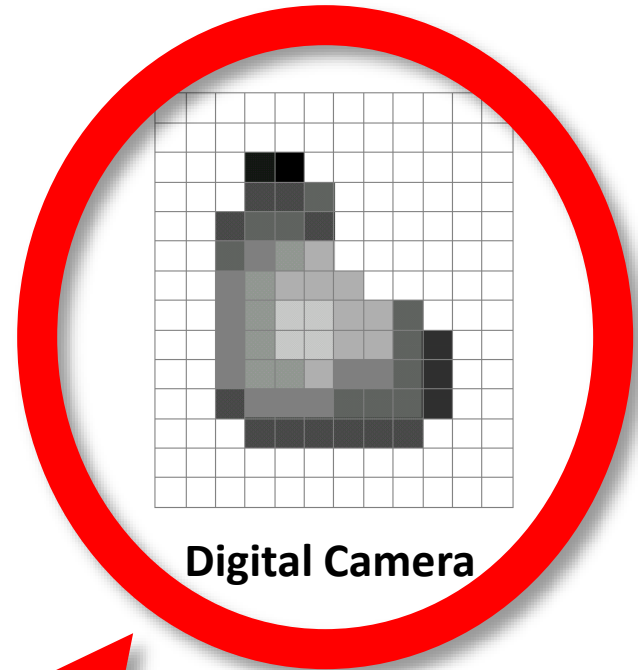
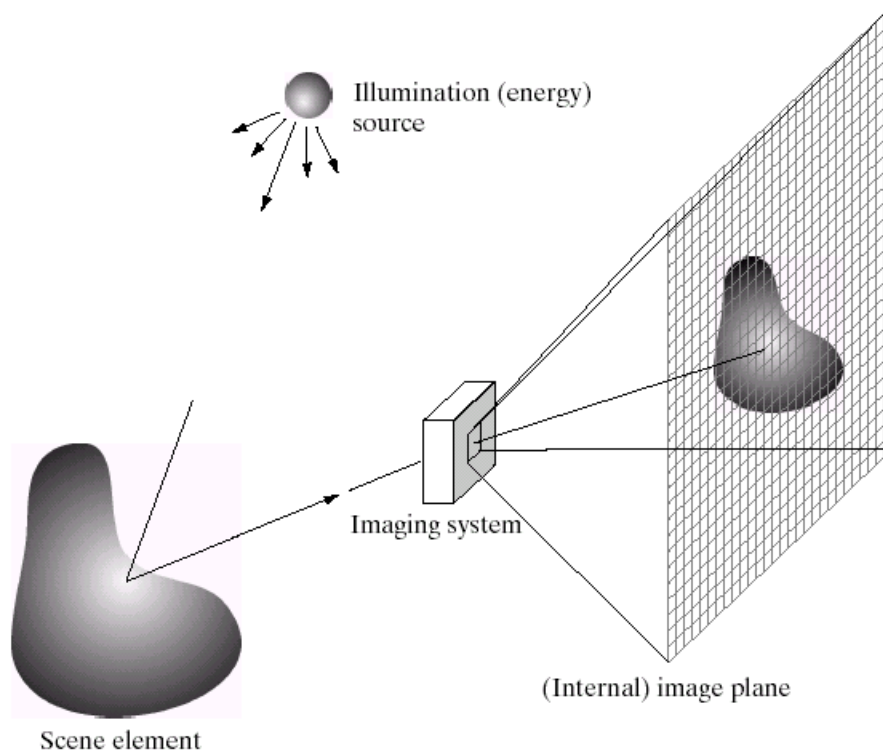
CS 231

Images and image filtering

What is an image?

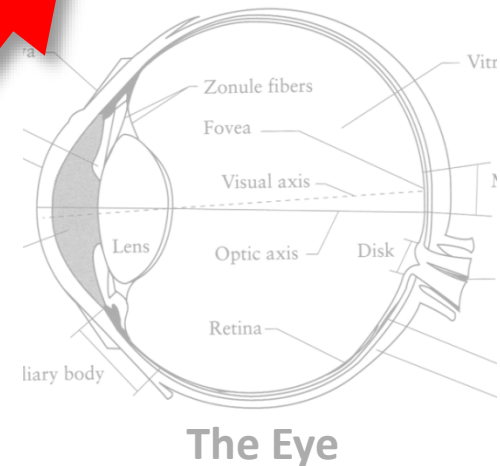


What is an image?



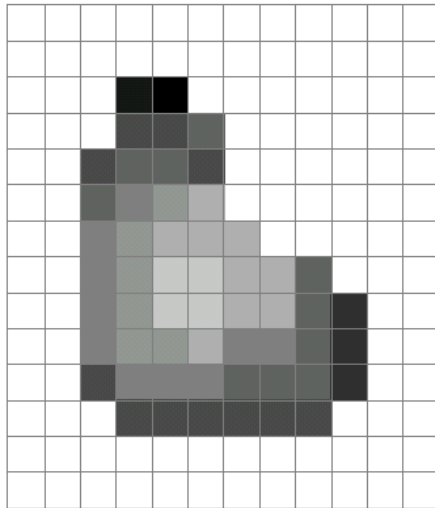
We'll focus on these in this class

(More on this process later)



What is an image?

- A grid (matrix) of intensity values



=

255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255	255	255	255
255	255	127	145	145	175	127	127	95	47	255	255	255	255	255
255	255	74	127	127	127	95	95	95	47	255	255	255	255	255
255	255	255	74	74	74	74	74	74	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255

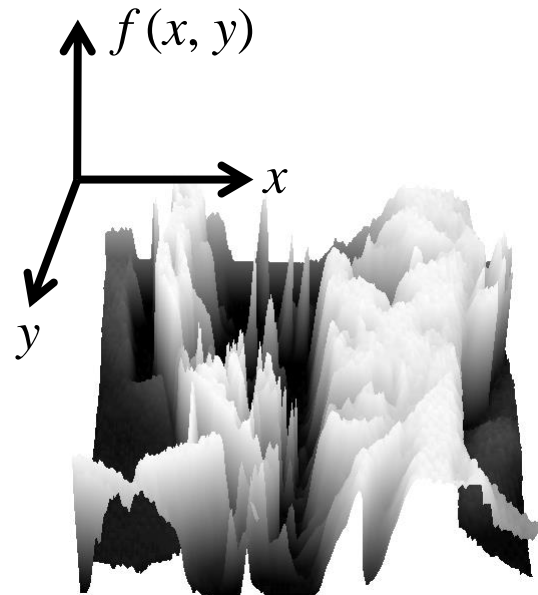
(common to use one byte per value: 0 = black, 255 = white)

What is an image?

- We can think of a (grayscale) image as a **function**, f , from \mathbb{R}^2 to \mathbb{R} :
 - $f(x,y)$ gives the **intensity** at position (x,y)



[snoop](#)



[3D view](#)

- A **digital** image is a discrete (**sampled, quantized**) version of this function

Image transformations

- As with any function, we can apply operators to an image



$$g(x,y) = f(x,y) + 20$$



$$g(x,y) = f(-x,y)$$

- We'll talk about a special kind of operator, *convolution* (linear filtering)

Question: Noise reduction

- Given a camera and a still scene, how can you reduce noise?



Take lots of images and average them!

What's the next best thing?

Image filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function

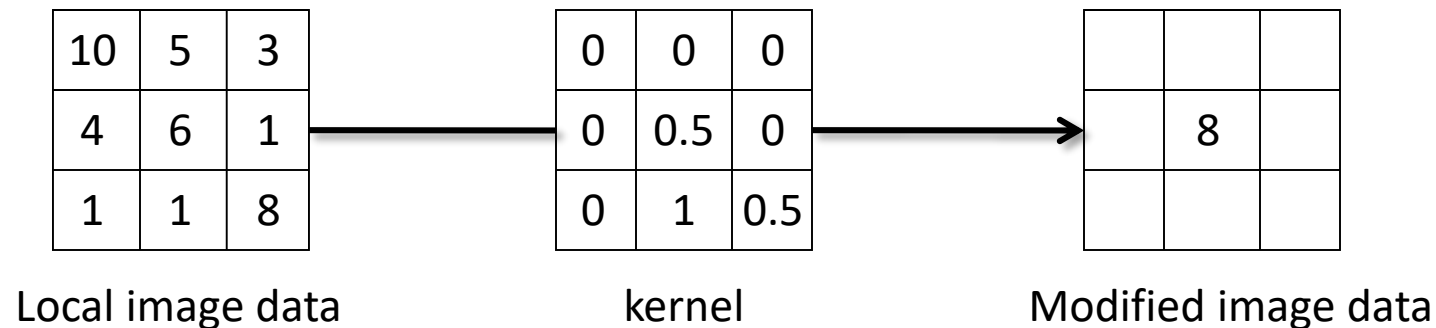


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Modified image data

Linear filtering

- One simple version: linear filtering (cross-correlation, convolution)
 - Replace each pixel by a linear combination (a weighted sum) of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)



Cross-correlation

Let F be the image, H be the kernel (of size $2k+1 \times 2k+1$), and G be the output image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

- Can think of as a “dot product” between local neighborhood and kernel for each pixel

Convolution

- Same as cross-correlation, except that the kernel is “flipped” (horizontally and vertically)

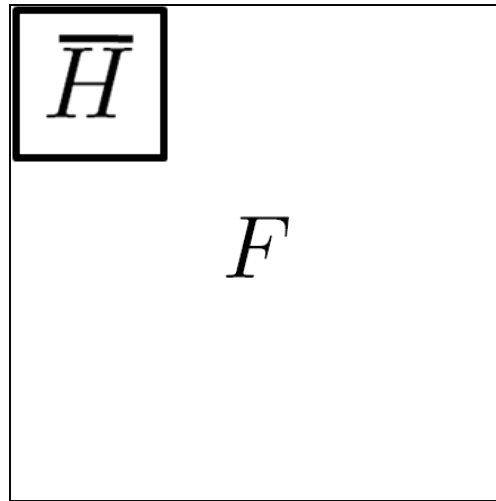
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

This is called a **convolution** operation:

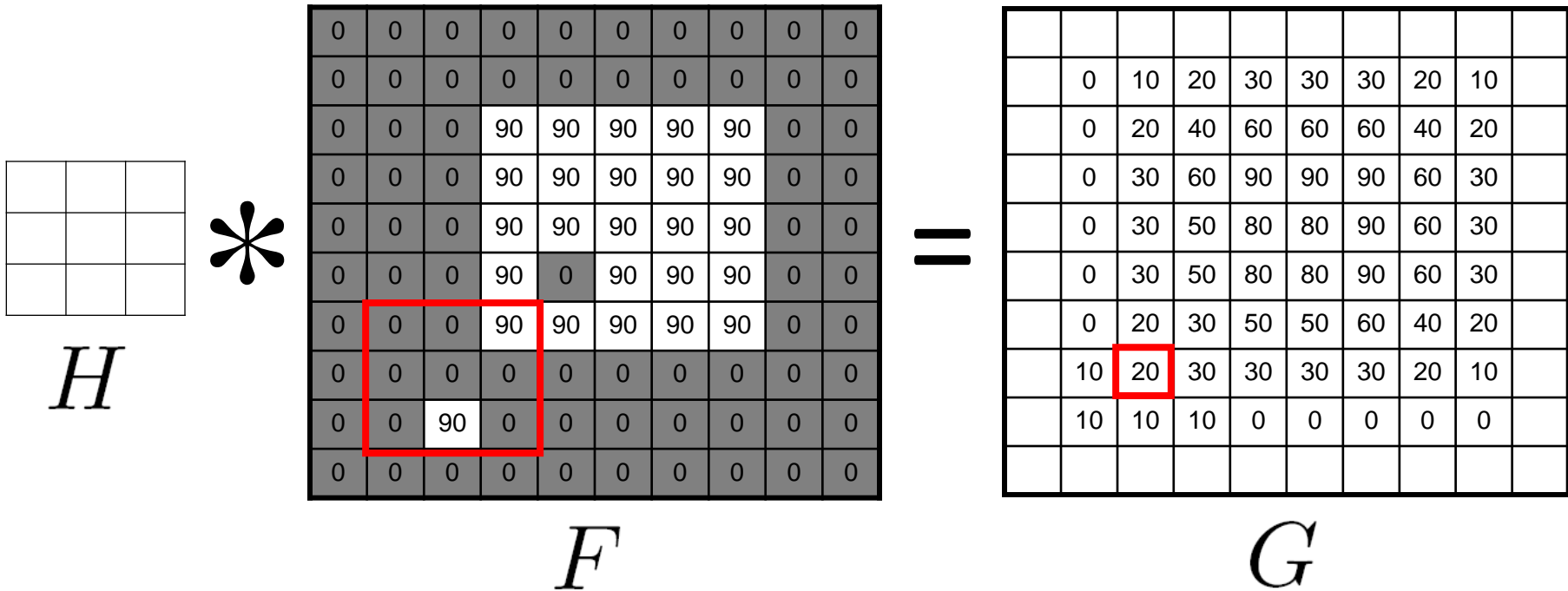
$$G = H * F$$

- Convolution is **commutative** and **associative**

Convolution



Mean filtering



Linear filters: examples



Original



0	0	0
0	1	0
0	0	0



Identical image

Linear filters: examples



Original



0	0	0
1	0	0
0	0	0



Shifted left
By 1 pixel

Linear filters: examples

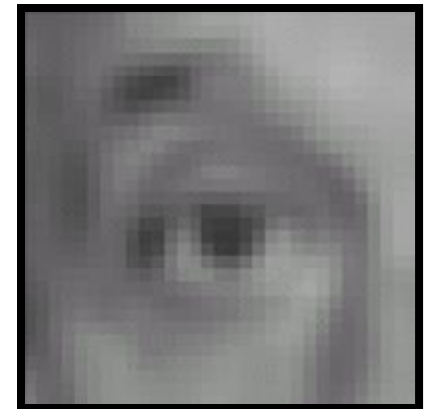


Original



$\frac{1}{9}$

1	1	1
1	1	1
1	1	1



Blur (with a mean filter)

Linear filters: examples



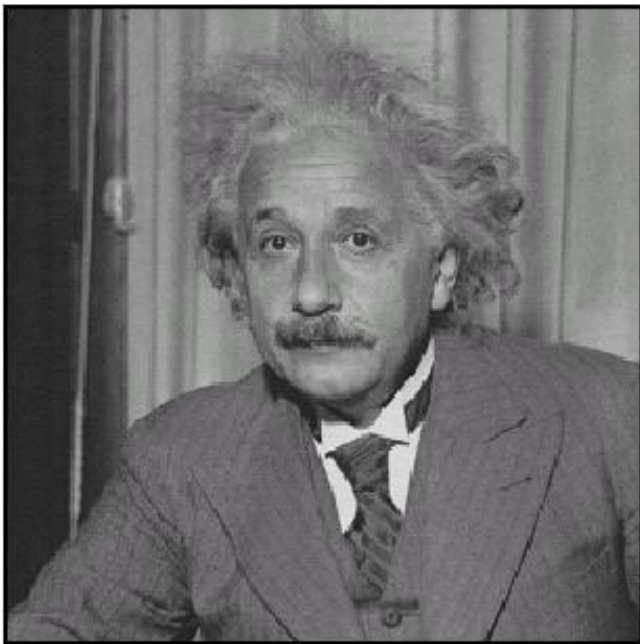
Original

$$* \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$$

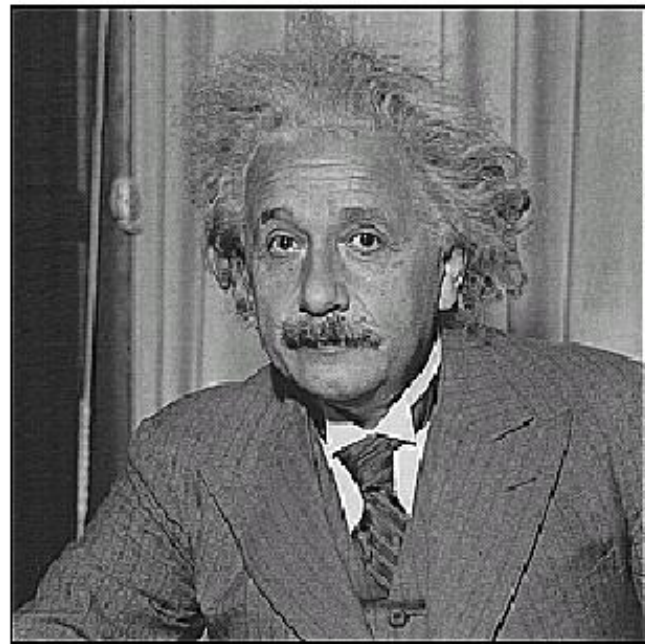


Sharpening filter
(accentuates edges)

Sharpening

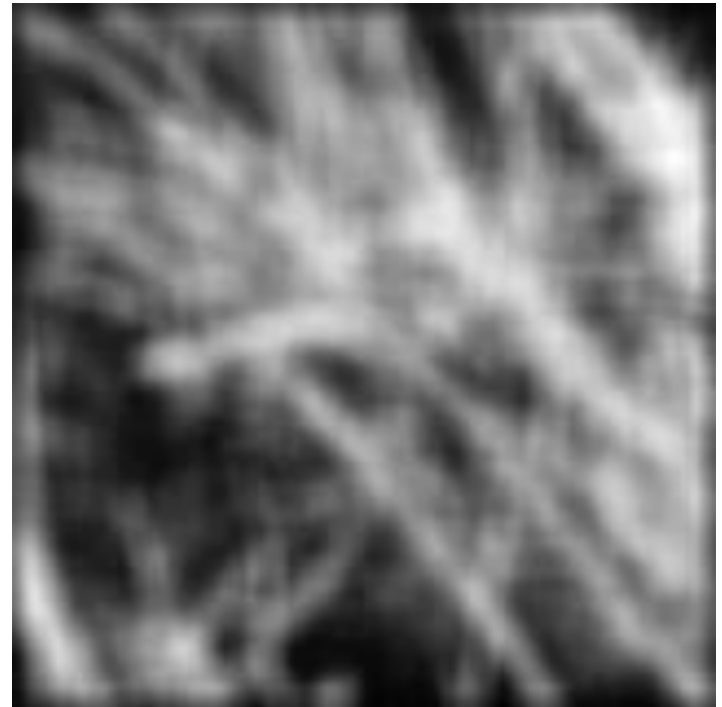
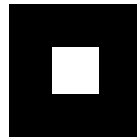


before

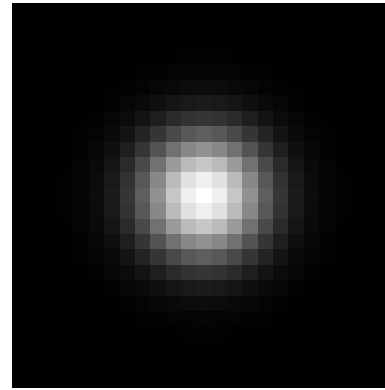
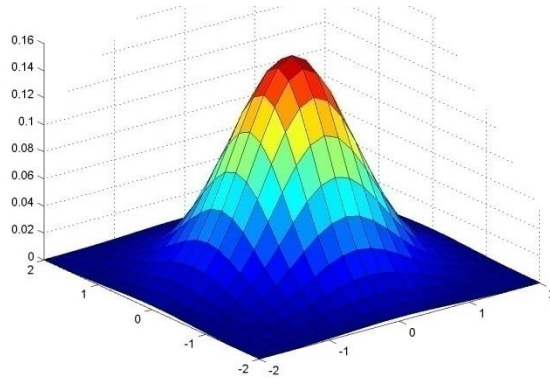


after

Smoothing with box filter revisited

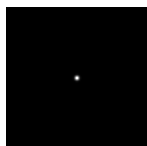
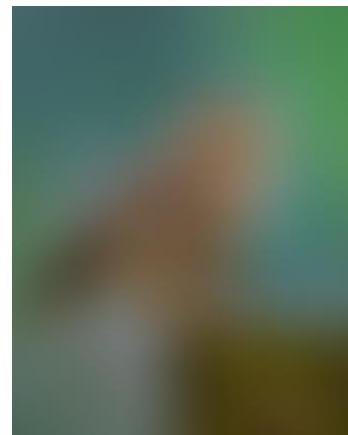
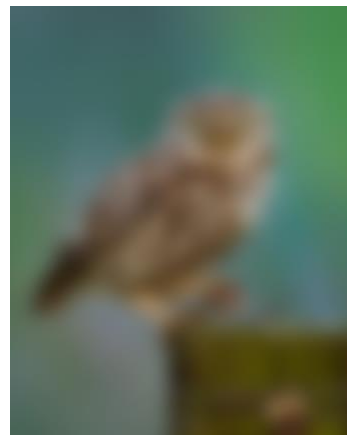


Gaussian Kernel

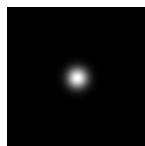


$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

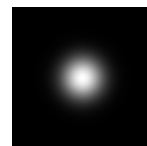
Gaussian filters



$\sigma = 1$ pixel



$\sigma = 5$ pixels

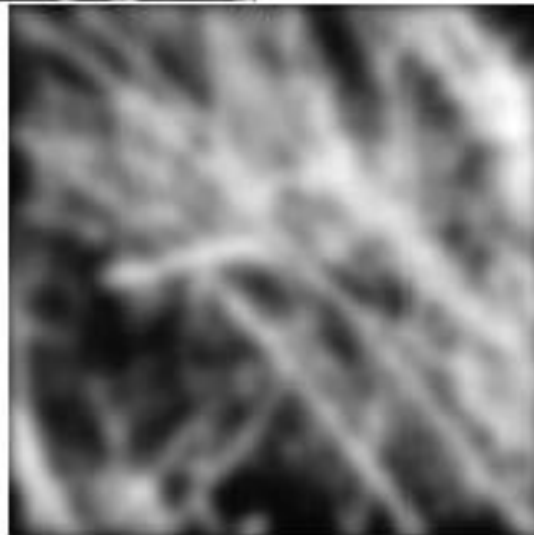
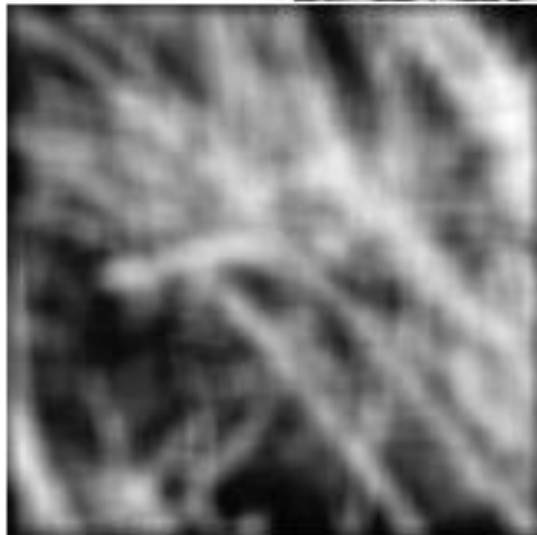


$\sigma = 10$ pixels



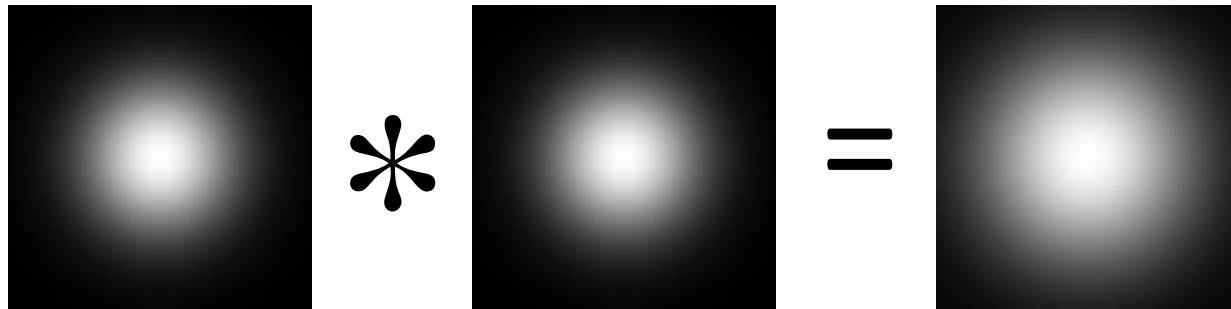
$\sigma = 30$ pixels

Mean vs. Gaussian filtering



Gaussian filter

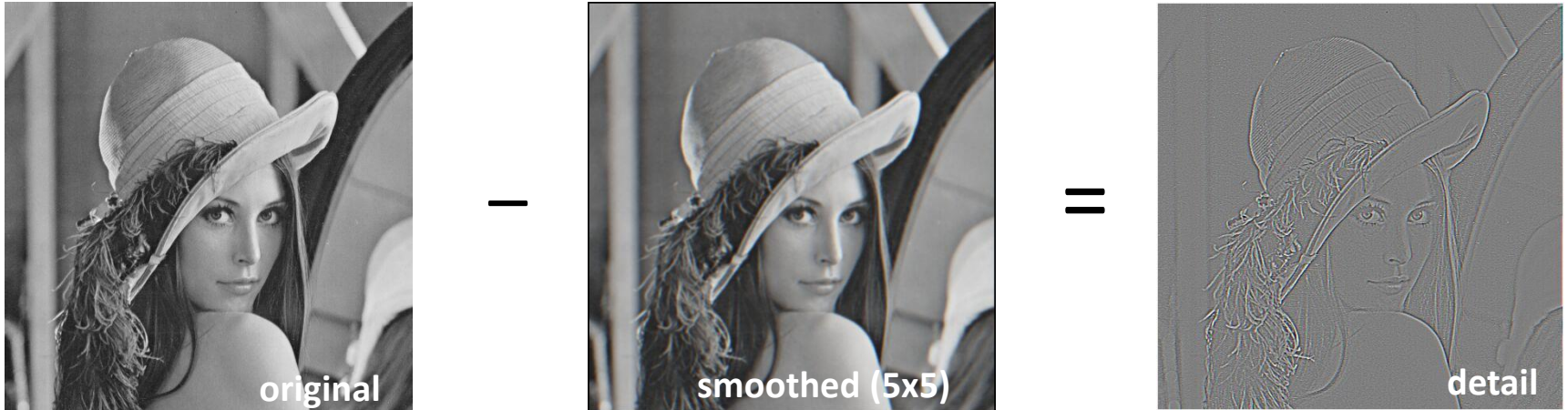
- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian



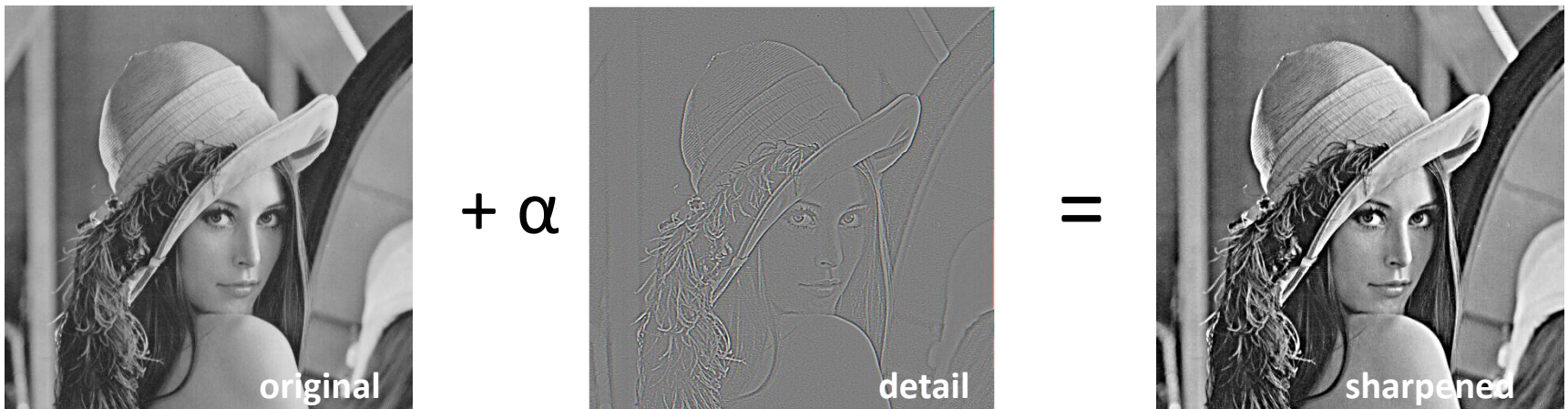
- Convolving twice with Gaussian kernel of width σ
= convolving once with kernel of width $\sigma\sqrt{2}$

Sharpening revisited

- What does blurring take away?



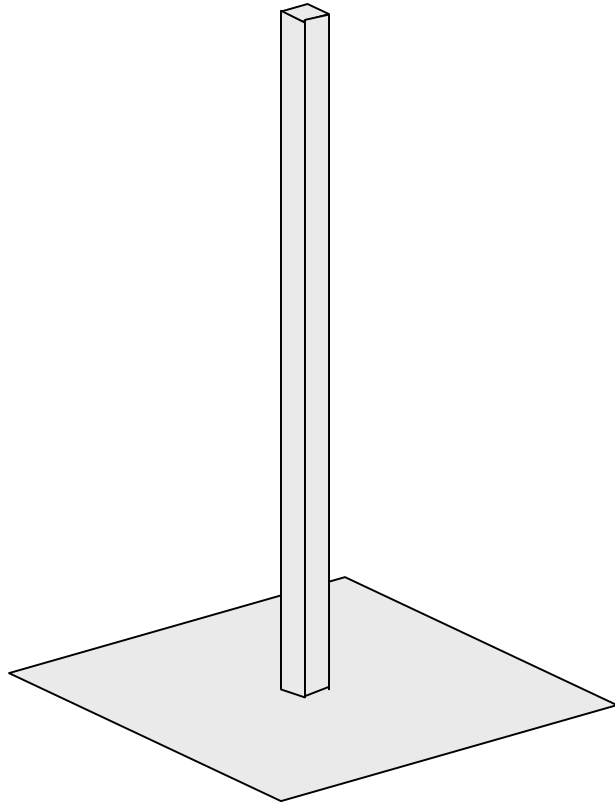
Let's add it back:



Sharpen filter

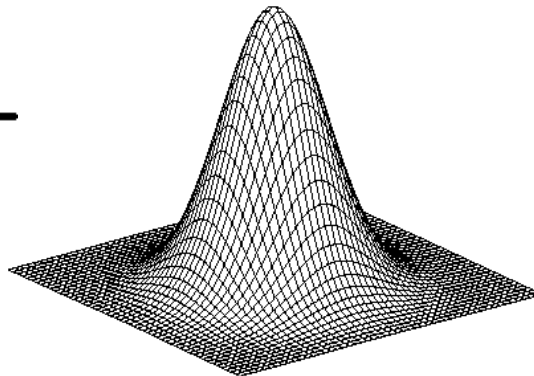
$$\underset{\substack{\uparrow \\ \text{image}}}{F} + \alpha \left(F - \underbrace{F * H}_{\substack{\text{blurred} \\ \text{image}}} \right)$$

\uparrow
unit impulse
(identity)



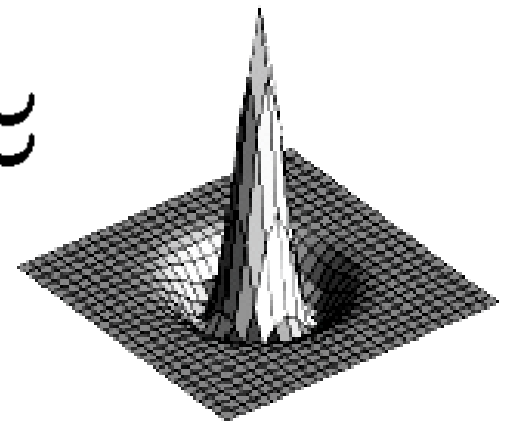
scaled impulse

—



Gaussian

\approx



Laplacian of Gaussian

Sharpen filter



“Optical” Convolution

Camera shake



Source: Fergus, *et al.* “Removing Camera Shake from a Single Photograph”, SIGGRAPH 2006

Bokeh: Blur in out-of-focus regions of an image.



Source: <http://lullaby.homepage.dk/diy-camera/bokeh.html>