DEPARTAMENTO DE COMPUTACION
Facultad de Ciencias Exactas y Naturales - UBA

Práctica 4

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Algoritmos y Estructuras de Datos 1

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4. Práctica 4

4.1. Ejercicio 1

1. True
2. $\{b \neq 0\}$
3. $\{b \neq 0 \wedge \frac{a}{b} \geq 0\}$
4. $\{0 \leq i < |A|\}$
5. $\{0 \leq i + 2 < |A|\}$
6. True
7. $i \neq |A|$

4.2. Ejercicio 2

Rdo. **Axioma 1 asignación**: $wp(x := E, Q) \equiv def(E) \wedge_L Q^x_E$

1. $\{def(a + 1) \wedge_L a + 1 \geq 0\} \equiv \{a \geq -1\}$
2. $\{def(\frac{a}{b}) \wedge_L \frac{a}{b} \geq 0\} \equiv \{b \neq 0 \wedge_L \frac{a}{b} \geq 0\}$
3. $\{def(A[i]) \wedge_L A[i] \geq 0\} \equiv 0 \leq \{i < |A| \wedge_L A[i] \geq 0\}$
4. $\{def(b \cdot b) \wedge_L b \cdot b \geq 0\} \equiv True$
5. $\{def(b + 1) \wedge_L a + 1 \geq 0\} \equiv \{True \wedge_L a \geq -1\} \equiv \{a \geq -1\}$

4.3. Ejercicio 3

Rdo. **Axioma 3 secuenciación**: $wp(S1; S2, Q) \equiv wp(S1, wp(S2, Q))$

4.3.A. Pregunta i

$$
\begin{aligned}
wp(a := a + 1; b = \frac{a}{2}, b \geq 0) &\equiv wp(a := a + 1, wp(b := \frac{a}{2}, b \geq 0)) \\
&\equiv wp(a := a + 1, def(\frac{a}{2}) \wedge_L \frac{a}{2} \geq 0) \\
&\equiv wp(a := a + 1, a \geq 0) \\
&\equiv \{def(a + 1) \wedge_L a + 1 \geq 0\} \\
&\equiv \{a \geq -1\}
\end{aligned}
$$

4.3.B. Pregunta ii

$$
\begin{aligned}
wp(a := A[i] + 1; b := a.a, b \neq 2) &\equiv wp(a := A[i] + 1, wp(b := a.a, b \neq 2)) \\
&\equiv wp(a := A[i] + 1, \{def(a.a) \wedge_L a.a \neq 2\}) \\
&\equiv wp(a := A[i] + 1, \{a \neq \pm\sqrt{2}\}) \\
&\equiv \{def(A[i] + 1) \wedge_L A[i] + 1 \neq \sqrt{2}\} \\
&\equiv \{0 \leq i < |A| \wedge_L A[i] \neq \sqrt{2} - 1\}
\end{aligned}
$$

4.3.C. Pregunta iii

$$\begin{aligned}
wp(a := A[i] + 1; a := b.b, a \geq 0) &\equiv wp(a := A[i] + 1, wp(a := b.b, a \geq 0)) \\
&\equiv wp(a := A[i] + 1, \{def(b.b) \wedge_L b.b \geq 0\}) \\
&\equiv wp(a := A[i] + 1, \{True\}) \\
&\equiv \{def(A[i] + 1) \wedge_L True\} \\
&\equiv \{0 \leq i < |A|\}
\end{aligned}$$

4.3.D. Pregunta iv

$$\begin{aligned}
wp(a := a - b; b := a + b, (a \geq 0 \wedge b \geq 0)) &\equiv wp(a := a - b, wp(b := a + b, (a \geq 0 \wedge b \geq 0))) \\
&\equiv wp(a := a - b, \{a \geq 0 \wedge a + b \geq 0\}) \\
&\equiv \{a - b \geq 0 \wedge a - b + b \geq 0\} \\
&\equiv \{a \geq b \wedge a \geq 0\} \\
&\equiv \{0 \leq b \leq a\}
\end{aligned}$$

4.4. Ejercicio 4

Rdo. **asignación a una secuencia**: $b[i] := E \equiv b := setAt(b, i, E)$

Sea $Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \geq 0)$

En todo lo que sigue considero que $|A| \equiv |setAt(A, i, E)|$

Pregunta i

$$\begin{aligned}
wp(A[i] := 0, Q) \equiv& wp(setAt(A, i, 0), Q) \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, 0)[j] \geq 0)\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L setAt(A, i, 0)[i] \geq 0) \wedge \\
& \qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L setAt(A, i, 0)[j] \geq 0))\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L 0 \geq 0) \wedge \\
& \qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\} \\
\equiv& \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}$$

Pregunta ii

$$
\begin{aligned}
wp(A[i+2] := 0; Q) &\equiv wp(A := setAt(A, i+2, 0), Q)\\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i+2, 0)[j] \geq 0)\}\\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L setAt(A, j, 0)[j] \geq 0 \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L setAt(A, i+2, 0)[j] \geq 0))\}\\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L 0 \geq 0 \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0))\}\\
&\equiv \{0 \leq i+2 < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

Pregunta iii

$$
\begin{aligned}
wp(A[i+2] := -1, Q) &\equiv wp(A := setAt(A, i+2, -1), Q)\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i+2, -1)[j] \geq 0)\}\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i+2) \longrightarrow_L -1 \geq 0) \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i+2) \longrightarrow_L A[j] \geq 0))\}\\
&\equiv \{0 \leq i < |A| \wedge_L False\}\\
&\equiv \{False\}
\end{aligned}
$$

Pregunta iv

$$
\begin{aligned}
wp(A[i] := 2 \cdot A[i], Q) &\equiv wp(A := setAt(A, i, 2 \cdot A[i]), Q)\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, 2 \cdot A[i])[j] \geq 0)\}\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L setAt(A, j, 2 \cdot A[j])[j] \geq 0) \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\}\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L A[j] \geq 0) \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\}\\
&\equiv \{0 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \wedge \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

Pregunta v

$$
\begin{aligned}
wp(A[i] := A[i-1], Q) &\equiv wp(setAt(A, i, A[i-1]), Q)\\
&\equiv \{(0 \leq i < |A| \wedge 0 \leq i-1 < |A|) \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A, i, A[i-1])[j] \geq 0)\}\\
&\equiv \{1 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})(((0 \leq j < |A| \wedge j = i) \longrightarrow_L A[j-1] \geq 0) \wedge\\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0))\}\\
&\equiv \{1 \leq i < |A| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq 0)\}
\end{aligned}
$$

4.5. Ejercicio 5

4.5.A. Pregunta i

$$\begin{aligned}
wp(S,Q) &\equiv wp(i := i+1, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(i+1) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)\} \\
&\equiv \{(\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.B. Pregunta ii

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[0] := 4, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv wp(A := setAt(A,0,4), (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(setAt(A,0,4)) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A,0,4)[j] \neq 0)\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 0) \longrightarrow_L setAt(A,0,4)[0] \neq 0) \wedge \\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L setAt(A,0,4)[j] \neq 0))\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 0) \longrightarrow_L 4 \neq 0) \wedge \\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L A[j] \neq 0))\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})((0 \leq j < |A| \wedge j \neq 0) \longrightarrow_L A[j] \neq 0)\} \\
&\equiv \{|A| > 0 \wedge_L (\forall j:\mathbb{Z})(1 \leq j < |A| \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.C. Pregunta iii

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[2] := 4, (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv wp(A := setAt(A,2,4), (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L A[j] \neq 0)) \\
&\equiv \{def(setAt(A,2,4)) \wedge_L (\forall j:\mathbb{Z})(0 \leq j < |A| \longrightarrow_L setAt(A,2,4)[j] \neq 0)\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 2) \longrightarrow_L setAt(A,2,4)[2] \neq 0) \wedge \\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L setAt(A,2,4)[j] \neq 0))\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})(((0 \leq j < |A| \wedge j = 2) \longrightarrow_L 4 \neq 0) \wedge \\
&\qquad\qquad ((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L A[j] \neq 0))\} \\
&\equiv \{|A| > 2 \wedge_L (\forall j:\mathbb{Z})((0 \leq j < |A| \wedge j \neq 2) \longrightarrow_L A[j] \neq 0)\}
\end{aligned}$$

4.5.D. Pregunta iv

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[i] := A[i+1]-1, (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \geq A[j-1])) \\
&\equiv wp(A := setAt(A,i,A[i+1]-1), (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \geq A[j-1])) \\
&\equiv \{def(setAt(A,i,A[i+1]-1)) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \geq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{(0 \leq i < |A| \wedge 0 \leq i+1 < |A|) \wedge_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \geq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{0 \leq i < |A|-1 \wedge_L (\forall j:\mathbb{Z})(((0 < j < |A| \wedge j = i) \longrightarrow_L A[j+1]-1 \geq A[j-1]) \wedge \\
&\qquad\qquad ((0 < j < |A| \wedge j \neq i) \longrightarrow_L A[j] \geq A[j-1]))\}
\end{aligned}$$

4.5.E. Pregunta v

$$\begin{aligned}
wp(S,Q) &\equiv wp(A[i] := A[i+1]-1, (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \leq A[j-1])) \\
&\equiv wp(A := setAt(A,i,A[i+1]-1), (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L A[j] \leq A[j-1])) \\
&\equiv \{def(setAt(A,i,A[i+1]-1)) \land_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \leq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{(0 \leq i < |A| \land 0 \leq i+1 < |A|) \land_L (\forall j:\mathbb{Z})(0 < j < |A| \longrightarrow_L \\
&\qquad\qquad setAt(A,i,A[i+1]-1)[j] \leq setAt(A,i,A[i+1]-1)[j-1])\} \\
&\equiv \{0 \leq i < |A|-1 \land_L (\forall j:\mathbb{Z})(((0 < j < |A| \land j = i) \longrightarrow_L A[j+1]-1 \leq A[j-1]) \land \\
&\qquad ((0 < j < |A| \land j-1 = i) \longrightarrow_L A[j] \leq A[j]-1) \land \\
&\qquad ((0 < j < |A| \land j \neq i \land j-1 \neq i) \longrightarrow_L A[j] \leq A[j-1]))\} \\
&\equiv \{0 \leq i < |A|-1 \land_L (\forall j:\mathbb{Z})(((0 < j < |A| \land j = i) \longrightarrow_L A[j+1]-1 \leq A[j-1]) \land \\
&\qquad False \land \\
&\qquad ((0 < j < |A| \land j \neq i \land j-1 \neq i) \longrightarrow_L A[j] \leq A[j-1]))\} \\
&\equiv \{0 \leq i < |A|-1 \land_L False\} \\
&\equiv \{False\}
\end{aligned}$$

4.6. Ejercicio 6

4.6.A. Pregunta i

$S \equiv a := a+2$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$\begin{aligned}
wp(a := a+2, a = a_0+2) &\equiv \{def(a+2) \land_L a+2 = a_0+2\} \\
&\equiv \{a = a_0\}
\end{aligned}$$

Por lo tanto tengo que probar que $Pre \implies \{a = a_0\}$

$$Pre \implies \{a = a_0\} \iff \{a = a_0 \land a \geq 0\} \implies \{a = a_0\}$$

Que es verdadero, pues $a = a_0 \implies a = a_0$

4.6.B. Pregunta ii

$S \equiv b := a+3$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$\begin{aligned}
wp(b := a+3, b = a+3) &\equiv \{def(a+3) \land_L a+3 = a+3\} \\
&\equiv \{True \land_L True\} \\
&\equiv \{True\}
\end{aligned}$$

Y dado que $\{a \neq 0\} \implies \{True\}$ el programa es correcto.

4.6.C. Pregunta iii

$S \equiv c := a+b$

Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$\begin{aligned} wp(c := a+b, c = a+b) &\equiv \{def(a+b) \wedge_L a+b = a+b\} \\ &\equiv \{True \wedge_L True\} \\ &\equiv \{True\} \end{aligned}$$

Y dado que $\{True\} \implies \{True\}$ el programa es correcto.

4.6.D. Pregunta iv

$S \equiv result := 2.a[i]$
Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$\begin{aligned} wp(result := 2.a[i], result = 2.a[i]) &\equiv \{def(2.a[i]) \wedge_L 2.a[i] = 2.a[i]\} \\ &\equiv \{0 \leq i < |a| \wedge_L True\} \\ &\equiv \{0 \leq i < |a|\} \end{aligned}$$

Y dado que $\{0 \leq i < |a|\} \implies \{0 \leq i < |a|\}$ el programa es correcto.

4.6.E. Pregunta v

$S \equiv result := a[i] + a[i+1]$
Luego busco probar la tripla $\{Pre\}S\{Post\} \iff Pre \implies wp(S;Post)$

$$\begin{aligned} wp(result := a[i]+a[i+1], result = a[i]+a[i+1]) &\equiv \{def(a[i]+a[i+1]) \wedge_L a[i]+a[i+1] = a[i]+a[i+1]\} \\ &\equiv \{(0 \leq i < |a| \wedge 0 \leq i+1 < |a|) \wedge_L True\} \\ &\equiv \{0 \leq i < |a| \wedge 0 \leq i+1 < |a|\} \\ &\equiv \{0 \leq i < |a| - 1\} \end{aligned}$$

Luego $Pre \equiv \{0 \leq i \wedge i+1 < |A|\} \implies \{0 \leq i < |a|-1\}$ por lo tanto el programa es correcto.

4.7. Ejercicio 7

Rdo. **axióma 4 condicional**: $wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q) \equiv def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))$

4.7.A. Pregunta i

Defino,

- $B \equiv a < 0$
- $S_1 \equiv b := a$
- $S_2 \equiv b := -a$
- $Q \equiv b = -|a|$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(a<0) \wedge_L (((a<0) \wedge wp(b:=a, b=-|a|)) \vee (a \geq 0 \wedge wp(b:=-a, b=-|a|)))\}\\
&\equiv \{((a<0) \wedge def(a) \wedge_L a=-|a|) \vee (a \geq 0 \wedge def(-a) \wedge_L -a=-|a|)\}\\
&\equiv \{((a<0) \wedge a=-|a|) \vee (a \geq 0 \wedge -a=-|a|)\}\\
&\equiv \{(a<0 \implies a=-|a|) \wedge (a \geq 0 \implies -a=-|a|)\}\\
&\equiv \{True \wedge True\}\\
&\equiv \{True\}
\end{aligned}$$

4.7.B. Pregunta ii

Defino,

- $B \equiv a < 0$
- $S_1 \equiv b := a$
- $S_2 \equiv b := -a$
- $Q \equiv b = |a|$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(a<0) \wedge_L (((a<0) \wedge wp(b:=a, b=|a|)) \vee (a \geq 0 \wedge wp(b:=-a, b=|a|)))\}\\
&\equiv \{((a<0) \wedge def(a) \wedge_L a=|a|) \vee (a \geq 0 \wedge def(-a) \wedge_L -a=|a|)\}\\
&\equiv \{(a<0 \wedge a=|a|) \vee (a \geq 0 \wedge -a=|a|)\}\\
&\equiv \{False \vee (a=0)\}\\
&\equiv \{a=0\}
\end{aligned}$$

4.7.C. Pregunta iii

Defino,

- $B \equiv i > 0$
- $S_1 \equiv s[i] := 0$
- $S_2 \equiv s[0] := 0$
- $Q \equiv (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0)$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(i>0) \wedge_L ((i>0 \wedge wp(s[i] := 0, Q)) \vee (\neg(i>0) \wedge wp(s[0] := 0, Q)))\}\\
&\equiv \{(i>0 \wedge wp(s := setAt(s,i,0),Q)) \vee (\neg(i>0) \wedge wp(s := setAt(s,0,0),Q))\}\\
&\equiv \{(i>0 \wedge def(setAt(s,i,0)) \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] \geq 0))\vee\\
&\quad\; (i \leq 0 \wedge def(setAt(s,0,0)) \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,0,0)[j] \geq 0))\}\\
&\equiv \{(i>0 \wedge 0 \leq i < |s| \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] \geq 0))\vee\\
&\quad\; (i \leq 0 \wedge |s| > 0 \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,0,0)[j] \geq 0))\}\\
&\equiv \{(i>0 \wedge 0 \leq i < |s| \wedge_L (\forall j \in \mathbb{Z})((0 \leq j < |s| \wedge j \neq i) \longrightarrow_L s[j] \geq 0))\vee\\
&\quad\; (i \leq 0 \wedge |s| > 0 \wedge_L (\forall j \in \mathbb{Z})((0 \leq j < |s| \wedge j \neq 0) \longrightarrow_L s[j] \geq 0))\}
\end{aligned}$$

4.7.D. Pregunta iv

Defino,

- $B \equiv i > 1$
- $S_1 \equiv s[i] := s[i-1]$
- $S_2 \equiv s[i] := 0$
- $Q \equiv (\forall j \in \mathbb{Z})(1 \leq j < |s| \longrightarrow_L s[j] < s[j-1])$

Luego,

$$\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(i>1) \wedge_L ((i>1 \wedge wp(s[i] := s[i-1], Q)) \vee (\neg(i>1) \wedge wp(s[i] := 0, Q)))\}\\
&\equiv \{(i>1 \wedge wp(s := setAt(s,i,s[i-1]),Q))\vee\\
&\quad\; (i \leq 1 \wedge wp(s := setAt(s,i,0),Q))\}\\
&\equiv \{(i>1 \wedge def(setAt(s,i,s[i-1])) \wedge_L (\forall j \in \mathbb{Z})(1 \leq j < |s| \longrightarrow_L setAt(s,i,s[i-1])[j] < setAt(s,i,s[i-1])[j-1]))\vee\\
&\quad\; (i \leq 1 \wedge def(setAt(s,i,0)) \wedge_L (\forall j \in \mathbb{Z})(1 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] < setAt(s,i,0)[j-1]))\}\\
&\equiv \{(i>1 \wedge 0 \leq i < |s| \wedge 0 \leq i-1 < |s| \wedge_L (\forall j \in \mathbb{Z})(1 \leq j < |s| \longrightarrow_L setAt(s,i,s[i-1])[j] < setAt(s,i,s[i-1])[j-1]))\vee\\
&\quad\; (i \leq 1 \wedge 0 \leq i < |s| \wedge_L (\forall j \in \mathbb{Z})(1 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] < setAt(s,i,0)[j-1]))\}
\end{aligned}$$

4.7.E. Pregunta v

- $B \equiv s[i] < 0$
- $S_1 \equiv s[i] := -s[i]$
- $S_2 \equiv skip$
- $Q \equiv (0 \leq i < |s| \longrightarrow_L s[i] \geq 0)$

Luego,

$$
\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(s[i] < 0) \wedge_L ((s[i] < 0 \wedge wp(s[i] := -s[i], Q)) \vee (\neg(s[i] < 0) \wedge wp(skip, Q)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] < 0 \wedge wp(s := setAt(s,i,-s[i]), Q)) \vee\\
&\qquad\qquad (s[i] \geq 0 \wedge wp(skip, Q)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] < 0 \wedge def(setAt(s,i,-s[i])) \wedge_L (0 \leq i < |s| \longrightarrow_L setAt(s,i,-s[i])[i] \geq 0)) \vee\\
&\qquad\qquad (s[i] \geq 0 \wedge (0 \leq i < |s| \longrightarrow_L s[i] \geq 0)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] < 0 \wedge 0 \leq i < |s| \wedge_L (0 \leq i < |s| \longrightarrow_L setAt(s,i,-s[i])[i] \geq 0)) \vee\\
&\qquad\qquad (s[i] \geq 0 \wedge (0 \leq i < |s| \longrightarrow_L s[i] \geq 0)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] < 0 \wedge (s[i] \leq 0)) \vee\\
&\qquad\qquad (s[i] \geq 0))\}\\
&\equiv \{0 \leq i < |s|\}
\end{aligned}
$$

4.7.F. Pregunta vi

- $B \equiv s[i] > 0$
- $S_1 \equiv s[i] := -s[i]$
- $S_2 \equiv skip$
- $Q \equiv (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0)$

Luego,

$$
\begin{aligned}
wp(S,Q) &\equiv wp(\textsf{if } B \textsf{ then } S_1 \textsf{ else } S_2 \textsf{ fi}, Q)\\
&\equiv \{def(B) \wedge_L ((B \wedge wp(S_1,Q)) \vee (\neg B \wedge wp(S_2,Q)))\}\\
&\equiv \{def(s[i] > 0) \wedge_L ((s[i] > 0 \wedge wp(s[i] := -s[i], Q)) \vee (\neg(s[i] > 0) \wedge wp(skip, Q)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] > 0 \wedge wp(s := setAt(s,i,-s[i]), Q)) \vee\\
&\qquad\qquad (s[i] \leq 0 \wedge wp(skip, Q)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] > 0 \wedge def(setAt(s,i,-s[i])) \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,i,-s[i])[j] \geq 0)) \vee\\
&\qquad\qquad (s[i] \leq 0 \wedge (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] > 0 \wedge 0 \leq i < |s| \wedge_L (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L setAt(s,i,-s[i])[j] \geq 0)) \vee\\
&\qquad\qquad (s[i] \leq 0 \wedge (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L ((s[i] > 0 \wedge False) \vee\\
&\qquad\qquad (s[i] < 0 \wedge False) \vee\\
&\qquad\qquad (s[i] = 0 \wedge (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0)))\}\\
&\equiv \{0 \leq i < |s| \wedge_L (s[i] = 0 \wedge (\forall j \in \mathbb{Z})(0 \leq j < |s| \longrightarrow_L s[j] \geq 0))\}
\end{aligned}
$$

4.8. Ejercicio 8

4.8.A. Pregunta i

Programa

```
a := a + s[i]
```

Corrección

$$wp(a := a + s[i], Post) \equiv def(a + s[i]) \wedge_L a + s[i] = \sum_{j=0}^{i} s[j]$$
$$\equiv 0 \leq i < |s| \wedge_L a = (\sum_{j=0}^{i} s[j]) - s[i]$$
$$\equiv 0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]$$

Por lo tanto, para verificar que el programa es correcto, tengo que probar que $Pre \implies wp(a := a + s[i], Post)$

Luego alcanza probar que $\{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]\} \implies \{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]\}$

Que es verdadero, por lo tanto el programa es correcto.

4.8.B. Pregunta ii

Programa

```
a := a − s[0]
```

Corrección

$$wp(a := a - s[0], Post) \equiv def(a - s[0]) \wedge_L a - s[0] = \sum_{j=1}^{i} s[j]$$
$$\equiv |s| > 0 \wedge_L a = \sum_{j=0}^{i} s[j]$$

Por lo tanto, para verificar que el programa es correcto, tengo que probar que,

$$Pre \implies wp(a := a - s[0], Post)$$
$$0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i} s[j] \implies |s| > 0 \wedge_L a = \sum_{j=0}^{i} s[j]$$

Que es verdadero, por lo tanto el programa es correcto.

4.8.C. Pregunta iii

Programa S

```
res := false;
if (s[i] >= 0) then {
    res := true;
} else {
    skip
}
```

Corrección

$$
\begin{aligned}
wp(S, Post) &\equiv wp(res := false, wp(\textsf{if } s[i] >= 0 \textsf{ then } res := true \textsf{ else } skip \textsf{ fi}, Post)) \\
&\equiv wp(res := false, def(s[i] >= 0) \wedge_L (s[i] >= 0 \wedge (true = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)))\vee \\
&\qquad (s[i] < 0 \wedge (res = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)))) \\
&\equiv def(false) \wedge_L 0 \leq i < |s| \wedge_L (s[i] \geq 0 \wedge (true = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)))\vee \\
&\qquad (s[i] < 0 \wedge (false = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0))) \\
&\equiv 0 \leq i < |s| \wedge_L (s[i] \geq 0 \wedge (true = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)))\vee \\
&\qquad (s[i] < 0 \wedge (false = true \iff (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)))
\end{aligned}
$$

Por lo tanto alcanza probar que:

- $s[i] \geq 0 \implies (\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)$
- $s[i] < 0 \implies \neg(\forall j : \mathbb{Z})(0 \leq j \leq i \longrightarrow_L s[j] > 0)$

Pero el primero es válido ya que en para $0 \leq j < i$ vale por Pre y se que $s[i] \geq 0$

El segundo por lo mismo se que $s[i] < 0$ y por lo tanto el para todo es falso.

Luego $Pre \implies wp(S, Post)$ y por lo tanto el programa es correcto.

4.8.D. Pregunta iv

Programa S

```
if (s[i] != 0) then {
    a := a + 1
} else {
    skip
}
```

Corrección

$$
\begin{aligned}
wp(S, Post) &\equiv wp(\textsf{if } s[i] \neq 0 \textsf{ then } a := a + 1 \textsf{ else } skip \textsf{ fi}, Post) \\
&\equiv def(s[i]! = 0) \wedge_L ((s[i] \neq 0 \wedge def(a+1) \wedge_L a + 1 = \sum_{j=0}^{i} \textsf{if } s[j] \neq 0 \textsf{ then } 1 \textsf{ else } 0 \textsf{ fi})\vee \\
&\qquad (s[i] = 0 \wedge a = \sum_{j=0}^{i} \textsf{if } s[j] \neq 0 \textsf{ then } 1 \textsf{ else } 0 \textsf{ fi})) \\
&\equiv 0 \leq i < |s| \wedge_L ((s[i] \neq 0 \wedge a + 1 = \sum_{j=0}^{i} \textsf{if } s[j] \neq 0 \textsf{ then } 1 \textsf{ else } 0 \textsf{ fi})\vee \\
&\qquad (s[i] = 0 \wedge a = \sum_{j=0}^{i} \textsf{if } s[j] \neq 0 \textsf{ then } 1 \textsf{ else } 0 \textsf{ fi}))
\end{aligned}
$$

Por lo tanto alcanza probar que $Pre \implies$ cada rama del if.

En ambas se ve que la sumatoria vale desde $j = 0$ hasta $i - 1$ por Pre, queda ver que pasa con el valor de $s[i]$

En la rama true del if, $s[i] \neq 0 \implies$ la sumatoria hasta i es igual a $a + 1$

En la rama false, $s[i] = 0 \implies$ la sumatoria hasta i es igual a a

Luego $Pre \implies wp(S, Post)$ y por lo tanto el programa es correcto.

4.8.E. Pregunta v

Programa S

```
if (s[0] != 0) then {
    a := a - 1
} else {
    skip
}
```

Queda probar la correctitud usando wp, similar al del ejercicio anterior.