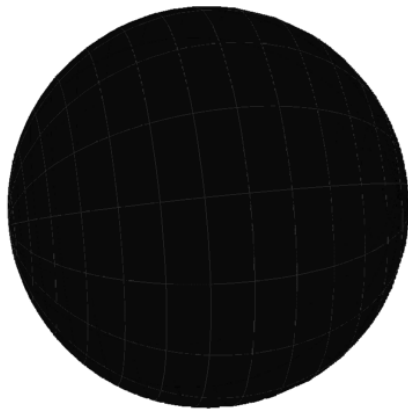
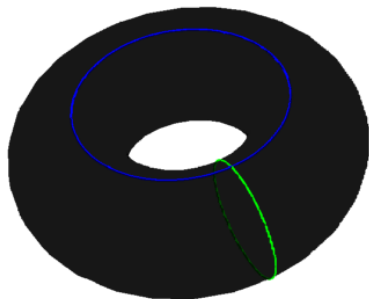


# Topological uniqueness for Lefschetz fibrations

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RSI 2018

# What is topology?



# What is a Lefschetz fibration?

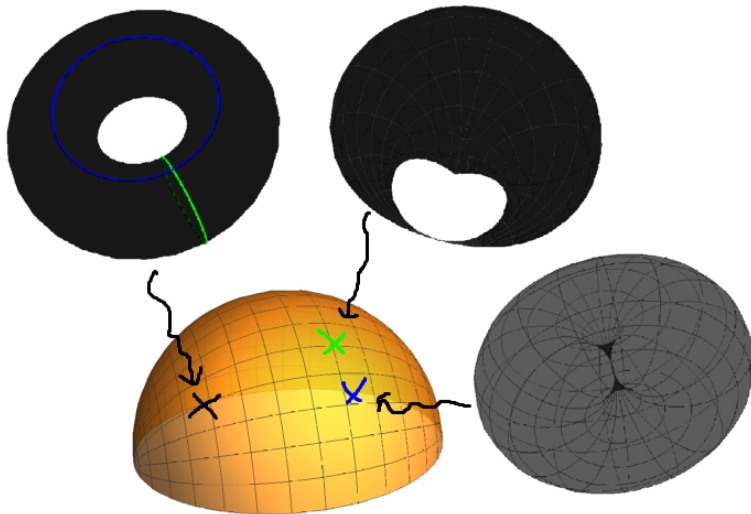
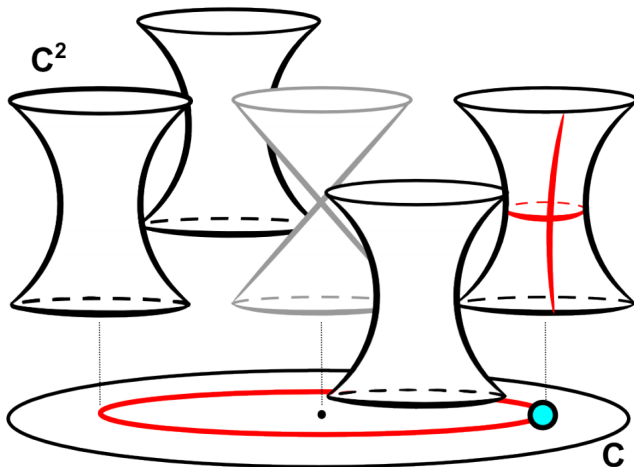


Figure: Pictures due to Jonny Evans



# Dehn twist

Figure: Pictures due to Jonny Evans

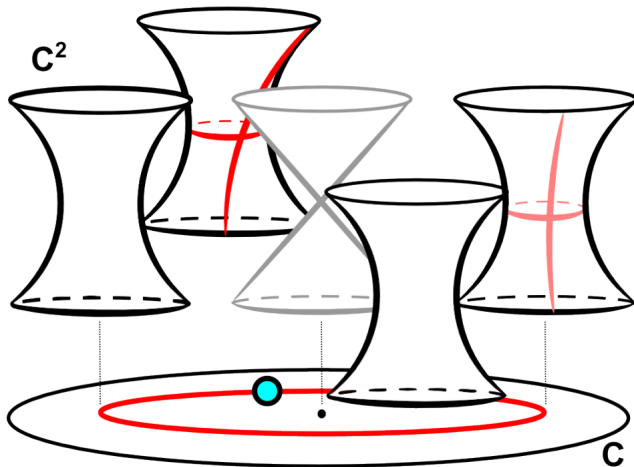


Figure: Pictures due to Jonny Evans

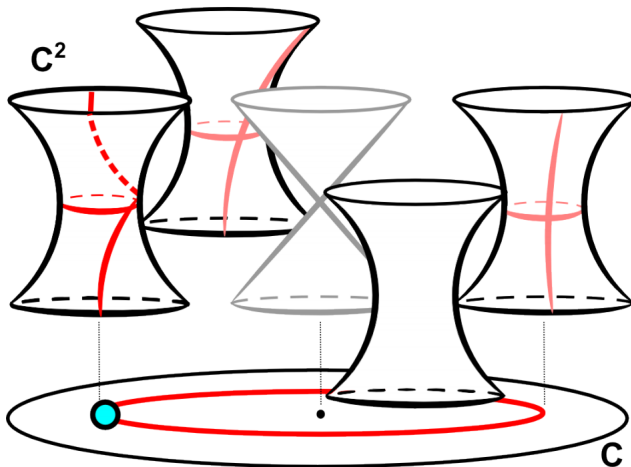
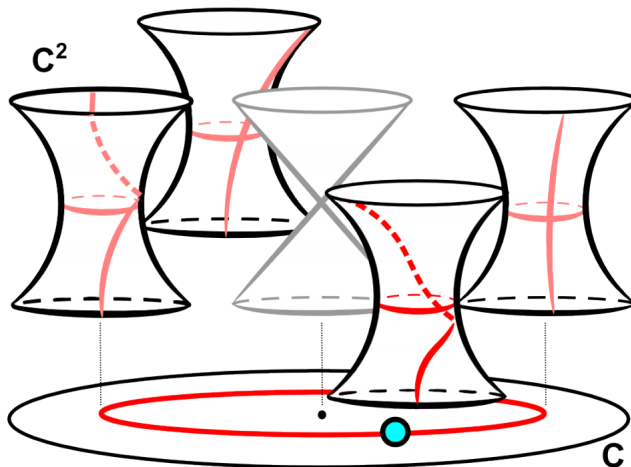
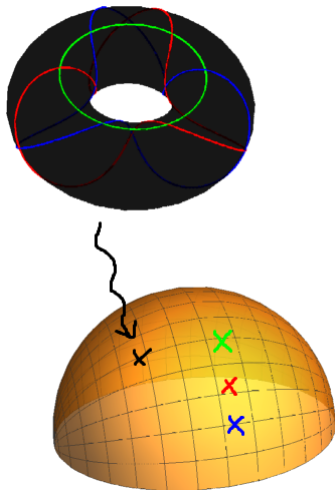


Figure: Pictures due to Jonny Evans



# Elliptic surfaces

**Figure:** The elliptic surface given by the equation  
$$y^2 = x^3 - 3(1 + 24v^3)x + 2(1 + 36v^3 + 216v^6)$$



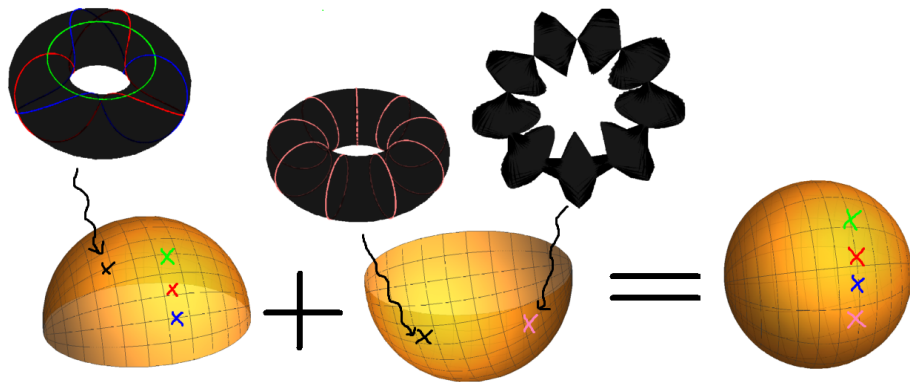


# Type of Lefschetz fibration

## Definition

A Lefschetz fibration of type  $(k, l, m, n)$  is a Lefschetz fibration that has 4 singular fibers such that the first one is a chain of  $k$  spheres, the second one is a chain of  $l$  spheres, etc.

# Example of type $(1, 1, 1, 9)$



# Topological uniqueness

## Theorem (K.)

*Any two Lefschetz fibrations of type  $(k, l, m, n)$  with  $k + l + m + n = 12$  are topologically equivalent.*

Remark: the sum  $k + l + m + n$  is always a multiple of 12.

Remark: topological uniqueness may fail for Lefschetz fibrations with more than 4 singular fibers (e.g. type  $(1, 1, 1, 1, 8)$ ).

- Our proof is inspired by **homological mirror symmetry**, a certain duality in string theory.
- HMS relates two strings theories, compactified on different spaces (thus relating the geometry of two spaces).
- Questions about Lefschetz fibrations over the disc can thus be converted into questions about del Pezzo surfaces.

- Prove or disprove topological uniqueness for Lefschetz fibrations of type  $(k, l, m, n)$  with  $k + l + m + n = 24$ .
- Classify Lefschetz fibrations with more than 4 singular fibers. Are they all algebraic?

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