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1. COMPARISONS BETWEEN NUMERICAL AND EOS

The EOS provides us the following quantities:

$$s, P, \left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T, \left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P, \left.\frac{\partial \ln s}{\partial \ln P}\right|_T, \left.\frac{\partial \ln s}{\partial \ln T}\right|_P, \nabla_{\rm ad} \tag{1}$$

over a regular grid in (T, ρ) space.

We obtained the following derivatives through taking finite differences of S and P:

$$\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T, \left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho, \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T, \left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho. \tag{2}$$

We then manipulate these derivatives to obtain new quantities corresponding to the raw outputs from the EOS. Full derivations are given in Section 2.

1.1. *He*

Figure 1 below shows the relative difference between the partials reported by the EOS and those we numerically computed (Eqns 3-6). The QMD region between $\log_{10}(\rho/\mathrm{g\,cm^{-3}}) \approx -1$ and 2 show deviations of tens of percent.

This one is the one Sunny
says is probably okay

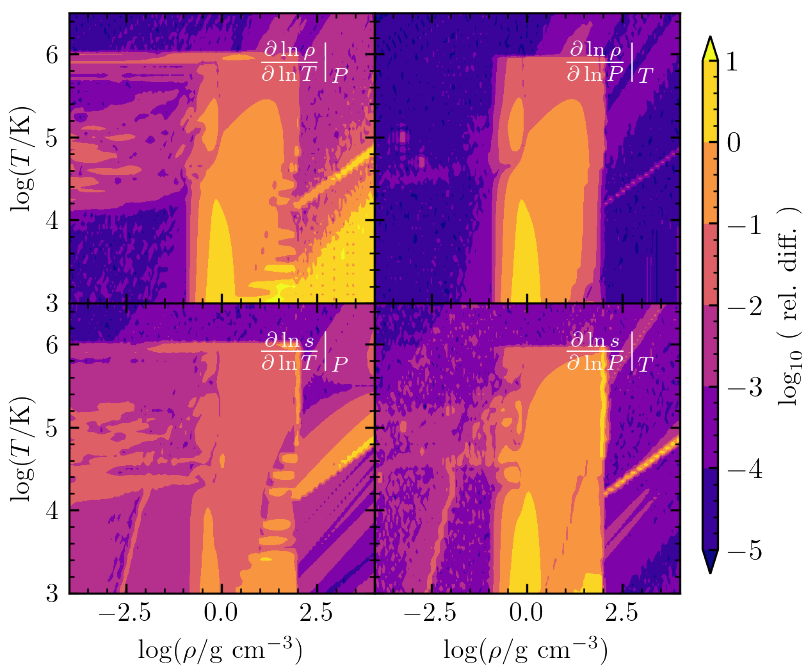

Figure 1. Relative differences between partials reported by the He-EOS and those we calculated.

Figure 2 shows the relative difference between the $\nabla_{\rm ad}$ reported by the EOS and those we computed (Eqn 9). The same discrepancy shows up in the QMD region.

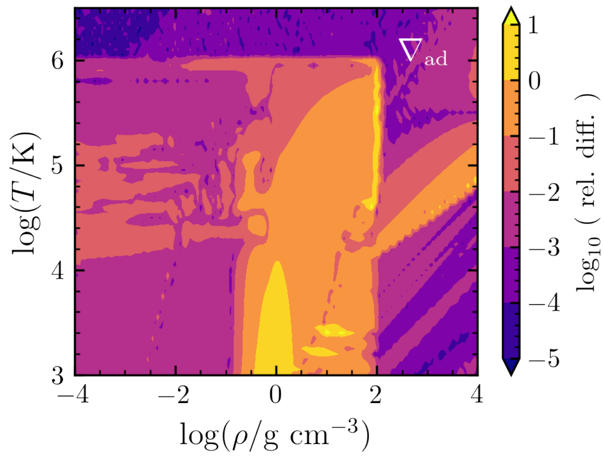

Figure 2. Relative differences between $\nabla_{\rm ad}$ reported by the He-EOS and those we calculated.

Figure 3 shows the impact of the $\nabla_{\rm ad}$ discrepancy. In the top panel, the red line shows the location in $T-\rho$ space of an isentrope of $S/(N_{\rm A}k_{\rm B})=3.5$, where $N_{\rm A}$ is Avogadro's number, and $k_{\rm B}$ is Boltzmann's constant. We picked a (T,ρ) pair along this isentrope, and integrated forward using two $\nabla_{\rm ad}$'s, one reported by the EOS and one we calculated. This resulting trajectories are shown in green (from the EOS $\nabla_{\rm ad}$) and orange (from our numerical $\nabla_{\rm ad}$) dashed lines. The trajectory following the EOS $\nabla_{\rm ad}$ does not coincide with the isentrope, and we illustrate this via the middle panel, which shows the entropy corresponding to the (T,ρ) trajectories of the dashed lines in the top panel. In the bottom panel, we show the $\nabla_{\rm ad}$'s reported by the EOS and from our calculation, that correspond to the isentrope of $S/(N_{\rm A}k_{\rm B})=3.5$. Again, there is a large discrepancy between the two.

In short, the $\nabla_{\rm ad}$ reported by the He-EOS appears to be inconsistent with constant entropy, in the QMD region between $\log_{10}(\rho/{\rm g\,cm^{-3}})\approx-1$ and 2. The same discrepancy is present in the other partials reported by the He-EOS. As we have no way to know which is the "true adiabatic," we are writing to you.

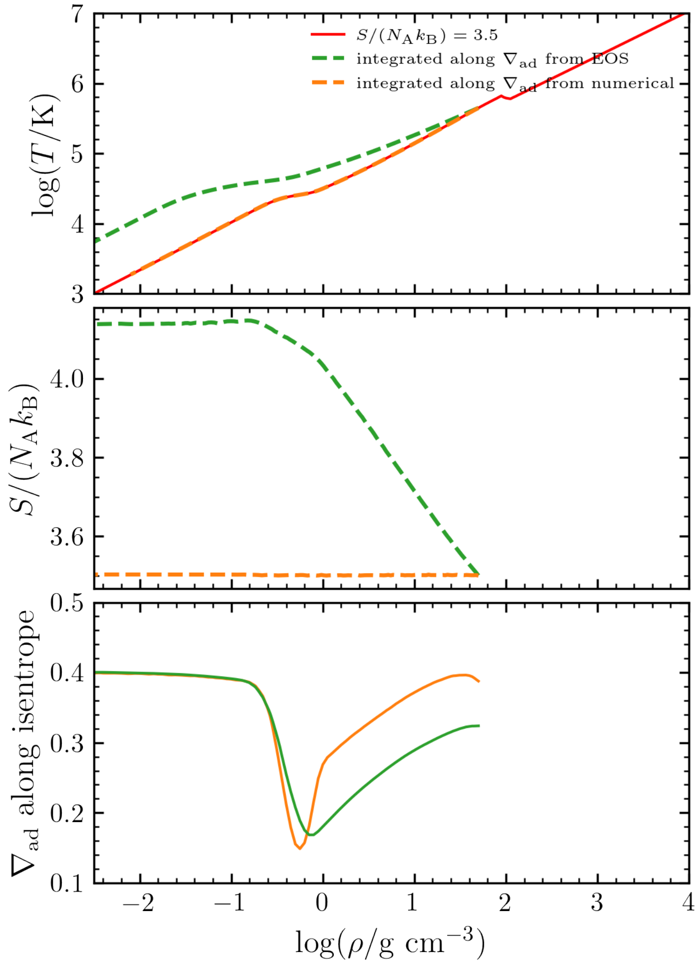

Figure 3. Top: $T-\rho$ trajectories along an isentrope (red solid line), and integrated forward using the $\nabla_{\rm ad}$'s reported by the EOS (green dashed line) and calculated by us (orange dashed line). Middle: Entropy values along the $(T-\rho)$ trajectories integrated using the $\nabla_{\rm ad}$'s reported by the EOS (green dashed line) and calculated by us (orange dashed line). Bottom: $\nabla_{\rm ad}$'s along the $S/(N_{\rm A}k_{\rm B}) = 3.5$ isentrope, one as reported by the EOS (orange line) and one we calculated (green line).

1.2. *H*

We did the same check for the pure-H EOS in Figures 4 and 5. There does not appear to be a severe discrepancy; most discrepancies appear in the blending regions located at $\log_{10}(\rho/{\rm g\,cm^{-3}}) \approx -1$ and 1.

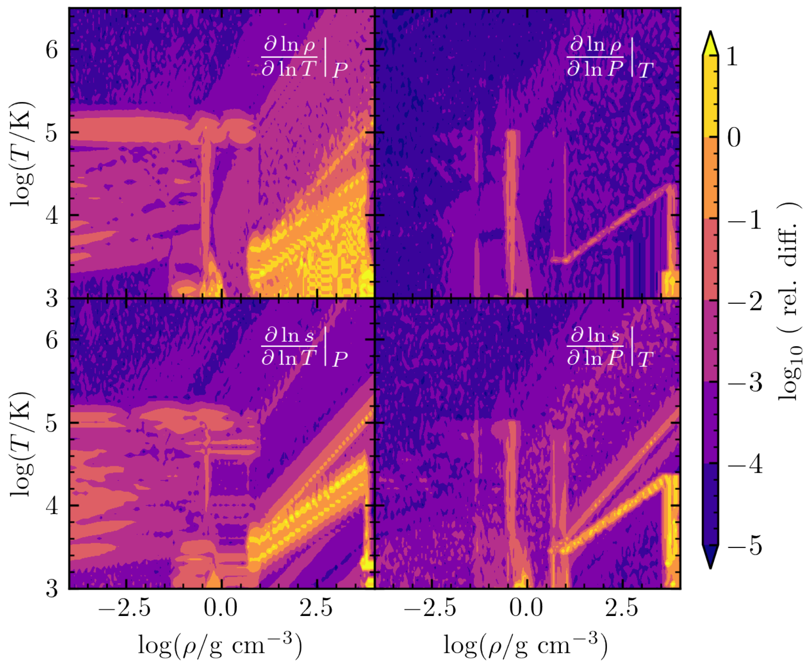

Figure 4. Relative differences between partials reported by the H-EOS and those we calculated.

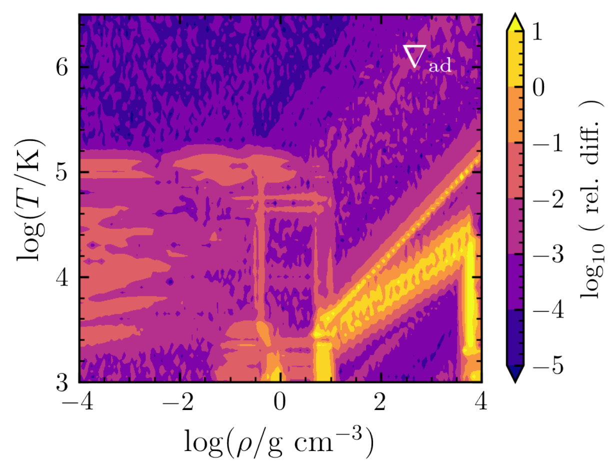

Figure 5. Relative differences between ∇_{ad} reported by the H-EOS and those we calculated.

2. DERIVATIONS

The numbered equations below are the ones used to derive $\left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T$, $\left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P$, $\left.\frac{\partial \ln s}{\partial \ln P}\right|_T$, $\left.\frac{\partial \ln s}{\partial \ln T}\right|_P$, ∇_{ad} from $\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T$, $\left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho$, $\left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T$, $\left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho$, where the latter are obtained through taking finite differences of S and P.

For partials with respect to $\ln P$, we have

$$\left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T = 1/\left(\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T\right) \tag{3}$$

and

$$0 = d\ln P = \left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T d\ln\rho + \left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho d\ln T$$

$$\implies \left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P = -\left(\left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho\right) / \left(\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T\right) \tag{4}$$

For partials with respect to $d\ln s$, we have

$$\begin{aligned}
d\ln P &= \left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T d\ln\rho + \left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho d\ln T \\
\implies d\ln\rho &= \left(d\ln P - \left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho d\ln T\right) / \left(\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T\right) \\
\implies d\ln s &= \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T (d\ln\rho) + \left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho d\ln T \\
&= \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T \frac{\left(d\ln P - \frac{\partial \ln P}{\partial \ln T}\big|_\rho d\ln T\right)}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T} + \left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho d\ln T \\
&= \left(\frac{\frac{\partial \ln s}{\partial \ln \rho}\big|_T}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T}\right) d\ln P + \left(\left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho - \frac{\frac{\partial \ln P}{\partial \ln T}\big|_\rho}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T} \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T\right) d\ln T
\end{aligned}$$

$$\implies \left.\frac{\partial \ln s}{\partial \ln T}\right|_P = \left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho - \frac{\frac{\partial \ln P}{\partial \ln T}\big|_\rho}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T} \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T \tag{5}$$

$$\implies \left.\frac{\partial \ln s}{\partial \ln P}\right|_T = \frac{\frac{\partial \ln s}{\partial \ln \rho}\big|_T}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T}. \tag{6}$$

Finally, for $\nabla_{\rm ad}$, we have

$$0 = d\ln s = \left.\frac{\partial \ln s}{\partial \ln P}\right|_T d\ln P + \left.\frac{\partial \ln s}{\partial \ln T}\right|_P d\ln T$$

$$\nabla_{\rm ad} \equiv \left.\frac{\partial \ln T}{\partial \ln P}\right|_s \tag{7}$$

$$= -\left.\frac{\partial \ln s}{\partial \ln P}\right|_T / \left.\frac{\partial \ln s}{\partial \ln T}\right|_P \tag{8}$$

$$= -\left(\frac{\frac{\partial \ln s}{\partial \ln \rho}\big|_T}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T}\right) / \left(\left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho - \frac{\frac{\partial \ln P}{\partial \ln T}\big|_\rho}{\frac{\partial \ln P}{\partial \ln \rho}\big|_T} \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T\right)$$

$$= -\left(\left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T\right) / \left(\left.\frac{\partial \ln P}{\partial \ln \rho}\right|_T \left.\frac{\partial \ln s}{\partial \ln T}\right|_\rho - \left.\frac{\partial \ln P}{\partial \ln T}\right|_\rho \left.\frac{\partial \ln s}{\partial \ln \rho}\right|_T\right) \tag{9}$$