Note of Fermi Liquid

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1 Introduction and Motivation

As Fermi Liquid system can be described into a many-particle system, it's difficult to describe it. But in low temperature, the whole system can be regarded as some elementary exitations(or 'quasiparticles'), it's easy to imagine that when the system in low temperature, such perturbation of system can be regarded as the action from some 'quasiparticles', like phonons in solid. For the proving and describe this picture, we need to describe it in term of field, or more precise, *Quantum Field Theory* (QFT). Also some knowledge of renormalization theory.

2 Compact introduction of QFT

During the learning of classical field theory, we may have a nature question: *What is field?*

Actually, field is just like a 'map'.(I want to say 'function' here, but this will be a contradiction between 'function' and the definition of 'action', the latter is defined with 'variation') For some given space-time position x (actually we write x_μ in field theory), field will give some amplitude. In the classical field theory, we use Lagrange density to connect two sides of the 'map'.

And we have some classification of field here: Scalar field, Vector field and Spinor field. From the definition of them, the equation of scalar fields is *Klein-Gorden euqation*, which is used for describing the behavior of spin-0 particles. The equation of vector fields is *Proca equation*, which is used for describing the behavior of spin-1 particles with nonzero mass. If some spin-1 particles' mass is zero, the *Proca equation will* degenerate to *Maxwell equation.* The equation of spinor field is *Dirac Equation*, which is used for the behavior of spin-$\frac{1}{2}$ pariticles.

2.1 Scalar field

Statement: All of our calculations are under Minkowski space

now we first consider a Non-interactive scalar field. From Quantum Mechanics we have the Schrödinger equation

$$\frac{\vec{p}^2}{2m}\psi = i\hbar\frac{\partial}{\partial t}\psi \tag{1}$$

it doesn't satisfy the special relativity(Why? some comments here). We replace the kinetic part to the relativity energy:

$$E = \sqrt{\vec{p}^2c^2 + m^2c^4} \tag{2}$$

Then we have:

$$\begin{aligned}
\frac{\sqrt{\vec{p}^2c^2 + m^2c^4}}{2m}\psi &= i\hbar\frac{\partial}{\partial t}\psi \\
\Rightarrow \sqrt{-\frac{\hbar^2\nabla^2}{c^2} + m^2}\psi &= \frac{2im}{c^2}\frac{\partial}{\partial t}\psi \\
\Rightarrow (m^2 - \frac{\hbar^2\nabla^2}{c^2})^2\psi &= (i\frac{2m}{c^2}\frac{\partial}{\partial t})^2\psi \quad {}^* \\
\Rightarrow (\Box^2 - \mu^2)\psi &= 0
\end{aligned} \tag{3}$$

Here we can find the reason that why we will get two roots each condition, but one of them doesn't have physical mean.(actually it has, that's the prediction of antiparticle)

Although we can also use the Lagrange density to get K-G equation, from the Lagrange density:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \tag{4}$$

From the definition of action, we have:

$$S = \int_{t_1}^{t_2} dt \int d^3x\mathcal{L} \tag{5}$$

Then we put these two equation in Euler-Lagrange Equation(whose solutions are the functions for which a given functional is stationary):

$$\frac{\partial\mathcal{L}}{\partial\phi} - \frac{\partial}{\partial x_\mu}\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)} = 0 \tag{6}$$

Then we can get K-G equation conveniently.

2.2 Spionr field

For being used for describe the behavior of spin-$\frac{1}{2}$ particles, this field is more like to match condense matter physics.

[Latex Transfer Waiting]
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[1]* here we have consider the square of ψ become a new field but they have simple relationship