the conditional probability of A given B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A and B are independent

$$P(A \cap B) = P(A)P(B)$$

chain rule

Extend to three events:

$$P(A,B,C) = P(A \cap B \cap C) = P(A|B,C)P(B,C)$$
$$= P(A|B,C)P(B|C)P(C)$$

X is conditionally independent of Y given Z

if and only if

$$P(X|Y \cap Z) = P(X|Z)$$

$$P(X \cap Y|Z) = P(X|Z)P(Y|Z)$$

Bayes' Theorem

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

Linearity of Expectation

$$\mathbb{E}[Z] = \mathbb{E}[X_1 + \cdots + X_n] = \mathbb{E}[X_1] + \cdots + \mathbb{E}[X_n] = n \cdot p$$

$$\mathbb{E}(aX + bY) = a \cdot \mathbb{E}(X) + b \cdot \mathbb{E}(Y)$$

無意識統計學家定律

$$\mathbb{E}[g(X)] = \sum_{\omega \in S} g(X(\omega)) \cdot P(\omega)$$

Variance

$$\mathrm{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] = \sum_x p_X(x) \cdot (x - \mathbb{E}[X])^2$$

$$\mathrm{Var}(a \cdot X + b) = a^2 \cdot \mathrm{Var}(X)$$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$
$$= \sum X^2 P(X) - \mathbb{E}[X]^2$$

Uniform Distribution

Notation: $X \sim \mathrm{Unif}(a, b)$

PMF: $\mathrm{P}(X = i) = \frac{1}{b-a+1}$

Expectation: $\mathbb{E}[X] = \frac{a+b}{2}$

Variance: $\mathrm{Var}(X) = \frac{(b-a)(b-a+2)}{12}$

Bernoulli

Notation: $X \sim \mathrm{Ber}(p)$

PMF: $P(X = 1) = p,\ P(X = 0) = 1 - p$

Expectation: $\mathbb{E}[X] = p$ Note: $\mathbb{E}[X^2] = p$

Variance: $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = p - p^2 = p(1-p)$

Binomial

Notation: $X \sim \mathrm{Bin}(n, p)$

PMF: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Expectation: $\mathbb{E}[X] = np$

Variance: $\mathrm{Var}(X) = np(1-p)$

Geometric

Notation: $X \sim \mathrm{Geo}(p)$

PMF: $P(X = k) = (1-p)^{k-1}p$

Expectation: $\mathbb{E}[X] = \frac{1}{p}$

Variance: $\mathrm{Var}(X) = \frac{1-p}{p^2}$

Hypergeometric

Notation: $X \sim \mathrm{HypGeo}(N, K, n)$

PMF: $P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$

Expectation: $\mathbb{E}[X] = n\frac{K}{N}$

Variance: $\mathrm{Var}(X) = n\frac{K(N-K)(N-n)}{N^2(N-1)}$

$$\overset{np}{\boxed{\mathbb{E}(X) = n \cdot \frac{K}{N}}} \quad \overset{npq \times adjustment}{\boxed{\mathrm{Var}(X) = n \cdot \frac{K}{N} \cdot \frac{N-K}{N} \cdot \boxed{\frac{N-n}{N-1}}}}$$

Poisson

$$P(X = x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$$

$$\mathbb{E}[X] = \lambda$$

$$\mathrm{Var}(X) = \lambda$$