Note of QFT

Zhiyua He

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1 Review

1.1 Interacting Fields

From the Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \sum_{n\geq 3}\frac{\lambda}{n!}\phi^n$$

where $\sum_{n\geq 3}\frac{\lambda}{n!}\phi^n$ is the interacting part
Then we use the dimensionless analysis and got

$$[\lambda_n] = 4 - n \ [\phi] = [m] = 1 \ [\mathcal{L}] = 4$$

we want to use perturbation theory, so we need to know the limitation of the coefficient λ
than we describe 3 situations

n=3, interating part:

$$\frac{\lambda_3}{3!}\phi^3$$

we try to dimensionless the coefficient:

$$\lambda_3 \to \frac{\lambda_3}{E} \ (because[\lambda_3] = 1)$$

then we get: when $\lambda_3 << E \to$ small perturbation, $\lambda_3 >> E \to$ high perturbation

n =4, $[\lambda_4] = 0$,dimensionless the coefficient:

$$\lambda_4 \to \lambda_4$$

if $\lambda_4 << 1 \to$ small perturbation $\to$ *marginal*

n $\geq$ 5, $[\lambda_n] < 0$,dimensionless the coefficient:

$$\lambda_n \to \lambda_n E^{n-4}$$

hence $\lambda_n << E \rightarrow highperturbation, \lambda_n >> E \rightarrow smallperturbation$
there are some quastions:

Q1:Why we need to dimensionless the coefficient? (in coupling part)
because we just want to let the coefficient in coupling part becomes a independent part, which we can just focus the field part.

Q2:These three situations, which is important?
relavant part *marginal* part is important, but irrelavant parts is unnecessary. Because for infinite interacting terms, we just need few parts to describe, in low energy, we can just ignore them

1.2 Some examples

1.2.1 weakly coupled theories

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$
$$(where\lambda \leq 1)$$

the way to solve the Lagrangian in the equation is to rewrite: $\phi^4 \rightarrow \widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}^\dagger, \widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}^\dagger\widehat{a}_{\vec{p}}, ...$
and we can find that the paticles relavant to $\widehat{a}_{\vec{p}}^\dagger$ maybe not conserved.

1.2.2 Scalar Yukawa Theory

$$\mathcal{L} = \partial_\mu\psi^*\partial^\mu\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - M^2\psi^*\psi - \frac{1}{2}m^2\phi^2 - g\psi^*\psi\phi$$

physical mean of this Lagrangian: paticles relavant to ϕ, will create a pair of another particle(relavant to ψ) and its anti-particle(relavant to ψ^*)

1.3 Two pictures of the interacting fields are the same

from *Schrödinger* picture: $i\frac{d|\psi\rangle_S}{dt} = \widehat{H}|\psi\rangle_S$
and *Heisenberg* picture: $\mathcal{O}_H(t) = e^{i\widehat{H}t}\mathcal{O}_S e^{-i\widehat{H}t}, |\psi\rangle_H = e^{i\widehat{H}t}|\psi\rangle_S$
and we assume the Hamiltonian of the system: $\widehat{H} = \widehat{H}_0 + \widehat{H}_{int}$
(where $\widehat{H}_0$ is time dependent , $\widehat{H}_{int} istimeindependent$)
we can get:

$$i\frac{d|\psi\rangle_I}{dt} = \widehat{H}_I(t)|\psi\rangle_I$$

so we proved two pictures are the same

2 Preview

2.1 Dyson's Formula

We want to solve the equation of the picture:

$$i\frac{d|\psi\rangle_I}{dt} = \widehat{H}_I(t)|\psi\rangle_I$$

we assume the solution has the form:

$$|\psi(t)\rangle_I = \widehat{U}(t,t_0)|\psi(t_0)\rangle_I$$

Note: this operator $\widehat{U}(t,t_0)$ has some properties:

$$\widehat{U}(t_1,t_2)\cdot\widehat{U}(t_2,t_3) = \widehat{U}(t_1,t_3)$$
$$\widehat{U}(t,t) = 1$$

(*Question*: why $\widehat{U}$ have these properties?) hence we get:

$$i\frac{dU}{dt} = \widehat{H}_I(t)U$$
$$U(t,t_0) = e^{-i\int_{t_0}^{t}\widehat{H}_I(t')dt'}$$

But this assumption solution will have some trouble
use Taylor series to rewrite $U(t,t_0)$:

$$e^{-i\int_{t_0}^{t}\widehat{H}_I(t')dt'} = 1 - i(\int_{t_0}^{t}\widehat{H}_I(t')dt) + \frac{(-i)^2}{2}(\int_{t_0}^{t}\widehat{H}_I(t')dt) + ...$$
$$\frac{dU}{dt} = ... - \frac{1}{2}(\int_{t_0}^{t}\widehat{H}_I(t')dt)H_I(t) - \frac{1}{2}H_I(t)(\int_{t_0}^{t}\widehat{H}_I(t')dt) + ...$$

so we find this part cannot equal to the left part of the equation, means $[H_I(t),\dot{H}_I(t)] \neq 0$
hence the assumption solution will be more complicate
Dyson solve this, the assumption solution should be:

$$U(t,t_0) = Te^{-i\int_{t_0}^{t}\widehat{H}_I(t')dt'}$$

$$T(\mathcal{O}_1(t_1),\mathcal{O}_2(t_2)) = \begin{cases}\mathcal{O}_1(t_1)\mathcal{O}_2(t_2), t_1 > t_2\\ \mathcal{O}_2(t_2)\mathcal{O}_1(t_1), t_1 < t_2\end{cases} \tag{1}$$

then the solution:

$$\begin{aligned}U(t,t_0) = 1 - i(\int_{t_0}^{t}H_I(t')dt) + \frac{(-i)^2}{2}[\int_{t_0}^{t}dt'\int_{t'}^{t}dt''H_I(t'')H_I(t') +\\ \int_{t_0}^{t}dt'\int_{t_0}^{t'}dt''H_I(t')H_I(t'')] + ...\\ = 1 - i(\int_{t_0}^{t}H_I(t')dt) + (-i)^2\int_{t_0}^{t}dt'\int_{t_0}^{t'}dt''H_I(t')H_I(t'') + ...\end{aligned}$$

ine the real situation, only when H_I is samll that it can be easly solved

2.2 Scattering

recall the Scalar Yukawa Theory:

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - M^2 \psi^* \psi - \frac{1}{2} m^2 \phi^2 - g\psi^* \psi \phi$$

in this Lagrangian density, $\widehat{H}_{int} = -g\psi^*\psi\phi$
and we define some notations:

$$\begin{cases} \phi \sim \widehat{a} + \widehat{a}^\dagger \\ \psi \sim \widehat{b} + \widehat{c}^\dagger \\ \psi \sim \widehat{b}^\dagger + \widehat{c} \end{cases} \tag{2}$$

with these notations we can describe the process like: $\widehat{c}^\dagger \widehat{b}^\dagger \widehat{a} \rightarrow$ kill a meson and create a nucleon pair($\phi \rightarrow \psi\bar{\psi}$)
Note: we have assumed the interaction is sudden and $|initial state\rangle, |final state\rangle is the eigenstate of \widehat{H}_{int}$

2.3 An example: Meson Decay

Maybe hand-writing !