Production Function of a Non-profit Service Delivery Organization

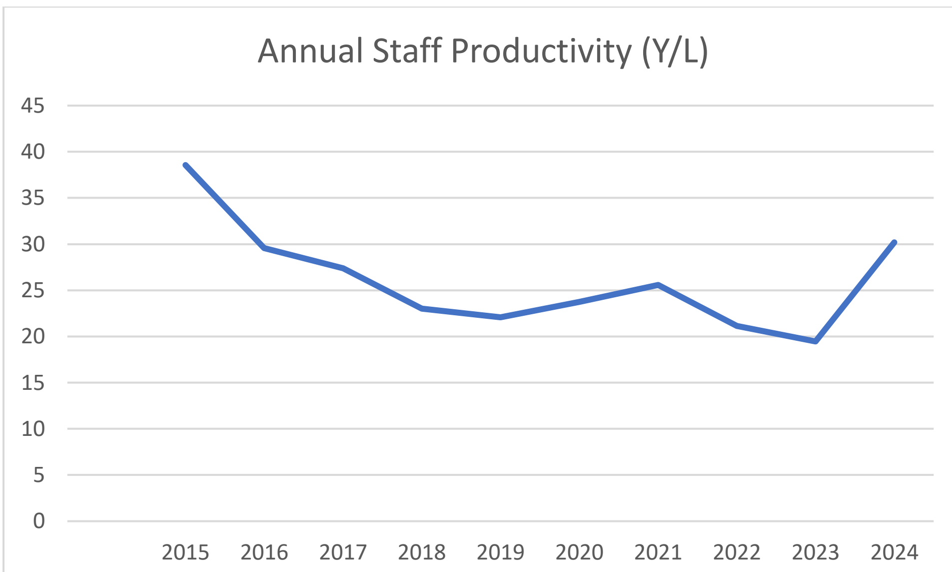
In this project, the task was to estimate staff productivity given total dollar value of the output as well as total value of staffing costs in a department X of a Non-profit Service Delivery Organization that distributes international aid. All data used in this write-up is synthetic.

Department X was hierarchical, with a department director in charge of several managers and advisors that play a role in the planning, distribution, and logistics of aid resources. All other downstream productivity was outside the scope of the model.

The issue with this organization was that the data was limited. This analysis relied on theoretical techniques from economics and the model was validated against real data over time. The estimated output elasticity deviated from actual output by about +/- 6% a year.

1) The Initial Question: How do we measure staff productivity?

a. **Approach**: Take the value of total dept X output and divide by dept X labor input.
b. **Interpretation**: $1 spent on staff yields an annual average of $12 in program output in FY22.



Fiscal Year	TE Staff Total	Productivity	Avg Productivity of Staff
2015	12,116,570	467,265,370	39
2016	14,584,816	470,511,433	30
2017	14,268,961	426,156,182	27
2018	17,936,742	450,411,906	23
2019	19,005,049	457,801,871	22
2020	18,690,220	484,223,265	24
2021	20,262,634	565,619,548	26
2022	26,663,739	615,080,333	21
2023	32,925,387	640,824,163	19
2024	21,190,628	639,562,173	30

Note: These are actual reported values. The dollar figures are nominal (not adjusted for inflation). The 2024 spike reflects a change in management in the organization, simulated by synthetic data. In general, staff productivity was declining year-on-year.

2) Secondary Questions: Productivity seems to be declining. Does this mean efficiency is also declining? Is that bad?

a. **Answer:** Not necessarily. Productivity per worker generally declines for mature organizations, i.e. "too many cooks in the kitchen". ***Efficiency is a question of how best to optimize (maximize) for production with respect to staff levels and subject to a budget constraint.***

b. Production increases but at a decreasing rate. Department X's average productivity may be an indication of decreasing returns to scale.

3) How do we find the optimal levels?

I) Estimate Department X share of productivity:

Regression Statistics	
Multiple R	0.809002
R Square	0.654484
Adjusted R Square	0.611295
Standard Error	0.098381
Observations	10

ANOVA

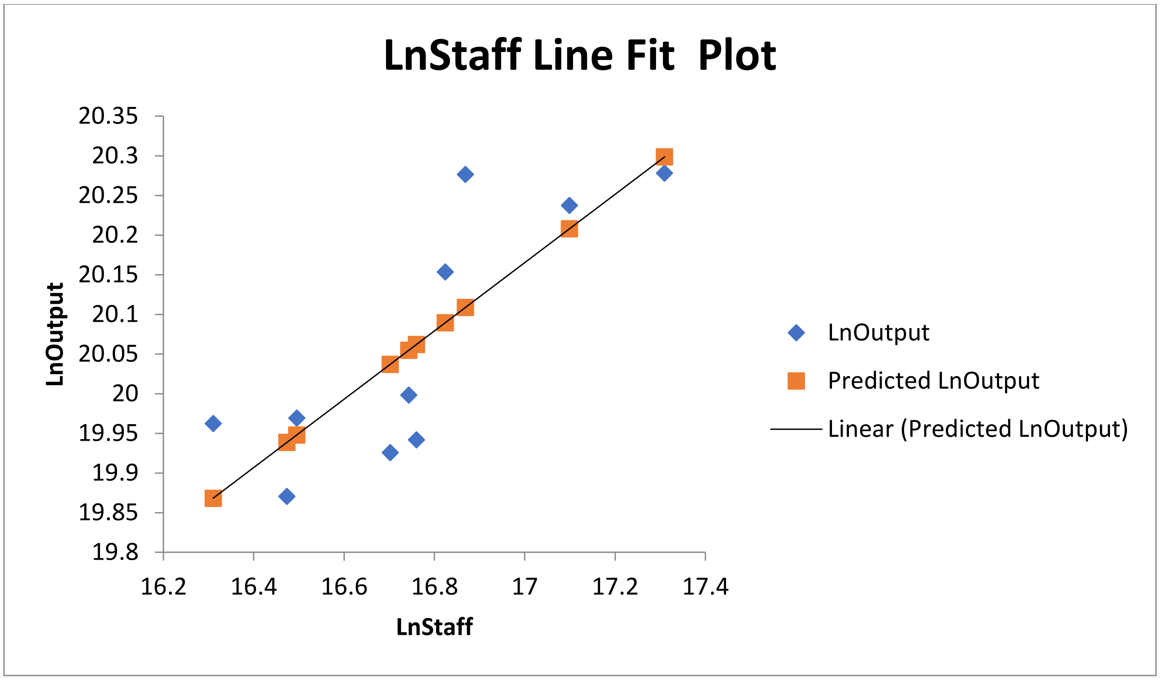
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.14667	0.14667	15.1538	0.004591
Residual	8	0.07743	0.009679		
Total	9	0.2241			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	12.84507	1.854004	6.928285	0.000121	8.569728	17.12041	8.569728	17.12041
LnStaff	0.430596	0.110614	3.892789	0.004591	0.17552	0.685671	0.17552	0.685671

i. **Regress productivity (output) on staff budget (input). Here we regress LogY on LogX to estimate the output elasticity (slope) directly.**

ii. **Interpret the slope:** The slope (see coefficient) describes how an incremental %change in X is associated with some incremental %change in Y. This is also known as the derivative.

iii. **This estimate suggests:** Every additional **1%** in TE staff budget is associated with **0.43%** increase in CRRD program spending.



II) Fit into a production function:

i. The slope describes the current level of staff input and productive outputs. We need to project this out to higher levels of staffing and optimize the function subject to a budget constraint.

ii. Use a common production function, Cobb-Douglas: $Y=f(A,K,L)= AL^{a}K^{(1-a)}$

iii. Where *Y* is productive output, *A* is a scalar representing technological or other gains in productivity, *L* is labor input, *K* is capital input*, a* is output elasticity or the slope (0.3843).

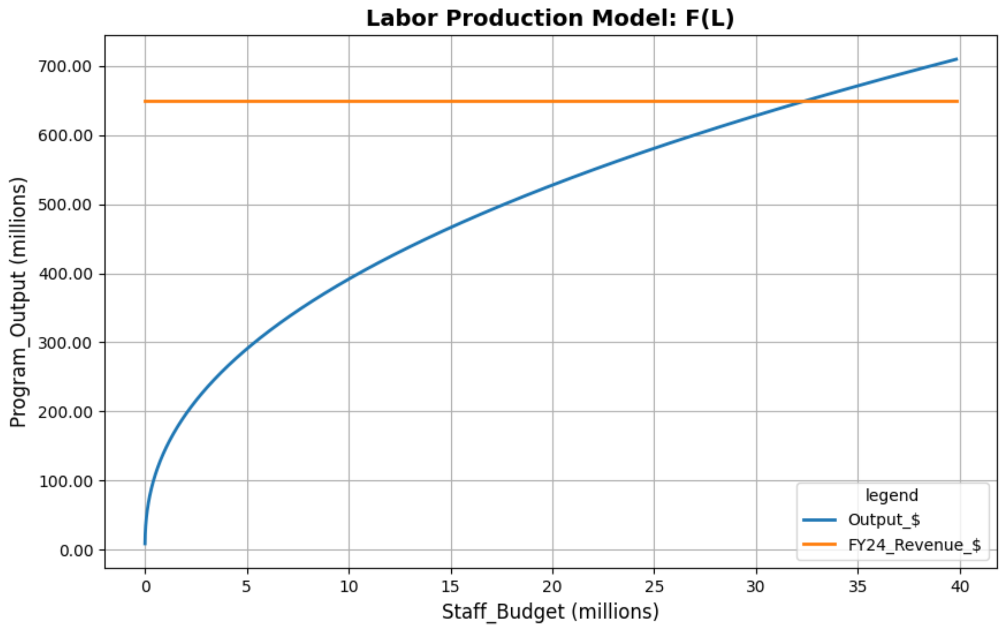
iv. To simplify in Cobb-Douglas log form: Log Y = a log L

III) Find a budget constraint

i. Usually, we'd have to use a revenue function (called the isoprofit curve). But to simplify, I'll just mark down FY24 budget.

IV) Estimate the optimal value

i. This is where the lines touch. Usually, it would be where the revenue function is tangent to the production function. But we shall simplify here since we don't have a function for revenue.



ii. The blue curve represents the production function with Department X staffing as the only input. The orange line is the FY24 revenue, using $650 million as an example

iii. Convert log units into dollar figures

iv. **According to the model, Department X staffing levels of $32 million is required to fulfill a $650 million Department X production budget, this is where the lines meet.**

Model values	
Department X Staff Budget (Labor Input)	**Department X Program Expenditure (Output)**
11,072,775.55	409,042,155.49
12,115,546.25	425,205,176.59
13,124,614.57	440,107,698.27
14,076,256.50	453,575,235.30
15,096,900.26	467,454,886.35
16,030,440.44	479,689,288.15
17,021,707.51	492,243,893.22
18,074,271.12	505,127,082.06
19,000,958.82	516,120,271.85
20,175,912.49	529,628,362.08
21,210,353.65	541,154,778.52
22,075,964.60	550,556,264.68
23,207,823.51	562,538,139.59
24,154,952.75	572,311,119.05
25,140,735.08	582,253,884.56
26,166,747.94	592,369,385.81
…	…
32,605,775.72	651,229,051.68

v. Note: these are nominal dollar figures
vi. Staff budget does not include Measurement or PQSU
vii. Highlighted are expanded to explore FY23.

4) Additional things to think about:

a. What is IRC optimizing for?
    i. The link is a theoretical utility maximizing argument for non-profits by Henry Hansmann at Yale.
b. How does IRC use funds? Does the model help in thinking about productivity?
c. What mix of funding would best help IRC achieve optimal productivity?
    i. Department X awards portfolio seem to be skewed, in that few donors contribute to large proportion of total awards. Is this good? Does this increase the quantity or quality of Department X deliveries?
    ii. What are the risk profiles of winning big awards as opposed to many small awards? Could an awards portfolio of many small and low risk awards be preferable to high risk and large awards?
    iii. Can an understanding of labor inputs lead to more efficient composition of awards? That is, can we reduce the amount of refunds if we can hire more staff to administer those funds? This is where the input/output chart may be useful.

5) Some Explanations and Caveats

a. The model is a very simplified version of a well-known theoretical production function. We do not have much insight to any other inputs. Also, this model purposely oversimplifies IRC's complexities. This might help clear the way, but also a lot of information is not on display here.
b. Mathematical models can be useful when data is scarce and can serve as a guide on what can be expected until the data becomes available. But almost anything can fit into a model.
c. Mathematical models also help with rational thinking and problem solving.
d. The short-term past is most useful in predicting the short-term future.
e. Too much focus on optimization can also lead to unintended consequences (good and bad). Think of second order effects.
f. And also expect a leaky bucket in real life.
g. Don't take modeling too seriously!