Exercise 2.2 (An Economy with Endogenous Labor Supply)

Consider a small open economy populated by a large number of households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where U is a period utility function given by

$$U(c_t, h_t) = \frac{1}{2}[(\bar{c} - c_t)^2 + h_t^2], \tag{2}$$

where $\bar{c} \geq 0$ is a satiation point. The household's budget constraint is given by

$$d_t = (1 - r)d_{t-1} + c_t - y_t, \tag{3}$$

where d_t denotes real debt acquired in period t and due in period $t + 1$, and $r > 0$ denotes the world interest rate.

To avoid inessential dynamics, we impose

$$\beta(1 + r) = 1. \tag{4}$$

The variable y_t denotes output, which is assumed to be produced by the linear technology

$$y_t = Ah_t. \tag{5}$$

Households are also subject to the no-Ponzi-game constraint

$$\lim_{j\to\infty} Ed_{t+j}/(1 + r)^j \leq 0. \tag{6}$$

1. Compute the equilibrium laws of motion of consumption, debt, the trade balance, and the current account.

2. Assume that in period 0, unexpectedly, the productivity parameter A increases permanently to $A' > A$. Establish the effect of this shock on output, consumption, the trade balance, the current account, and the stock of debt.

Answer:

1. Household solves:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}((\bar{c} - c_t)^2 + h_t^2)\right] \tag{7}$$

subject to

$$c_t + (1 + r)d_{t-1} = y_t + d_t, \tag{8}$$

$$y_t = Ah_t, \tag{9}$$

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0. \tag{10}$$

Lagrangian of this problem can be written as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(-\frac{1}{2}((\bar{c}-c_t)^2 + h_t^2) \right) - \lambda_t \left(c_t + (1+r)d_{t-1} - Ah_t - d_t \right) \right] \tag{11}$$

First order conditions:

$$(\bar{c}-c_t) - \lambda_t = 0 \tag{12}$$

$$-h_t + \lambda_t A = 0 \tag{13}$$

$$\lambda_t - E_t\beta(1+r)\lambda_{t+1} = 0 \tag{14}$$

This yields Euler Equation and optimal labor supply condition:

$$\bar{c} - c_t = E_t\beta(1+r)(\bar{c}-c_{t+1}) \tag{15}$$

$$h_t = A(\bar{c}-c_t) \tag{16}$$

Recall that $(1+r)\beta = 1$, then Euler Equation becomes:

$$c_t = E_t c_{t+1} \tag{17}$$

$$h_t = A(\bar{c}-c_t) \tag{18}$$

Intertemporal budget constraint is given by:

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t(y_{t+j} - c_{t+j})}{(1+r)^j} \tag{19}$$

From the optimality condition for labor and the budget constraint:

$$h_t = \frac{A}{A^2+1}[\bar{c} + rd_{t-1}] \tag{20}$$

$$tb_t = y_t - c_t = rd_{t-1} \tag{21}$$

$$ca_t = tb_t - rd_{t-1} = 0 \tag{22}$$

2. Recall that:

$$c_t = \frac{1}{A^2+1}[A^2\bar{c} - rd_{t-1}] \tag{23}$$

$$y_t = Ah_t = \frac{A^2}{A^2+1}[\bar{c} + rd_{t-1}] \tag{24}$$

$$tb_t = y_t - c_t - rd_{t-1} \tag{25}$$

$$ca_t = tb_t - rd_{t-1} = 0 \tag{26}$$

Therefore, consumption will increase once and for all at period $t = 0$, output will also increase. Trade balance and current account will not change.