Generic 2D Inverse Kinematics for Rovers

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Robot Model

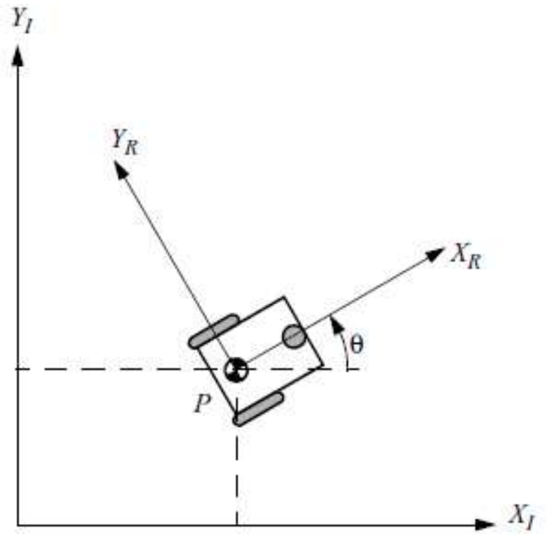

Figure 3.1
The global reference frame and the robot local reference frame.

Position and orientation of rover in world

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Linear and angular velocities of rover in world

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

Transformation from world to rover

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Velocities transformed from world to rover

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I$$

Figures and equations from
Introduction to Autonomous Mobile Robots, Siegwart, Nourbakhsh, Scaramuzza
https://mitpress.mit.edu/books/introduction-autonomous-mobile-robots-second-edition

Steered Wheel Constraints

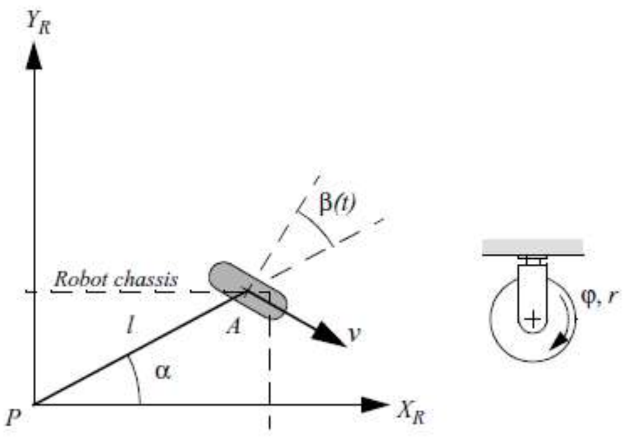

Figure 3.5
A steered standard wheel and its parameters.

Rolling Constraint

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \quad (3.15)$$

No Sliding Constraint

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0 \quad (3.16)$$

Equations 3.15 & 3.16 are used to first compute the steering angle β with (3.16) and then the wheel speed $\dot{\varphi}$ with (3.15) for each wheel.

We want to steer the rover with respect to the rover frame. Therefore:

$$\theta = 0 \quad \text{and} \quad R(0) = I_3$$

$$\dot{\xi}_R = R(0)\,\dot{\xi}_I = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix}$$

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Steer Wheel Constraints

The equations (3.15, 3.16) simplify to:

$$\sin(\alpha + \beta)\, \dot{x_R} \; - \cos(\alpha + \beta)\dot{y_R} \; - l\cos(\beta)\,\dot{\theta_R} = r\dot{\varphi}$$

$$\cos(\alpha + \beta)\dot{x_R} + \cos(\alpha + \beta)\,\dot{y_R} + l\sin(\beta)\,\dot{\theta_R} = 0$$

Solving for β using [1]:

$$\boxed{\beta = \mathrm{atan2}(-(\sin(\alpha)\,\dot{y_R} + \cos(\mathrm{alpha})\dot{x_R}), l\dot{\theta_R} + \cos(\alpha)\,\dot{y_R} - \sin(\alpha)\dot{x_R})}$$

And $\dot{\varphi}$ with β now known:

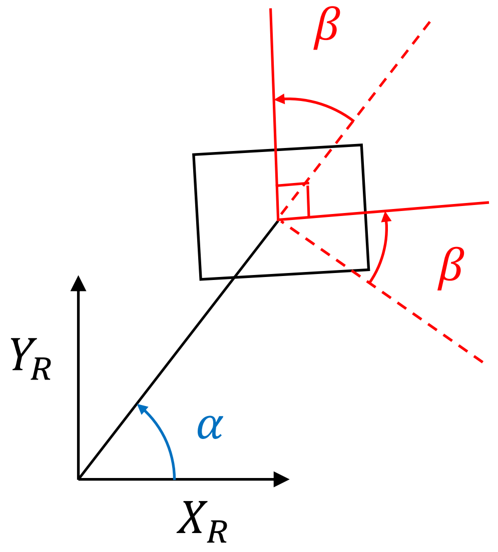
$$\boxed{\dot{\varphi} = \frac{\sin(\alpha + \beta)\,\dot{x_R} - \cos(\alpha + \beta)\,\dot{y_R} - l\cos(\beta)\dot{\theta_R}}{r}}$$

[1]: $\mathrm{s}(\alpha + \beta) = \mathrm{s}(\alpha)\mathrm{c}(\beta) + \mathrm{c}(\alpha)\mathrm{s}(\beta)$ and $\mathrm{c}(\alpha + \beta) = \mathrm{c}(\alpha)\mathrm{c}(\beta) - \mathrm{s}(\alpha)\mathrm{s}(\beta)$

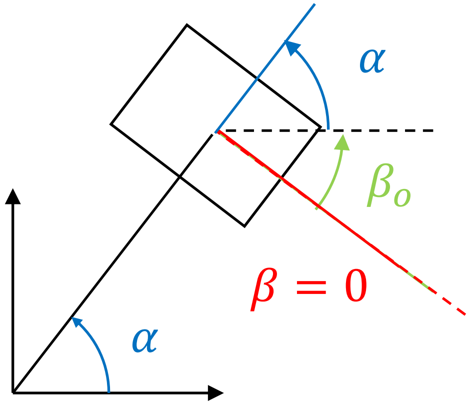
Steering Angle Corrections

As shown in figure 3.5, β defines the wheel angle in respect the rover origin-normal.

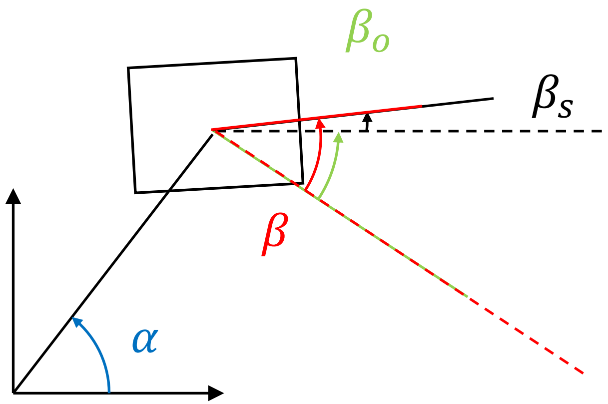
For practical purposes the angle β_s in respect to the forward direction x is required.



$$\beta_o = \frac{\pi}{2} - \alpha$$



$$\beta_s = \beta - \beta_o$$



Steering Angle Requirements

β_s should be set to minimize the wheel steering from its current position β_{curr}.

The wheel steering also has to stay within the motor limits:

$$-100° \leq \beta_s \leq +100°$$

Whenever β_s is changed by factors of $180°$ the sign of the wheel velocity $\dot{\varphi}$ has to be flipped.

$$\dot{\varphi} = -\dot{\varphi}$$

Steering Angle Correction Procedure

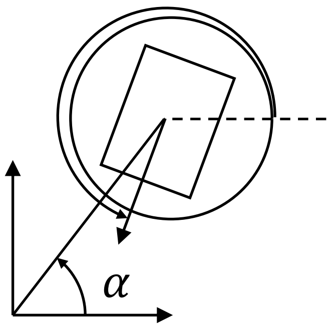
1. Limit β_s to $-360° \leq \beta_s \leq +360°$
2. Limit β_s to $-180° \leq \beta_s \leq +180°$
   - Flip $\dot{\varphi}$ if applied
3. Limit β_s to motor limits $-100° \leq \beta_s \leq +100°$
   - Flip $\dot{\varphi}$ if applied
4. If there are alternative configurations possible:

$$-100° \leq \beta_s \pm 180\,° \leq +100°$$

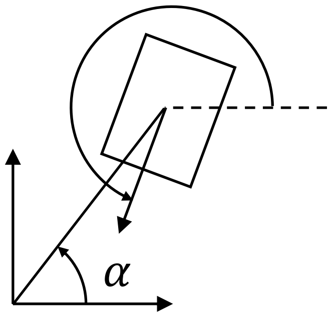
5. Set β_s to closestest one to β_{curr}
   - Flip $\dot{\varphi}$ if applied

Steering Angle Correction Example

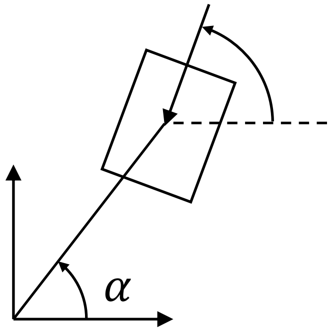
0. $\beta_s = 620°$ $\dot{\varphi} = \frac{10°}{s}$



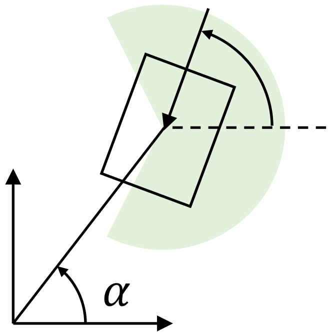
1. $\beta_s = 260°$ $\dot{\varphi} = \frac{10°}{s}$



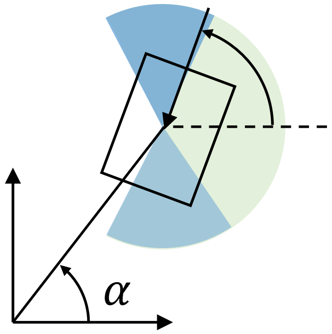
2. $\beta_s = 80°$ $\dot{\varphi} = -\frac{10°}{s}$



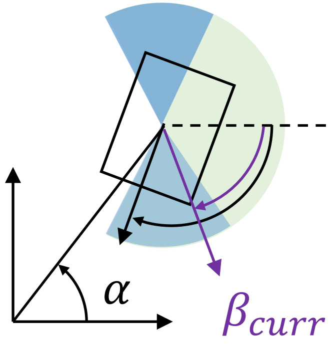
3. $\beta_s = 80°$ $\dot{\varphi} = -\frac{10°}{s}$



4. $\beta_s = 80°$ $\dot{\varphi} = -\frac{10°}{s}$



5. $\beta_s - 100°$ $\dot{\varphi} = \frac{10°}{s}$



Sources

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