Here are the derivations without the scaling factor.

$\mathbf{x}_i, \mathbf{v}_i$ correspond to the i^{th} row of X and V respectively.

First, we assume each entry r_{ij} is drawn i.i.d. with mean 0 and standard deviation 1.

Consider the column vector $\mathbf{x}_1, \mathbf{x}_2, \mathbf{r} \in \mathbb{R}^p$.

Denoting $v_1 = \langle \mathbf{x}_1, \mathbf{r} \rangle$ and $v_2 = \langle \mathbf{x}_2, \mathbf{r} \rangle$, we have:

$$\mathbb{E}[v_1^2] = \|\mathbf{x}_1\|_2^2 \tag{0.1}$$
$$\mathbb{E}[v_2^2] = \|\mathbf{x}_2\|_2^2 \tag{0.2}$$
$$\mathbb{E}[(v_1 - v_2)^2] = \|\mathbf{x}_1 - \mathbf{x}_2\|_2^2 \tag{0.3}$$
$$\mathbb{E}[v_1 v_2] = \langle \mathbf{x}_1, \mathbf{x}_2 \rangle \tag{0.4}$$

Therefore, for `option` 1 and 2, where we simulate r_{ij} i.i.d. from $N(0,1)$ and $\{-1, 1\}$ with probability $\frac{1}{2}$ respectively, it suffices to compute:

$$\begin{aligned}
\frac{1}{k}\|\mathbf{v}_1\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_1\|_2^2 \\
\frac{1}{k}\|\mathbf{v}_2\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_2\|_2^2 \\
\frac{1}{k}\|\mathbf{v}_1 - \mathbf{v}_2\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_1 - \mathbf{x}_2\|_2^2 \\
\frac{1}{k}\langle \mathbf{v}_1, \mathbf{v}_2 \rangle &\qquad \text{as an estimate for } \langle \mathbf{x}_1, \mathbf{x}_2 \rangle
\end{aligned}$$

For the Sparse Bernoulli distribution (`option` 3), we computed $V = \frac{1}{\sqrt{s}} XR$ instead of $V = XR$.

Thus, this implies we need to compute:

$$\begin{aligned}
\frac{s}{k}\|\mathbf{v}_1\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_1\|_2^2 \\
\frac{s}{k}\|\mathbf{v}_2\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_2\|_2^2 \\
\frac{s}{k}\|\mathbf{v}_1 - \mathbf{v}_2\|_2^2 &\qquad \text{as an estimate for } \|\mathbf{x}_1 - \mathbf{x}_2\|_2^2 \\
\frac{s}{k}\langle \mathbf{v}_1, \mathbf{v}_2 \rangle &\qquad \text{as an estimate for } \langle \mathbf{x}_1, \mathbf{x}_2 \rangle
\end{aligned}$$