Exercise 2.4 (An Open Economy With Habit Formation, II)

Section 2.2 characterizes the equilibrium dynamics of a small open economy with time separable preferences driven by stationary endowment shocks. It shows that a positive endowment shock induces an improvement in the trade balance on impact. This prediction, we argued, was at odds with the empirical evidence presented in Chapter 1.

Consider now a variant of the aforementioned model economy in which the representative consumer has time nonseparable preferences described by the utility function:

$$-\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j[c_{t+j}-\alpha\bar{c}_{t+j-1}-\bar{c}]^2, \quad t\geq 0, \tag{1}$$

where c_t denotes consumption in period t, $\bar{c}_t$ denotes the cross-sectional average level of consumption in period t, E_t denotes the mathematical expectations operator conditional on information available in period t, and $\beta\in(0,1)$, $\alpha\in(-1,1)$, and $\bar{c}>0$ are parameters. The case $\alpha=0$ corresponds to time separable preferences, which is studied in the main text.

Households take as given the evolution of $\bar{c}_t$. Households can borrow and lend in international financial markets at the constant interest rate r. For simplicity, assume that $(1+r)\beta$ equals unity. In addition, each period $t=0,1,2,\dots$, the household is endowed with an exogenous and stochastic amount of goods y_t. The endowment stream follows an AR(1) process of the form:

$$y_{t+1}=\rho y_t+\epsilon_{t+1}, \tag{2}$$

where $\rho\in[0,1)$ is a parameter and ϵ_t is a mean-zero i.i.d. shock. Households are subject to the no-Ponzi-game constraint:

$$\lim_{j\to\infty}\frac{E_t d_{t+j}}{(1+r)^{-j}}\leq 0, \tag{3}$$

where d_t denotes the representative household's net debt position at date t. At the beginning of period 0, the household inherits a stock of debt equal to d_{-1}.

1. Derive the initial equilibrium response of consumption to a unit endowment shock in period 0.

The equilibrium conditions of this model are:

$$
\begin{aligned}
x_t &= E_t x_{t+1}, && (4)\\
x_t &= c_t - \alpha \bar{c}_{t-1}, && (5)\\
d_t &= (1+r)d_{t-1} - c_t + y_t, && (6)\\
\lim_{j\to\infty} E_t d_{t+j}(1+r)^{-j} &= 0. && (7)
\end{aligned}
$$

From (2.1EX) and (2.2EX) we get:

$$
E_t c_{t+j} = \alpha(1-\alpha)^{j-1} c_{t-1} + \frac{1-\alpha^{j+1}}{1-\alpha} x_t. \tag{8}
$$

It follows that:

$$
E_t \sum_{j=0}^{\infty} \beta^j c_{t+j} = \frac{\alpha}{1-\alpha\beta} c_{t-1} + \left[\frac{1}{1-\beta} - \frac{\alpha\beta}{(1-\alpha\beta)(1-\beta)}\right] x_t. \tag{9}
$$

From (2.3EX) and (2.4EX) we get:

$$
(1+r)d_{t-1} = \sum_{j=0}^{\infty} \beta^j y_{t+j} - \sum_{j=0}^{\infty} \beta^j c_{t+j}. \tag{10}
$$

Solving for consumption response:

$$
\frac{dc_t}{dy_t} = \frac{(1-\beta)(1-\alpha\beta)}{1-\rho\beta}. \tag{11}
$$

2. Discuss conditions under which a positive output shock leads to a deterioration of the trade balance.

For $\frac{d\mathrm{tb}_t}{dy_t}$ to be negative, we require the above expression to be greater than unity:

$$
\alpha < \frac{\rho - 1}{1-\beta}. \tag{12}
$$

Thus, α must be negative. As $\rho \to 1$, $\alpha < 0$ is sufficient.