Exercise 2.4 (An Open Economy With Habit Formation, II)

Section 2.2 characterizes the equilibrium dynamics of a small open economy with time separable preferences driven by stationary endowment shocks. It shows that a positive endowment shock induces an improvement in the trade balance on impact. This prediction, we argued, was at odds with the empirical evidence presented in Chapter 1. Consider now a variant of the aforementioned model economy in which the representative consumer has time nonseparable preferences described by the utility function

$$-\frac{1}{2}E_t \sum_{j=0}^{\infty} \beta^j [c_{t+j} - \alpha \bar{c}_{t+j-1} - \bar{c}]^2, \quad t \geq 0,$$

where c_t denotes consumption in period t, $\bar{c}_t$ denotes the cross-sectional average level of consumption in period t, E_t denotes the mathematical expectations operator conditional on information available in period t, and $\beta \in (0,1)$, $\alpha \in (-1,1)$, and $\bar{c} > 0$ are parameters. The case $\alpha = 0$ corresponds to time separable preferences, which is studied in the main text. Households take as given the evolution of $\bar{c}_t$. Households can borrow and lend in international financial markets at the constant interest rate r. For simplicity, assume that $(1+r)\beta$ equals unity. In addition, each period $t = 0, 1, \ldots$ the household is endowed with an exogenous and stochastic amount of goods y_t. The endowment stream follows an AR(1) process of the form

$$y_{t+1} = \rho y_t + \epsilon_{t+1}.$$

where $\rho \in [0, 1)$ is a parameter and ε_t is a mean-zero i.i.d. shock. Households are subject to the no-Ponzi-game constraint

$$\lim_{j \to \infty} \mathbb{E}_t \left(\frac{d_{t+j}}{(1+r)^j} \right) \leq 0,$$

where d_t denotes the representative household's net debt position at date t. At the beginning of period 0, the household inherits a stock of debt equal to d_{-1}.

1. Derive the initial equilibrium response of consumption to a unit endowment shock in period 0.

2. Discuss conditions (i.e., parameter restrictions), if any, under which a positive output shock can lead to a deterioration of the trade balance.

Answer:

1. The equilibrium conditions of this model are:

$$
\begin{aligned}
x_t &= \mathbb{E}_t x_{t+1} \quad (2.1\text{EX}) && (1)\\
x_t &\equiv c_t - \alpha c_{t-1} \quad (2.2\text{EX}) && (2)\\
d_t &= (1+r)d_{t-1} + c_t - y_t \quad (2.3\text{EX}) && (3)\\
\lim_{j\to\infty} \mathbb{E}_t \left(\frac{d_{t+j}}{(1+r)^j}\right) &= 0 \quad (2.4\text{EX}) && (4)
\end{aligned}
$$

From (2.1EX) and (2.2EX) we get

$$\mathbb{E}_t c_{t+j} = \alpha^{j+1} c_{t-1} + \frac{1-\alpha^{j+1}}{1-\alpha} x_t$$

It follows that

$$\mathbb{E}_t \left(\sum_{j=0}^{\infty} \beta^j c_{t+j}\right) = \frac{\alpha}{1-\alpha\beta} c_{t-1} + \left[\frac{1}{1-\beta} - \frac{\alpha}{1-\alpha\beta}\right] \frac{x_t}{1-\alpha} = \frac{\alpha}{1-\alpha\beta} c_{t-1} + \frac{1}{(1-\beta)(1-\alpha\beta)} x_t$$

$$= \frac{\alpha}{1-\alpha\beta} c_{t-1} + \frac{1}{(1-\beta)(1-\alpha\beta)} x_t$$

From (2.3EX) and (2.4EX) we get

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \beta^j y_{t+j} - \sum_{j=0}^{\infty} \beta^j c_{t+j}$$

$$= \frac{1}{1-\rho\beta} y_t - \frac{\alpha}{1-\alpha\beta} c_{t-1} - \frac{1}{(1-\beta)(1-\alpha\beta)} (c_t - \alpha c_{t-1})$$

$$= \frac{1}{1-\rho\beta} y_t + \frac{\alpha\beta}{(1-\alpha\beta)(1-\beta)} c_{t-1} - \frac{1}{(1-\beta)(1-\alpha\beta)} c_t$$

So we have

$$\frac{dc_t}{dy_t} = \frac{(1-\beta)(1-\alpha\beta)}{1-\rho\beta}$$

2. For dtb_t/dy_t to be negative, we need the above expression to be larger than unity. This requires

$$\alpha < \frac{\rho - 1}{1-\beta}$$

So α must be negative. As $\rho \to 1$, $\alpha < 0$ is enough.