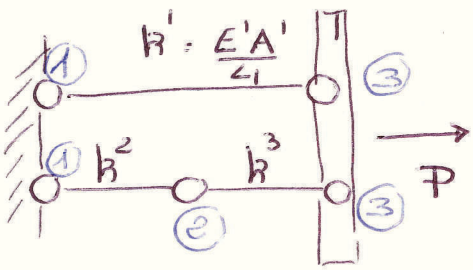
Problem



- nodal displ ?
- nodal forces ?
-

Element stiffness matrices are

$$\underline{\underline{K}}^{(1)} = k^{(1)} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{matrix} (1) \\ (3) \end{matrix} \qquad \text{columns } (1)\ (3)$$

↑ global node numbers

$$\underline{\underline{K}}^{(2)} = k^{(2)} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \begin{matrix} (1) \\ (2) \end{matrix} \qquad \text{columns } (1)\ (2)$$

$$\underline{\underline{K}}^{(3)} = k^{(3)} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \qquad \text{columns } (2)\ (3$$

Assembly $\Rightarrow$ global stiffness matrix

$$\underline{\underline{K}} = \sum_e \underline{\underline{K}}^e$$

$$\underline{\underline{K}} = \begin{vmatrix} k^1 + k^2 & -k^2 & -k^1 \\ -k^{(2)} & k^2 + k^3 & -k^3 \\ -k^{(1)} & -k^3 & k^{(1)} + k^3 \end{vmatrix} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \qquad \text{columns } (1)\ (2)\ (3)$$

boundary conditions

$u_1 = 0 \qquad F_2 = 0 \qquad F_3 = P$

$$\underline{d}_E = \left| 0 \right| \ (\leftarrow u_1) \qquad \underline{F}_E = \left| F_1 \right| \qquad \underline{d}_F = \begin{vmatrix} u_2 \\ u_3 \end{vmatrix} \qquad \underline{F}_F = \begin{vmatrix} 0 \\ P \end{vmatrix}$$

$$\underline{\underline{K}} = \left| \begin{array}{c|cc} k_1 + k_2 & -k_2 & -k_1 \\ \hline -k_2 & k_2 + k_3 & -k_3 \\ [illegible] & [illegible] & [illegible] \end{array} \right| \begin{matrix} (1) \\ (2) \\ \end{matrix}$$

(columns (1), (2), (3); K_{EE} marks the (1),(1) block, K_{EF} marks the (1),(2)–(3) block)

$$\underline{d}_F = \underline{\underline{K}}_{FF}^{-1} \left(\underline{F}_F - \underline{\underline{K}}_{FE} \underline{d}_E \right)$$

$$= \frac{1}{h_1 h_2 + h_2 h_3 + h_1 h_3} \begin{vmatrix} h_1 + h_3 & h_3 \\ h_3 & h_2 + h_3 \end{vmatrix} \left(\begin{vmatrix} 0 \\ P \end{vmatrix} - \begin{vmatrix} -h_2 \\ -h_1 \end{vmatrix} \cdot 0 \right)$$

$$= \frac{1}{h_1 h_2 + h_2 h_3 + h_1 h_3} \begin{vmatrix} h_3 P \\ h_2 + h_3 P \end{vmatrix} = \begin{vmatrix} u_2 \\ u_3 \end{vmatrix}$$

$$\underline{F}_E = \underline{\underline{K}}_{EE} \underline{d}_E + \underline{\underline{K}}_{EF} \underline{d}_F$$

$$= (h_1 + h_2) \cdot 0 + \begin{bmatrix} -h_2 & -h_1 \end{bmatrix} \begin{bmatrix} h_3 \\ h_2 + h_3 \end{bmatrix} \frac{P}{h_1 h_2 + h_2 h_3 + h_1 h_3}$$

$$= \frac{-h_2 h_3 - h_1 h_2 - h_3 h_1}{h_1 h_2 + h_2 h_3 + h_1 h_3} \cdot P = -P \quad \underline{\underline{\text{OK}}} !$$