# Deep Learning Basics

Lecture 4: Convolutional Neural Networks

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WARNING: 본 교육 콘텐츠의 지식재산권은 재단법인 네이버커넥트에 귀속됩니다. 본 콘텐츠를 어떠한 경로로든 외부로 유출 및 수정하는 행위를 엄격히 금합니다. 다만, 비영리적 교육 및 연구활동에 한정되어 사용할 수 있으나 재단의 허락을 받아야 합니다. 이를 위반하는 경우, 관련 법률에 따라 책임을 질 수 있습니다.



Continuous convolution

$$(f * g)(t) = \int f(\tau)g(t - \tau)d\tau = \int f(t - \tau)g(t)d\tau$$

Discrete convolution

$$(f * g)(t) = \sum_{i=-\infty}^{\infty} f(i)g(t-i) = \sum_{i=-\infty}^{\infty} f(t-i)g(i)$$

2D image convolution

$$(I * K)(i,j) = \sum_{m} \sum_{n} I(m,n)K(i-m,j-n) = \sum_{m} \sum_{n} I(i-m,i-n)K(m,n)$$

K (3x3 filter)

K <sub>11</sub>	K <sub>12</sub>	K <sub>13</sub>
K <sub>21</sub>	K <sub>22</sub>	K <sub>23</sub>
K <sub>31</sub>	K <sub>32</sub>	K <sub>33</sub>

I (7x7 image)

I <sub>11</sub>	I <sub>12</sub>	I <sub>13</sub>	I <sub>14</sub>	I <sub>15</sub>	I <sub>16</sub>	I <sub>17</sub>
I <sub>21</sub>	I <sub>22</sub>	I <sub>23</sub>	I <sub>24</sub>	I <sub>25</sub>	I <sub>26</sub>	I <sub>27</sub>
I <sub>31</sub>	I <sub>32</sub>	I <sub>33</sub>	I <sub>34</sub>	I <sub>35</sub>	I <sub>36</sub>	I <sub>37</sub>
I <sub>41</sub>	I <sub>42</sub>	I <sub>43</sub>	$I_{44}$	$I_{45}$	I <sub>46</sub>	I <sub>47</sub>
I <sub>51</sub>	I <sub>52</sub>	I <sub>53</sub>	I <sub>54</sub>	I <sub>55</sub>	I <sub>56</sub>	I <sub>57</sub>
I <sub>61</sub>	I <sub>62</sub>	I <sub>63</sub>	I <sub>64</sub>	I <sub>65</sub>	I <sub>66</sub>	I <sub>67</sub>
I <sub>71</sub>	I <sub>72</sub>	I <sub>73</sub>	I <sub>74</sub>	I <sub>75</sub>	I <sub>76</sub>	I <sub>77</sub>

Output (5x5)

0 <sub>11</sub>	0 <sub>12</sub>	0 <sub>13</sub>	0 <sub>14</sub>	0 <sub>15</sub>
021	022	023	024	0 <sub>25</sub>
031	032	033	034	035
041	042	043	044	045
0 <sub>51</sub>	052	0 <sub>53</sub>	0 <sub>54</sub>	0 <sub>55</sub>



K (3x3 filter)

K <sub>11</sub>	K <sub>12</sub>	K <sub>13</sub>
K <sub>21</sub>	K <sub>22</sub>	K <sub>23</sub>
K <sub>31</sub>	K <sub>32</sub>	K <sub>33</sub>

I (7x7 image)									
$I_{11}^{K_{11}}$		$I_{12}^{K_1}$	2	$I_{13}^{K_1}$	3	I <sub>14</sub>	I <sub>15</sub>	I <sub>16</sub>	I <sub>17</sub>
$I_{21}^{K_{21}}$		$I_{22}^{K_2}$	2	$I_{23}^{K_2}$	3	I <sub>24</sub>	I <sub>25</sub>	I <sub>26</sub>	I <sub>27</sub>
$I_{31}^{K_{31}}$		$I_{32}^{K_3}$	2	$I_{33}^{K_3}$	3	I <sub>34</sub>	I <sub>35</sub>	I <sub>36</sub>	I <sub>37</sub>
I <sub>41</sub>		I <sub>42</sub>		I <sub>43</sub>		I <sub>44</sub>	I <sub>45</sub>	I <sub>46</sub>	I <sub>47</sub>
I <sub>51</sub>		I <sub>52</sub>		I <sub>53</sub>		I <sub>54</sub>	I <sub>55</sub>	I <sub>56</sub>	I <sub>57</sub>
I <sub>61</sub>		I <sub>62</sub>		I <sub>63</sub>		I <sub>64</sub>	I <sub>65</sub>	I <sub>66</sub>	I <sub>67</sub>
I <sub>71</sub>		I <sub>72</sub>		I <sub>73</sub>		I <sub>74</sub>	I <sub>75</sub>	I <sub>76</sub>	I <sub>77</sub>

Output (5x5)

r					
	011	012	013	014	015
	0 <sub>21</sub>	022	023	024	025
	031	032	033	034	035
	041	042	043	044	045
	0 <sub>51</sub>	052	0 <sub>53</sub>	0 <sub>54</sub>	0 <sub>55</sub>

 $O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$ 



K (3x3 filter)

K <sub>11</sub>	K <sub>12</sub>	K <sub>13</sub>
K <sub>21</sub>	K <sub>22</sub>	K <sub>23</sub>
K <sub>31</sub>	K <sub>32</sub>	К <sub>33</sub>

I (7x7 image)

(7711 1113.60)								
I <sub>11</sub>	$K_{1/1_{1/2}}$	$K_{12}$	K <sub>13</sub>	I <sub>15</sub>	I <sub>16</sub>	I <sub>17</sub>		
I <sub>21</sub>		$K_{\frac{12}{1_{23}}}$	K <sub>23</sub>	I <sub>25</sub>	I <sub>26</sub>	I <sub>27</sub>		
I <sub>31</sub>	$K_{3_{1_{32}}}$	K <sub>32</sub> I <sub>33</sub>	K <sub>33</sub>	I <sub>35</sub>	I <sub>36</sub>	I <sub>37</sub>		
I <sub>41</sub>	I <sub>42</sub>	I <sub>43</sub>	I <sub>44</sub>	I <sub>45</sub>	I <sub>46</sub>	I <sub>47</sub>		
I <sub>51</sub>	I <sub>52</sub>	I <sub>53</sub>	I <sub>54</sub>	I <sub>55</sub>	I <sub>56</sub>	I <sub>57</sub>		
I <sub>61</sub>	I <sub>62</sub>	I <sub>63</sub>	I <sub>64</sub>	I <sub>65</sub>	I <sub>66</sub>	I <sub>67</sub>		
I <sub>71</sub>	I <sub>72</sub>	I <sub>73</sub>	I <sub>74</sub>	I <sub>75</sub>	I <sub>76</sub>	I <sub>77</sub>		

Output (5x5)

011	0 <sub>12</sub>	0 <sub>13</sub>	0 <sub>14</sub>	0 <sub>15</sub>
021	022	O <sub>23</sub>	024	025
031	032	033	034	035
041	042	043	044	045
0 <sub>51</sub>	052	0 <sub>53</sub>	0 <sub>54</sub>	0 <sub>55</sub>

$$O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$$

$$O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$$



K (3x3 filter)

K <sub>11</sub>	K <sub>12</sub>	K <sub>13</sub>
K <sub>21</sub>	K <sub>22</sub>	K <sub>23</sub>
K <sub>31</sub>	K <sub>32</sub>	K <sub>33</sub>

...

_		_					
	<i>I</i> <sub>11</sub>	$I_{12}$	K <sub>1/1</sub>	$K_{12}$ $I_{14}$	$K_{13}$	I <sub>16</sub>	I <sub>17</sub>
	I <sub>21</sub>	I <sub>22</sub>	$K_{2_{l_{23}}^{1}}$	K <sub>22</sub>	$K_{23}$	I <sub>26</sub>	I <sub>27</sub>
	I <sub>31</sub>	I <sub>32</sub>	$K_{3_{1_{33}}^{1}}$	$K_{\beta 2}$	$K_{33} I_{35}$	I <sub>36</sub>	I <sub>37</sub>
	I <sub>41</sub>	I <sub>42</sub>	I <sub>43</sub>	I <sub>44</sub>	I <sub>45</sub>	I <sub>46</sub>	I <sub>47</sub>
	I <sub>51</sub>	I <sub>52</sub>	I <sub>53</sub>	I <sub>54</sub>	I <sub>55</sub>	I <sub>56</sub>	I <sub>57</sub>
	I <sub>61</sub>	I <sub>62</sub>	I <sub>63</sub>	I <sub>64</sub>	I <sub>65</sub>	I <sub>66</sub>	I <sub>67</sub>
	I <sub>71</sub>	I <sub>72</sub>	I <sub>73</sub>	I <sub>74</sub>	I <sub>75</sub>	I <sub>76</sub>	I <sub>77</sub>

Output (5x5)

011	0 <sub>12</sub>	013	014	0 <sub>15</sub>
021	022	023	$O_{24}$	025
031	032	033	034	035
041	042	043	044	045
0 <sub>51</sub>	052	0 <sub>53</sub>	0 <sub>54</sub>	0 <sub>55</sub>

$$O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$$

$$O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$$

$$O_{13} = I_{13}K_{11} + I_{14}K_{12} + I_{15}K_{13} + I_{23}K_{21} + I_{24}K_{22} + I_{25}K_{23} + I_{33}K_{31} + I_{34}K_{32} + I_{35}K_{33} + bias$$



K (3x3 filter)

K <sub>11</sub>	K <sub>12</sub>	K <sub>13</sub>
K <sub>21</sub>	K <sub>22</sub>	K <sub>23</sub>
K <sub>31</sub>	K <sub>32</sub>	K <sub>33</sub>

	$I_{12}$ $I_{13}$ $K_{11}$ $K_{12}$ $K_{13}$ $I_{17}$ $I_{22}$ $I_{23}$ $K_{21}$ $K_{22}$ $K_{23}$ $I_{24}$ $I_{25}$ $I_{26}$ $I_{27}$ $I_{32}$ $I_{33}$ $K_{31}$ $K_{32}$ $K_{33}$ $K_{33}$ $I_{34}$ $I_{42}$ $I_{43}$ $I_{44}$ $I_{45}$ $I_{46}$ $I_{47}$ $I_{52}$ $I_{53}$ $I_{54}$ $I_{55}$ $I_{56}$ $I_{57}$					
I <sub>11</sub>	I <sub>12</sub>	I <sub>13</sub>	$K_{\frac{11}{14}}$	<sup>1</sup> 15	$K_{13}$	I <sub>17</sub>
I <sub>21</sub>	I <sub>22</sub>	I <sub>23</sub>	$K_{\frac{21}{I_{24}}}$	$K_{\frac{2}{1}_{2}}$	$K_{23}$	I <sub>27</sub>
I <sub>31</sub>	I <sub>32</sub>	I <sub>33</sub>	$K_{\frac{31}{I_{34}}}$	$K_{\frac{32}{I_{35}}}$	K <sub>33</sub>	I <sub>37</sub>
I <sub>41</sub>	I <sub>42</sub>	I <sub>43</sub>	I <sub>44</sub>	I <sub>45</sub>	I <sub>46</sub>	I <sub>47</sub>
I <sub>51</sub>	I <sub>52</sub>	I <sub>53</sub>	I <sub>54</sub>	I <sub>55</sub>	I <sub>56</sub>	I <sub>57</sub>
I <sub>61</sub>	I <sub>62</sub>	I <sub>63</sub>	I <sub>64</sub>	I <sub>65</sub>	I <sub>66</sub>	I <sub>67</sub>
I <sub>71</sub>	I <sub>72</sub>	I <sub>73</sub>	I <sub>74</sub>	I <sub>75</sub>	I <sub>76</sub>	I <sub>77</sub>

1 (7v7 image)

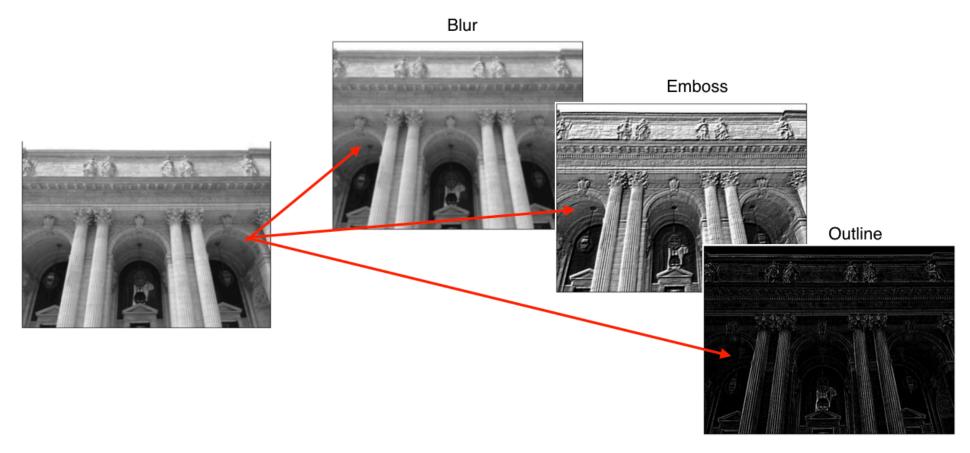
Output (5x5)

0 <sub>11</sub>	0 <sub>12</sub>	0 <sub>13</sub>	0 <sub>14</sub>	O <sub>15</sub>	
021	022	0 <sub>23</sub>	024	$O_{25}$	
031	032	033	034	035	
041	042	043	044	045	
0 <sub>51</sub>	052	0 <sub>53</sub>	0 <sub>54</sub>	055	

 $O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$   $O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$   $O_{13} = I_{13}K_{11} + I_{14}K_{12} + I_{15}K_{13} + I_{23}K_{21} + I_{24}K_{22} + I_{25}K_{23} + I_{33}K_{31} + I_{34}K_{32} + I_{35}K_{33} + bias$   $O_{14} = I_{14}K_{11} + I_{15}K_{12} + I_{16}K_{13} + I_{24}K_{21} + I_{25}K_{22} + I_{26}K_{23} + I_{34}K_{31} + I_{35}K_{32} + I_{36}K_{33} + bias$ 

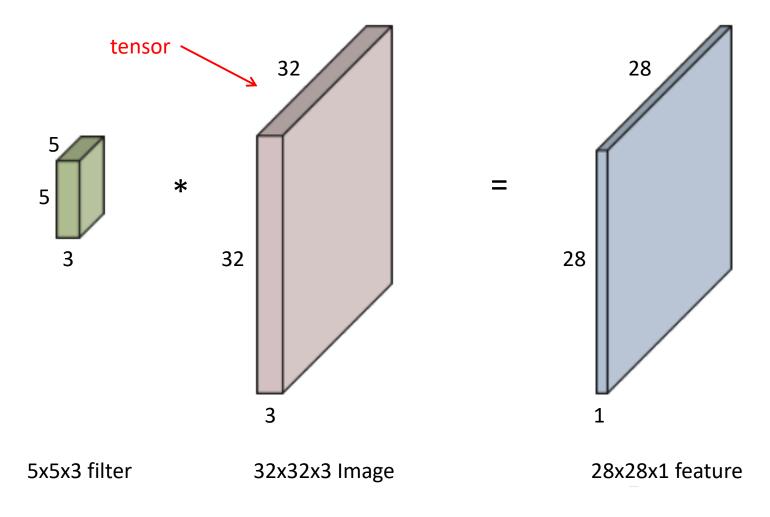


2D convolution in action



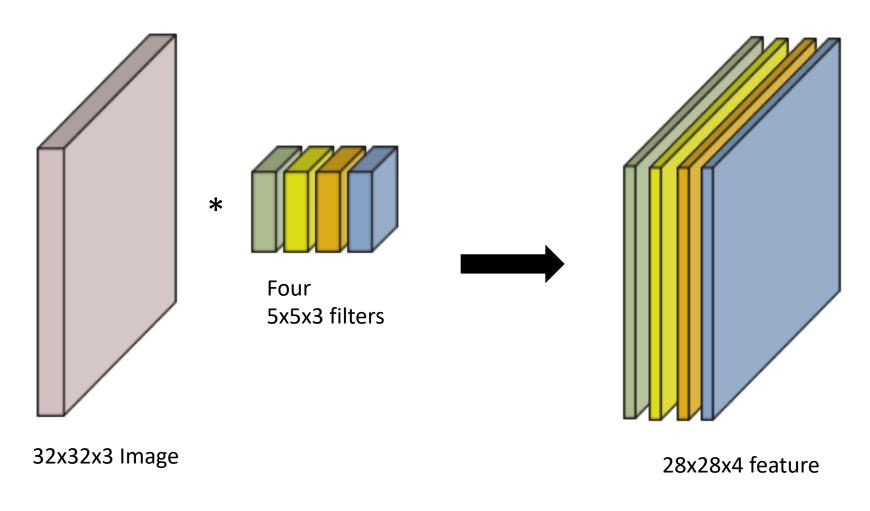


# **RGB** Image Convolution



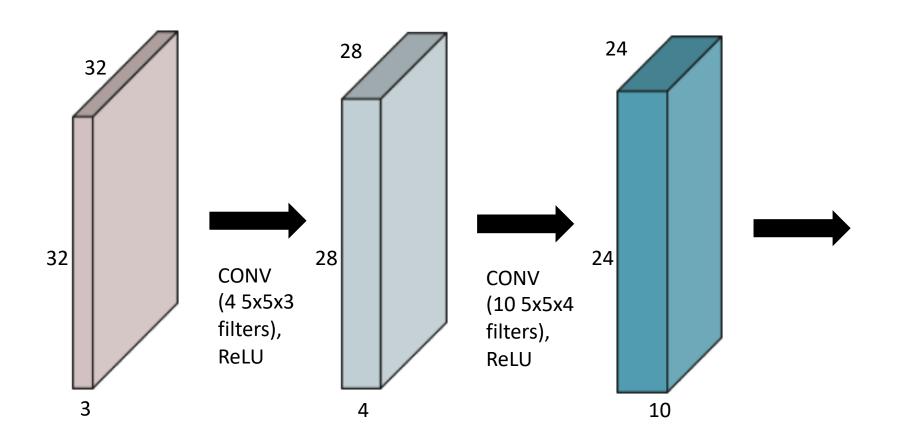


# **RGB** Image Convolution





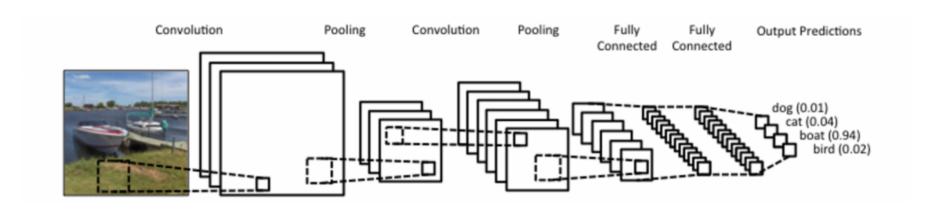
## Stack of Convolutions





#### Convolutional Neural Networks

- CNN consists of convolution layer, pooling layer, and fully connected layer.
  - Convolution and pooling layers: feature extraction
  - Fully connected layer: decision making (e.g., classification)



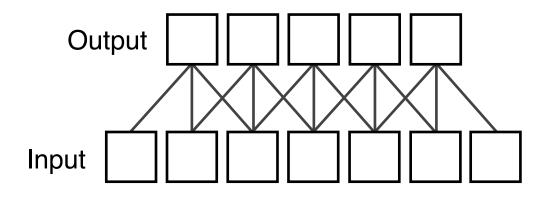


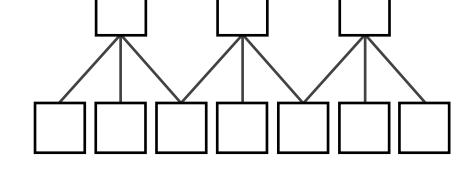
## Convolution Arithmetic (of GoogLeNet)

	Stride	Channel	_						#	param	
type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	$7 \times 7/2$	$112{\times}112{\times}64$	1							2.7K	34M
max pool	$3\times3/2$	$56 \times 56 \times 64$	0								
convolution	$3\times3/1$	$56 \times 56 \times 192$	2		64	192				112K	360M
max pool	$3\times3/2$	$28 \times 28 \times 192$	0								
inception (3a)		$28 \times 28 \times 256$	2	64	96	128	16	32	32	159K	128M
inception (3b)		$28 \times 28 \times 480$	2	128	128	192	32	96	64	380K	304M
max pool	$3\times3/2$	14×14×480	0								
inception (4a)		$14 \times 14 \times 512$	2	192	96	208	16	48	64	364K	73M
inception (4b)		$14 \times 14 \times 512$	2	160	112	224	24	64	64	437K	88M
inception (4c)		$14 \times 14 \times 512$	2	128	128	256	24	64	64	463K	100M
inception (4d)		$14 \times 14 \times 528$	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	$3\times3/2$	$7 \times 7 \times 832$	0								
inception (5a)		$7 \times 7 \times 832$	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	$1\times1\times1024$	0								
dropout (40%)		$1\times1\times1024$	0								
linear		1×1×1000	1							1000K	1M
softmax		$1\times1\times1000$	0								



## Stride



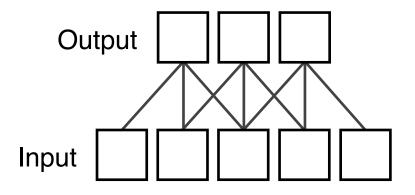


Stride = 1

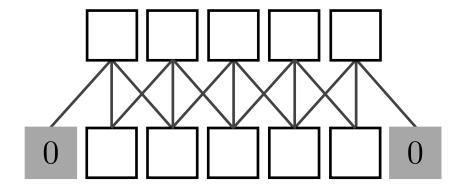
Stride = 2



# Padding



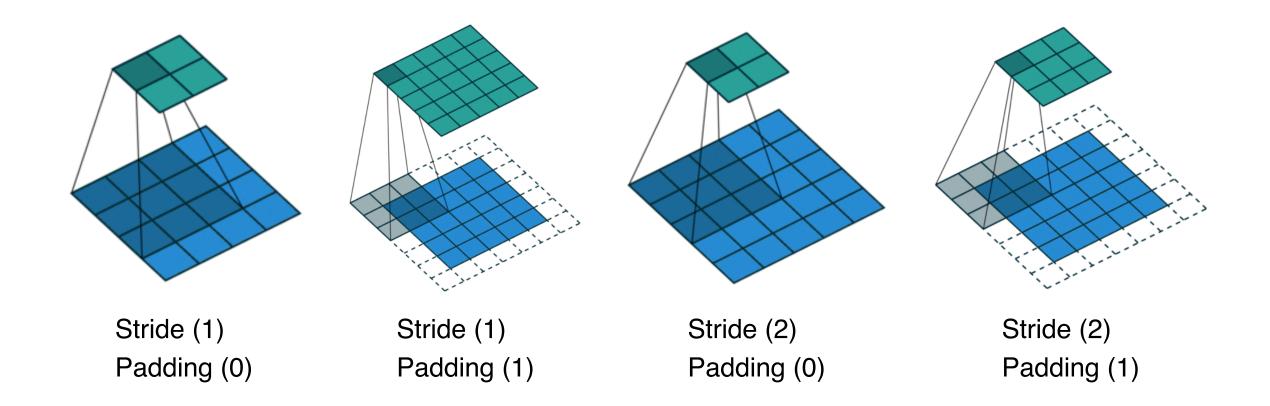
No padding (stride=1)



Zero padding (stride=1)



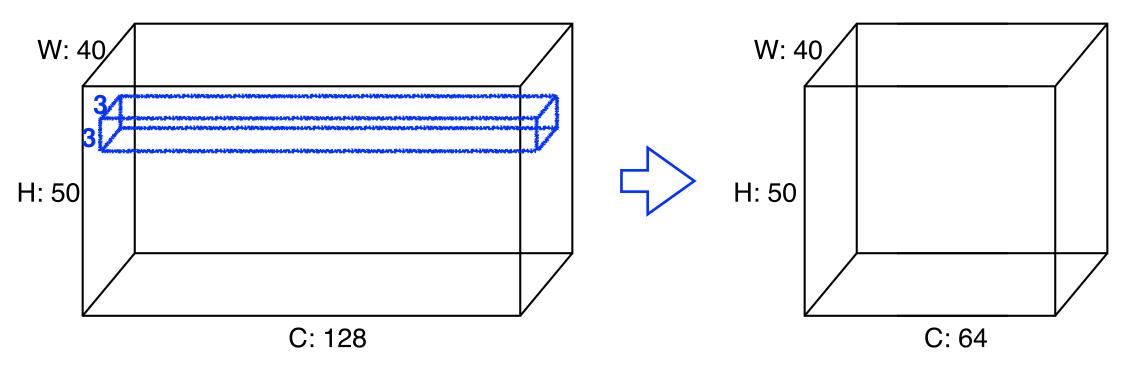
# Stride? Padding?





### **Convolution Arithmetic**

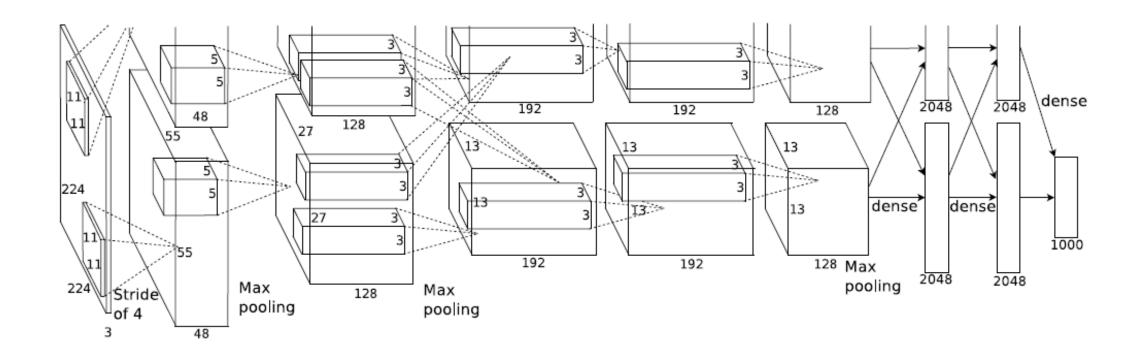
 $\bullet$  Padding (1), Stride (1),  $3 \times 3$  Kernel



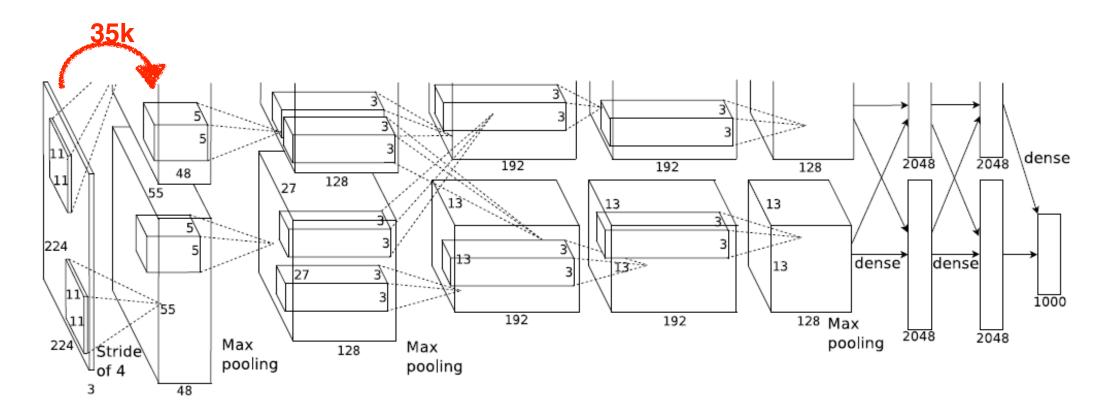
What is the **number of parameters** of this model?

The answer is  $3 \times 3 \times 128 \times 64 = 73,728$ 



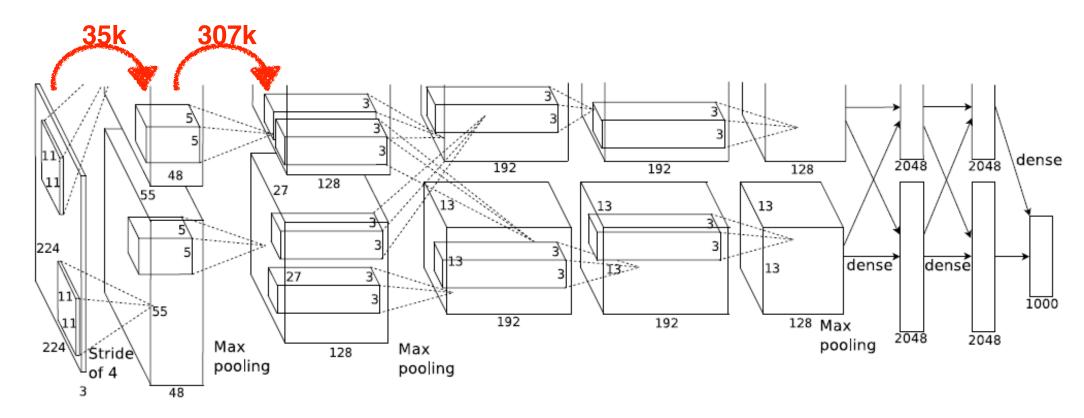






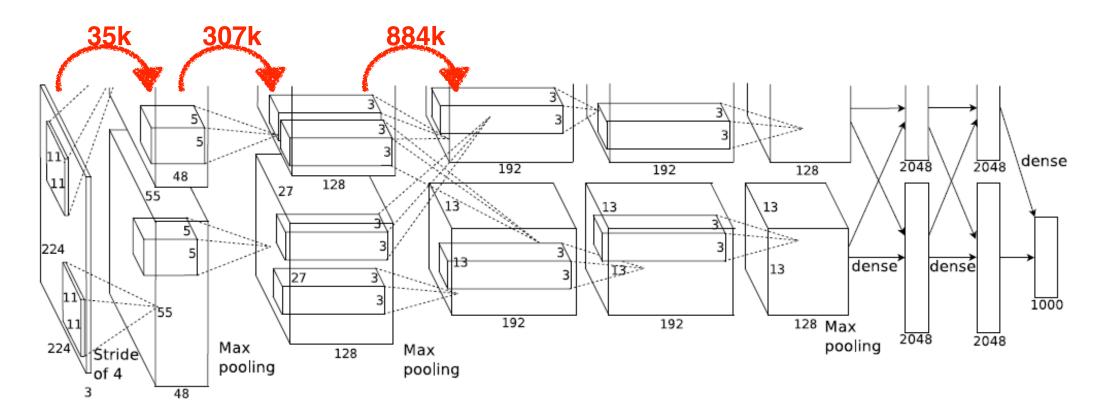
$$11 \times 11 \times 3 \times 48 * 2 \approx 35k$$





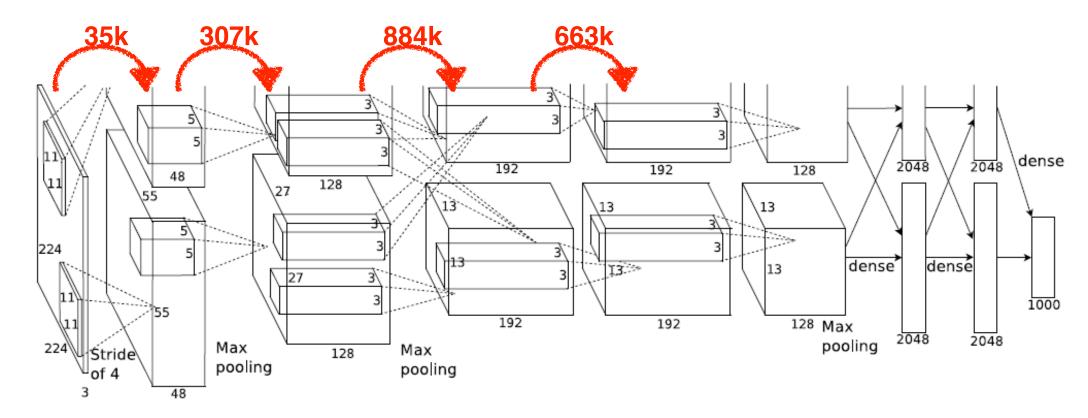
$$5 \times 5 \times 48 \times 128 * 2 \approx 307k$$





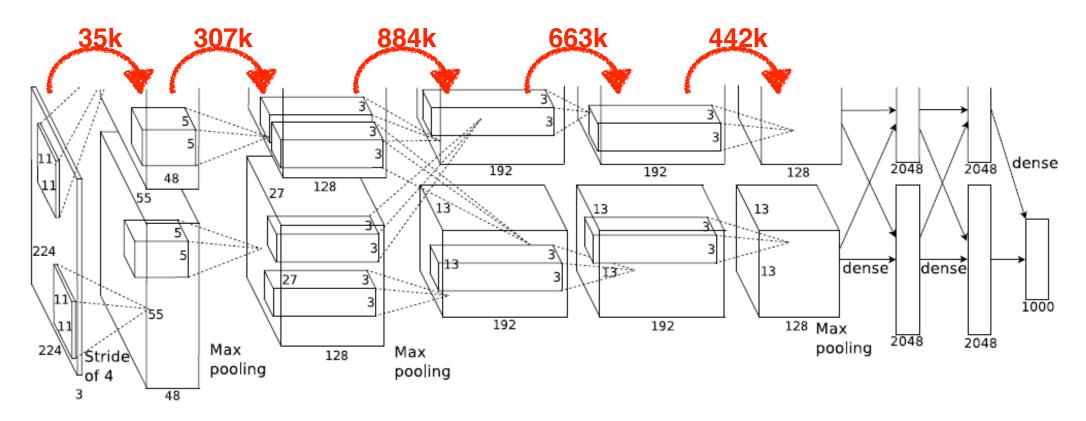
$$3 \times 3 \times 128 * 2 \times 192 * 2 \approx 884k$$





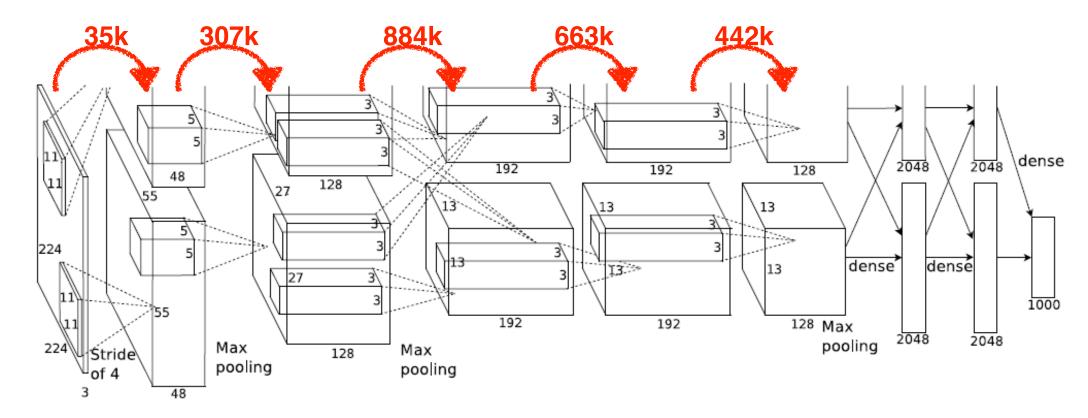
$$3 \times 3 \times 192 \times 192 * 2 \approx 663k$$





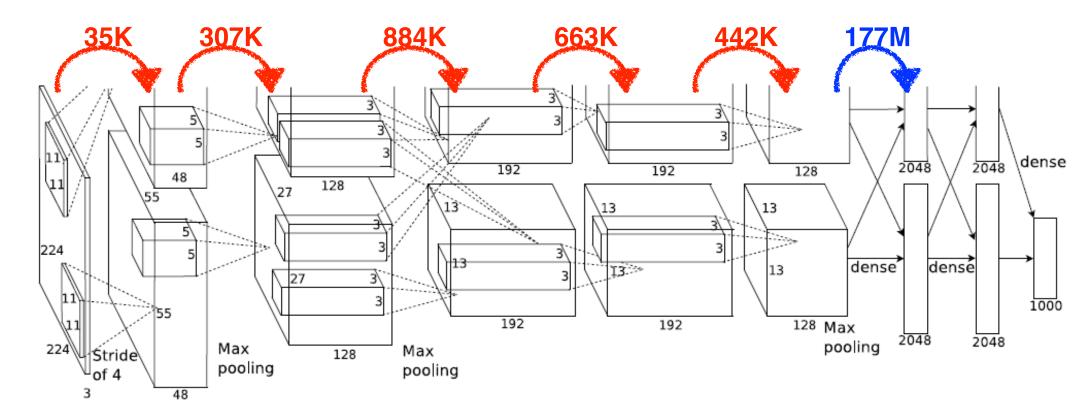
$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$





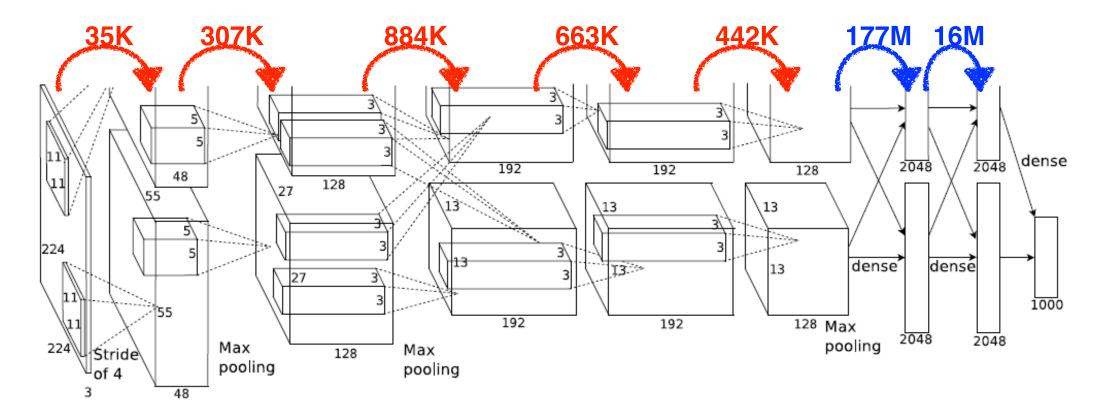
$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$





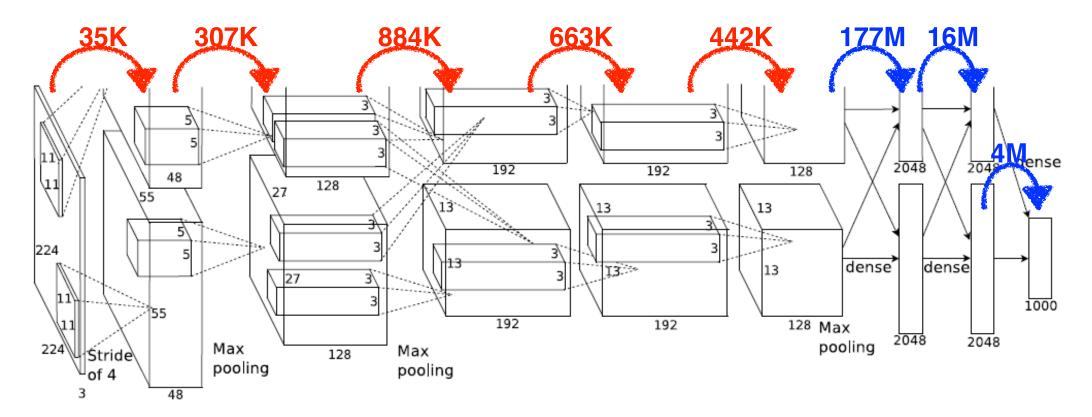
$$13 * 13 * 128 * 2 \times 2048 * 2 \approx 177M$$





$$2048 * 2 \times 2048 * 2 \approx 16M$$

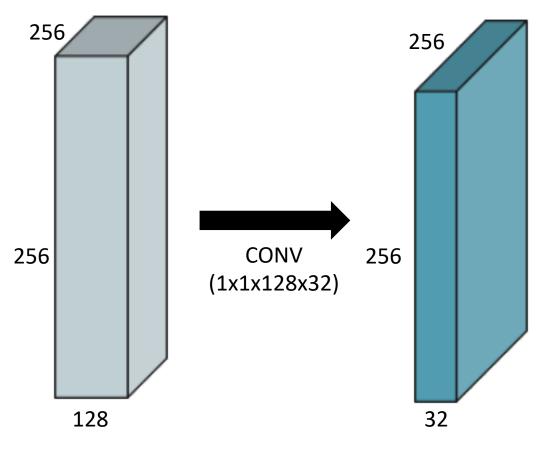




$$2048 * 2 \times 1000 \approx 4M$$



#### 1x1 Convolution



Why?

- Dimension reduction
- To reduce the number of parameters while increasing the depth
- e.g., bottleneck architecture



# Thank you for listening

