Matrix Analysis Project

Question 2

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Table of Contents

Table of Contents

Analytical Solution

The given problem is : A circle C_1 passes through a point P

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix}$$

and this circle C_1 intersects another circle C_2 orthogonally given by the equation

$$C_2 : \mathbf{x}^T \mathbf{x} = 4$$

The goal is to **find the locus of the center of the circle**.

Center of Circle

Let the circle C_2 be :

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{G}^T\mathbf{x} + C = 0$$

Since the point $\mathbf{P}$ lies on the circle we get by substitution:

$$\mathbf{P}^T\mathbf{P} + 2\mathbf{G}^T\mathbf{P} + C = 0 \tag{1}$$

Now for orthogonality for two circles of the form:

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{G_1}^T\mathbf{x} + C_1 = 0 \tag{2}$$
$$\mathbf{x}^T\mathbf{x} + 2\mathbf{G_2}^T\mathbf{x} + C_2 = 0 \tag{3}$$

we have the condition

$$2\mathbf{G_1}^T\mathbf{G_2} = C_1 + C_2$$

Now after substituting we get

$$C_1 - 4 = 0 \implies C_1 = 4$$

Therefore the equation of the circle after substituting C_1 in (1) becomes

$$\mathbf{P}^T\mathbf{P} + 2\mathbf{G}^T\mathbf{P} + 4 = 0$$

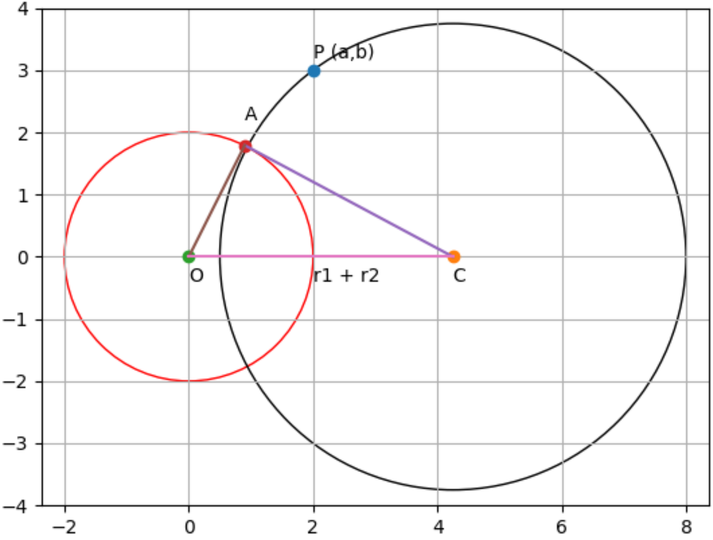
Now we know that the centre is $-\mathbf{G}$ and hence the equation becomes

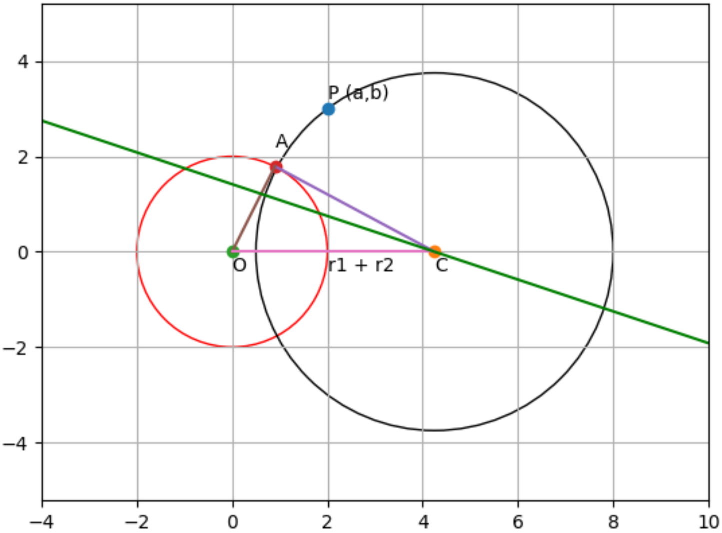
$$2\mathbf{x}^T\mathbf{P} - (\mathbf{P}^T\mathbf{P} + 4) = 0 \implies 2\mathbf{P}^T\mathbf{x} - (\mathbf{P}^T\mathbf{P} + 4) = 0 \tag{4}$$

This is the required locus equation of the centre of the circle.

Table of Contents

Graphical verification





P (a,b)
A
O
r1 + r2
C
4
2
0
−2
−4
−4
−2
0
2
4
6
8
10

Thank You!