NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2014/2015

MA1101R Linear Algebra 1 **Tutorial 4**

1. Let
$$A = \{(1+t, 1+2t, 1+3t) | t \in \mathbf{R}\}$$
be a subset of $\mathbf{R}^3$.
(a) Describe A geometrically.
(b) Show that
$$A = \{(x, y, z) | x + y - z = 1 \quad \text{and} \quad x - 2y + z = 0\}.$$
(c) Write down a matrix equation $M\mathbf{x} = \mathbf{b}$ where M is a 3×3 matrix and $\mathbf{b}$ is a 3×1 matrix such that its solution set is A.
(Textbook, p. 118, Problem 5)

2. Determine whether the following subsets of $\mathbf{R}^4$ are equal to each other.
$$S = \{(p, q, r, s) | p, q \in \mathbf{R}\}$$
$$T = \{(x, y, z, w) | x + y - z - w = 0\}$$
$$V = \left\{ (a, b, c, d) \middle| \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ a & b & c & d \end{vmatrix} = 0 \right\}.$$
Briefly explain why one subset is equal (or not equal) to another subset.
(Textbook, p. 118, Problem 6)

3. Let
$$V = \{(x, y, z) | x - y - z = 0\}$$
be a subset of $\mathbf{R}^3$.
(a) Let $S = \{(1, 1, 0), (5, 2, 3)\}$. Show that $\text{span}(S) = V$.
(b) Let $S' = \{(1, 1, 0), (5, 2, 3), (0, 0, 1)\}$. Show that $\text{span}(S') = \mathbf{R}^3$.

(Textbook, p. 119, Problem 10)

4. Let u, v, w be vectors in $\mathbf{R}^n$ and let

$$S_1 = \{u, v\},\ S_2 = \{u - v, v - w, w - u\},\ S_3 = \{u - v, v - w, u + w\}$$
$$S_4 = \{u, u + v, u + v + w\},\ S_5 = \{u + v, v + w, u + w, u + v + w\}.$$

Suppose that $n = 3$ and $\text{span}\{u, v, w\} = \mathbf{R}^3$. Determine which of the above sets span $\mathbf{R}^3$.

(Textbook, p. 120, Problem 13)

5. Determine which of the following statements are true. Justify your answers.
   (a) If u is a nonzero vector in $\mathbf{R}$, then $\text{span}\{u\} = \mathbf{R}$
   (b) If u, v are nonzero vectors in $\mathbf{R}^2$, such that $u \neq v$, then $\text{span}\{u, v\} = \mathbf{R}^2$.
   (c) If S_1 and S_2 are two subsets of $\mathbf{R}^n$, then $\text{span}(S_1 \cap S_2) = \text{span}(S_1) \cap \text{span}(S_2)$.
   (d) If S_1 and S_2 are two subsets of $\mathbf{R}^n$, then $\text{span}(S_1 \cup S_2) = \text{span}(S_1) \cup \text{span}(S_2)$.

(Textbook, p. 120, Problem 14)