## Objective(s):

This activity aims to perform regression analysis using polynomial regression

## Intended Learning Outcomes (ILOs):

- Demonstrate how to build a regression model to predict the outcome using polynomial regression.
- Evaluate the performance of the regression model using polynomial regression

## Resources:

- Jupyter Notebook
- internet_traffic_hist.csv

## Procedure:

## Using numpy polyfit to perform polynomial regression

Import the libraries and the data

In [127…

```
from google.colab import drive

drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call driv
e.mount("/content/drive", force_remount=True).
```

In [128…

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

In [129…

```
from sklearn.metrics import r2_score
from scipy.optimize import curve_fit
```

Load the dataset

In [130…

```
internet = '/content/drive/MyDrive/DATASETS/internet_traffic_hist-2.csv'

df_hist = pd.read_csv(internet)
df_hist.head(11)
```

Out[130]:

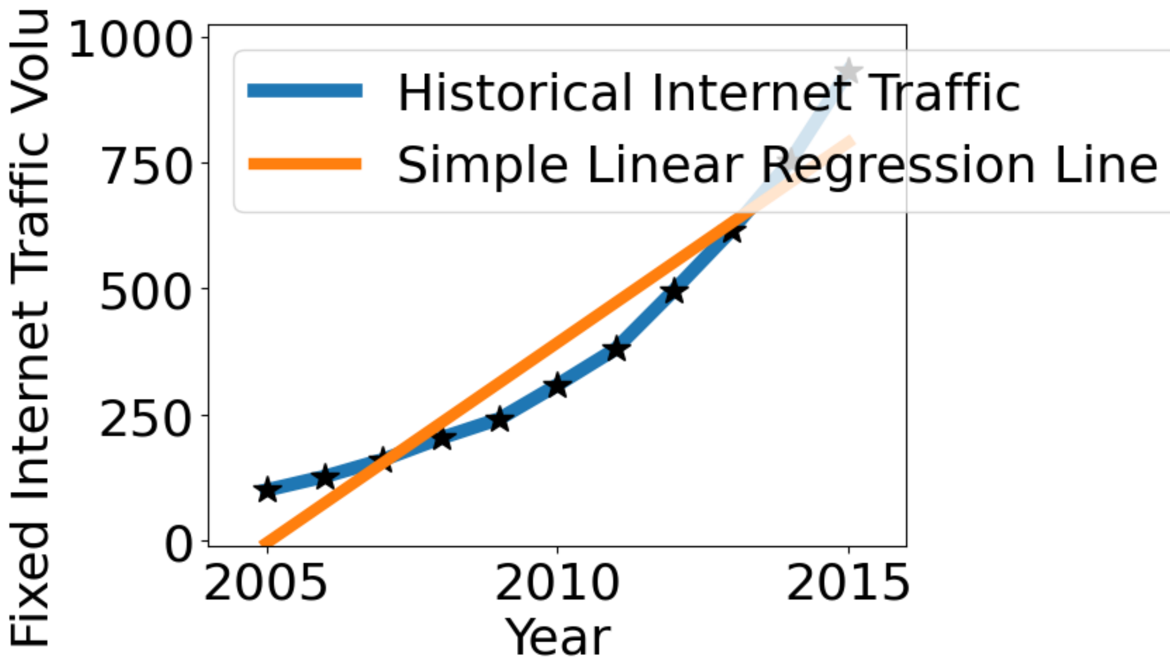
| | traffic | year |
|---|---|---|
| 0 | 100.000000 | 2005 |
| 1 | 126.933755 | 2006 |
| 2 | 160.303757 | 2007 |
| 3 | 203.390603 | 2008 |
| 4 | 241.292566 | 2009 |
| 5 | 308.791823 | 2010 |
| 6 | 379.980659 | 2011 |
| 7 | 495.840568 | 2012 |
| 8 | 616.207252 | 2013 |
| 9 | 752.103483 | 2014 |
| 10 | 931.200929 | 2015 |

Build the first order polynomial using numpy polyfit

In [131…

```
order = 1

# XY Plot of year and traffic
x = df_hist.year
y = df_hist.traffic

m, b = np.polyfit(x,y,order)

plt.plot(x, y, label = 'Historical Internet Traffic', linewidth = 7)
plt.plot(x, y,'*k', markersize = 15, label ='')
plt.plot(x, m*x + b, '-', label = 'Simple Linear Regression Line', linewidth = 6)

print ('The slope of line is {}.'.format(m))
print ('The y intercept is {}.'.format(b))
print ('The best fit simple linear regression line is {}x + {}.'.format(m,b))


#Increase sligthly the axis sizes to make the plot more clear
plt.axis([x.iloc[0]-1, x.iloc[-1]+1, y.iloc[0]*-0.1, y.iloc[-1]*1.1])

# Add axis labels
plt.xlabel('Year')
plt.ylabel('Fixed Internet Traffic Volume')
plt.legend(loc = 'upper left')


# Increase default font size
plt.rcParams.update({'font.size': 26})
plt.show()
```

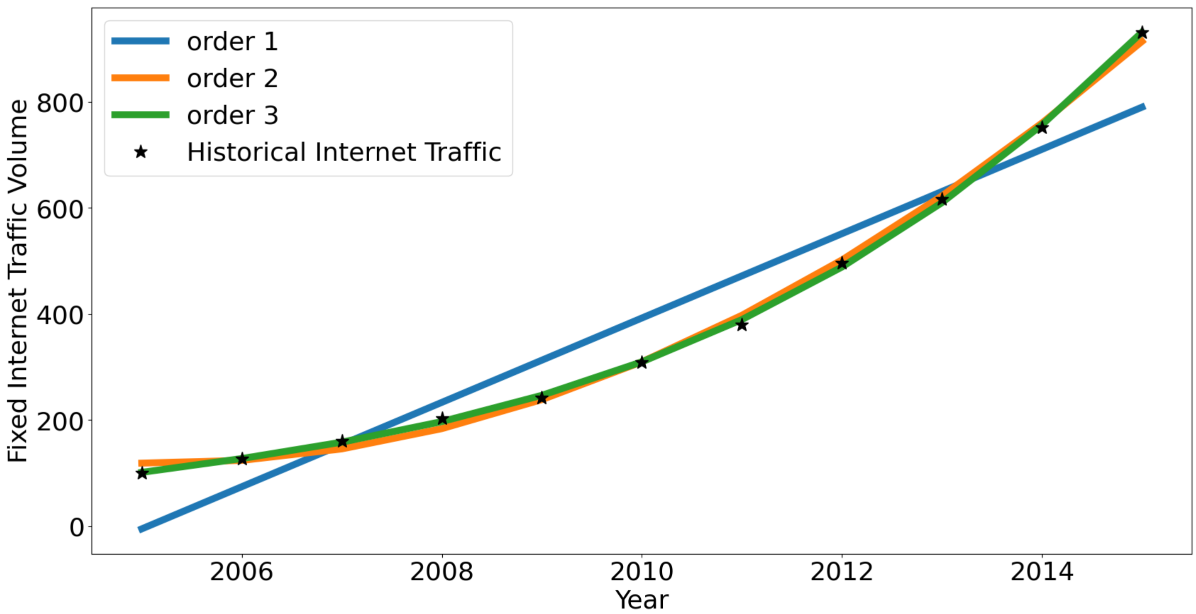
```
The slope of line is 79.52710966244513.
The y intercept is -159457.12265833947.
The best fit simple linear regression line is 79.52710966244513x + -159457.12265833
947.
```




Build the model using Higher Order Polynomial (1 to 4)

In [132…

```
models = []        # to store polynomial model parameters (list of poly1d objects)
errors_hist = []  # to store the absolute errors for each point (2005-2015) and for
mse_hist = []     # to store the MSE for each model (list of numpy floats)

#Try polynomial models with increasing order
for order in range(1,4):
    # Fit polynomial model
    p = (np.poly1d(np.polyfit(x, y, order)))
    models.append(p)

plt.figure(figsize = (20,10))

# Visualize polynomial models fit
for model in models[0:3]:
    plt.plot(x, model(x), label = 'order ' + str(len(model)), linewidth = 7)

plt.plot(x, y, '*k', markersize = 14, label = 'Historical Internet Traffic', linewi
plt.legend(loc = 'upper left')

# Add axis labels
plt.xlabel('Year')
plt.ylabel('Fixed Internet Traffic Volume')

plt.show()
```



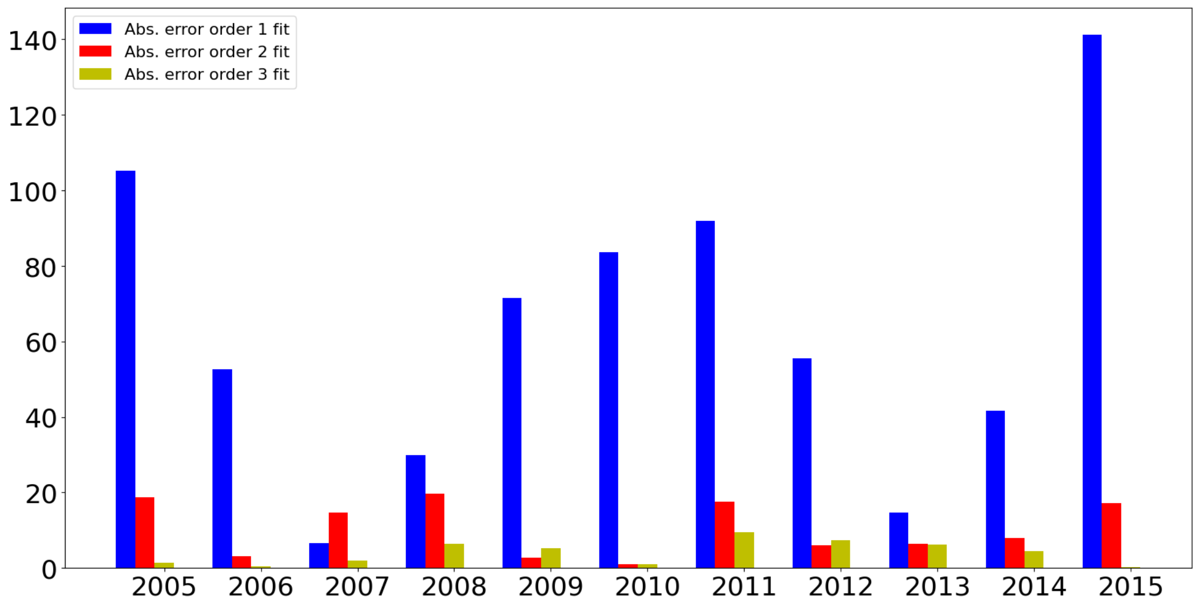

Calculate the error for each order

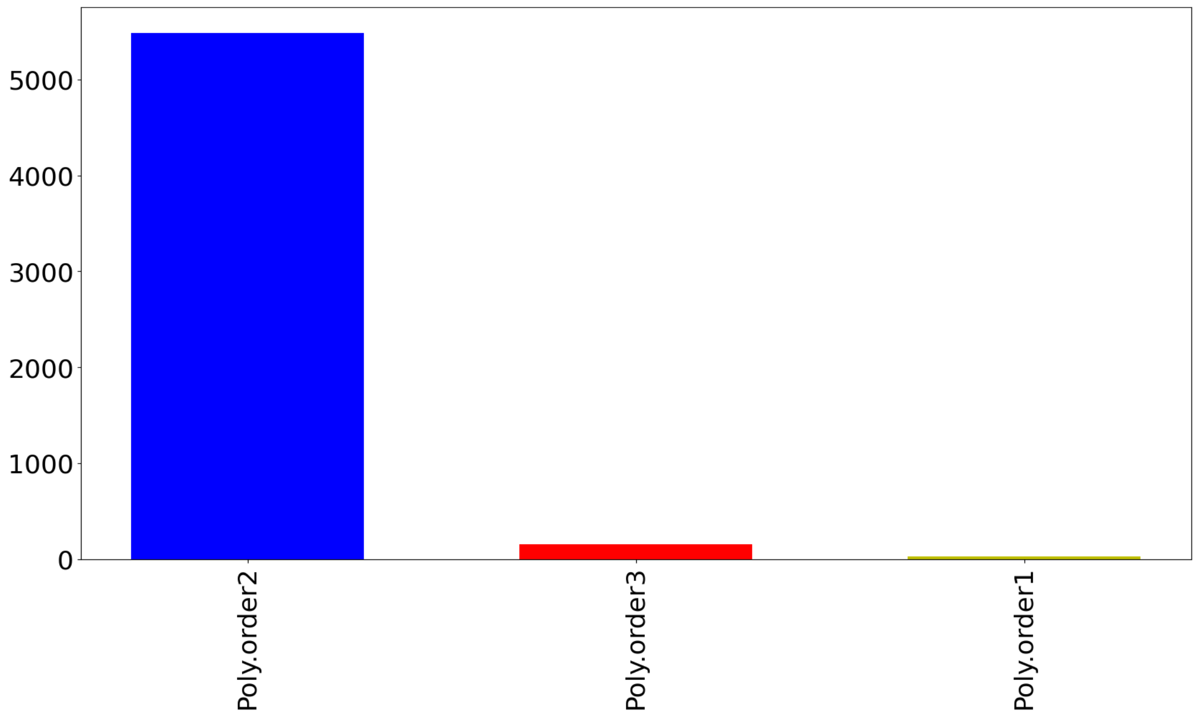
In [133…

```
# Calculate and store the erros
models = []       # to store polynomial model parameters (list of poly1d objects)
errors_hist = []  # to store the absolute errors for each point (2005-2015) and for
mse_hist = []     # to store the MSE for each model (list of numpy floats)

#Try polynomial models with increasing order
for order in range(1,4):
    # Fit polynomial model
    p = (np.poly1d(np.polyfit(x, y, order)))
    models.append(p)

    e = np.abs(y-p(x))        # absolute error
    mse = np.sum(e**2)/len(df_hist) # mse

    errors_hist.append(e)   #Store the absolute errors
    mse_hist.append(mse) # Store the mse
```

In [134...

```
# Visualize fit error for each year

x = df_hist.year
width = 0.2   #size of the bar

fig = plt.figure(figsize=(20,10))
ax = fig.add_subplot(111)

p1 = ax.bar( x, errors_hist[0], width, color = 'b', label = 'Abs. error order 1 fit
p2 = ax.bar( x + width, errors_hist[1], width, color = 'r', label = 'Abs. error ord
p3 = ax.bar( x + 2*width, errors_hist[2], width, color = 'y', label = 'Abs. error o

# "Prettyfy" the bar graph
ax.set_xticks(x+2*width)
ax.set_xticklabels(x)
plt.legend(loc = 'upper left', fontsize =16)
plt.show()

#Visualise MSE for each model
fig = plt.figure(figsize=(20,10))
ax = fig.add_subplot(111)

x = np.array([0,1,2,3])
width = .6   #size of the bar

p1 = ax.bar( x[0], mse_hist[0], width, color = 'b', label = 'pred. error order 1 fi
p2 = ax.bar( x[1], mse_hist[1], width, color = 'r', label = 'pred. error order 2 fi
p3 = ax.bar( x[2], mse_hist[2], width, color = 'y', label = 'pred. error order 3 fi

ax.set_xticks(ticks=[0.0,1.0,2.0],labels={'Poly.order1', 'Poly.order2', 'Poly.order
plt.show()
```






Interpret the result of the fit error for each year

my interpretation of the fit error per order in the our polynomial regression is that as the o**rder of the polynomial regression (from the code above) increases** the **error between the model and the true value decreases**

In [135…

```
# Polynomial function order
order = 3

x = df_hist.year.values      # regressor
y = df_hist.traffic.values   # regressand

# Fit the model, return the polynomial parameter values in a numpy array such that
# y = p[0]*x**order + p[1]*x*(order-1) ...

p_array = np.polyfit(x,y,order)

print(type(p_array), p_array)

# poly1d is a convenience class, used to encapsulate "natural" operations on polyno
# so that said operations may take on their customary form in code

# wrap the p_array in a poly1 object
p = np.poly1d(p_array)

print(type(p), p)

# use the poly1d object to evaluate the value of the polynomial in a specific point
print('The value of the polynomial for x = 2020 is : {} '.format(p(2020)))

# compute the absolute error for each value of x and the MSE error for the estimate
e = np.abs(y-p(x))
mse = np.sum(e**2)/len(x)

print('The estimated polynomial parameters are: {}'.format(p))
print('The errors for each value of x, given the estimated polynomial parameters ar
print('The MSE is :{}'.format(mse))
```

```
<class 'numpy.ndarray'> [ 4.83129404e-01 -2.90500578e+03  5.82252085e+06 -3.8900538
7e+09]
<class 'numpy.poly1d'>         3        2
0.4831 x - 2905 x + 5.823e+06 x - 3.89e+09
The value of the polynomial for x = 2020 is : 2328.5784521102905
The estimated polynomial parameters are:         3        2
0.4831 x - 2905 x + 5.823e+06 x - 3.89e+09
The errors for each value of x, given the estimated polynomial parameters are:
 [1.30743027 0.39125264 2.02722693 6.32983208 5.28394403 0.93069802
 9.41692212 7.34010081 6.27729748 4.48133933 0.16291521]
The MSE is :25.17218620372407
```

## Using sklearn to perform polynomial regression

Import the necessary libraries

In [136…

```
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
```

Training the Polynomial Regression model using degree 3

In [137…

```
poly_reg = PolynomialFeatures(degree=3)
X_poly = poly_reg.fit_transform(x.reshape(-1, 1))
```

In [138…
```
lin_reg = LinearRegression()
lin_reg.fit(X_poly,y)
```

Out[138]:
```
▾ LinearRegression

LinearRegression()
```

Predict the result using polynomial regression model

In [139…
```
y_pred = lin_reg.predict(X_poly)
```

In [140…
```
df = pd.DataFrame({'Real Values': y, 'Predicted Values':y_pred})
```

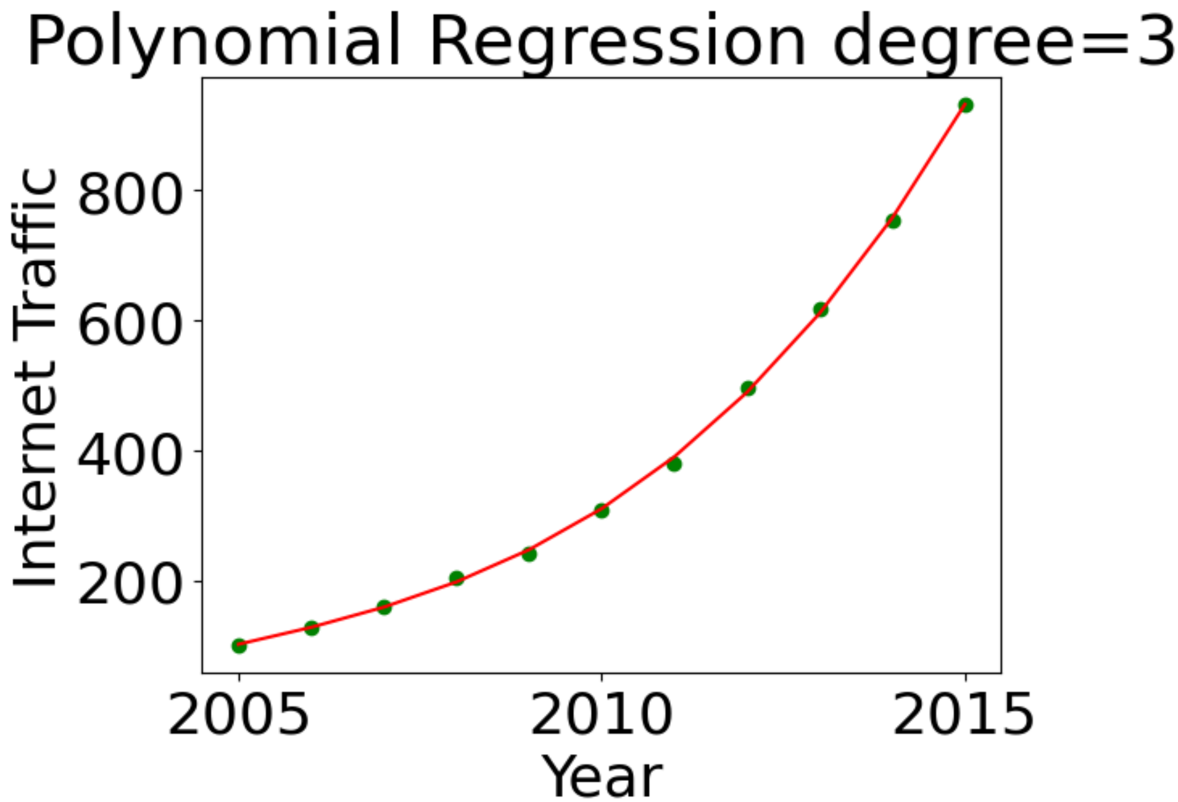
In [141…
```
df
```

Out[141]:

| | Real Values | Predicted Values |
|---|---|---|
| **0** | 100.000000 | 101.241620 |
| **1** | 126.933755 | 127.295559 |
| **2** | 160.303757 | 158.270686 |
| **3** | 203.390603 | 197.068289 |
| **4** | 241.292566 | 246.589652 |
| **5** | 308.791823 | 309.736069 |
| **6** | 379.980659 | 389.408823 |
| **7** | 495.840568 | 488.509205 |
| **8** | 616.207252 | 609.938500 |
| **9** | 752.103483 | 756.598000 |
| **10** | 931.200929 | 931.388989 |

Visualize the Polynomial Regression results

In [142…
```
plt.scatter(x, y, color='green')
plt.plot(x, y_pred, color = 'red')

plt.title("Polynomial Regression degree=3")
plt.xlabel('Year')
plt.ylabel('Internet Traffic')


plt.show();
```




Supplementary Activity:

- Choose your own dataset
- Import the dataset
- Perform polynomial regression using sklearn and polyfit
- Measure the performance for each polynomial degree.
- Plot the performance of the model for each polynomial degree.

In [143…

```
suppath = '/content/drive/MyDrive/DATASETS/Advertising Budget and Sales.csv'
suppdf = pd.read_csv(suppath)
suppdf.head()
```

Out[143]:

| | Unnamed: 0 | TV Ad Budget ($) | Radio Ad Budget ($) | Newspaper Ad Budget ($) | Sales ($) |
|---|---|---|---|---|---|
| **0** | 1 | 230.1 | 37.8 | 69.2 | 22.1 |
| **1** | 2 | 44.5 | 39.3 | 45.1 | 10.4 |
| **2** | 3 | 17.2 | 45.9 | 69.3 | 9.3 |
| **3** | 4 | 151.5 | 41.3 | 58.5 | 18.5 |
| **4** | 5 | 180.8 | 10.8 | 58.4 | 12.9 |

In [144…

```
# cleaning the data
suppdf.drop(columns=['Unnamed: 0'], inplace=True)
suppdf.head()
```

Out[144]:

| | TV Ad Budget ($) | Radio Ad Budget ($) | Newspaper Ad Budget ($) | Sales ($) |
|---|---|---|---|---|
| **0** | 230.1 | 37.8 | 69.2 | 22.1 |
| **1** | 44.5 | 39.3 | 45.1 | 10.4 |
| **2** | 17.2 | 45.9 | 69.3 | 9.3 |
| **3** | 151.5 | 41.3 | 58.5 | 18.5 |
| **4** | 180.8 | 10.8 | 58.4 | 12.9 |

In [145…

```
# renaming columns
suppdf.rename(columns={'TV Ad Budget ($)':'TVBudget(USD)',
                       'Radio Ad Budget ($)':'RadioBudget(USD)',
                       'Newspaper Ad Budget ($)':'NewspaperBudget(USD)',
                       'Sales ($)':'Sales(USD)'}, inplace=True)
suppdf.head()
```

Out[145]:

| | TVBudget(USD) | RadioBudget(USD) | NewspaperBudget(USD) | Sales(USD) |
|---|---|---|---|---|
| **0** | 230.1 | 37.8 | 69.2 | 22.1 |
| **1** | 44.5 | 39.3 | 45.1 | 10.4 |
| **2** | 17.2 | 45.9 | 69.3 | 9.3 |
| **3** | 151.5 | 41.3 | 58.5 | 18.5 |
| **4** | 180.8 | 10.8 | 58.4 | 12.9 |

In [146…

```
# checking for null values
suppdf.isnull().sum()
```

Out[146]:

| | 0 |
|---|---|
| **TVBudget(USD)** | 0 |
| **RadioBudget(USD)** | 0 |
| **NewspaperBudget(USD)** | 0 |
| **Sales(USD)** | 0 |

**dtype:** int64

In [147…

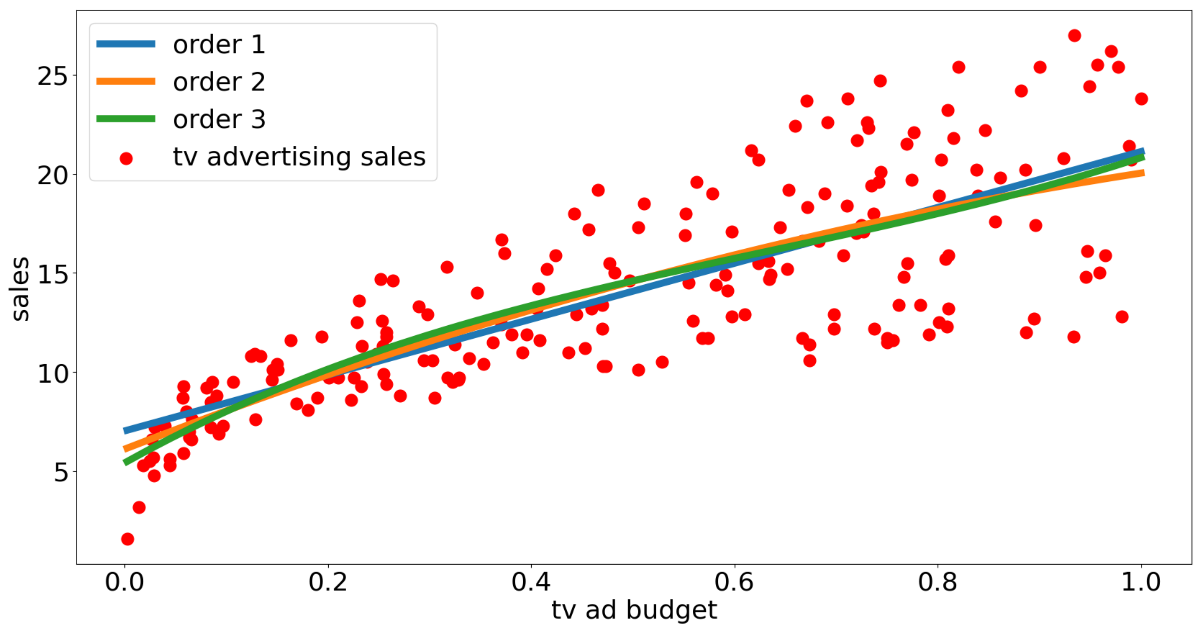
```
# getting the independent variable and dependent variable
x = suppdf['TVBudget(USD)'].values/np.max(suppdf['TVBudget(USD)'].values)
y = suppdf['Sales(USD)']
```

In [148…

```
#fitting and graphing the model
models = []       # to store polynomial model parameters (list of poly1d objects)

for order in range(1,4):# loop for fitting the polynomial regression per degree
    p = (np.poly1d(np.polyfit(x, y, order)))
    models.append(p)

plt.figure(figsize = (20,10))

for model in models[0:3]: # loop for graphing the model per order in polynomial reg
    plt.plot(sorted(x), model(x)[np.argsort(x.ravel())], label = 'order ' + str(len

plt.scatter(x, y, label = 'tv advertising sales', linewidth = 7,color='red')
plt.legend(loc = 'upper left')

# Adding axis labels
plt.xlabel('tv ad budget')
plt.ylabel('sales')
```

Out[148]:

```
Text(0, 0.5, 'sales')
```




In [149…

```
mse_hist = []      # to store the MSE for each model (list of numpy floats)

#Try polynomial models with increasing order
for model in models[0:3]:
    mse = np.sum(e**2)/len(df_hist) # get the mean error of every order model
    mse_hist.append(mse) # store the mse in the list
```

In [166…

```
ord = 1
for order in range(0,3):
  print(f'order {ord} mean error:', mse_hist[order])
  ord += 1
```

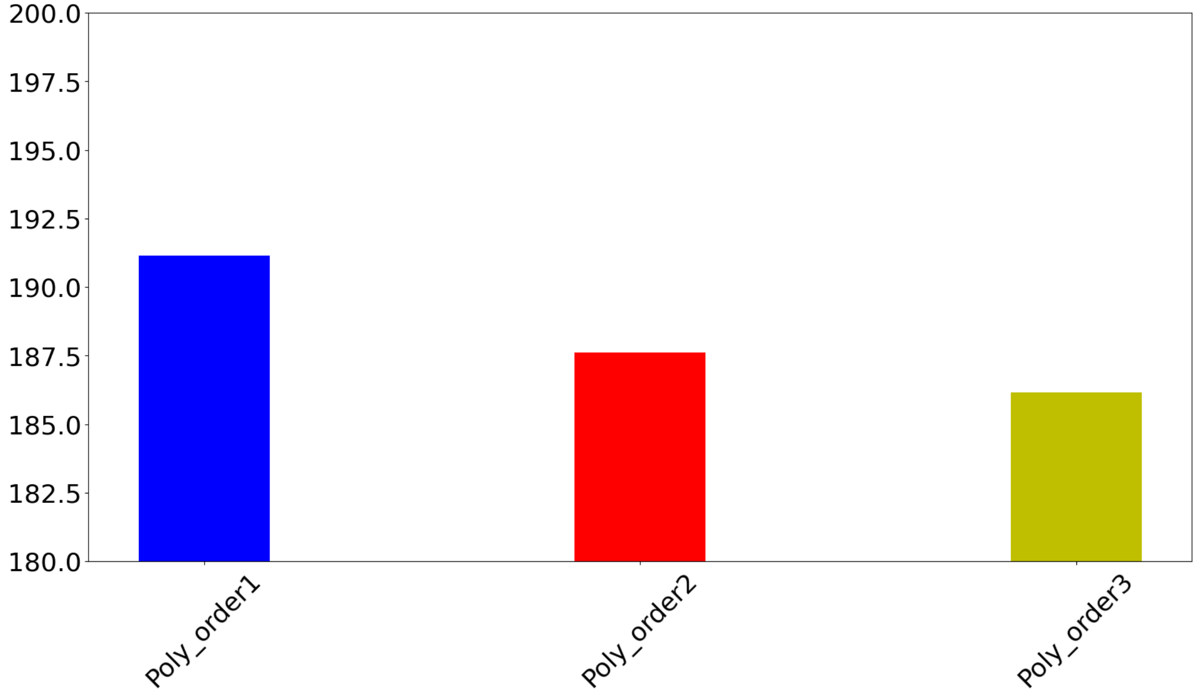
```
order 1 mean error: 191.13914392103197
order 2 mean error: 187.61205509549927
order 3 mean error: 186.15210094717514
```

In [168…

```
#graphing the mean error of the model we created
fig = plt.figure(figsize=(20,10))
ax = fig.add_subplot(111)

x = np.array([0,1,2])
p1 = ax.bar( x[0], mse_hist[0], width, color = 'b', label = 'pred. error order 1 fi
p2 = ax.bar( x[1], mse_hist[1], width, color = 'r', label = 'pred. error order 2 fi
p3 = ax.bar( x[2], mse_hist[2], width, color = 'y', label = 'pred. error order 3 fi

plt.ylim(180,200) #zooms in the value of y limit
ax.set_xticks(ticks=[0.0,1.0,2.0],labels=['Poly_order1', 'Poly_order2', 'Poly_order
```

Out[168]:

```
[<matplotlib.axis.XTick at 0x7b6fbbc8edd0>,
 <matplotlib.axis.XTick at 0x7b6fbbc8d9f0>,
 <matplotlib.axis.XTick at 0x7b6fbbc8e560>]
```




Conclusion:

what I learn today in this activity is about polynomial regression, how to model it, evaluate it by getting the mean error, and the absolute error, I learned that by fitting the polynomial comes with the value called order the value of the order of the poly regression is inversely proportional to the error of the model, I also concluded that as order of the poly fit increases the more it joins the trend of the datapoints of our dataset