Homework 6

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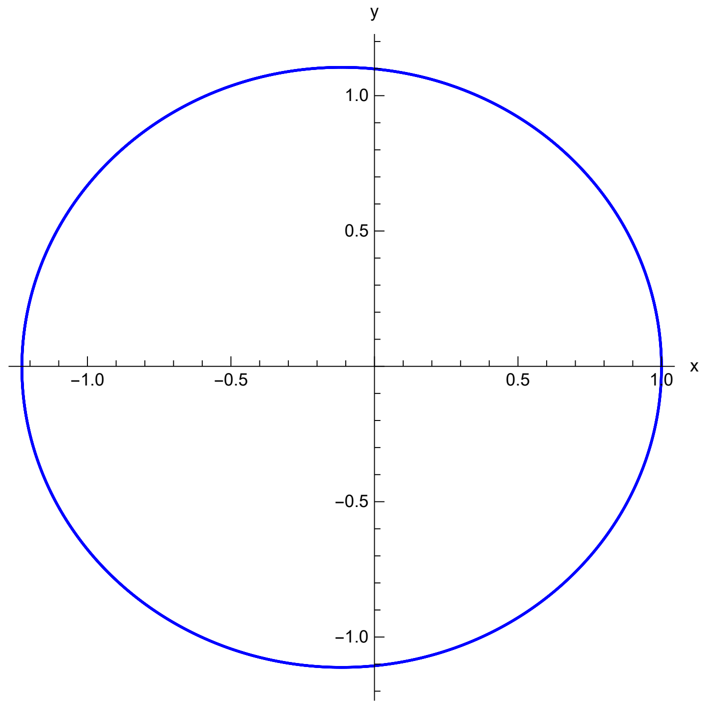
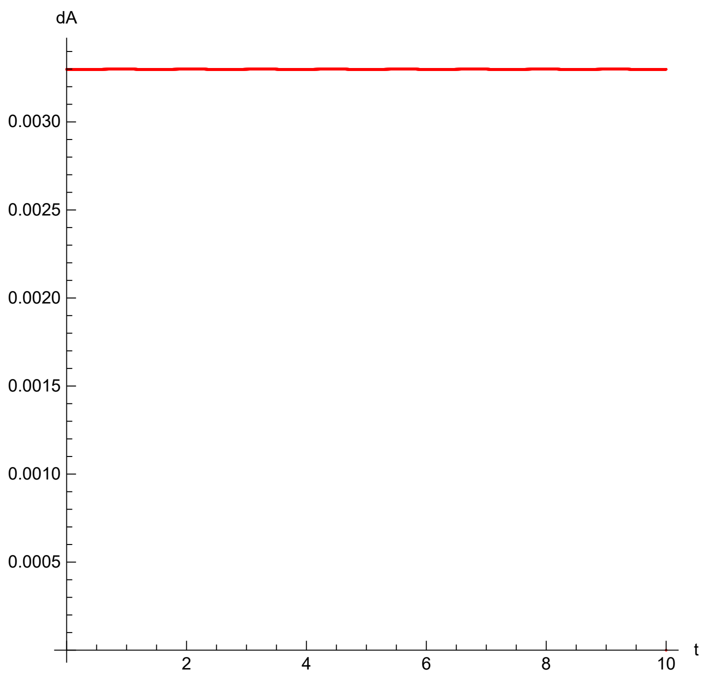
Problem 1 (Section1: 3)

```
ClearAll["Global`*"];
 tmax = 10;
dt = 0.001;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = 1.0; y0 = 0.0;
vx0 = 0.0; vy0 = 2.1 Pi;
Tab
vxList = Table[0, {i, lt}]; vyList = vxList; xList = vxList;
yList = vxList;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;

dthetaList = 0 tL;
dA = 0 tL;

Do[
  rNorm = Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  dthetaList[[nt - 1]] =
   VectorAngle[{xList[[nt]], yList[[nt]]}, {xList[[nt - 1]], yList[[nt - 1]]}] ;
  dA[[nt - 1]] = 0.5 * rNorm^2 * dthetaList[[nt - 1]]
  , {nt, 2, lt}];
xylist = Table[{xList[[j]], yList[[j]]}, {j, lt}];
dAtlist = Table[{tL[[j]], dA[[j]]}, {j, lt}];
p1 = ListPlot[xylist, PlotStyle → Blue, AspectRatio → 1, AxesLabel → {"x", "y"}]
p2 = ListPlot[dAtlist, PlotStyle → Red,
  AspectRatio → 1, PlotRange → All, AxesLabel → {"t", "dA"}]
(*As you can see in the final plot which is time
 (where the increments are dt) versus the change in
 area: it is obvious that the change of area is constant with time which verify kepler
   second law as this means that the planet sweep same area for same time interval*)
(*I used velocity which 2.1 Pi to make it eeliptical oribit. anyother
 velocity might be used except 2 pi as this makes the orbit circular*)
```

Out[•]=



Out[•]=



Out[•]=

Problem 2 (Section1: (5) Circular)

In[•]:=

```
(*Circular*)
ClearAll["Global`*"];
```

```
 tmax = 10;
me = 1; (*put mass of earth as 1: it wouldn't matter as it
  will cancel which comparing potential or kientic or total energy*)
dt = 0.001;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = 1.0; y0 = 0.0;
vx0 = 0.0; vy0 = 2 Pi;
Tab
vxList = Table[0, {i, lt}]; vyList = vxList; xList = vxList;
yList = vxList;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;

dthetaList = 0 tL;
dA = 0 tL;
ke = 0 tL;
pe = 0 tL;
totale = 0 tL;
angmomentum = 0 tL;
Do[
  rNorm =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]]  = vxList[[nt - 1]]  - 4 Pi^2 dt xList[[nt - 1]] / rNorm^3;
  vyList[[nt]]  = vyList[[nt - 1]]  - 4 Pi^2 dt yList[[nt - 1]] / rNorm^3;
  xList[[nt]]  =  xList[[nt - 1]]  + vxList[[nt]] dt;
  yList[[nt]]  =  yList[[nt - 1]]  + vyList[[nt]] dt;
  ke[[nt - 1]] = 0.5 me (vxList[[nt - 1]]^2 + vyList[[nt - 1]]^2);
  pe[[nt - 1]] = (-4 Pi^2 me)/rNorm;
  totale[[nt - 1]] = ke[[nt - 1]] + pe[[nt - 1]];
  angmomentum[[nt - 1]] = rNorm me Sqrt[vxList[[nt - 1]]^2 + vyList[[nt - 1]]^2] *
    VectorAngle[{xList[[nt - 1]], yList[[nt - 1]]}, {vxList[[nt - 1]], vyList[[nt - 1]]}];

  dthetaList[[nt - 1]] =
   VectorAngle[{xList[[nt]], yList[[nt]]}, {xList[[nt - 1]], yList[[nt - 1]]}] ;
  dA[[nt - 1]] = 0.5 * rNorm^2 * dthetaList[[nt - 1]]
  , {nt, 2, lt}];
xylist = Table[{xList[[j]], yList[[j]]}, {j, lt}];
dAtlist = Table[{tL[[j]], dA[[j]]}, {j, lt}];
keList = Table[{tL[[j]], ke[[j]]}, {j, lt}];
peList = Table[{tL[[j]], pe[[j]]}, {j, lt}];
totaleList = Table[{tL[[j]], totale[[j]]}, {j, lt}];
angmomentumList = Table[{tL[[j]], angmomentum[[j]]}, {j, lt}];
p1 = ListPlot[xylist, PlotStyle → Blue, AspectRatio → 1,  AxesLabel → {"x", "y"}]
p2 = ListPlot[keList, PlotStyle → Red, AspectRatio → 1, PlotRange → All,
   AxesLabel → {"t"}, PlotLegends → PointLegend[{"kinetic energy"}]];
p3 = ListPlot[peList, PlotStyle → Green, AspectRatio → 1, PlotRange → All,
   PlotLegends → PointLegend[{"potential energy"}]];
p4 = ListPlot[totaleList, PlotStyle → Orange, AspectRatio → 1,
   PlotRange → All, PlotLegends → PointLegend[{"total energy"}]];
Show[p2, p3, p4, PlotRange → All, PlotLabel →  "energy vs time in circular orbits"]
```

```
p5 = ListPlot[angmomentumList, PlotStyle → Red, AspectRatio → 1,
  PlotRange → All, PlotLabel → "angular momentum vs time in circular orbits"]
```

Out[∘]=

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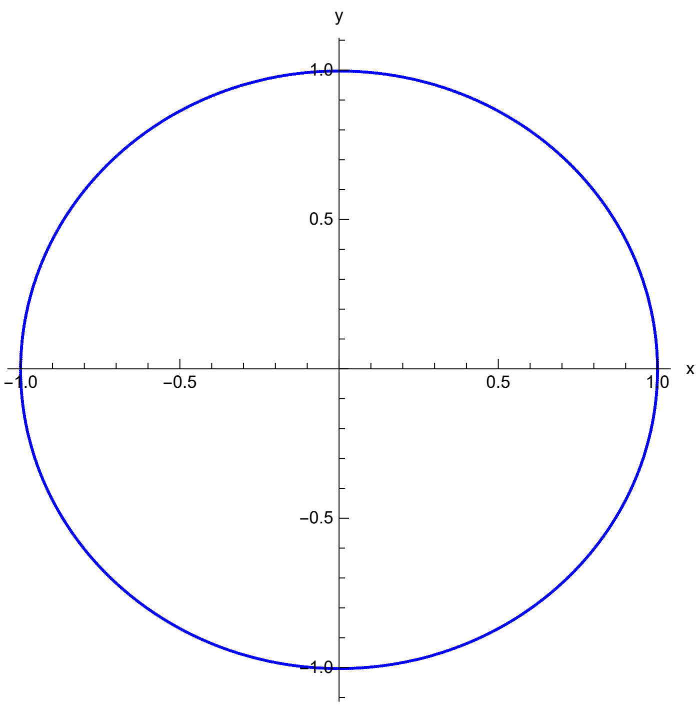
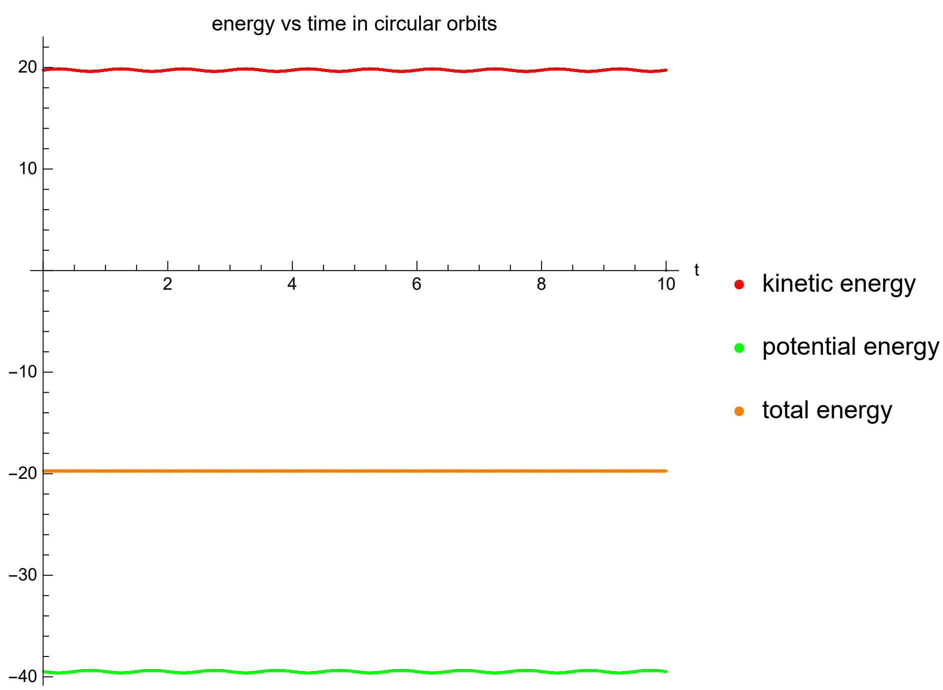


Out[∘]=



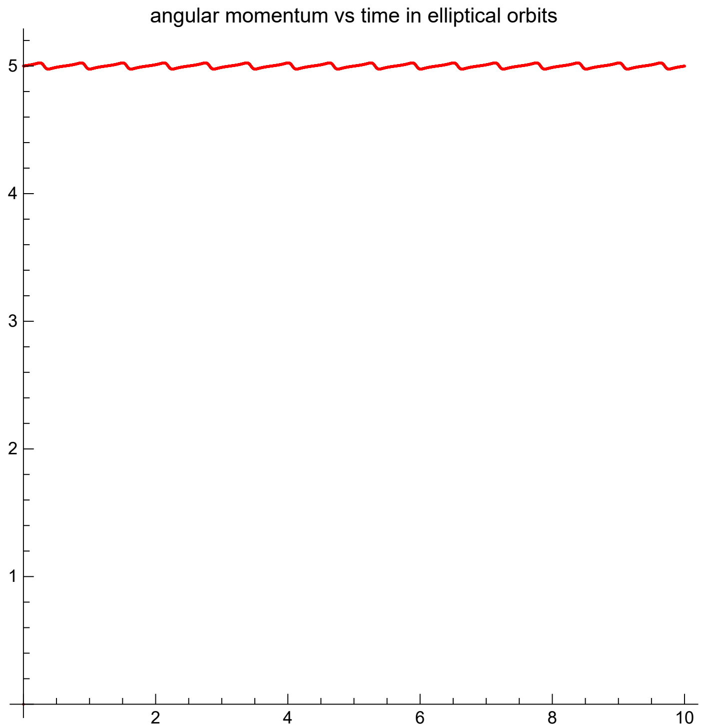
Out[ ]=

```
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  ke[[nt]] = 0.5 me (vxList[[nt]]^2 + vyList[[nt]]^2);
  pe[[nt]] = (-4 Pi^2 me)/rNorm;
  totale[[nt]] = ke[[nt]] + pe[[nt]];
  angmomentum[[nt]] = rNorm * me * Sqrt[vxList[[nt]]^2 + vyList[[nt]]^2] *
    Sin[VectorAngle[{xList[[nt]], yList[[nt]]}, {vxList[[nt]], vyList[[nt]]}]];


  , {nt, 2, lt}];
xylist = Table[{xList[[j]], yList[[j]]}, {j, lt}];
dAtlist = Table[{tL[[j]], dA[[j]]}, {j, lt}];
keList = Table[{tL[[j]], ke[[j]]}, {j, lt}];
peList = Table[{tL[[j]], pe[[j]]}, {j, lt}];
totaleList = Table[{tL[[j]], totale[[j]]}, {j, lt}];
angmomentumList = Table[{tL[[j]], angmomentum[[j]]}, {j, lt}];
p1 = ListPlot[xylist, PlotStyle → Blue, AspectRatio → 1, AxesLabel → {"x", "y"}]
p2 = ListPlot[keList, PlotStyle → Red, AspectRatio → 1, PlotRange → All,
   AxesLabel → {"t"}, PlotLegends → PointLegend[{"kinetic energy"}]];
p3 = ListPlot[peList, PlotStyle → Green, AspectRatio → 1, PlotRange → All,
   PlotLegends → PointLegend[{"potential energy"}]];
p4 = ListPlot[totaleList, PlotStyle → Blue, AspectRatio → 1,
   PlotRange → All, PlotLegends → PointLegend[{"total energy"}]];
Show[p2, p3, p4, PlotRange → All, PlotLabel → "energy vs time in elliptical orbits"]
p5 = ListPlot[angmomentumList, PlotStyle → Red, AspectRatio → 1,
  PlotRange → All, PlotLabel → "angular momentum vs time in elliptical orbits"]
```

Out[ ]=

Out[ ]=

Out[ ]=



Problem 4 (4.8)

In[ ]:=
```
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)
(*earth*) (*all other planets change the a and  e and plug *)
 tmax = 10;
dt = 0.001;
a = 1; e = 0.017;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = vxList; xList = vxList;
yList = vxList; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]] = x0;  yList[[1]] = y0;
Do[rNorm[[nt]] = Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]] = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]] = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];
```

```
d = FindPeaks[xList][[2]][[1]];
T = tL[[d]];
T^2/a^3

(*mercury*)
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)

 tmax = 10;
dt = 0.001;
a = 0.39; e = 0.206;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = 0 tL; xList = 0 tL;
yList = 0 tL; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;
Do[rNorm[[nt]] =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];

d = FindPeaks[xList][[2]][[1]];
T = tL[[d]];
T^2/a^3
(*mars*)
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)

 tmax = 10;
dt = 0.001;
a = 0.52; e = 0.093;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = 0 tL; xList = 0 tL;
yList = 0 tL; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
```

```
xList[[1]]  = x0;  yList[[1]]  = y0;
Do[rNorm[[nt]] =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];

d = FindPeaks[xList][[2]][[1]];
T = tL[[d]];
T^2 / a^3

(*venus*)
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)

 tmax = 10;
dt = 0.001;
a = 0.72; e = 0.007;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = 0 tL; xList = 0 tL;
yList = 0 tL; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;
Do[rNorm[[nt]] =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];

d = FindPeaks[xList][[2]][[1]];
T = tL[[d]];
T^2 / a^3

(*Jupitar*)
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)

 tmax = 100;
dt = 0.001;
a = 5.2; e = 0.048;
```

```
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = 0 tL; xList = 0 tL;
yList = 0 tL; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;
Do[rNorm[[nt]] =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];

d = FindPeaks[xList][[2]][[1]];
T = tL[[d]];
T^2 / a^3

(*saturn*)
ClearAll["Global`*"];
(*earth x0=1,mercury 0.39, venus=0.72, mars=1.52, jupitar 5.2, saturn 9.54 *)

 tmax = 100;
dt = 0.001;
a = 9.54; e = 0.054;
tL = Range[0, tmax, dt]; lt = Length[tL];
 x0 = a * (e + 1); y0 = 0.0;
vx0 = 0.0; vy0 = Sqrt[(4 Pi^2 * (1 - e))/(a (1 + e))];
theta = 0 tL;
vxList = Table[0, {i, lt}]; vyList = 0 tL; xList = 0 tL;
yList = 0 tL; rNorm = 0 tL;
rNorm[[1]] = x0;
vxList[[1]] = vx0; vyList[[1]] = vy0;
xList[[1]]  = x0;  yList[[1]]  = y0;
Do[rNorm[[nt]] =  Sqrt[xList[[nt - 1]]^2 + yList[[nt - 1]]^2];
  vxList[[nt]] = vxList[[nt - 1]] - 4 Pi^2 dt xList[[nt - 1]] / rNorm[[nt]]^3;
  vyList[[nt]] = vyList[[nt - 1]] - 4 Pi^2 dt yList[[nt - 1]] / rNorm[[nt]]^3;
  xList[[nt]]  = xList[[nt - 1]] + vxList[[nt]] dt;
  yList[[nt]]  = yList[[nt - 1]] + vyList[[nt]] dt;
  theta[[nt]] = VectorAngle[{x0, 0}, {xList[[nt]], yList[[nt]]}]
  , {nt, 2, lt}];

d = FindPeaks[xList][[2]][[1]];
```

```
T = tL[[d]];
T^2/a^3
```

Out[ ]= 1.

Out[ ]= 0.995448

Out[ ]= 1.00012

Out[ ]= 0.996924

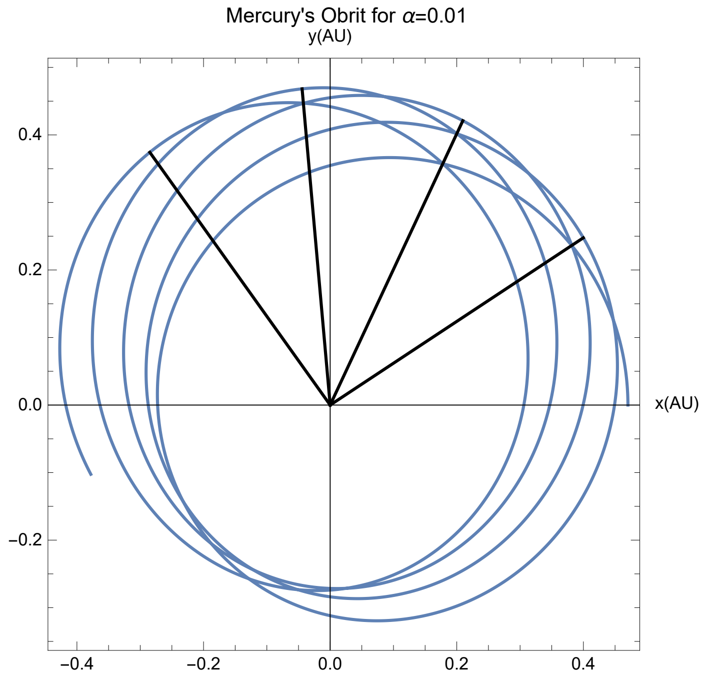
Out[ ]= 0.999861

Out[ ]= 0.999994

Problem5

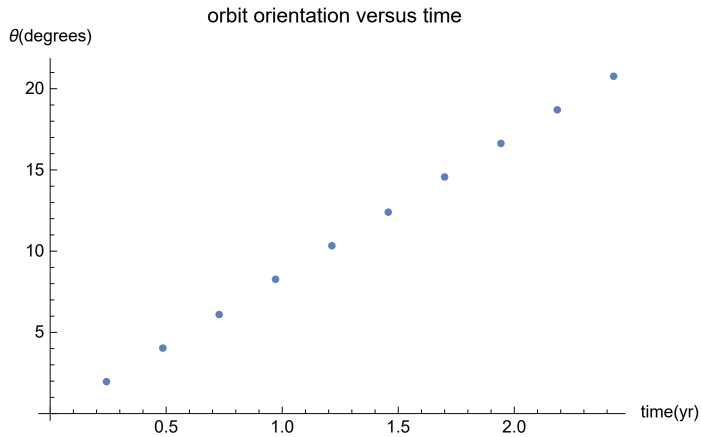
In[ ]:=
```
ClearAll["Global`*"]
k = 4 * π^2;
a = 0.39;  e = 0.206;
x0 = a * (e + 1); y0 = 0;
vx0 = 0; vy0 = Sqrt[(k * (1 - e))/(a (1 + e))];
α = 0.01;
dt = 0.0001; tmax = 1; t = Range[0, tmax, dt];
x = 0 * t; y = 0 * t;
vx = 0 * t; vy = 0 * t;

x[[1]] = x0; y[[1]] = y0;
vx[[1]] = vx0; vy[[1]] = vy0;
apx = List[];  apy = List[];
norm = 0 * t;

 norm[[1]] = Sqrt[(x[[1]]^2 + y[[1]]^2)];
For[i = 2, i ≤ Length[t], i++,
norm[[i]] = Sqrt[(x[[i - 1]]^2 + y[[i - 1]]^2)];
norm3 = norm[[i]]^3;

vx[[i]] = vx[[i - 1]] - (k x[[i - 1]])/norm3 * (1 + α/norm[[i]]^2) dt;
x[[i]] = x[[i - 1]] + vx[[i]] dt;

vy[[i]] = vy[[i - 1]] - (k y[[i - 1]])/norm3 * (1 + α/norm[[i]]^2) dt;
y[[i]] = y[[i - 1]] + vy[[i]] dt;
  ];
For[i = 1, i ≤ Length[t], i++,
If[norm[[i]] > norm[[i - 1]] && norm[[i]] > norm[[i + 1]],
AppendTo[apx, x[[i]]]; AppendTo[apy, y[[i]]]]];

apline = 0 * apx; aplinePlot = 0 * apx;
For[i = 1, i ≤ Length[apx], i++,  (*plotting vectors to the aphelions*)
apline[[i]] = {apx[[i]], apy[[i]]};
aplinePlot[[i]] = ListLinePlot[{{0, 0}, {apx[[i]], apy[[i]]}}, PlotStyle → Black]
];
xylist = Table[{x[[i]], y[[i]]}, {i, Length[t]}];

Quiet[Show[ListLinePlot[xylist, AspectRatio → 1, PlotLabel → " Mercury's Obrit for α=0.01",
    AxesLabel → {"x(AU)", "y(AU)"}, Frame -> True],
   aplinePlot[[i]] /. i → Range[1, Length[apx]]]]
```

Out[]=



```
In[•]:= ClearAll["Global`*"]
k = 4 * π^2;
a = 0.39;  e = 0.206;
x0 = a * (e + 1); y0 = 0;
vx0 = 0; vy0 = Sqrt[(k * (1 - e))/(a (1 + e))];
α = 0.0008;
dt = 0.0001; tmax = 2.5; t = Range[0, tmax, dt];
x = 0 * t; y = 0 * t;
vx = 0 * t; vy = 0 * t;

x[[1]] = x0; y[[1]] = y0;
vx[[1]] = vx0; vy[[1]] = vy0;
apx = List[];  apy = List[];  apt = List[];
norm = 0 * t;

 norm[[1]] = Sqrt[(x[[1]]^2 + y[[1]]^2)];
For[i = 2, i ≤ Length[t], i++,
norm[[i]] = Sqrt[(x[[i - 1]]^2 + y[[i - 1]]^2)];
norm3 = norm[[i]]^3;

vx[[i]] = vx[[i - 1]] - (k x[[i - 1]])/norm3 * (1 + α/norm[[i]]^2) dt;
x[[i]] = x[[i - 1]] + vx[[i]] dt;

vy[[i]] = vy[[i - 1]] - (k y[[i - 1]])/norm3 * (1 + α/norm[[i]]^2) dt;
y[[i]] = y[[i - 1]] + vy[[i]] dt;
  ];
For[i = 1, i ≤ Length[t], i++,
If[norm[[i]] > norm[[i - 1]] && norm[[i]] > norm[[i + 1]],
AppendTo[apx, x[[i]]]; AppendTo[apy, y[[i]]]; AppendTo[apt, t[[i]]]]];
θ = 0 * apx;
For[i = 1, i ≤ Length[apx], i++,
θ[[i]] = 180/π * VectorAngle[{x[[1]], y[[1]]}, {apx[[i]], apy[[i]]}]
];
θvstlist = Table[{apt[[i]], θ[[i]]}, {i, Length[apx]}];
ListPlot[θvstlist, PlotLabel → " orbit orientation versus time",
 AxesLabel → {"time(yr)", "θ(degrees)"}]
```

Out[•]=

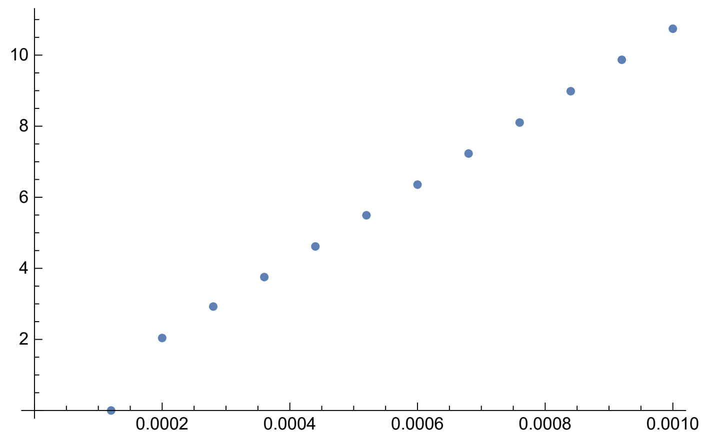


In[•]:=

```
ClearAll["Global`*"]
k = 4 * π^2;
a = 0.39;  e = 0.206;
x0 = a * (e + 1); y0 = 0;
vx0 = 0; vy0 = Sqrt[(k * (1 - e))/(a (1 + e))];
α = Range[0.001, 0.00008, -0.00008];
dθdt = 0 * α;
dt = 0.0001; tmax = 2.5; t = Range[0, tmax, dt];
x = 0 * t; y = 0 * t;
vx = 0 * t; vy = 0 * t;

For[j = 1, j < Length[α], j++,
  x[[1]] = x0; y[[1]] = y0;
  vx[[1]] = vx0; vy[[1]] = vy0;
  apx = List[]; apy = List[]; apt = List[];
  norm = 0 * t;

   norm[[1]] = Sqrt[(x[[1]]^2 + y[[1]]^2)];
  For[i = 2, i ≤ Length[t], i++,
norm[[i]] = Sqrt[(x[[i - 1]]^2 + y[[i - 1]]^2)];
norm3 = norm[[i]]^3;

vx[[i]] = vx[[i - 1]] - (k x[[i - 1]])/norm3 * (1 + α[[j]]/norm[[i]]^2) dt;
x[[i]] = x[[i - 1]] + vx[[i]] dt;

vy[[i]] = vy[[i - 1]] - (k y[[i - 1]])/norm3 * (1 + α[[j]]/norm[[i]]^2) dt;
y[[i]] = y[[i - 1]] + vy[[i]] dt;
  ];
  For[i = 1, i ≤ Length[t], i++,
If[norm[[i]] > norm[[i - 1]] && norm[[i]] > norm[[i + 1]],
AppendTo[apx, x[[i]]];
```

```
      AppendTo[apy, y[[i]]];
      AppendTo[apt, t[[i]]]]];
    θ = 0 * apx;
    For[i = 1, i ≤ Length[apx], i++,  (*plotting vectors to the aphelions*)
  θ[[i]] = 180/π * VectorAngle[{x[[1]], y[[1]]}, {apx[[i]], apy[[i]]}]
  ];
    θvstlist = Table[{apt[[i]], θ[[i]]}, {i, Length[apx]}];
    dθdt[[j]] = m /. FindFit[θvstlist, m * time, m, time][[1]]
   ];
  αvsdθdt = Table[{α[[i]], dθdt[[i]]}, {i, Length[α]}];
  ListPlot[αvsdθdt]
  Ω = c /. FindFit[αvsdθdt, c * time, c, time][[1]];
  precessionRate = Ω * 3600 * 1.1 * 10^-8 * 100
```

Out[•]=



Out[•]= 42.0872