Matrix Analysis Project

Question 2

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Analytical Solution

The given problem is : A circle passes through two points, let be A and B

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

and the centre of the circle lies on the line given by the equation

$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} + 3 = 0$$

The goal is to **find the radius of the circle**, so first we **find the centre of the circle** and then proceed on to find the radius of the circle.

Center of Circle

Let D be the midpoint of the chord AB

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

By using the fact that the centre lies on the perpendicular bisector of the chord AB, we get equation of the perpendicular bisector to be:

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{D}$$

where $\mathbf{n}$ is the direction vector of $\mathbf{AB}$ *or* $\mathbf{m}$ which is

$$\mathbf{m} = \mathbf{T}_{AB} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \textit{ where } \mathbf{T}_{AB} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}$$

Hence we get $\mathbf{n}$ to be

$$\mathbf{n} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Now we have two line equations passing through the centre

$$\begin{pmatrix}-1 & 4\end{pmatrix}\mathbf{x} = -3 \tag{1}$$

$$\begin{pmatrix}2 & 2\end{pmatrix}\mathbf{x} = 14 \tag{2}$$

Let $\mathbf{N}$ be

$$\mathbf{N} = \begin{pmatrix}-1 & 4 \\ 2 & 2\end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix}-3 \\ 14\end{pmatrix}$$

On combining the above equations we get

$$\begin{pmatrix}-1 & 4 \\ 2 & 2\end{pmatrix}\mathbf{x} = \begin{pmatrix}-3 \\ 14\end{pmatrix}$$

Clearly

$$\mathbf{x} = \mathbf{N}^{-1}\mathbf{b}$$

Hence we get

$$\mathbf{x} = \begin{pmatrix}6.2 \\ 0.8\end{pmatrix}$$

which is the centre of the circle

Radius of Circle

Let $\mathbf{T}_{AC}$ be

$$\begin{pmatrix} \mathbf{A} & \mathbf{C} \end{pmatrix}$$

$$\mathbf{T}_{\mathbf{vec}} = \mathbf{T}_{AC} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\mathbf{T}_{\mathbf{vec}} = \begin{pmatrix} 4.2 \\ -2.2 \end{pmatrix}$$

Now,

$$(\mathbf{T}_{\mathbf{vec}})^T (\mathbf{T}_{\mathbf{vec}}) = r^2$$

Upon substitution we get

$$r^2 = 22.48 \implies r = 4.74 \; approx.$$

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Graphical verification

Now we plot the given circle and the two lines which intersect at the centre of the circle to verify the correctness of the procedure adopted above.
The equation of the given line

$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} = -3$$

can be alternatively written as

$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 & 4 \end{pmatrix} \begin{pmatrix} 4\lambda - 1 \\ \lambda \end{pmatrix} 3 = 0$$

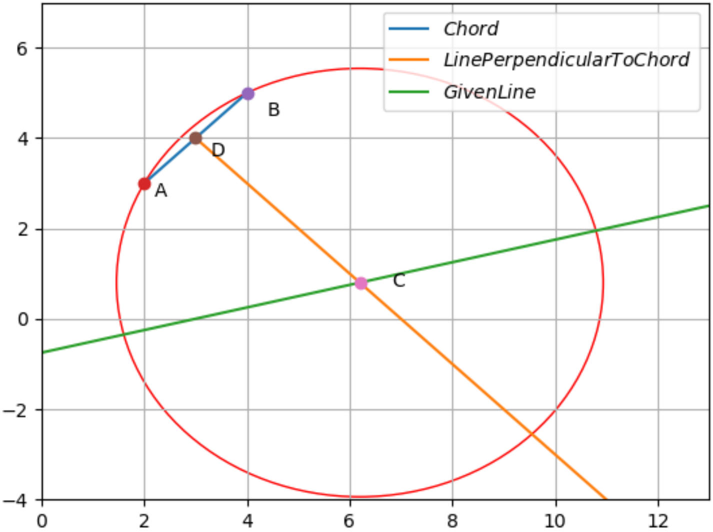
where λ is a parameter; or alternatively

$$\mathbf{x} = -3 \begin{pmatrix} 4\lambda - 1 \\ \lambda \end{pmatrix} \implies \mathbf{x} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -12 \\ -3 \end{pmatrix}$$

Now this form can be easily plotted and for the second line passing through D and C (center) we can use

$$\mathbf{x} = \mathbf{D} + \lambda(\mathbf{C} - \mathbf{D})$$

Using the above lines the graph plotted is as follows:



Thank You!