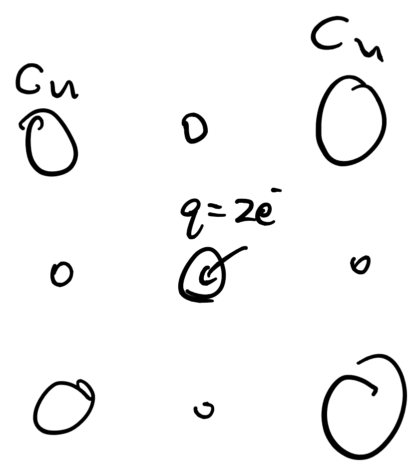
Review:

we are talking about the weakly correlated electronic liquid.

and one of ~~the~~ basic phenomenon is Thomas-Fermi Theory:

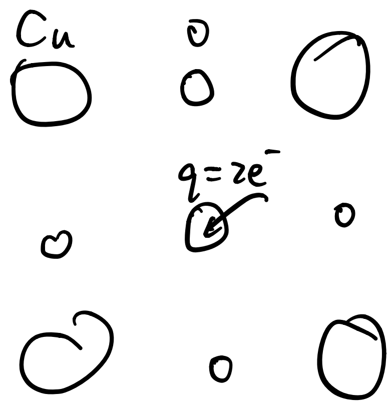


we have calculated the effect of the defect's ionic potential is screened away for distances $r > r_{TF} = \left(4\pi^2 g(E_F)\right)^{-\frac{1}{2}}$

Thomas-Fermi screening

what this will cause? → Mott trasition

we consider the same example in CuO material.



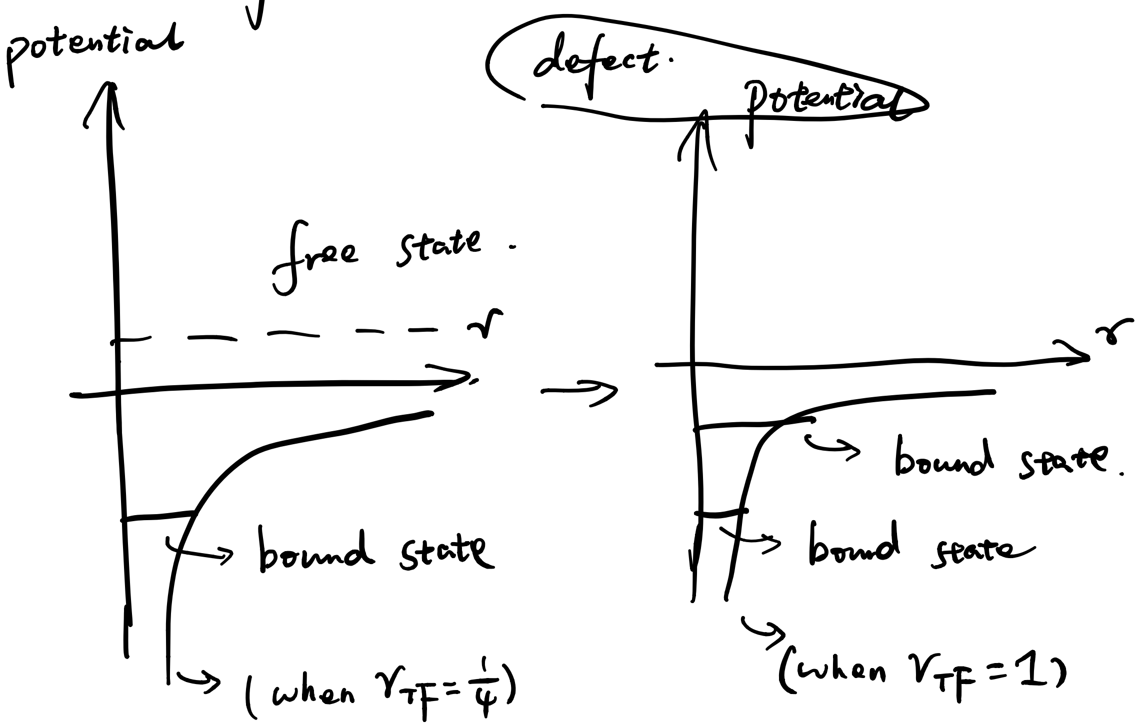
when the screening length ↓

the bound state energy will rise up

but if in snm weak metal, which valence state is barely free, the reduction of carriers (electrons) will increase the screening length. → they have the relationship

$$r_{TF} \sim n^{-1/6}.$$

This means, with the reduction of carriers, the range of the potential will extend and more free valance state will be bound.



Then this phenomenon called Mott transition when we reducing the density of carriers.

Some metal will abrupt transform to Insulator

Fermi Liquid:

we add interactions in free Fermions in a box

(This theory aimed to describe gaseous phase, but actually it's useful in ~~the theories of~~ super-conductivity and super fluidity)

Fermi Liquid theory has 3 basic tenants.

① momentum and spin remain good qutum number to describe the quasi-particles.

② over some time t, particle-particle sys will turn to interacting sys in adiabatically sys.

③ excitations $\xrightarrow{\text{described by}}$ quasi-particles

with lifetimes $\gg t$

we first introduce quasi-particles.

<1> particles & Holes.

For $T=0$, non-interacting, free Fermions system

we'll have the result.

eigenstates of single particle state:

$$\begin{cases} \psi_p(\vec{r}) = \dfrac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{r}/\hbar} \\ E = \sum\limits_p n_p \dfrac{p^2}{2m} \\ n_p = \theta(p - p_F) \\ \dfrac{1}{3\pi^2}\left(\dfrac{p_F}{\hbar}\right)^3 = \dfrac{N}{V} \end{cases}$$

if we add a particle to the lowest available

State $p = p_F$, then, in $T=0$:

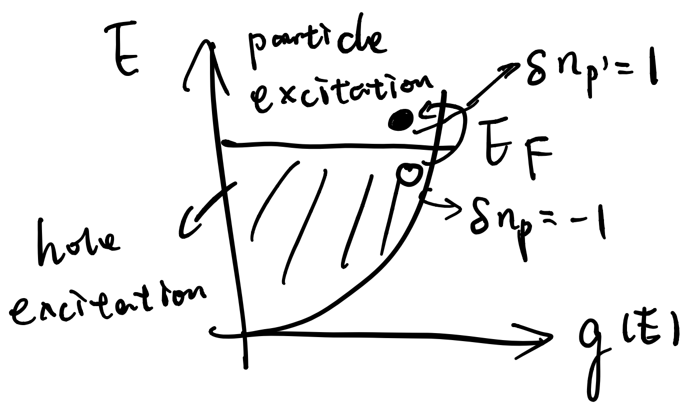
$$\mu = E_0(N+1) - E_0(N) = \frac{\partial E_0}{\partial N} = \frac{p_F^2}{2m}$$

if we excite sys now, there will be some holes below the Fermi surface and some particles across the Fermi surface.

These elementary excitations are quantified by:

$$\delta n_p = \begin{cases} \delta_{p,p'} & \text{for a particle } p' > p_F \\ -\delta_{p,p'} & \text{for a hole } p' < p_F \end{cases}$$

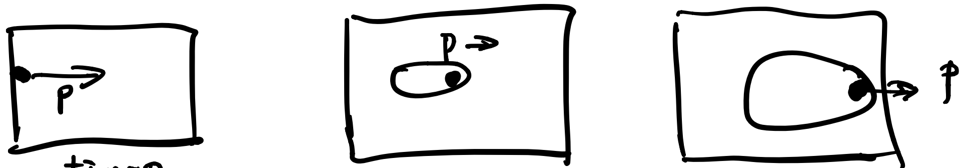
$$\delta n_p = n_p - n_p^0$$



Hence the energy of system can be rewrited as:

$$E - E_0 = \sum_p \frac{p^2}{2m}(n_p - n_p^0) = \sum_p \frac{p^2}{2m}\delta n_p$$

<2> quasi-particles and quasiholes in $T = 0$.

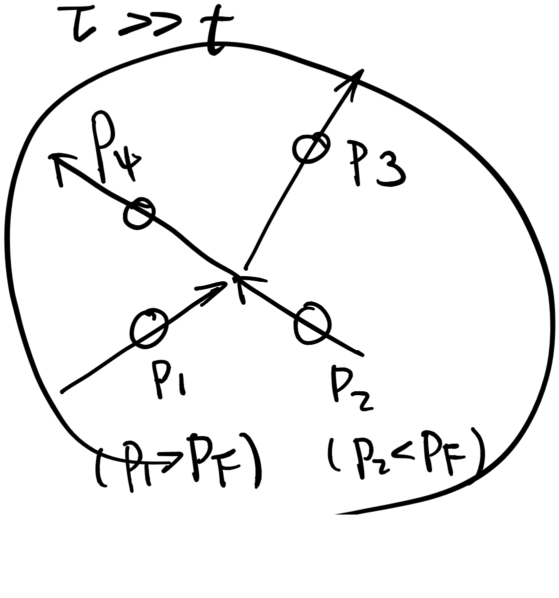


time = 0, $U = 0$ non-interacting Fermi Liquid at time 0

time = t, $U = \left(\frac{e^2}{a}\right)e^{-a/r_{TF}}$ (a — average distance between particles)

(we have assumed life time of quasiparticle $\tau \gg t$)

prove



$(P_3, P_4 > P_F)$

<3> energy of qusiparticles.

in ideal sys:

$$E - E_0 = \sum_p \frac{p^2}{2m} \delta n_p$$

in Interacting sys:

$$E[n_p] = E_0 + \sum_p \varepsilon_p \delta n_p + \mathcal{O}(\delta n_p^2)$$

$$\varepsilon_p = \frac{\delta E}{\delta n_p}$$

we consider the ε_p near the Fermi surface, wher δn_p is finite. Hence we approximate

$$\varepsilon_p \approx \underbrace{\mu}_{\frac{p_F^2}{2m}} + (\vec{p} - \vec{p}_F) \cdot \underbrace{\nabla_p \varepsilon_p}_{= v_{\vec{p}}} \Big|_{p_F}$$

(some special situation: $T=0$. ground states

$$\varepsilon_p = \varepsilon_F = \mu = \frac{\partial E_0}{\partial N}$$)

First quantization:

consider the enviroment classically

(particle → wave)

$$\sum_{\nu} |\nu\rangle\langle\nu| = I \quad \text{(one-particle sys)}$$

$$\psi(\ldots\vec{r}_j\ldots\vec{r}_K\ldots) = \psi(\ldots\vec{r}_K\ldots\vec{r}_j) \quad \text{(bosons)}$$

$$\ldots\ldots = - \ldots \quad \text{(Fermions)}$$

Second quantization

(wave → particle)

① why we do this?

because now we focus on the many body system. we consider the sys states by considering the occupation of different States.

For Bosons

$$a_i | N_1, \ldots, N_i, \ldots \rangle = \sqrt{N_i} \, | N_1 \ldots (N_i - 1), \ldots \rangle$$

$$a_i^\dagger | N_1 \ldots N_i \ldots \rangle = \sqrt{N_i + 1} \, | N_1 \ldots (N_i + 1) \ldots \rangle$$

Commutation:

$$[a_i, a_j] = 0 = [a_i^\dagger, a_j^\dagger] \qquad [a_i, a_j^\dagger] = \delta_{ij}$$

Fermions

$$c_j | N_1, \ldots N_j = 0, \ldots \rangle = 0$$

$$c_j | N_1 \ldots N_j = 1, \ldots \rangle = (-1)^{(N_1 + \ldots + N_{j-1})} | N_1 \ldots N_j = 0, \ldots \rangle$$

$$c_j^\dagger |n_1, n_2 \ldots n_j=0, \ldots\rangle = (-1)^{(n_1+\ldots+n_j-1)} |n_1 \ldots n_j=1, \ldots\rangle$$

$$c_j^\dagger |n_1 \ldots n_j=1 \ldots\rangle = 0$$

commutation:

Same with Bosons.

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proof $|1_1, 1_2\rangle = (\psi_1\psi_2 + \psi_2\psi_1)/\sqrt{2}$.

$$b_1^\dagger |1_1, 1_2\rangle = \frac{1}{\sqrt{2}}(b_1^\dagger \psi_1\psi_2 + b_1^\dagger \psi_2\psi_1)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}\psi_1 \otimes_+ \psi_1\psi_2 + \frac{1}{\sqrt{3}}\psi_1 \otimes_+ \psi_2\psi_1\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}(\psi_1\psi_1\psi_2 + \psi_1\psi_1\psi_2 + \psi_1\psi_2\psi_1) + \frac{1}{\sqrt{3}}(\psi_1\psi_2\psi_1 + \psi_2\psi_1\psi_1 + \psi_2\psi_1\psi_1)\right)$$

$$= \sqrt{2}\, |2_1, 1_2\rangle$$

$$C_1^\dagger |1_1, 1_2\rangle = \frac{1}{\sqrt{2}}\left(C_1^\dagger \psi_1 \psi_2 - C_1^\dagger \psi_2 \psi_1 \right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}} \psi_1 \otimes_- \psi_1 \psi_2 - \frac{1}{\sqrt{3}} \otimes_- \psi_2 \psi_1 \right)$$