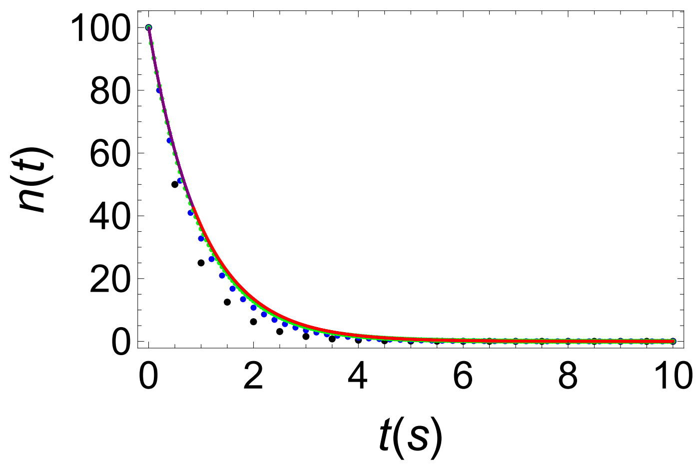
Homework 4

- Name: Eslam Muhammed Ahmed Zenhom
- ID: 201700788

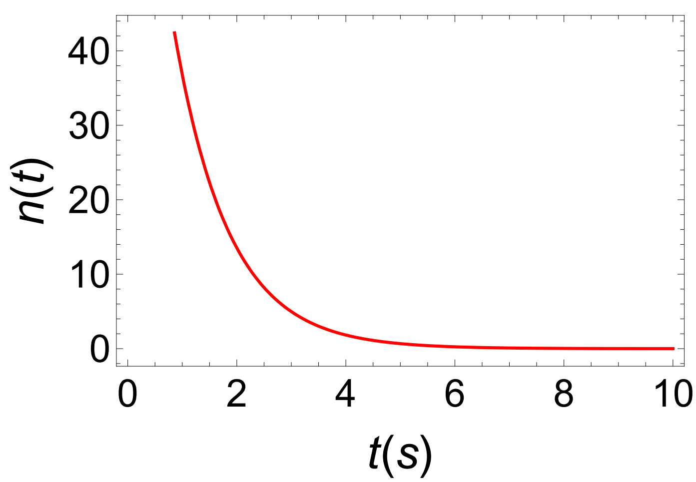
Problem 1

```
ClearAll["Global`*"];
n[t_, n0_, τ_ ] := n0 ⅇ^(-t/τ) (* Exact solution *)
tau = 1.0;  tStart = 0.0; tMax = 10;  n0 = 100;
dt = {0.5, 0.2, 0.05, 0.001}; (* different time steps *)

ldt = Length[dt];
color = {Black, Blue, Green, Purple};
pList = Array[plot, ldt];

Do[ (* over the values of dt *)
  tList = Range[tStart, tMax, dt[[ndt]]];
  lt = Length[tList];
  nt = Table[0, {i, lt}];
  nt[[1]] = n0;
  const = (1 - dt[[ndt]] / tau);

   (* Calculation *)
  Do[nt[[i]] = const nt[[i - 1]], {i, 2, lt}];
  (* Euler method *)ntList = Table[{tList[[i]], nt[[i]]}, {i, lt}];

  (* Results *)
  plot[ndt] = ListPlot[ntList, PlotStyle → color[[ndt]]];
  , {ndt, ldt}];

p2 = Plot[n[t, n0, tau ], {t, tStart, tMax}, PlotStyle → Red];
Show[{pList, p2}, PlotRange -> All, Frame → True,
 FrameLabel → {Style[t[s], Large], Style[n[t], Large]}, LabelStyle → 20]
Show[{p2}, PlotRange -> All, Frame → True,
 FrameLabel → {Style[t[s], Large], Style[n[t], Large]}, LabelStyle → 20]

(*The exact solution comes from DSolve[{n'[t]==-n[t]/τ,n[0]==1},n[t],t] *)
(*the purple line is almost above the red oneas the red one is the
 exact while the purple has a step of 0.001 and the pattern is clear as we
 decrease the step we find the curves converges to the red exact solution.*)
```

Out[•]=



Out[•]=



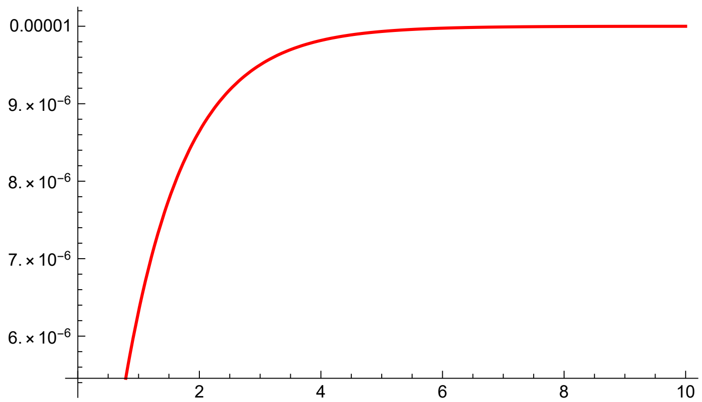
Problem 2

```
clearAll["Global`*"];
v = 10; R = 10^6;  c = 10^-6;
 sol = DSolve[{v == q'[t] * R + q[t] / c, q[0] == 0}, q[t], t] (* Exact solution *)
n1  = q[t] /. sol
dt  = {0.5, 0.2, 0.05, 0.001}; (* different time steps *)

ldt  = Length[dt];
color  = {Black, Blue, Green,  Yellow};
pList  = Array[plot, ldt];

Do[ (* over the values of dt *)
  tList  = Range[tStart, tMax, dt[[qdt]]];
  lt  = Length[tList];
  qt  = Table[0, {i, lt}];
  qt[[1]]  = 0;
  const  =  (1 - dt[[qdt]]);

   (* Calculation *)
  Do[qt[[i]]  =  const qt[[i - 1]] + dt[[qdt]] * v/R, {i, 2, lt}];
  (* Euler method *)ntList  = Table[{tList[[i]], qt[[i]]}, {i, lt}];

  (* Results *)
  plot[qdt]  = ListPlot[ntList, PlotStyle → color[[qdt]]];
  , {qdt, ldt}];

p2  = Plot[n1, {t, tStart, tMax}, PlotStyle → Red]
Show[{pList, p2}, PlotRange -> All, Frame  → True,
 FrameLabel → {Style[t[s], Large], Style[q[t], Large]}, LabelStyle → 20]

(*The exact solution comes from DSolve[{n'[t]==-n[t]/τ,n[0]==1},n[t],t]  *)
(*For part b as you can see I plotted for four values of
 Delta t and when the step decreases to the lowest one (0.001) the
 yellow graph it converges to the exact value in the red graph*)
```

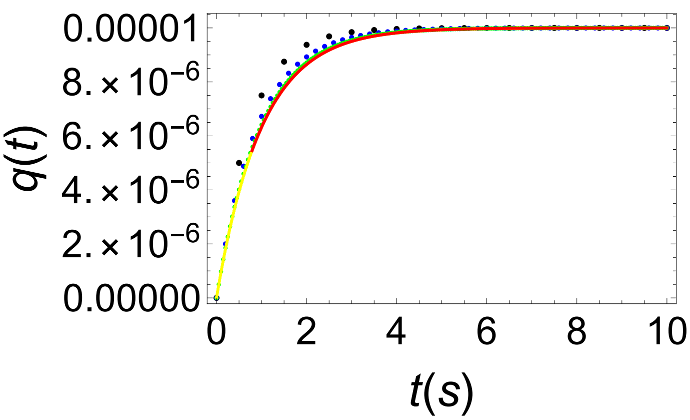
Out[ ]= $\left\{\left\{q[t] \to \frac{e^{-t}\left(-1+e^{t}\right)}{100000}\right\}\right\}$

Out[ ]= $\left\{\frac{e^{-t}\left(-1+e^{t}\right)}{100000}\right\}$

Out[ ]=


0.00001
9. × 10⁻⁶
8. × 10⁻⁶
7. × 10⁻⁶
6. × 10⁻⁶
2
4
6
8
10

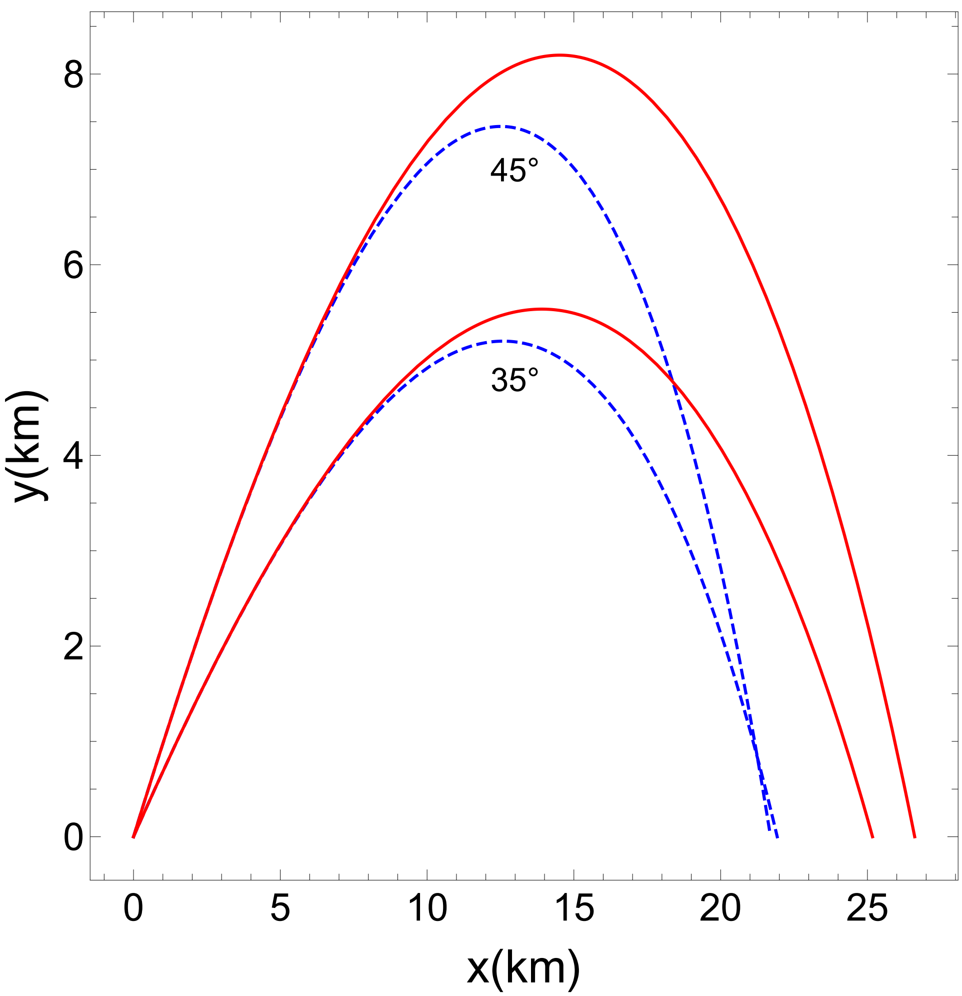
Out[ ]=


0.00001
8. × 10⁻⁶
6. × 10⁻⁶
4. × 10⁻⁶
2. × 10⁻⁶
0.00000
q(t)
0
2
4
6
8
10
t(s)

Problem 3

In[ ]:=
```
clearAll["Global`*"];
y0 = 10^4;
grav = 9.8; tmax = 1000; v0 = 700;
B2m = 4.0×10^-5 * e^(-y[t]/y0); (* B2m = B2/m *)
θ = π/180.0 {35, 45} ; lθ = Length[θ];
pWDrag = Table[0, {i, lθ }]; (* Trajectory plots for diff angles *)
Do[vx0 = v0 Cos[θ[[nθ]]]; vy0 = v0 Sin[θ[[nθ]]];
 solProj = NDSolve[{x'[t] == vx[t], vx'[t] == -B2m vx[t] Sqrt[vx[t]^2 + vy[t]^2], y'[t] == vy[t],
    vy'[t] == -grav - B2m vy[t] Sqrt[vx[t]^2 + vy[t]^2], x[0] == 0., y[0] == 0., vx[0] == vx0,
    vy[0] == vy0, WhenEvent[y[t] == 0.0, end = t]}, {x, y}, {t, tmax}];
pWDrag[[nθ]] = ParametricPlot[Evaluate[{x[t], y[t]}/1000. /. solProj],
   {t, 0, end}, PlotStyle → Red], {nθ, lθ }]
pWDall = Show[pWDrag, PlotRange -> All, Frame → True,
   FrameLabel → {Style["x(km)", Large], Style["y(km)", Large]},
   LabelStyle → 20, ImageSize → 500 , AspectRatio -> 1,
   Epilog → { Inset[Style["35°", 17] , {13, 4.8}], Inset[Style["45°", 17], {13, 7}]}];


C2m = 4.0×10^-5;
pWDrag2 = Table[0, {i, lθ }]; (* Trajectory plots for diff angles *)
Do[vx0 = v0 Cos[θ[[nθ]]]; vy0 = v0 Sin[θ[[nθ]]];
 solProj = NDSolve[{x'[t] == vx[t], vx'[t] == -C2m vx[t] Sqrt[vx[t]^2 + vy[t]^2], y'[t] == vy[t],
    vy'[t] == -grav - C2m vy[t] Sqrt[vx[t]^2 + vy[t]^2], x[0] == 0., y[0] == 0., vx[0] == vx0,
    vy[0] == vy0, WhenEvent[y[t] == 0.0, end = t]}, {x, y}, {t, tmax}];
pWDrag2[[nθ]] = ParametricPlot[Evaluate[{x[t], y[t]}/1000. /. solProj],
   {t, 0, end}, PlotStyle → {Blue, Dashed}], {nθ, lθ }]
pWDall = Show[pWDrag2, pWDrag, PlotRange → All, Frame → True,
  FrameLabel → {Style["x(km)", Large], Style["y(km)", Large]},
  LabelStyle → 20, ImageSize → 500 , AspectRatio -> 1,
  Epilog → { Inset[Style["35°", 17] , {13, 4.8}], Inset[Style["45°", 17], {13, 7}]}]
```

Out[ ]=

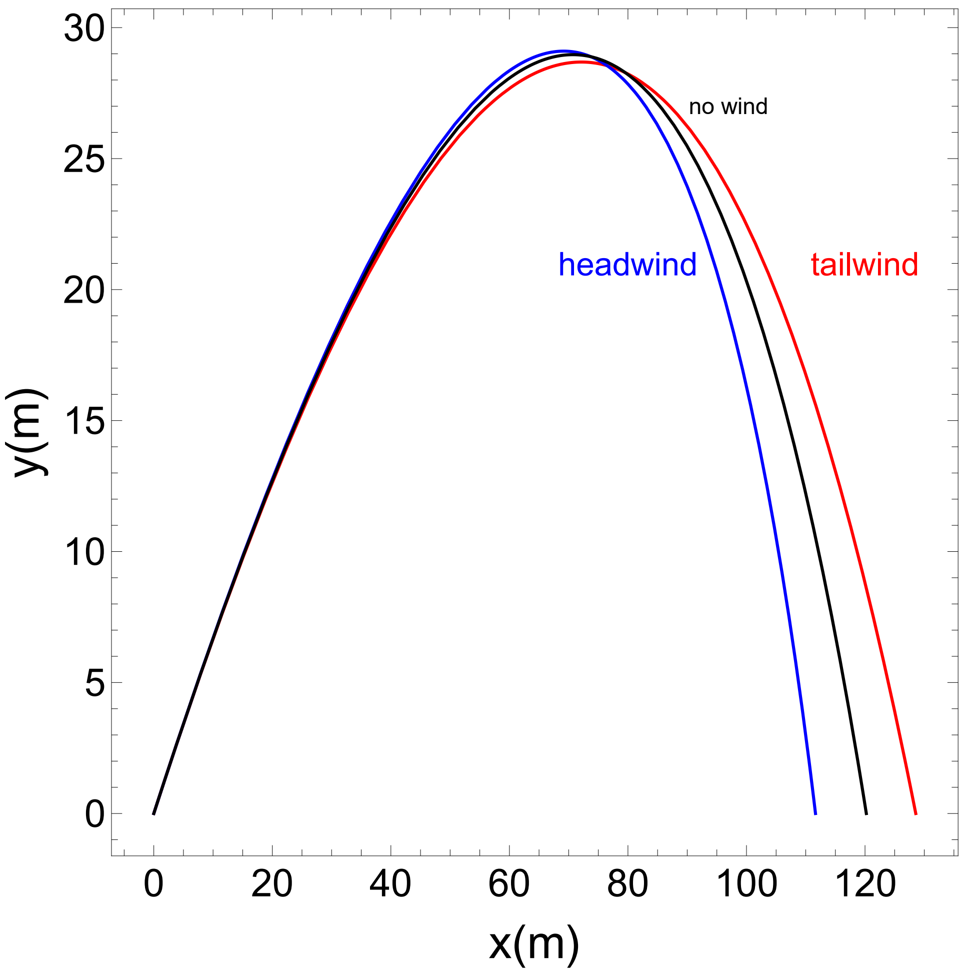


Problem 4

In[ ]:=
```
clearAll["Global`*"];
(*Tail wind*)

grav = 9.8; tmax = 1000; v0 = 49.1744; vwind = 4.4704;
B2m = 0.0039 + 0.0058 / (1 + Exp[((Sqrt[vx[t]^2 + vy[t]^2] - vwind) - 35) / 5]); (* B2m = B2/m *)
θ = π/180.0 {35} ; lθ = Length[θ];
pWDrag1 = Table[0, {i, lθ }]; (* Trajectory plots for diff angles *)
Do[vx0 = v0 Cos[θ[[nθ]]]; vy0 = v0 Sin[θ[[nθ]]];
 solProj = NDSolve[{x'[t] == vx[t],
    vx'[t] == -B2m (vx[t] - vwind) * Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], y'[t] == vy[t],
    vy'[t] == -grav - B2m vy[t] Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], x[0] == 0., y[0] == 0.,
    vx[0] == vx0, vy[0] == vy0, WhenEvent[y[t] == 0.0, end = t]}, {x, y}, {t, tmax}];
pWDrag1[[nθ]] = ParametricPlot[Evaluate[{x[t], y[t]} /. solProj],
   {t, 0, end}, PlotStyle → Red], {nθ, lθ }]
```

```
(*Head wind*)
grav = 9.8; tmax = 1000; v0 = 49.1744; vwind = -4.4704;
B2m = 0.0039 + 0.0058 / (1 + Exp[((Sqrt[vx[t]^2 + vy[t]^2] - vwind) - 35) / 5]); (* B2m = B2/m *)
θ = π/180.0 {35} ; lθ = Length[θ];
pWDrag2 = Table[0, {i, lθ }]; (* Trajectory plots for diff angles *)
Do[vx0 = v0 Cos[θ[[nθ]]]; vy0 = v0 Sin[θ[[nθ]]];
 solProj = NDSolve[{x'[t] == vx[t],
    vx'[t] == -B2m (vx[t] - vwind) * Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], y'[t] == vy[t],
    vy'[t] == -grav - B2m vy[t] Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], x[0] == 0., y[0] == 0.,
    vx[0] == vx0, vy[0] == vy0, WhenEvent[y[t] == 0.0, end = t]}, {x, y}, {t, tmax}];
pWDrag2[[nθ]] = ParametricPlot[Evaluate[{x[t], y[t]} /. solProj],
   {t, 0, end}, PlotStyle → Blue], {nθ, lθ }]


(*No wind*)

grav = 9.8; tmax = 1000; v0 = 49.1744; vwind = 0;
B2m = 0.0039 + 0.0058 / (1 + Exp[((Sqrt[vx[t]^2 + vy[t]^2] - vwind) - 35) / 5]); (* B2m = B2/m *)
θ = π/180.0 {35} ; lθ = Length[θ];
pWDrag3 = Table[0, {i, lθ }]; (* Trajectory plots for diff angles *)
Do[vx0 = v0 Cos[θ[[nθ]]]; vy0 = v0 Sin[θ[[nθ]]];
 solProj = NDSolve[{x'[t] == vx[t],
    vx'[t] == -B2m (vx[t] - vwind) * Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], y'[t] == vy[t],
    vy'[t] == -grav - B2m vy[t] Abs[ (Sqrt[vx[t]^2 + vy[t]^2] - vwind)], x[0] == 0., y[0] == 0.,
    vx[0] == vx0, vy[0] == vy0, WhenEvent[y[t] == 0.0, end = t]}, {x, y}, {t, tmax}];
pWDrag3[[nθ]] = ParametricPlot[Evaluate[{x[t], y[t]} /. solProj],
   {t, 0, end}, PlotStyle → Black], {nθ, lθ }]

pWDall = Show[pWDrag1, pWDrag2, pWDrag3, PlotRange → All,
  Frame → True, FrameLabel → {Style["x(m)", Large], Style["y(m)", Large]},
  LabelStyle → 20, ImageSize → 500 , AspectRatio -> 1, Epilog →
   { Inset[Style["tailwind", 17, Red] , {120, 21}], Inset[Style["headwind", 17, Blue] ,
     {80, 21}], Inset[Style["no wind", 12, Black] , {97, 27}]}]
```

Out[•]=



no wind
headwind
tailwind
y(m)
x(m)
0
5
10
15
20
25
30
0
20
40
60
80
100
120