ECE256 Statistical Signal Processing Spring 2022

UNIVERSITY OF CALIFORNIA, SANTA CRUZ

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Problem Set #1

Due Wednesday, 6 April, 2022

1. Problem 2.1 from Kay, Vol. 2.
2. Problem 1.1 from Kay, Vol. 1.
3. Problem 1.4 from Kay, Vol. 1.
4. Problem 1.1 from Kay, Vol. 2.
5. Problem 1.2 from Kay, Vol. 2.
6. Suppose that $x \sim \mathcal{N}(5, 2)$ and $y = 2x + 4$. Find $E(y)$, var(y), and the probability density function $p_y(y)$.
7. Suppose that $x \sim \mathcal{N}(0, \sigma_x^2)$ and $w \sim \mathcal{N}(0, \sigma_w^2)$ and $y = ax + w$. If w and x are independent, what is mean and covariance matrix for the Gaussian vector $\mathbf{z} = [x, y]^T$? Hint: Note that $E[\mathbf{z}]$ should be a 2-dimensional column vector and var($\mathbf{z}$) should be a 2×2 matrix.

1. In a radar system an estimator of round trip delay τ_0 has the PDF $\hat{\tau}_0 \sim \mathcal{N}(\tau_0, \sigma_{\hat{\tau}_0}^2)$, where τ_0 is the true value. If the range is to be estimated, propose an estimator $\hat{R}$ and find its PDF. Next determine the standard deviation $\sigma_{\hat{\tau}_0}$ so that 99% of the time the range estimate will be within 100 m of the true value. Use $c = 3 \times 10^8$ m/s for the speed of electromagnetic propagation.

4. It is desired to estimate the value of a DC level A in WGN or

$$x[n] = A + w[n] \qquad n = 0, 1, \ldots, N-1$$

where $w[n]$ is zero mean and uncorrelated, and each sample has variance $\sigma^2 = 1$. Consider the two estimators

$$\hat{A} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$$

$$\check{A} = \frac{1}{N+2}\left(2x[0] + \sum_{n=1}^{N-2} x[n] + 2x[N-1]\right).$$

Which one is better? Does it depend on the value of A?