

Matrix Analysis Project

Question 2

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Table of Contents

- 1 Analytical Solution
- 2 Graphical verification

Table of Contents

1 Analytical Solution

2 Graphical verification

Analytical Solution

The given problem is : A circle C_1 passes through a point P

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix}$$

and this circle C_1 intersects another circle C_2 orthogonally given by the equation

$$C_2 : \mathbf{x}^T \mathbf{x} = 4$$

The goal is to **find the locus of the center of the circle.**

Center of Circle

Let the circle C_2 be :

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{G}^T \mathbf{x} + C = 0$$

Since the point \mathbf{P} lies on the circle we get by substitution:

$$\mathbf{P}^T \mathbf{P} + 2\mathbf{G}^T \mathbf{P} + C = 0 \quad (1)$$

Now for orthogonality for two circles of the form:

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{G}_1^T \mathbf{x} + C_1 = 0 \quad (2)$$

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{G}_2^T \mathbf{x} + C_2 = 0 \quad (3)$$

we have the condition

$$2\mathbf{G}_1^T \mathbf{G}_2 = C_1 + C_2$$

Now after substituting we get

$$C_1 - 4 = 0 \implies C_1 = 4$$

Therefore the equation of the circle after substituting C_1 in (1) becomes

$$\mathbf{P}^T \mathbf{P} + 2\mathbf{G}^T \mathbf{P} + 4 = 0$$

Now we know that the centre is $-\mathbf{G}$ and hence the equation becomes

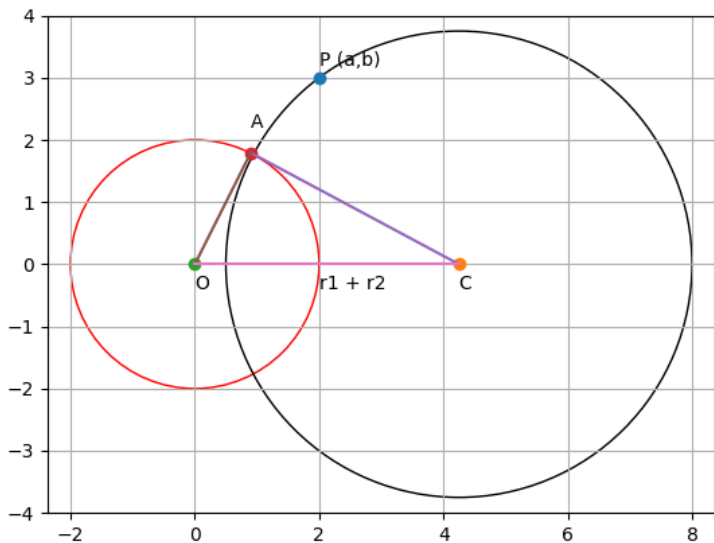
$$2\mathbf{x}^T \mathbf{P} - (\mathbf{P}^T \mathbf{P} + 4) = 0 \implies 2\mathbf{P}^T \mathbf{x} - (\mathbf{P}^T \mathbf{P} + 4) = 0 \quad (4)$$

This is the required locus equation of the centre of the circle.

Table of Contents

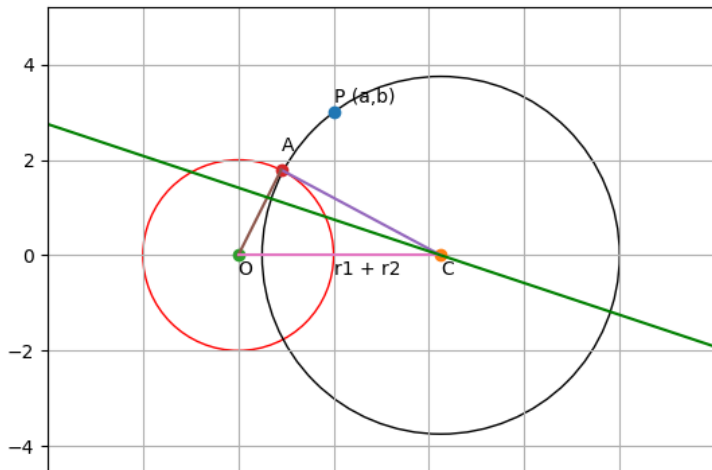
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Graphical verification

For plotting \mathbf{P} is taken to be $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and thereby C is obtained



Thank You!