NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1 **Tutorial 6**

1. Determine which of the following are subspaces of $\mathbf{R}^4$. Justify your answer.

   (a) $\{(w, x, y, z) | w + x + y = z^2\}$

   (b) $\{(w, x, y, z) | w = 0 \quad \text{and} \quad y = 0\}$

   (c) $\{(w, x, y, z) | w = 0 \quad \text{or} \quad y = 0\}$

   (d) $\{(w, x, y, z) | w = 1 \quad \text{and} \quad y = 0\}$

   (Textbook, p. 120, Problem 16)

2. Let u, v, w be vectors in $\mathbf{R}^3$ such that $V = \text{span}\{u, v\}$ and $W = \text{span}\{u, w\}$ are planes in $\mathbf{R}^3$. Find $V \cap W$ if

   (a) u, v, w are linearly independent.

   (b) u, v, w are not linearly independent.

   (Textbook, p. 123, Problem 29)

3. All vectors in this question are written as column vectors. Let $u_1, u_2, \cdots, u_k$ be vectors in $\mathbf{R}^n$ and P a square matrix of order n.

   (a) Show that if $Pu_1, \cdots, Pu_k$ are linearly independent then $u_1, \cdots, u_k$ are linearly independent.

   (b) Suppose thta $u_1, \cdots, u_k$ are linearly independent.

      (i) Show that if P is invertible, then $Pu_1, \cdots, Pu_k$ are linearly independent.

      (ii) If P is not invertible, are $Pu_1, \cdots, Pu_k$ linearly independent?

   (Textbook, p. 123, Problem 30)

4. Let
$$V = \{(a+b, a+c, c+d, b+d) | a, b, c, d \in \mathbf{R}\}$$
and
$$S = \{(1,1,0,0), (1,0,-1,0), (0,-1,0,1)\}.$$
   (a) Show that V is a subspace of $\mathbf{R}^4$ and S is a basis for V.
   (b) find the coordinate vector of $u = (1,2,3,2)$ relative to S.
   (c) Find a vector v such that $(v)_S = (1,3,-1)$.

(Textbook, p. 124, Problem 35)

5. Find a basis and determine the dimension of each of the following subspaces of $\mathbf{R}^4$:
   (a) the subspace containing all vectors of the form (w, x, y, z) with $w = 2x = 3y$.
   (b) the solution space of
$$\begin{aligned} 2w + 3x + y + z &= 0 \\ -3w + x + 4y - 7z &= 0 \\ w + 2x + y &= 0 \end{aligned}$$
   (c) the subspace $\{(w, x, y, z) | y = w + x \quad \text{and} \quad z = w - x\}$.

(Textbook, p. 124, Problem 37)