

NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1

Tutorial 1

1. Each equation in the following linear system represents a line in the xy -plane:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$a_3x + b_3y = c_3$$

where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are constants and for each $i = 1, 2, 3$, a_i, b_i are not both zero. Discuss the relative positions of the three lines when the system

- (a) has no solution,
- (b) has only one solution,
- (c) has infinitely many solutions.

(Textbook, p. 25, Problem 7)

2. Determine whether the following augmented matrices are row equivalent to each other.

$$A = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 6 & 0 \end{array} \right) \quad B = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 6 & 9 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right) \quad C = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right).$$

(Textbook, p. 27, Problem 13)

3. For the following linear system, determine the values of a such that the system has (i) no solution, (ii) only one solution and (iii) infinitely many solutions.

$$x + y + z = 1$$

$$2x + ay + 2z = 2$$

$$4x + 4y + a^2z = 2a$$

(Textbook, p. 30, Problem 22(b))

4. Consider the homogeneous linear system

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0$$

$$a_3x + b_3y + c_3z = 0$$

where $a_i, b_i, c_i, 1 \leq i \leq 3$ are constants. Determine all possible reduced row echelon forms of the augmented matrix of the system and describe the geometrical meaning of the solutions obtained from various reduced row-echelon forms.

(Textbook, p. 31, Problem 28)

5. The following is the reduce row-echelon form of the augmented matrix of a linear system:

$$\left(\begin{array}{ccc|c} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & k \end{array} \right)$$

where $a, b, c, d, e, f, g, h, k$ are constants. Suppose the solution set of this system is represented by a line that passes through $(0, 0, 0)$ and the point $(1, 1, 1)$. Find the values of $a, b, c, d, e, f, g, h, k$. Justify your answers.

(Textbook, p. 31, Problem 29)