

Q1

- a) No. A does not functionally determine B since for $A=a_2$, the tuples $t_3(a_2, b_3, c_1, d_3)$ and $t_4(a_2, b_3, c_2, d_3)$ have two different values for C namely c_1, c_2 .
- b) Yes. B functionally determines D.
 $t_3(a_2, b_3, c_1, d_3)$, $t_4(a_2, b_3, c_2, d_3)$, and $t_5(a_3, b_3, c_2, d_3)$ have the same values for B for the same values for D.
- c) No. BCD does not functionally determine A since $t_4(a_2, b_3, c_2, d_3)$ and $t_5(a_3, b_3, c_2, d_3)$ have all the same BCD values namely b_3, c_2, d_3 , but they have different A values namely a_2, a_3 .
- d) $(A)^+ = \{A, D\}$
 $(B)^+ = \{B, D\}$
- e) No. AB is neither a candidate key nor a super key of this relation. Since:
 $(AB)^+ = \{A, B, D\}$
so AB cannot uniquely identify C.

Q2

a)

$$A^+ = \{ADE\}$$

b)

$$CF^+ = \{ABCDEF\}$$

c)

$$AB \rightarrow C$$

$$A \rightarrow D$$

$$F \rightarrow A$$

$$D \rightarrow F$$

$$BE \rightarrow F$$

$$AC \rightarrow B$$

$$DB^+ \rightarrow DBE$$

so $DB \rightarrow C$ holds on R

Q3

a) A is the candidate key of the relation because all the attributes can be reached according to relation F.

$$A \rightarrow A \quad A \rightarrow D \quad A \rightarrow BD \quad B \rightarrow C$$

$$A^+ \rightarrow ABCD$$

b) No, it does not satisfy BCNF. Since except one of the left hand side, B, D is a super key

$$A^+ \rightarrow ABCD$$

$$B^+ \rightarrow BC$$

$$D^+ \rightarrow BCD$$

c) No, it does not satisfy 3NF since three functional dependencies violate 3NF which are:

$$A^+ \rightarrow ABCD$$

$$B^+ \rightarrow BC$$

$$D^+ \rightarrow BCD$$

Except one of the left hand side, attribute sets

B, D is a super key.

Only one of the r.h.s. attribute sets ABCD is a part of the candidate key A.

Q4

a) Primary key of the relation is (AF) since
 $(AF) = \{A, D, F, B, C, E\}$

$A \rightarrow D$ is a partial functional dependency on it
is violating 2NF.

So the relation is not in BCNF. Moreover it is in 1NF.

b) Removing partial functional dependency, we convert it into 2NF.

2NF decomposition is :

(A, D)

(A, B, C, F, E)

removing transitive functional dep: $BC \rightarrow E$ we get

3NF decomposition:

(A, D)

(B, C, E)

(A, B, C, F)

} this is the BCNF we are asked.

c) All the dependencies are covered by some relation
For example relation (A, D) covers $A \rightarrow D$,
relation (B, C, E) covers $BC \rightarrow E$, and (A, B, C, F)
covers $AF \rightarrow BC$. So Yes, the BCNF dependency is preserving.

Q4 continue

d) The decomposition is lossy since the attribute D is in common in both R1 and R2 and D is not a key of R1 nor R2.

e) R1(A, B, C, D): $A \rightarrow D$
R2(D, E, F): no dependency

$BC \rightarrow E$ is not covered in any decomposed relation
So relation R1 and R2 is not dependency preserving.

Q5

a) we need to check if $B \rightarrow C$ since $B \rightarrow C$ must preserve for D to be extraneous in $BD \rightarrow C$

$$B^+ = \{B, E, A, D, C\}$$

$$B \rightarrow E$$

$$E \rightarrow AD$$

$$A \rightarrow BC$$

$$\text{so } B \rightarrow C$$

and D is extraneous in $BD \rightarrow C$

b)

$$A \rightarrow B$$

$$B \rightarrow E$$

$$E \rightarrow AD$$

$$AD \rightarrow CE$$

$$\text{so, } A^+ = \{A, B, E, D, C\}$$

so E is extraneous in $A \rightarrow BC$.

c) Minimal Cover

$$F = \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \end{array} \quad B \rightarrow E \quad BD \rightarrow C \quad \begin{array}{l} AD \rightarrow C \\ AD \rightarrow E \end{array} \quad \begin{array}{l} E \rightarrow A \\ E \rightarrow D \end{array} \right\}$$

check if $A \rightarrow B$ reducible $\left\{ \begin{array}{l} \text{check if } A \rightarrow C \text{ reducible} \\ A^+ = \{C\} \\ \text{so it is not reducible} \end{array} \right. \left\{ \begin{array}{l} A^+ = \{B, E, A, D, C\} \\ \text{so it is reducible} \end{array} \right.$

check if $B \rightarrow E$ reducible $\left\{ \begin{array}{l} \text{check if } BD \rightarrow C \text{ reducible} \\ B^+ = \{B, E, A, D, C\} \\ \text{so it is not reducible} \end{array} \right. \left\{ \begin{array}{l} \text{yes it is reducible} \end{array} \right.$

Q5

C continue

check if $AD \rightarrow C$ reducible } check if $AD \rightarrow E$ reduc.

$$AD^+ = \{A B C E D\}$$

$$A^+ = \{A B C D E\}$$

So it is reducible

Result

$$F = \{A \rightarrow B, B \rightarrow E, A \rightarrow C, E \rightarrow A, E \rightarrow D\}$$
