NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1 **Tutorial 10**

1. Let B be a 4×4 matrix and $\{u_1, u_2 u_3, u_4\}$ a basis for $\mathbf{R}^4$. Suppose
$$Bu_1 = 2u_1, Bu_2 = 0, Bu_3 = u_4, Bu_4 = u_3.$$
   (a) Write down all the eigenvalues of B.
   (b) For each eigenvalue of B, write down one eigenvector associated with it.
   (c) Is B a diagonalizable matrix? Justify your answer.

   (Textbook, p. 203, Problem 14)

2. A square matrix $(a_{ij})_{n\times n}$ is called a stochastic matrix if all the entries are non-negative and the sum of entries of each column is 1,i.e.,
$$a_{1i} + a_{2i} + \cdots + a_{ni} = 1$$
for $i = 1, 2, \cdots, n$. Let A be a stochastic matrix.
   (a) Show that 1 is an eigenvalue of A.
   (b) If λ is an eigenvalue of A, then $|\lambda| \leq 1$.

   (Textbook, p. 204, Problem 16)

3. Let A be a symmetric matrix. If u and v are two eigenvectors of A associated with eigenvalues λ and μ, respectively, where $\lambda \neq \mu$, show that $u \cdot v = 0$.

   (Textbook, p. 206, Problem 26)

4. Let A be a 3×3 symmetric matrix with two eigenvalues 1 and -1. Suppose the eigenspace associated with the eigenvalue 1 represents the plane $x + y - z = 0$. Determine the matrix A.

   (Textbook, p. 207, Problem 27)

5. (a) Let

$$S_1 = \{(1,0),(0,1)\}, S_2 = \{(1,-2),(2,1)\} \quad \text{and} \quad S_3 = \{(1/\sqrt{2}, 1/\sqrt{2}), (-1/\sqrt{2}, 1/\sqrt{2})\}.$$

Clearly, S_1, S_2 and S_3 are three bases for $\mathbf{R}^2$. Let $u = (1,4)$ and $v = (-1,1)$. Compute $(u)_{S_i}, (v)_{S_i}$ and $(u)_{S_i} \cdot (v)_{S_i}$ for $i = 1,2,3$. What do you observe?

(b) Prove that if S and T are two orthonormal bases for a vector space V, then for any vectors $u, v \in V$, $(u)_S \cdot (v)_S = (u)_T \cdot (v)_T$.

(Textbook, p. 175, Problem 31)