NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1 **Tutorial 7**

1. Give an example of a family of subspaces $V_1, \cdots, V_n$ of $\mathbf{R}^n$ such that $\dim(V_i) = i$ for $i = 1, 2, \cdots, n$ and
$$V_1 \subset V_2 \subset \cdots \subset V_n.$$
Justify your answer.

(Textbook, p. 125, Problem 39)

2. Let
$u_1 = (1, 0, 1, 1), u_2 = (-3, 3, 7, 1), u_3 = (-1, 3, 9, 3), u_4 = (-5, 3, 5, -1)$
and let
$$S = \{u_1, u_2, u_3, u_4\}$$
and $V = \text{span}(S)$.
   (a) Find a non trivial solution to the equation
$$au_1 + bu_2 + cu_3 + du_4 = 0.$$
   (b) Express u_3 adn u_4 separately as linear combinations of u_1 and u_2.
   (c) Find a basis for V and determine the dimension of V.
   (d) Find a subspace W of $\mathbf{R}^4$ such that $\dim(W) = 3$ and $\dim(W \cap V) = 2$. Justify your answer.

(Textbook, p. 125, Problem 40)

3. Let $S = \{u_1, u_2, u_3\}$ be a basis for $\mathbf{R}^3$ and $T = \{v_1, v_2, v_3\}$ where
$$v_1 = u_1 + u_2 + u_3, v_2 = u_2 + u_3 \quad \text{and} \quad v_3 = u_2 - u_3.$$
   (a) Show that T is a basis for $\mathbf{R}^3$.
   (b) Find the transition matrix from S to T.

(Textbook, p. 126, Problem 49)

4. Let $A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix}$ be a 4×5 matrix such that the columns a_1, a_2, a_3 are linearly independent while $a_4 = a_1 - 2a_2 + a_3$ and $a_5 = a_2 + a_3$.
   (a) Determine the reduced row echelon form of A.
   (b) Find a basis for the row space of A and a basis for the column space of A.

   (Textbook, p. 143, Problem 10)

5. For each of the following cases, write down a matrix with the required property or explain why no such matrix exists.
   (a) The column space contains vectors $(1,0,0)^T, (0,0,1)^T$ and the row space contains vectors $(1,1), (1,2)$.
   (b) The column space is $\mathbf{R}^4$ and the row space is $\mathbf{R}^3$.
   (c) The column space = row space = $\text{span}\{(1,2,3)\}$.

   (Textbook, p. 144, Problem 12)