DEPARTAMENTO DE COMPUTACION
Facultad de Ciencias Exactas y Naturales - UBA

Práctica 4

1er cuatrimestre 2022 — Algoritmos y Estructuras de Datos 1

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4. Práctica 4

4.1. Ejercicio 1

1. True
2. $\{b \neq 0\}$
3. $\{b \neq 0 \wedge \frac{a}{b} \geq 0\}$
4. $\{0 \leq i < |A|\}$
5. $\{0 \leq i + 2 < |A|\}$
6. True
7. $i \neq |A|$

4.2. Ejercicio 2

Rdo. **Axioma 1 asignación**: $wp(x := E, Q) \equiv def(E) \wedge_L Q^x_E$

1. $\{def(a+1) \wedge_L a + 1 \geq 0\} \equiv \{a \geq -1\}$
2. $\{def(\frac{a}{b}) \wedge_L \frac{a}{b} \geq 0\} \equiv \{b \neq 0 \wedge_L \frac{a}{b} \geq 0\}$
3. $\{def(A[i]) \wedge_L A[i] \geq 0\} \equiv 0 \leq \{i < |A| \wedge_L A[i] \geq 0\}$
4. $\{def(b \cdot b) \wedge_L b \cdot b \geq 0\} \equiv True$
5. $\{def(b+1) \wedge_L a + 1 \geq 0\} \equiv \{True \wedge_L a \geq -1\} \equiv \{a \geq -1\}$

4.3. Ejercicio 3

Rdo. **Axioma 3 secuenciación**: $wp(S1; S2, Q) \equiv wp(S1, wp(S2, Q))$

4.3.A. Pregunta i

$$\begin{aligned}
wp(a := a+1; b = \frac{a}{2}, b \geq 0) &\equiv wp(a := a+1, wp(b := \frac{a}{2}, b \geq 0)) \\
&\equiv wp(a := a+1, def(\frac{a}{2}) \wedge_L \frac{a}{2} \geq 0) \\
&\equiv wp(a := a+1, a \geq 0) \\
&\equiv \{def(a+1) \wedge_L a + 1 \geq 0\} \\
&\equiv \{a \geq -1\}
\end{aligned}$$

4.3.B. Pregunta ii

$$\begin{aligned}
wp(a := A[i]+1; b := a.a, b \neq 2) &\equiv wp(a := A[i]+1, wp(b := a.a, b \neq 2)) \\
&\equiv wp(a := A[i]+1, \{def(a.a) \wedge_L a.a \neq 2\}) \\
&\equiv wp(a := A[i]+1, \{a \neq \pm\sqrt{2}\}) \\
&\equiv \{def(A[i]+1) \wedge_L A[i] + 1 \neq \sqrt{2}\} \\
&\equiv \{0 \leq i < |A| \wedge_L A[i] \neq \sqrt{2} - 1\}
\end{aligned}$$

4.3.C. Pregunta iii

$$\begin{aligned}
wp(a := A[i]+1; a := b.b, a \geq 0) &\equiv wp(a := A[i]+1, wp(a := b.b, a \geq 0)) \\
&\equiv wp(a := A[i]+1, \{def(b.b) \wedge_L b.b \geq 0\}) \\
&\equiv wp(a := A[i]+1, \{True\}) \\
&\equiv \{def(A[i]+1) \wedge_L True\} \\
&\equiv \{0 \leq i < |A|\}
\end{aligned}$$

4.3.D. Pregunta iv

$$\begin{aligned}
wp(a := a-b; b := a+b, (a \geq 0 \wedge b \geq 0)) &\equiv wp(a := a-b, wp(b := a+b, (a \geq 0 \wedge b \geq 0))) \\
&\equiv wp(a := a-b, \{a \geq 0 \wedge a+b \geq 0\}) \\
&\equiv \{a-b \geq 0 \wedge a-b+b \geq 0\} \\
&\equiv \{a \geq b \wedge a \geq 0\} \\
&\equiv \{0 \leq b \leq a\}
\end{aligned}$$

4.4. Ejercicio 4

Rdo. **asignación a una secuencia**: $b[i] := E \equiv b := setAt(b, i, E)$

Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \geq 0)$

En todo lo que sigue considero que $|A| \equiv |setAt(A, i, E)|$

Pregunta i

$$\begin{aligned}
wp(A[i] := 0, Q) \equiv& wp(setAt(A, i, 0), Q) \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, 0)[j] \geq 0)\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L setAt(A, i, 0)[i] \geq 0) \wedge \\
& \qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L setAt(A, i, 0)[j] \geq 0))\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L 0 \geq 0) \wedge \\
& \qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}$$

Pregunta ii

$$
\begin{aligned}
wp(A[i+2] := 0; Q) &\equiv wp(A := setAt(A, i+2, 0), Q) \\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i+2, 0)[j] \geq 0)\} \\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L setAt(A, j, 0)[j] \geq 0 \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L setAt(A, i+2, 0)[j] \geq 0))\} \\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L 0 \geq 0 \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0))\} \\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

Pregunta iii

$$
\begin{aligned}
wp(A[i+2] := -1, Q) &\equiv wp(A := setAt(A, i+2, -1), Q) \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i+2, -1)[j] \geq 0)\} \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L -1 \geq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0))\} \\
&\equiv \{0 \leq i < |A| \wedge_L False\} \\
&\equiv \{False\}
\end{aligned}
$$

Pregunta iv

$$
\begin{aligned}
wp(A[i] := 2 \cdot A[i], Q) &\equiv wp(A := setAt(A, i, 2 \cdot A[i]), Q) \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, 2 \cdot A[i])[j] \geq 0)\} \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L setAt(A, j, 2 \cdot A[j])[j] \geq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\} \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L A[j] \geq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\} \\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \wedge \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

Pregunta v

$$
\begin{aligned}
wp(A[i] := A[i-1], Q) &\equiv wp(setAt(A, i, A[i-1]), Q) \\
&\equiv \{(0 \leq i < |A| \wedge 0 \leq i-1 < |A|) \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, A[i-1])[j] \geq 0)\} \\
&\equiv \{1 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L A[j-1] \geq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\} \\
&\equiv \{1 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

4.5. Ejercicio 5

4.5.A. Pregunta i

$$\begin{aligned}
wp(S,Q) &\equiv wp(i := i+1, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(i+1) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)\} \\
&\equiv \{(\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.B. Pregunta ii

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[0] := 4, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv wp(A := setAt(A,0,4), (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(setAt(A,0,4)) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A,0,4)[j] \neq 0)\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 0) \longrightarrow_L setAt(A,0,4)[0] \neq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L setAt(A,0,4)[j] \neq 0))\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 0) \longrightarrow_L 4 \neq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L A[j] \neq 0))\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L A[j] \neq 0)\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(1 \leq j < |A| \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.C. Pregunta iii

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[2] := 4, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv wp(A := setAt(A,2,4), (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(setAt(A,2,4)) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A,2,4)[j] \neq 0)\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 2) \longrightarrow_L setAt(A,2,4)[2] \neq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L setAt(A,2,4)[j] \neq 0))\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 2) \longrightarrow_L 4 \neq 0) \wedge \\
&\qquad ((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L A[j] \neq 0))\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.D. Pregunta iv

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[i] := A[i+1]-1, (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \geq A[j-1])) \\
&\equiv wp(A := setAt(A,i,A[i+1]-1), (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \geq A[j-1])) \\
&\equiv \{def(setAt(A,i,A[i+1]-1)) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad setAt(A,i,A[i+1]-1)[j] \geq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{(0 \leq i < |A| \wedge 0 \leq i+1 < |A|) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad setAt(A,i,A[i+1]-1)[j] \geq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{0 \leq i < |A|-1 \wedge_L (\forall j:\mathbb{Z})(((0 < j < |A| \wedge j = i) \longrightarrow_L A[j+1]-1 \geq A[j-1]) \wedge \\
&\qquad ((0 < j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq A[j-1]))\}
\end{aligned}$$

4.5.E. Pregunta v

$$
\begin{aligned}
wp(S,Q) &\equiv wp(A[i] := A[i+1]-1, (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \leq A[j-1])) \\
&\equiv wp(A := setAt(A,i,A[i+1]-1), (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \leq A[j-1])) \\
&\equiv \{def(setAt(A,i,A[i+1]-1)) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \leq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{(0 \leq i < |A| \wedge 0 \leq i+1 < |A|) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \leq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{0 \leq i < |A|-1 \wedge_L (\forall j:\mathbb{Z})(((0 < j < |A| \wedge j = i) \longrightarrow_L A[j+1]-1 \leq A[j-1]) \wedge \\
&\qquad\qquad ((0 < j < |A| \wedge j-1 = i) \longrightarrow_L A[j] \leq A[j]-1) \wedge \\
&\qquad\qquad ((0 < j < |A| \wedge j \neq i \wedge j-1 \neq i) \longrightarrow_L A[j] \leq A[j-1]))\} \\
&\equiv \{0 \leq i < |A|-1 \wedge_L (\forall j:\mathbb{Z})(((0 < j < |A| \wedge j = i) \longrightarrow_L A[j+1]-1 \leq A[j-1]) \wedge \\
&\qquad\qquad False \wedge \\
&\qquad\qquad ((0 < j < |A| \wedge j \neq i \wedge j-1 \neq i) \longrightarrow_L A[j] \leq A[j-1]))\} \\
&\equiv \{0 \leq i < |A|-1 \wedge_L False\} \\
&\equiv \{False\}
\end{aligned}
$$

4.6. Ejercicio 6

4.6.A. Pregunta i

$S \equiv a := a+2$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$
\begin{aligned}
wp(a := a+2, a = a_0+2) &\equiv \{def(a+2) \wedge_L a+2 = a_0+2\} \\
&\equiv \{a = a_0\}
\end{aligned}
$$

Por lo tanto tengo que probar que $Pre \implies \{a = a_0\}$

$$Pre \implies \{a = a_0\} \iff \{a = a_0 \wedge a \geq 0\} \implies \{a = a_0\}$$

Que es verdadero, pues $a = a_0 \implies a = a_0$

4.6.B. Pregunta ii

$S \equiv b := a+3$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$
\begin{aligned}
wp(b := a+3, b = a+3) &\equiv \{def(a+3) \wedge_L a+3 = a+3\} \\
&\equiv \{True \wedge_L True\} \\
&\equiv \{True\}
\end{aligned}
$$

Y dado que $\{a \neq 0\} \implies \{True\}$ el programa es correcto.

4.6.C. Pregunta iii

$S \equiv c := a+b$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S; Post)$

$$\begin{aligned} wp(c := a+b, c = a+b) &\equiv \{def(a+b) \wedge_L a+b = a+b\} \\ &\equiv \{True \wedge_L True\} \\ &\equiv \{True\} \end{aligned}$$

Y dado que $\{True\} \implies \{True\}$ el programa es correcto.

4.6.D. Pregunta iv

$S \equiv result := 2.a[i]$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S; Post)$

$$\begin{aligned} wp(result := 2.a[i], result = 2.a[i]) &\equiv \{def(2.a[i]) \wedge_L 2.a[i] = 2.a[i]\} \\ &\equiv \{0 \leq i < |a| \wedge_L True\} \\ &\equiv \{0 \leq i < |a|\} \end{aligned}$$

Y dado que $\{0 \leq i < |a|\} \implies \{0 \leq i < |a|\}$ el programa es correcto.

4.6.E. Pregunta v

$S \equiv result := a[i] + a[i+1]$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S; Post)$

$$\begin{aligned} wp(result := a[i] + a[i+1], result = a[i] + a[i+1]) &\equiv \{def(a[i] + a[i+1]) \wedge_L a[i] + a[i+1] = a[i] + a[i+1]\} \\ &\equiv \{(0 \leq i < |a| \wedge 0 \leq i+1 < |a|) \wedge_L True\} \\ &\equiv \{0 \leq i < |a| \wedge 0 \leq i+1 < |a|\} \\ &\equiv \{0 \leq i < |a| - 1\} \end{aligned}$$

Luego $Pre \equiv \{0 \leq i \wedge i+1 < |A|\} \implies \{0 \leq i < |a| - 1\}$ por lo tanto el programa es correcto.

4.7. Ejercicio 7

Rdo. **axióma 4 condicional**: $wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q) \equiv def(B) \wedge_L ((B \wedge wp(S_1, Q)) \vee (\neg B \wedge wp(S_2, Q)))$

4.7.A. Pregunta i

Defino,

- $B \equiv a < 0$
- $S_1 \equiv b := a$
- $S_2 \equiv b := -a$
- $Q \equiv b = -|a|$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(a<0) \wedge_L (((a<0) \wedge wp(b:=a, b=-|a|)) \vee (a \geq 0 \wedge wp(b:=-a, b=-|a|)))\}\\
&\equiv \{((a<0) \wedge def(a) \wedge_L a=-|a|) \vee (a \geq 0 \wedge def(-a) \wedge_L -a=-|a|)\}\\
&\equiv \{((a<0) \wedge a=-|a|) \vee (a \geq 0 \wedge -a=-|a|)\}\\
&\equiv \{(a<0 \implies a=-|a|) \wedge (a \geq 0 \implies -a=-|a|)\}\\
&\equiv \{True \wedge True\}\\
&\equiv \{True\}
\end{aligned}$$

4.7.B. Pregunta ii

Defino,

- $B \equiv a < 0$
- $S_1 \equiv b := a$
- $S_2 \equiv b := -a$
- $Q \equiv b = |a|$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(a<0) \wedge_L (((a<0) \wedge wp(b:=a, b=|a|)) \vee (a \geq 0 \wedge wp(b:=-a, b=|a|)))\}\\
&\equiv \{((a<0) \wedge def(a) \wedge_L a=|a|) \vee (a \geq 0 \wedge def(-a) \wedge_L -a=|a|)\}\\
&\equiv \{(a<0 \wedge a=|a|) \vee (a \geq 0 \wedge -a=|a|)\}\\
&\equiv \{False \vee (a=0)\}\\
&\equiv \{a=0\}
\end{aligned}$$