Summary of the research

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1 Introduction

The juncture of our work is the physical phenomena - entanglement. From the work between 1935$(EPR, PhysRev.477.777)$ and 2008 $(Quantum\ entanglement, Horodecki)$, the picture of the entanglement is clearer than before. And with the Bell inequalities we can try to solve some simple system accurately(like bipartite systems).

Then we find the mathematical manifestations of entanglement:
think about a system with n subsystem, the total system should be *Cartesian* product of subsystem. And the total Hilbert space should be a high dimension tensor, like:

$$\widehat{H} = \otimes_{l=1}^{n} \widehat{H}_l$$

and in this Hilbert space, the state of the total system should be:

$$|\psi\rangle = \sum_{\widehat{i}_n} C_{\widehat{i}_n} |\widehat{i}_n\rangle$$
$$|\widehat{i}_n\rangle = |i_1\rangle \otimes |i_2\rangle \otimes \cdots\cdots \otimes |i_n\rangle$$

But it has the property:

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots\cdots \otimes |\psi_n\rangle$$

the operator has the same property as the wavefunction.

And then we will have some naturally questions in this process. How can we judge a system is or not entanglement system? How we can quantify the strength between a entanglement system?

From the paper about correlation($Clauser, Horne, ShimonyandHolt, 1969$) and the paper about entropic inequalities($Schumacher, 1995$), we begin to find the way to calculate the correlation strength between subsystems. And at that time we knew we could begin to find some interesting systems' property with these models.
Before we begin our work we have recalled the:

Bell nonlocality $\subseteq$ *Quantum entanglement* $\subseteq$ *Quantum discord* $\subseteq$ *Quantum total correlation*

And our work is most about entanglement, and sometimes involves discord & correlation.

2 Work & Result

2.1 Quantum Mutual Information

From the definition of entropy, we can define the quantum mutual information in a many-body system, which should have the property - nonnegative. So we try to repeat the paper's ($PhysRevA.96.012332$) work.

The main work of this reference is to give an alternative definition of quantum mutual information. For understanding it quickly, learn the example of a set maybe the best way. for a set with N elements, the way to compute all the mutual information of this system is summing all the elements without single parts, like:

$$I(\rho_{123...N}) = \sum_{i=1}^{N} S(\rho_{1,2...i-1,i+1...n}) - (N-1)S(\rho_{123...N})$$

where the part (N - 1) is the increase of multiplex parts, and with increasing of the set's size N, this part will also increase.
This definition is different with another calssical Quantum Mutual Information *(QMI)* definition, such like:

Bipartite QMI

$$I_x(\rho_1 23...N) = \sum_{i=1}^{N} S(\rho_i) - S(\rho_{123...N}) = S(\rho_{123...N}||\rho_1 \otimes \rho_2 \otimes ... \otimes \rho_N)$$

Common information

$$I_c(A_1 : A_2 : ... : A_n) = \sum_{k=1}^{N} (-1)^{k+1} \sum_{A_1 A_2 ... A_k} SA_1 A_2 ... A_3$$

$$A\ spatial\ situation (N=3):$$
$$I_c(A:B:C) = [S(A)+S(B)+S(C)]-[S(AB)+S(AC)+S(BC)]+S(ABC)$$

And it also define the Quantum Discord:

$$\mathcal{D}_A(\rho_{ABC}) = I(\rho_{ABC}) - \max_{\Phi_A} I(\Phi_A(\rho_{ABC}))$$
$$\mathcal{D}_{AB}(\rho_{ABC}) = I(\rho_{ABC}) - \max_{\Phi_{AB}} I(\Phi_{AB}(\rho_{ABC}))$$
$$\mathcal{D}_{ABC}(\rho_{ABC}) = I(\rho_{ABC}) - \max_{\Phi_{ABC}} I(\Phi_{ABC}(\rho_{ABC}))$$

With these definitions, it selects three common entanglement states:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$
$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$
$$|\psi_{as}\rangle = \frac{1}{\sqrt{6}}(|123\rangle - |132\rangle + |231\rangle - |213\rangle + |312\rangle - |321\rangle)$$

(where 'as' means 'totally antisymmetric state')

And the result from our computation is really close to the paper.

2.2 Quantum Optics

For preparing the entanglement states, in experiment, a good way is to use photon. Hence it's useful do research the interacting between subsystems. In the quantum optics, people create a useful model to solve this complex problem - Dicke model.

The Dicke model describes a single bosonic mode which interacts collectively with a set of B two-level systems. We choose two representative papers $IntroductiontotheDickemodel(AQT.$201800043) and $PhysRevA.$69.042110 to do some research.
From the Dicke model, the Hamiltonian of the system is:

$$H = \omega_z \sum_{j=1}^{N} \sigma_j^z + \omega_e a^\dagger a + \frac{2\lambda}{N}(a^\dagger + a) \sum_j \sigma_j^x$$

where ω_c is the photon frequency, ω_z is the atomic energy splitting, λ is the photon-atom coupling, a and $a^\dagger$ are the annihilation and creation operators of the photon.
Then we can try to find the thermodynamic properties and the evolution in time. Hence we need to find two coherent states to perform this Hamiltonian. In the paper, they are defined as:

$$|\tau\rangle = (1+\tau\tau^*)^{-j} e^{\tau J_+} |j,-j\rangle$$
$$|\beta\rangle = e^{\frac{-\beta\beta^*}{2}} e^{\beta a^\dagger} |0\rangle$$

with:

$$\tau = \frac{q_1 + ip_1}{\sqrt{4j - ({q_1}^2 + {p_1}^2)}}$$
$$\beta = \frac{1}{\sqrt{2}}(q_2 + ip_2)$$

Then we can get the total density operator of the system, and use partial trace to get subsystems' density operator.