4190.310 Programming Language

The K- Language

1 Syntax

Expression e →	`unit`				unit
	|	*x* `:=` *e*			assignment
	|	*e* `;` *e*			sequence
	|	`if` *e* `then` *e* `else` *e*			branch
	|	`while` *e* `do` *e*			while loop
	|	`for` *x* `:=` *e* `to` *e* `do` *e*			for loop
	|	`read` *x*			input
	|	`write` *e*			output
	|	`let` *x* `:=` *e* `in` *e*			variable binding
	|	`let proc` *f*(*x*)`=` *e* `in` *e*			procedure binding
	|	*f*(*e*)			call by value
	|	*f*`<`*x*`>`			call by reference
	|	*n*			integer
	|	`true` | `false`			boolean
	|	*x*			identifier
	|	*e* `+` *e* | *e* `-` *e* | *e* `*` *e* | *e* `/` *e*			arithmetic operation
	|	*e* `<` *e* | *e* `=` *e* | `not` *e*			conditional operation

1.1 Program

A program is an expression.

1.2   Identifiers

Alpha-numeric identifiers are [a-zA-Z][a-zA-Z0-9_]*. Identifiers are case sensitive: z and Z are different. The reserved words cannot be used as identifiers: unit true false not if then else let in end proc while do for to read write

1.3   Numbers/Comments

Numbers are integers, optionally prefixed with -(for negative integer): -?[0-9]$^+$.

   A comment is any character sequence within the comment block (* *). The comment block can be nested.

1.4   Precedence/Associativity

In parsing K-- program text, the precedence of the K-- constructs in decreasing order is as follows. Symbols in the same set have identical precedence. Symbols with subscript L (respectively R) are left (respectively right) associative. Symbols without subscript are nonassociative.

$$\{\texttt{not}\}_R,$$
$$\{\texttt{*},\texttt{/}\}_L,$$
$$\{\texttt{+},\texttt{-}\}_L,$$
$$\{\texttt{=},\texttt{<}\}_L,$$
$$\{\texttt{write}\}_R,$$
$$\{\texttt{:=}\}_R,$$
$$\{\texttt{else}\},$$
$$\{\texttt{then}\},$$
$$\{\texttt{do}\},$$
$$\{\texttt{;}\}_L,$$
$$\{\texttt{in}\}$$

For example, K-- program

```
x := e1; e2               ⇒  (x := e1) ; e2
while e do e1; e2         ⇒  (while e do e1); e2
if e1 then e2 else e3; e4 ⇒  (if e1 then e2 else e3); e4
```

Rule of thumb: for your test programs, if your programs are hard to read (hence can be parsed not as you expected) then put parentheses around.

2   Domains

$$
\begin{array}{rccll}
n & \in & \mathbb{Z} & & \text{integer} \\
b & \in & \mathbb{B} & & \text{boolean} \\
v & \in & Val & = & \mathbb{Z} + \mathbb{B} + \{\cdot\} \\
\sigma & \in & Env & = & Id \overset{fin}{\to} Addr + Procedure \\
M & \in & Mem & = & Addr \overset{fin}{\to} Val \\
x, y & \in & Id & & \text{identifier} \\
l & \in & Addr & & \text{an index set} \\
& & Procedure & = & Id \times Expression \times Env
\end{array}
$$

3   Semantics

$$\text{TRUE} \; \frac{}{\sigma, M \vdash \mathtt{true} \Rightarrow true, M} \qquad \text{FALSE} \; \frac{}{\sigma, M \vdash \mathtt{false} \Rightarrow false, M}$$

$$\text{NUM} \; \frac{}{\sigma, M \vdash \mathtt{n} \Rightarrow n, M} \qquad \text{UNIT} \; \frac{}{\sigma, M \vdash \mathtt{unit} \Rightarrow \cdot, M}$$

$$\text{VAR} \; \frac{}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

$$\text{RECF} \; \frac{}{\sigma, M \vdash \mathtt{\{\}} \Rightarrow \cdot, M}$$

$$\text{RECT} \; \frac{\sigma, M \vdash e_k \Rightarrow v_k, M_k \qquad \sigma, M_k \vdash \{x_{k+1} \; \mathtt{:=} \; e_{k+1}, \cdots\} \Rightarrow v', M'}{\sigma, M \vdash \{x_k \; \mathtt{:=} \; e_k, x_{k+1} \; \mathtt{:=} \; e_{k+1}, \cdots\} \Rightarrow v', M'\{x_k \mapsto v_k\}}$$

$$\text{ADD} \; \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \; \mathtt{+} \; e_2 \Rightarrow n_1 + n_2, M''}$$

$$\text{SUB} \; \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \; \mathtt{-} \; e_2 \Rightarrow n_1 - n_2, M''}$$

$$\text{MUL} \; \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \; \mathtt{*} \; e_2 \Rightarrow n_1 * n_2, M''}$$

$$\text{DIV} \; \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \; \mathtt{/} \; e_2 \Rightarrow n_1/n_2, M''}$$

$$\text{EQUALT} \quad \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 \ \texttt{=}\ e_2 \Rightarrow \texttt{true}, M''} \quad \begin{array}{l} v_1 = v_2 = n \\ \vee\ v_1 = v_2 = b \\ \vee\ v_1 = v_2 = \cdot \end{array}$$

$$\text{EQUALF} \quad \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 \ \texttt{=}\ e_2 \Rightarrow \texttt{false}, M''} \quad \texttt{otherwise}$$

$$\text{LESS} \quad \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \ \texttt{<}\ e_2 \Rightarrow n_1 < n_2, M''}$$

$$\text{NOT} \quad \frac{\sigma, M \vdash e \Rightarrow b, M'}{\sigma, M \vdash \texttt{not}\ e \Rightarrow not\ b, M'}$$

$$\text{ASSIGN} \quad \frac{\sigma, M \vdash e \Rightarrow v, M'}{\sigma, M \vdash x \ \texttt{:=}\ e \Rightarrow v, M'\{\sigma(x) \mapsto v\}}$$

$$\text{RECASSIGN} \quad \frac{\sigma, M \vdash e_1 \Rightarrow r, M_1 \qquad \sigma, M_1 \vdash e_2 \Rightarrow v, M_2}{\sigma, M \vdash e_1.x \ \texttt{:=}\ e_2 \Rightarrow v, M_2\{r(x) \mapsto v\}}$$

$$\text{SEQ} \quad \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 \ \texttt{;}\ e_2 \Rightarrow v_2, M''}$$

$$\text{IFT} \quad \frac{\sigma, M \vdash e \Rightarrow true, M' \qquad \sigma, M' \vdash e_1 \Rightarrow v, M''}{\sigma, M \vdash \texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \Rightarrow v, M''}$$

$$\text{IFF} \quad \frac{\sigma, M \vdash e \Rightarrow false, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v, M''}{\sigma, M \vdash \texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \Rightarrow v, M''}$$

$$\text{WHILEF} \quad \frac{\sigma, M \vdash e_1 \Rightarrow false, M'}{\sigma, M \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 \Rightarrow \cdot, M'}$$

$$\text{WHILET} \quad \frac{\sigma, M \vdash e_1 \Rightarrow true, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_1, M_1 \qquad \sigma, M_1 \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 \Rightarrow v_2, M_2}$$

$$\text{FORT } \frac{\begin{array}{c} \sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M'' \\ \sigma, M''\{\sigma(x) \mapsto n_1 + 0\} \vdash e_3 \Rightarrow v_0, M_0 \\ \vdots \\ \sigma, M_{n_2-n_1-1}\{\sigma(x) \mapsto n_1 + (n_2 - n_1)\} \vdash e_3 \Rightarrow v_{n_2-n_1}, M_{n_2-n_1} \end{array}}{\sigma, M \vdash \texttt{for } x \texttt{ := } e_1 \texttt{ to } e_2 \texttt{ do } e_3 \Rightarrow \cdot, M_{n_2-n_1}} \; n_2 \geq n_1$$

$$\text{FORF } \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash \texttt{for } x \texttt{ := } e_1 \texttt{ to } e_2 \texttt{ do } e_3 \Rightarrow \cdot, M''} \; n_2 < n_1$$

$$\text{LETV } \frac{\begin{array}{c} \sigma, M \vdash e_1 \Rightarrow v, M' \\ \sigma\{x \mapsto l\}, M'\{l \mapsto v\} \vdash e_2 \Rightarrow v', M'' \end{array}}{\sigma, M \vdash \texttt{let } x \texttt{ := } e_1 \texttt{ in } e_2 \Rightarrow v', M''} \; l \notin Dom\, M'$$

$$\text{LETF } \frac{\sigma\{f \mapsto \langle (x_1, \cdots, x_n), e, \sigma \rangle\}, M \vdash e_2 \Rightarrow v, M'}{\sigma, M \vdash \texttt{let proc } f(x_1, \; \cdots, \; x_n) \texttt{ = } e_1 \texttt{ in } e_2 \Rightarrow v, M'}$$

$$\text{CALLV } \frac{\begin{array}{c} \sigma, M \vdash e_1 \Rightarrow v_1, M_1 \\ \sigma, M \vdash e_2 \Rightarrow v_2, M_2 \\ \vdots \\ \sigma, M \vdash e_n \Rightarrow v_n, M_n \\ \sigma'\{x_1 \mapsto l_1\} \cdots \{x_n \mapsto l_n\}\{f \mapsto \langle (x_1, \cdots, x_n), e', \sigma' \rangle\}, \\ M'\{l_1 \mapsto v_1\} \cdots \{l_n \mapsto v_n\} \vdash e' \Rightarrow v', M' \end{array}}{\sigma, M \vdash f(e_1, \; \cdots, \; e_n) \Rightarrow v', M'} \; \begin{array}{l} \sigma(f) = \langle (x_1, \cdots, x_n), e', \sigma' \rangle \\ l_i \notin Dom\, M' \end{array}$$

$$\text{CALLR } \frac{\sigma'\{x \mapsto \sigma(y)\}\{f \mapsto \langle x, e, \sigma' \rangle\}, M \vdash e \Rightarrow v, M'}{\sigma, M \vdash f\texttt{<}y\texttt{>} \Rightarrow v, M'} \; \sigma(f) = \langle x, e, \sigma' \rangle$$

$$\text{READ } \frac{}{\sigma, M \vdash \texttt{read } x \Rightarrow n, M\{\sigma(x) \mapsto n\}}$$

$$\text{WRITE } \frac{\sigma, M \vdash e \Rightarrow n, M'}{\sigma, M \vdash \texttt{write } e \Rightarrow n, M'}$$