• Çözüm Algoritması

Mühendislik Problemi >> Matematiksel Modeli >> Kodlama Sınama

↓

Uygulama

• $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

• ε: epsilon; hata oranı demektir.

• $\varepsilon_{bağıl} = \varepsilon_{mutlak} / f_{gerçek} = \varepsilon_{mutlak} / f_{gerçek}$

• $\varepsilon_{yaklaşım} = (f_{yeni} - f_{eski}) / f_{yeni}$

Matlab

• >> 3+7

ans = 10

• disp(3+7)

10

• >> disp([3,8])

3 8

• >> [5,2]

ans = 5 2

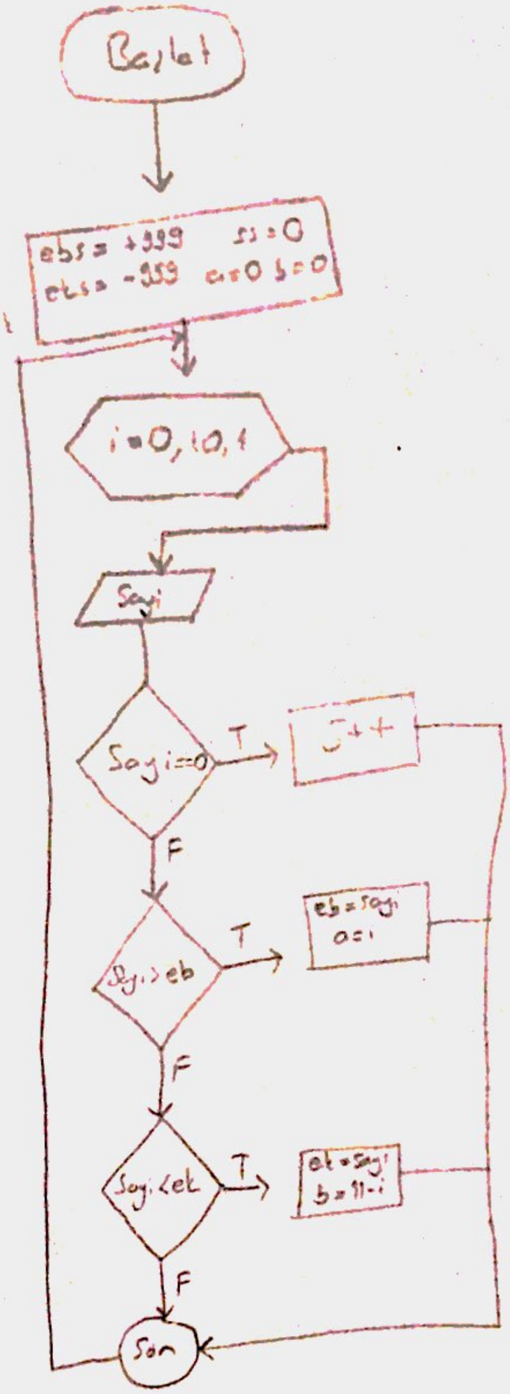
• a = [2,1,-1]

• b = [3,5,7]

• >> a+b

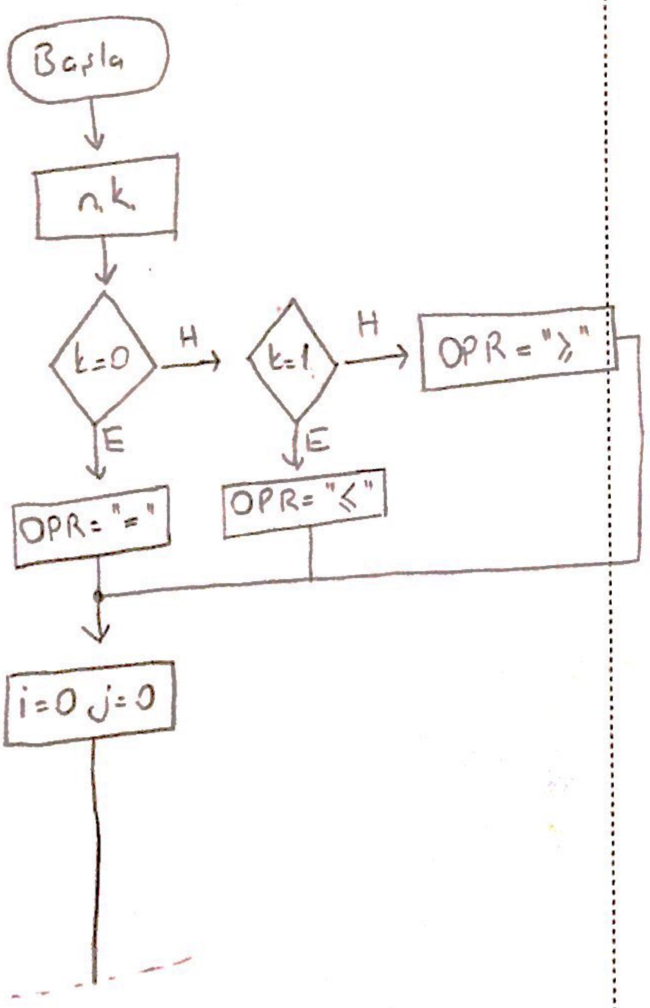
ans = 5 6 6

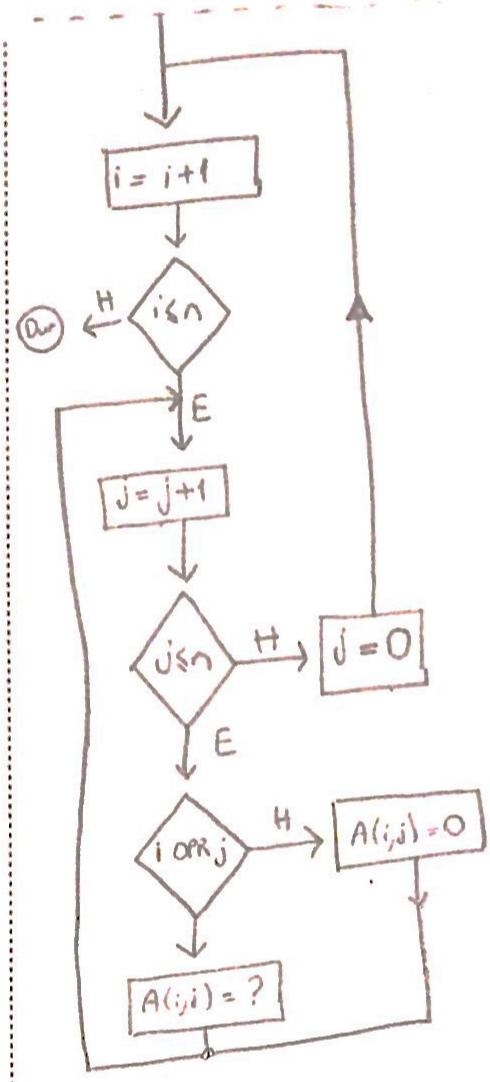
• Klavyeden girilen 10 adet sayıdan en büyüğünü baştan sırasını, en küçüğünü sondan sırasını ve girilen sıfırların sayısını bulan akış diyagramını çiziniz.



※MATRIS※

- $A(BC) = (AB)C$
- $\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$ which is "Köşegen Matris"
- $A^{-1} = \frac{A_{adj}}{A}$
- $\det(A) = 0 \Rightarrow$ Singüler Matris
- $a_{ij} = \begin{cases} \text{üst} & i<j \\ \text{alt} & i>j \\ \text{keş.} & i=j \end{cases} \quad k = \begin{cases} 1 \\ -1 \\ 0 \end{cases}$





Chio Yöntemi

$$\det A = \frac{1}{a_{11}^{(n-2)}} \begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \cdots & \begin{vmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{vmatrix} \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \cdots & \\ \vdots & & & \end{vmatrix}$$

Barış ŞENYERLİ

$$\det(A) = \begin{vmatrix} 1 & 3 & 5 & 6 \\ 2 & 4 & 7 & 8 \\ 1 & 1 & 2 & 2 \\ 3 & 3 & 5 & 6 \end{vmatrix} = \frac{1}{1^2} \begin{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} & \begin{vmatrix} 1 & 6 \\ 2 & 8 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 6 \\ 3 & 6 \end{vmatrix} \end{vmatrix} = \begin{vmatrix} -2 & -3 & -4 \\ -2 & -3 & -4 \\ -6 & -10 & -12 \end{vmatrix}$$

bir mertebe daha indirgenirse:

$$\begin{vmatrix} -2 & -3 & -4 \\ -2 & -3 & -4 \\ -6 & -10 & -12 \end{vmatrix} = \frac{1}{(-2)^1} \begin{vmatrix} \begin{vmatrix} -2 & -3 \\ -2 & -3 \end{vmatrix} & \begin{vmatrix} -2 & -4 \\ -2 & -4 \end{vmatrix} \\ \begin{vmatrix} -2 & -3 \\ -6 & -10 \end{vmatrix} & \begin{vmatrix} -2 & -4 \\ -6 & -12 \end{vmatrix} \end{vmatrix} = \frac{-1}{2} \begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} = 0$$ bulunmuş olur.

$$A = \begin{matrix} 2 & -1 & 3 & 0 \\ -3 & 1 & 0 & 4 \\ -2 & 1 & 4 & 1 \\ -1 & 3 & 0 & -2 \end{matrix}$$

$$\frac{1}{2^{2}} \begin{vmatrix} \begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ -3 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ -3 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ -1 & -2 \end{vmatrix} \end{vmatrix} = \frac{1}{4} \begin{vmatrix} 2-3 & 0+9 & 8-0 \\ 2-2 & 8+6 & 2-0 \\ 6-1 & 0+3 & -4+0 \end{vmatrix}$$

$$\frac{1}{4} \begin{vmatrix} -1 & 9 & 8 \\ 0 & 14 & 2 \\ 5 & 3 & -4 \end{vmatrix} = \frac{1}{4} \cdot \frac{1}{-1} \begin{vmatrix} \begin{vmatrix} -1 & 9 \\ 0 & 14 \end{vmatrix} & \begin{vmatrix} -1 & 8 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 9 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} -1 & 8 \\ 5 & -4 \end{vmatrix} \end{vmatrix} = -\frac{1}{4} \begin{vmatrix} -14 & -2 \\ -48 & -36 \end{vmatrix} = 48.2 - 14.36$$

$6(16-84)$

$= -68.6$

$= -408/4$

$= -102$

$$\begin{vmatrix}1&0&3&5&1\\0&1&5&1&0\\0&4&0&0&2\\2&3&1&2&0\\1&0&0&1&1\end{vmatrix}=\frac{1}{1}\begin{vmatrix}\begin{vmatrix}1&0\\0&1\end{vmatrix}&\begin{vmatrix}1&3\\0&5\end{vmatrix}&\begin{vmatrix}1&5\\0&1\end{vmatrix}&\begin{vmatrix}1&1\\0&0\end{vmatrix}\\\begin{vmatrix}1&0\\0&4\end{vmatrix}&\begin{vmatrix}1&3\\0&0\end{vmatrix}&\begin{vmatrix}1&5\\0&0\end{vmatrix}&\begin{vmatrix}1&1\\0&2\end{vmatrix}\\\begin{vmatrix}1&0\\2&3\end{vmatrix}&\begin{vmatrix}1&3\\2&1\end{vmatrix}&\begin{vmatrix}1&5\\2&2\end{vmatrix}&\begin{vmatrix}1&1\\2&0\end{vmatrix}\\\begin{vmatrix}1&0\\1&0\end{vmatrix}&\begin{vmatrix}1&3\\1&0\end{vmatrix}&\begin{vmatrix}1&5\\1&1\end{vmatrix}&\begin{vmatrix}1&1\\1&1\end{vmatrix}\end{vmatrix}$$

$$=\begin{vmatrix}1&5&1&0\\4&0&0&2\\3&-5&-8&-2\\0&-3&-4&0\end{vmatrix}=\frac{1}{1}\begin{vmatrix}\begin{vmatrix}1&5\\4&0\end{vmatrix}&\begin{vmatrix}1&1\\4&0\end{vmatrix}&\begin{vmatrix}1&0\\4&2\end{vmatrix}\\\begin{vmatrix}1&5\\3&-5\end{vmatrix}&\begin{vmatrix}1&1\\3&-8\end{vmatrix}&\begin{vmatrix}1&0\\3&-2\end{vmatrix}\\\begin{vmatrix}1&5\\0&-3\end{vmatrix}&\begin{vmatrix}1&1\\0&-4\end{vmatrix}&\begin{vmatrix}1&0\\0&0\end{vmatrix}\end{vmatrix}=\begin{vmatrix}-20&-4&2\\-20&-11&-2\\-3&-4&0\end{vmatrix}$$

$$\frac{1}{(-20)^1}\begin{vmatrix}\begin{vmatrix}-20&-4\\-20&-11\end{vmatrix}&\begin{vmatrix}-20&2\\-20&-2\end{vmatrix}\\\begin{vmatrix}-20&-4\\-3&-4\end{vmatrix}&\begin{vmatrix}-20&2\\-3&0\end{vmatrix}\end{vmatrix}=-\frac{1}{20}\begin{vmatrix}+20.7&-80\\+68&6\end{vmatrix}=-\frac{1}{20}\left((20.7.6)-(80.68)\right)$$

$$=-\frac{1}{\cancel{20}}\cancel{20}(42-272)$$

$$=+230$$

$$\begin{vmatrix}0&1&0&1\\-1&0&-1&3\\0&1&0&2\\-1&-3&-2&0\end{vmatrix}=\begin{vmatrix}1&0&0&1\\0&-1&-1&3\\1&0&0&2\\-3&-1&-2&0\end{vmatrix}\begin{vmatrix}\begin{vmatrix}1&0\\0&-1\end{vmatrix}&\begin{vmatrix}1&0\\0&-1\end{vmatrix}&\begin{vmatrix}1&1\\0&3\end{vmatrix}\\\begin{vmatrix}1&0\\1&0\end{vmatrix}&\begin{vmatrix}1&0\\1&0\end{vmatrix}&\begin{vmatrix}1&1\\1&2\end{vmatrix}\\\begin{vmatrix}1&0\\-3&-1\end{vmatrix}&\begin{vmatrix}1&0\\-3&-2\end{vmatrix}&\begin{vmatrix}1&1\\-3&0\end{vmatrix}\end{vmatrix}$$

$$=-1\begin{vmatrix}-1&-1&3\\0&0&1\\-1&-2&3\end{vmatrix}=\begin{vmatrix}\begin{vmatrix}-1&-1\\0&0\end{vmatrix}&\begin{vmatrix}-1&3\\0&1\end{vmatrix}\\\begin{vmatrix}-1&-1\\-1&-2\end{vmatrix}&\begin{vmatrix}-1&3\\-1&3\end{vmatrix}\end{vmatrix}=\begin{vmatrix}0&-1\\1&0\end{vmatrix}=0-(-1)$$

$$=1$$

Chio akış diyagramı

Örnek(1)

$2x_1 - 3x_2 + 2x_3 = -11$
$x_1 + x_2 - 2x_3 = 8$
$3x_1 - 2x_2 - x_3 = -1$

$$\begin{bmatrix} 2 & -3 & 2 \\ 1 & 1 & -2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} -11 \\ +8 \\ -1 \end{bmatrix}$$

① $$\left[\begin{array}{ccc|c} 1 & -1,5 & 1 & -5,5 \\ 1 & 1 & -2 & 8 \\ 3 & -2 & -1 & -1 \end{array}\right]$$

② adım uygulandı

② $$\left[\begin{array}{ccc|c} 1 & -1,5 & 1 & -5,5 \\ 0 & 2.5 & -3 & 13,5 \\ 0 & 2.5 & -4 & 15,5 \end{array}\right]$$

2. Satır 2.5'e bölünür.

③ $$\left[\begin{array}{ccc|c} 1 & -1,5 & +1 & -5,5 \\ 0 & 1 & -1,2 & -5,4 \\ 0 & 2,5 & -4 & +15,4 \end{array}\right]$$

④ $$\left[\begin{array}{ccc|c} 1 & -1,5 & 1 & -5,5 \\ 0 & 1 & -1,2 & -5,4 \\ 0 & 0 & -1 & 2 \end{array}\right]$$

⑤ $$\left[\begin{array}{ccc|c} 1 & -1,5 & 1 & -5,5 \\ 0 & 1 & -1,2 & -5,4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Buradan

$x_3 = -2$

$x_2 = 5.4 + 1.2x_3 = 5.4 + 1.2(-2) = 3$

$x_1 = -5.5 + 1.5x_2 - x_3 = -5.5 + 1.5 - 1(-2) = 1$

elde edilir.

$$\left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

② $a_{21} - (a_{21} \times 1)$
$a_{22} - (a_{21} \times a_{12})$
$a_{23} - (a_{21} \times a_{13})$

③ $a_{31} - (a_{31} \times 1)$
$a_{32} - (a_{31} \times a_{12})$
$a_{33} - (a_{31} \times a_{13})$

Örnek 2; $$= \begin{bmatrix} 4 & -2 & 1 \\ -3 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 8 \\ 13 \end{Bmatrix}$$

$$\begin{array}{l} \\ R_2 - (3/4) \times R_1 \rightarrow \\ R_3 - (1/4) \times R_1 \rightarrow \end{array} \left.\begin{array}{ccc|c} 4 & -2 & 1 & 15 \\ 0 & -4,5 & 4.75 & 19.25 \\ 0 & -0,5 & 2.75 & 9.25 \end{array}\right.$$

$$R_3 - (-0.5 \times -2.5) \times R_2 \rightarrow \begin{array}{ccc|c} 4 & -2 & 1 & 15 \\ 0 & -3,5 & 4.75 & 19.75 \\ 0 & 0 & 1.80 & 5.40 \end{array}$$

Örnek 3; $x_1 + x_2 - x_3 + x_4 = 2$

$$2x_2 + x_3 + x_4 = 5$$

$$x_1 \quad - x_3 + x_4 = 0$$

$$-x_1 - x_2 + x_3 \quad = -4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 1 & 5 \\ 1 & 0 & -1 & 1 & 0 \\ -1 & -1 & 1 & 0 & -4 \end{array}\right]$$

6. Hft

Gauss Jordan Yöntemi

$$\begin{matrix} 1 & -1 & 1 & 3 \\ 1 & 1 & -1 & 5 \\ -1 & 1 & 1 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 0 & 6 \\ -1 & 1 & 1 & 1 \end{matrix}$$

$$\begin{matrix} 2 & -2 & 0 & 2 \\ 0 & 2 & 0 & 6 \\ -1 & 1 & 1 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ -1 & 1 & 1 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

$x_3 = 2$

$x_2 = 3$

$x_1 = 4$

$$\begin{matrix} 6 & 2 & 1 & -5 \\ -1 & -3 & 2 & 1 \\ -2 & 1 & -3 & -5 \end{matrix}$$

$$\begin{matrix} 1 & 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & -1 & 5 \\ 1 & 0 & -1 & 1 & 0 \\ -1 & -1 & 1 & 0 & -4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & -1 & 1 & 2 \\ -1 & 1 & 2 & -1 & -1 \\ 1 & 0 & -1 & 1 & 0 \\ -1 & -1 & 1 & 0 & -4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & -4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & -4 \\ 1 & 0 & -1 & 1 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & -1 & +1 & 0 & -4 \\ 1 & 0 & -1 & 1 & 0 \end{matrix}$$

Cramer Yöntemi

$$\frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|} = x_1$$

$$\frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|} = x_2$$

$$\frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|} = x_3$$

```
>> R=[1 1 2; 2 3 1; 3 2 2]
>> E=[1 1 0]'
=E: [1
     1
     0]
>> MI1 = [E, R(:,[2 3])]
>> MI2 = [R(:,1) E R(:,3)]
>> MI3 = [R(:,[1 2]) E]
>> I = [det(MI1); det(MI2); det(MI3) / det(R)] = -2
                                                  1
                                                  2
```

1	2	1	-3
3	7	4	9
2	-1	3	12

$$\det(A) = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 7 & 4 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \\ 3 & 7 & 4 \end{vmatrix}$$

$-$: 14, -4, 18 → 28

$+$: 21, -3, 16 → 34

$34 - 28 = 6$

$$\det(A_1) = \begin{vmatrix} -3 & 2 & 1 \\ 9 & 7 & 4 \\ 12 & -1 & 3 \end{vmatrix} = -126$$

$$\det(A_2) = \begin{vmatrix} 1 & -3 & 1 \\ 3 & 9 & 4 \\ 2 & 12 & 3 \end{vmatrix} = 0$$

$$\det(A_3) = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 7 & 9 \\ 2 & -1 & 12 \\ 1 & 2 & -3 \\ 3 & 7 & 9 \end{vmatrix}$$

$-$ 42, $-$ 9, ~~36~~

84, 9, ~~36~~

$= 108$

$x_3 = \frac{108}{6} = 12$

$x_2 = 0$

$x_1 = -21$

Ters Matris

$AX = B \Rightarrow X = A^{-1} \cdot B$

- >> AT = inv(A)

  $\underset{\text{için}}{}$

- >> I = AT * A
- >> X = AT * B

LU (Ayrıştırma, Cholesky) Yöntemi

- $AX = B$ ve $A = LU \Rightarrow LUX = B$

- $\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

  L U

- $L \underbrace{UX}_{Z} = B \Rightarrow LZ = B$

- $Z = UX$ z bulunur.

$$\left[\begin{array}{ccc|c} 4 & 1 & 1 & 9 \\ 2 & -1 & 1 & 3 \\ 2 & 1 & 1 & 7 \end{array}\right]$$

$AX = B \qquad A = LU$

$$\begin{matrix} 4 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & 1 \end{matrix} = \begin{matrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{matrix} \times \begin{matrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{matrix}$$

- L ve U'yu bul.
- L ve B'yi kullanarak Z'yi bul.
- U ve Z'yi kullanarak X'i bul.

- >> [l, u] = lu(A)
- >> z = inv(l) * B
- >> x = inv(u) * z

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 1 & 1 & -1 & 5 \\ -1 & 1 & 1 & 1 \end{array}\right]$$

$\underbrace{}_{A}$ $\underbrace{}_{B}$

$$\underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}\cdot u_{11} & l_{21}\cdot u_{12}+u_{22} & l_{21}\cdot u_{13}+u_{23} \\ l_{31}\cdot u_{11} & l_{31}\cdot u_{12}+l_{32}\cdot u_{22} & l_{31}\cdot u_{13}+l_{32}\cdot u_{23}+u_{33} \end{bmatrix}}_{L\times U = A}$$

$u_{11}=1$

$l_{21}.u_{11}=1$ $\quad l_{21}=1$

$l_{31}.u_{11}=-1$ $\quad l_{31}=-1$

$u_{12}=-1$

$l_{21}\cdot u_{12}+u_{22}=1$ $\quad u_{22}=2$

$(1)(-1)$

$l_{31}\cdot u_{12}+l_{32}\cdot u_{22}=1$ $\quad l_{32}=0$

$(-1)(-1)+l_{32}\cdot(2)=1$

$u_{13}=1$

$l_{21}\cdot u_{13}+u_{23}=-1$ $\quad u_{23}=-2$

$(1)(1)$

$l_{31}\cdot u_{13}+l_{32}\cdot u_{23}+u_{33}=1$

$(-1)(1)+0+u_{33}=1$ $\quad u_{33}=2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

$z_1=3$

$z_1+z_2=5$ $\quad z_2=2$

$-z_1+z_3=1$ $\quad z_3=4$

$U \cdot X = Z$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

$x_1-x_2+x_3=3$

$2x_2-2x_3=2$

$2x_3=4$ $\quad x_3=2$

$2x_2-4=2$

$2x_2=6$

$x_2=3$

$x_1-3+2=3$

$x_1=4$

$$GK=\begin{Bmatrix} 4 \\ 3 \\ 2 \end{Bmatrix}$$

1 tane daha örnek sloytta

Jacobi iterasyon yöntemi

$X_1 = AX_0 + C$

$X_2 = AX_1 + C$

⋮

$X_k = AX_{k-1} + C \quad i = 1:n$

$$\max \left| \frac{x_i^k - x_i^k}{x^k} \right|$$

Örnek (1)

$x + y = 3$

$2x - y = 3$

x	y
3/2	3/2
2,25	0,75
1,875	1,125
⋮	⋮
2,000987	0,995023

$x(1) = (3+0)/2 = 1,5 \Rightarrow 1,5$

$x(2) = (3+1,5)/2 = 2,25 \Rightarrow 0,75$

⋮

$x(n) = (3+k)/2 = 2.000987 \Rightarrow 0,995023$

Örnek (2)

$$\begin{matrix} 4 & 1 & 1 & 0 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 1 & 0 & 4 & 1 & 0 \\ 0 & 1 & 1 & 4 & 1 \end{matrix}$$

$x_1 = \frac{1}{4}(1 - x_2 - x_3)$

$x_2 = \frac{1}{4}(2 - x_1 - x_4)$

$x_3 = \frac{1}{4}(-x_2 - x_4)$

$x_4 = \frac{1}{4}(1 - x_2 - x_3)$

Gauss-Seidel Yöntemi

• Jacobi yöntemini daha kısa yolla yapmıştır.

	x_1	x_2	x_3	x_4
1	0	0	0	0
2	0,25	0,4375	-0,0625	0,1563
⋮	⋮	⋮	⋮	⋮
7	0,1667	0,4167	-0,0834	0,1667

Aitken iterasyon yöntemi

• Jacobi yöntemini, Gaussdan daha kısa yolla yapmıştır.

$$x_i^k = x_i^k - \frac{(x_i^k - x_i^{k-1})^2}{x_i^k - 2 \cdot x_i^{k-1} + x_i^{k-2}}$$

	x_1	x_2	x_3	x_4
0	0	0	0	0
1	0,25	0,5	-0,125	0,125
2	⋮	⋮	⋮	⋮
3				
4	0,1667	0,4167	-0,0834	0,1667

Barış ŞENYERLİ

Örnek 1) $f(x) = x^3 - x - 1 = 0$ denkleminin $x_0 = 1.3$ civarında kökü olduğu bilindiğine göre, gerçek kökü $\varepsilon = 0{,}000001$ hassasiyetle basit iterasyon yöntemiyle bulunuz.

1. $f(x) = x^3 - x - 1$
2. $x = x^3 - 1$
3. $g(x) = x^3 - 1$
4. $x_0 = 1.3$
5. $g'(x_0) = 3x^2 > 1$ ıraksak
6. $x = (x+1)^{1/3}$
7. $g'(x_0) = \frac{1}{3}(x+1)^{-2/3} = 0{,}15 < 1$ yakınsak

x_0	$g(x)$	$h(x)$
1.3	1.320061	

Örnek 2) $f(x) = 2x^4 - 3x - 2 = 0$ fonksiyonun $x_0 = 1.3$ ve $x_0 = -0{,}5$ civarında kökleri olduğu bilindiğine göre

$x = \frac{2x^4}{3} - \frac{2}{3}$

$g'(x_0) = \frac{8x^3}{3} > 1$
$\downarrow$
1.3

$g'(x_0) = -\frac{1}{3} < 1$
$\downarrow$
$-0{,}5$

x_0	$h(x)$
$x_0 = -0{,}5$	
$x_1 = -0{,}5678$	
$x = -0{,}5875$	0,000001

Barış ŞENYERLİ

Yarılama (Bisection) Yöntemi

1. x_a, x_b ve $f(x_a), f(x_b)$ değerleri zıt işaretli olmalı.
2. $x_1 = \frac{x_a + x_b}{2}$ yeni bir x_1 bulunur.
3. $f(x_1), f(x_a)$ aynı işaretli,

$f(x) = x^3 - 6{,}5x^2 + 13x - 9$ $[a = 1{,}75, b = 2{,}5]$

$\varepsilon = 0{,}8$

$f(1{,}75) = 0{,}078125$
$f(2{,}5) = -0{,}25$
} Zıt işaretli kök mevcuttur.

$c_1 = \frac{a+b}{2} = \frac{1{,}75 + 2{,}5}{2} = 2{,}125$ } $f(2{,}125) = -1{,}13086$

$c_2 = \frac{a + c_1}{2} = \frac{1{,}75 + 2{,}125}{2} = 1{,}9375$ } $f(1{,}9375) = -0{,}93970$

Örnek 3) $f(x) = \exp(x) - x - 2 = 0$ $\quad 1 < x < 1{,}8$

$f(1) = -0{,}28171828$

$f(1{,}8) = 2{,}24964746$

$c_1 = \frac{1 + 1{,}8}{2} = 1{,}4 \Rightarrow f(1{,}4) = 0{,}655199967$

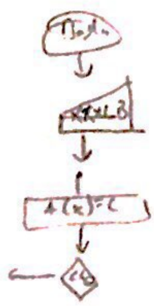
$c_2 = \frac{1{,}4 + 1}{2} = 1{,}2 \Rightarrow f(1{,}2) = 0{,}120116925$

$c_3 = \frac{1 + 1{,}2}{2} = 1{,}1 \Rightarrow f(1{,}1) = -0{,}0958233761$
$\Rightarrow x_a = 1{,}2$
$\Rightarrow \varepsilon$

$c_4 = \frac{1{,}2 + 1{,}1}{2} = 1{,}15 \Rightarrow f(1{,}15) = 0{,}00815250869$

$c_5 = \frac{1{,}1 + 1{,}15}{2} = 1{,}125 \Rightarrow f(1{,}125) = -0{,}044703$
$\downarrow$
en yakın kök

diyagramı



Kiriş (Secant) yöntemi:

① $y - y_0 = \frac{y_0 - y_1}{x_0 - x_1}(x - x_0)$

② $x_3 = x_0 - \frac{(x_2 - x_0)\, y_0}{y_2 - y_0}$

③ $x_{n+1} = x_n - \frac{(x_n - x_{n-1})}{(y_n - y_{n-1})} y_n$

Örnek 4) $f(x) = e^{-x} - x = 0$

$f(0) = 1.0 \Rightarrow f(1) = -0,632120558$

$\Rightarrow f(0) \cdot f(1) < 0$

olduğundan bu aralıkta kök vardır.

$x_0 = 0$, $y_0 =$

Örnek (1) Bir A sayısının istenilen duyarlılıkta karekökünün bulunması için Newton Raphson yöntemini kullanarak bir algoritma geliştiriniz. Buna göre 10'un karekökünü $x_0=1$ başlangıç değeri, $\varepsilon = 0{,}005$ mutlak hatayla bulunuz.

$$\sqrt{A} = x$$

$$A = x^2$$

$$f(x) = x^2 - A \rightarrow \sqrt{9} = 3$$

A gibi bir sayının karekökünü bulma $x^2 = A$ gibi bir problemdir.

$$x_1 = x_0 - \frac{x_0^2 - A}{2x_0} = \frac{1}{2}\left(x_0 + \frac{A}{x_0}\right)$$ elde edilir.

iterasyon için genelleme x_n bilinen kök ise hesaplanan kök

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right)$$ olarak elde edilir.

Örnek (2) $x^3 + 2x^2 + 6x + 3 = 0$ denkleminin $[-1, 0]$ arasında kökü olup olmadığını araştırınız. Varsa Newton-Raphson ile kökü yaklaşınız.

$f(-1) = -1 + 2 + (-6) + 3 = -2 < 0$

$f(0) = 3 > 0$ kök var

$x_0 = 0$ alalım.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{f(0)}{f'(0)} = \frac{-3}{6} = -0{,}5$$

$$x_2 = -0{,}5 - \frac{f(-0{,}5)}{f'(-0{,}5)} = -0{,}578$$

$$x_3 = -0{,}578 - \frac{f(-0{,}578)}{f'(-0{,}578)} = -0{,}5784$$

Örnek (3) $e^x - 3x = 0$ denkleminin $[0, 1]$ aralığı

$f(0) = e^0 - 0 = 1 - 0 = 1$ $\quad f'(x) = e^x - 3$

$f(1) = e - 3 < 0$ $\quad f''(x) = e^x$

$x_0 = 0$ olarak $\rightarrow$ $f''(0) = e^0 = 1 > 0$

$f(0) = 1 > 0$

değerler aynı olmalı

$f''(1) = e > 0$

$f(1) < 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{1}{-2} = 0{,}5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0{,}5 - \frac{f(0{,}5)}{f'(0{,}5)} = 0{,}61$$

Örnek (4) $x^3+6x^2+13x-20=0$

$f(2)=38>0$

$f'(x)>0$) kök vardır

$f''(x)>0$)

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{38}{49}=1{,}224$

$x_2=1{,}014$

$x_3=1{,}0005\cong 1$

4. Soru Ödev 7. Hafta 16. Sayfa

En küçük kareler yöntemi

$y=f(x)=mx+b$

$\sum=(y-mx-b)^2$