NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1 **Tutorial 5**

1. Let C and D be two $n \times n$ matrices that are row equivalent and $\underset{\sim}{b}$ be an $n \times 1$ column matrix. Suppose the linear system $C\underset{\sim}{x} = \underset{\sim}{b}$ has no solution. How many solutions does $D\underset{\sim}{x} = \underset{\sim}{b}$ have? Justify your answer.

   (Year 09/10, Problem 2(b))

2. Let $\underset{\sim}{v_1} = (1, 1, -1), \underset{\sim}{v_2} = (-1, 1, 1)$ and $\underset{\sim}{u_1} = (0, 1, 0), \underset{\sim}{u_2} = (1, 0, -1), \underset{\sim}{u_3} = (1, 1, 1)$.
   (i) Show that $\text{span}\{\underset{\sim}{v_1}, \underset{\sim}{v_2}\} \subset \text{span}\{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \underset{\sim}{u_3}\}$.
   (ii) Find two vectors in $\text{span}\{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \underset{\sim}{u_3}\}$ that are not in $\text{span}\{\underset{\sim}{v_1}, \underset{\sim}{v_2}\}$. Justify your answer.

   (Year 09/10, Problem 4(b))

3. Write down a 3×3 matrix B such that the homogeneous system
   $$B\underset{\sim}{x} = \underset{\sim}{0}$$
   has solution to the plane P given by $x - 2y + z = 0$.

   (Year 10/11, Problem 4(c))

4. Let $\underset{\sim}{v_1}, \underset{\sim}{v_2}, \underset{\sim}{v_3}$ be the three vectors in $(1, 0, 1, 2), (1, -2, 0, 1), (-3, 2, -2, -5)$ respectively. Suppose that $\underset{\sim}{w}$ is a linear combination of $\underset{\sim}{v_1}, \underset{\sim}{v_2}, \underset{\sim}{v_3}$. Is it possible to find positive numbers a, b, c such that
   $$\underset{\sim}{w} = a\underset{\sim}{v_1} + b\underset{\sim}{v_2} + c\underset{\sim}{v_3}?$$

   (Year 10/11, Problem 5(b))

5. Let A and B be square matrices such that
   $$AB = A + B.$$
   (i) Show that
   $$(A - I)^{-1} = B - I$$
   where I is the identity matrix.

(ii) If B is a singular matrix, show that A is also a singular matrix.

(Year 11/12, Problem 4(b))

6. (a) Let $A\underset{\sim}{x} = \underset{\sim}{0}$ denote a homogeneous system where A is an $n \times n$ matrix. Then the solution set of this system can be expressed in terms of the implicit set notation

$$S = \{\underset{\sim}{x} \in \mathbf{R}^n | A\underset{\sim}{x} = \underset{\sim}{0}\}.$$

Find S explicitly if (i) A is invertible; (ii) $A = O$ where O is the $n \times n$ zero matrix. Justify your answer.

(b) Let A be a 2×2 matrix. If $A^3 = O$, prove that $A^2 = O$.

(Year 12/13, Problem 4)