WIP: Work in Progress Title

Ludwig-Maximilians-University Munich
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Abstract

My abstract

Zusammenfassung

Meine Zusammenfassung

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Here is my introduction

Natural units and Minkowski metric

Chapter 1

Theory

This chapter starts with an outline of the basic principles and concepts of the Standard Model of Particle Physics (SM), the theoretical framework describing nature on the level of elementary particles. This is followed by an introduction to supersymmetry, a promising class of theories aiming to solve some of the shortcomings of the SM.

By no means intended to be a full description, this chapter merely tries to highlight the important relations and consequences of the SM and supersymmetry. The mathematical description of this chapter largely follows [1, 2] for the SM and [3] for supersymmetry.

1.1 The Standard Model of Particle Physics

By the end of the 1920s, quantum mechanics and general relativity had been relatively well established and the consensus among physicists was that matter was made of nuclear atoms consisting of electrons and protons. During the 1930s, a multitude of new experimental discoveries and theoretical puzzles excited physicists in three main fields of research: nuclear physics, cosmic rays and relativistic quantum mechanics. The following years and decades saw particle physics emerge as a result of these currents ultimately flowing together.

Since these early times of particle physics research, physicists have made extraordinary progress in describing nature at the subatomic scale. Today, a century later, the resulting theoretical framework, the Standard Model of Particle Physics, is the most fundamental theory of nature to date. It provides an extremely precise description of the interactions of elementary particles and—using the Large Electron Positron collider (LEP)—has been tested and verified to an unprecedented level of accuracy up to the electroweak (EWK) scale. Given the unprecedented success of SM, it is not surprising that its history is paved with numerous awards for both experimental and theoretical work. In 1964, the Nobel prize was awarded to Feynman, Schwinger and Tomonoga for their fundamental work in quantum electrodynamics (QED). This quantum field theory allows to precisely calculate fundamental processes as e.g. the anomalous magnetic moment of the electron to a relative experimental uncertainty of 2.3×10^{-10} [4]. In 1979, Glashow, Weinberg and Salam were awarded with the Nobel prize for their work towards electroweak unification. The most prominent recent progress is undoubtedly the discovery of the Higgs boson, not only resulting in the Nobel prize being awarded to Englert and Higgs,

Table 1.1: Names, electric charges and masses (rounded to three significant digits if known to that precision) of all observed fermions in the SM [5].

	generation	particle	electric charge [e]	mass
leptons	1	electron (e)	-1	511 keV
		electron neutrino (ν_e)	0	$<$ 2 eV
	2	muon (μ)	-1	106 MeV
		muon neutrino (ν_μ)	0	$<$ 0.19 MeV
	3	tau (τ)	-1	1.78 GeV
		tau neutrino (ν_τ)	0	$<$ 18.2 MeV
quarks	1	up (u)	$\frac{2}{3}$	2.3 MeV
		down (d)	$-\frac{1}{3}$	4.8 MeV
	2	charm (c)	$\frac{2}{3}$	1.28 GeV
		strange (s)	$-\frac{1}{3}$	95 MeV
	3	top (t)	$\frac{2}{3}$	173 GeV
		bottom (b)	$-\frac{1}{3}$	4.18 GeV

but also completing the SM, roughly 50 years after the existence of the Higgs boson had been theorised.

Couplings and masses are measured from experiment

1.1.1 Particle content of the SM

Neutrino masses not in SM!

The SM successfully describes ordinary matter as well as their interactions, namely the electromagnetic, weak and strong interactions. Gravity is the only fundamental force not described within the SM. The particles in the SM are classified into two main categories, depending on their spin. Particles with half-integer spin follow Fermi-Dirac statistics and are called fermions. As they are subject to the Pauli exclusion principle, they make up ordinary matter. Particles with integer spin follow Bose-Einstein statistics and mediate the fundamental interactions between fermions.

Need ref

Fermions are further divided into leptons and quarks, which each come in three generations with increasing masses[†]. The three electrically charged leptons are each associated with a corresponding neutral neutrino (more on this *association* in chapter). While the SM assumes massless neutrinos, the observation of neutrino oscillations [6] implies the existence of at least two massive neutrinos. By extending the SM to allow non-vanishing neutrino masses, neutrino oscillations can be introduced through lepton generation mixing, described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [7]. Apart from an electric charge, the six quarks also carry a colour charge. There are three types of colour charge: *red*, *green* and *blue* as well as their respective anti-colours. The mixing in the quark sector through the weak interaction can be described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [8, 9]. Finally, each fermion comes with its own anti-particle with same mass and spin, but inverted charge-like quantum numbers[§]. All fermions in the SM are listed in table 1.1.

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† Neutrinos might not exist in a normal mass hierarchy but could also have an inverted mass hierarchy.

§ The exact nature of anti-neutrinos is still an open question and ties into whether or not the neutrino mass matrix contains non-vanishing Majorana mass terms.

Table 1.2: Names, electric charges and masses (rounded to three significant digits if known to that precision) of all observed bosons in the SM [5].

particle	spin	electric charge [e]	mass
photon (γ)	1	0	0
gluon (g)	1	0	0
$W^{\pm}$	1	±1	80.4 GeV
Z^0	1	0	91.2 GeV
Higgs boson (H)	0	0	125 GeV

The fundamental forces described by the SM are propagated by bosons with spin $1\hbar$. The photon γ couples to electrically charged particles and mediates the electromagnetic interaction. As the photon is massless, the electromagnetic force has infinite range. The strong force is mediated by gluons carrying one unit of colour and one unit of anti-colour. Due to colour-confinement, colour charged particles like quarks and gluons cannot exist as free particles and instead will always form colour-neutral bound states. Although nine gluon states would theoretically be possible, only eight of them are realised in nature: the colour-singlet state $\frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle)$ would be colour-neutral result in long-range strong interactions, which have not been observed. Finally, the weak force is mediated by a total of three bosons, two charged W-bosons W^+ and W^-, and a neutral Z-boson. The mediators of the weak force are massive, resulting in a finitely ranged interaction. The $W^{\pm}$ and Z bosons gain their masses through the Higgs mechanism (discussed in chapter), resulting in a massive spin-0 boson, called the Higgs boson. All bosons known to the SM are listed in table 1.2.

Might want to explain this later once I introduced the gauge groups?

1.1.2 The SM as a gauge theory

Formally, the SM is a collection of a special type of quantum field theories, called gauge theories. Quantum field theory (QFT) is the application of quantum mechanics to dynamical systems of fields, just as quantum mechanics is the quantisation of dynamical systems of particles. QFT provides a uniform description of quantum mechanical particles and classical fields, while including special relativity.

In classical mechanics, the fundamental quantity is the action S, which is the time integral of the Lagrangian L, a functional characterising the state of a system of particles in terms of generalised coordinates $q_1, \ldots, q_n$. In field theory, the Lagrangian can be written as spatial integral of a Lagrangian density $\mathcal{L}(\phi_i, \partial_\mu\phi_i)$, that is a function of one or more fields ϕ_i and their spacetime derivates $\partial_\mu\phi_i$. For the action, this yields

$$S = \int L\,\mathrm{d}t = \int \mathcal{L}\left(\phi_i, \partial_\mu\phi_i\right)\mathrm{d}^4x. \tag{1.1}$$

In the following, the Lagrangian density $\mathcal{L}$ will simply be referred to as the *Lagrangian*.

Using the principle of least action $\delta S = 0$, the equation of motions for each field are given by the Euler-Lagrange-equation,

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi_i\right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0. \tag{1.2}$$

As opposed to the Hamiltonian formalism, the Lagrange formulation of field theory is especially well suited in this context, as it exhibits explicit Lorentz-invariance. This is a direct consequence of the principle of least action, since boosted extrema in the action will still be extrema for Lorentz-invariant Lagrangians.

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Symmetries are of central importance in the SM. As Emmy Noether has famously shown in 1918 [10] for classical mechanics, every continuous symmetry of the action has a corresponding conservation law. In the context of classical field theory, each generator of a continuous internal or spacetime symmetry transformation leads to a conserved current, and thus to a conserved charge. In QFTs, quantum versions of Noether's theorem, called Ward–Takahashi identities [11, 12] for Abelian theories and Slavnov–Taylor identities [13–15] for non-Abelian theories relate the conservation of quantum currents and charge-like quantum numbers to continuous global symmetries of the Lagrangian.

Explicitly derive the Euler-Lagrange equations? Cf. Peskins Ch.2.2.

From a theoretical point of view, the SM can be described by a non-Abelian Yang-Mills type gauge theory based on the symmetry group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,$$

where $U(n)$ ($SU(n)$) describes (special) unitary groups, i.e. the Lie groups of $n \times n$ unitary matrices (with determinant 1, if special). $SU(3)_C$ generates quantum chromodynamics (QCD), i.e. the interaction of particles with colour charge through exchange of gluons, and $SU(2)_L \otimes U(1)_Y$ generates the electroweak interaction. Here, the subscript Y represents the weak hypercharge, while the L indicates that $SU(2)_L$ only couples to left-handed particles (right-handed antiparticles).

Check correctness of formulation

Gauge principle

The gauge principle is fundamental to the SM and dictates that the existence of gauge fields is directly related to symmetries under local gauge transformations. QED, being the simplest gauge theory, can be taken to illustrate this important principle. The free Dirac Lagrangian for a single, non-interacting fermion with mass m is given by

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi} \left(i\gamma^\mu \partial_\mu - m \right) \psi, \tag{1.3}$$

where ψ is a four-component complex spinor field, $\bar{\psi} = \psi^\dagger \gamma^0$, and γ^μ with $\mu = 0, 1, 2, 3$ are the Dirac matrices with the usual anticommutation relations generating a matrix representation of the Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} \mathbb{1}_4. \tag{1.4}$$

It is worth noting that the free Dirac Lagrangian is invariant under a global $U(1)$ transformation

$$\psi \to e^{i\theta} \psi, \tag{1.5}$$

where the phase θ is spacetime independent and real. In order to produce the physics of electromagnetism, the free Dirac Lagrangian however has to be invariant under *local* $U(1)$ phase transformations, which is not the case, as the transformed Lagrangian picks up an additional term from the spacetime derivative of the phase $\partial_\mu \theta(x)$.

In order for the Dirac Lagrangian to become invariant under a local gauge transformation, a new vector field $A_\mu(x)$ has to be introduced and the partial derivative has to be replaced with the covariant derivative[†]

$$\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu, \tag{1.6}$$

where e is the coupling of the fermion field to the gauge field A_μ and can be identified with the elementary charge. This leads to a Lagrangian that is invariant under the transformations

$$\psi \to e^{i\theta(x)}\psi, \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x). \tag{1.7}$$

The modified Lagrangian now includes a term for interactions between the gauge field and the fermion field

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{int}} \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - \left(e\bar{\psi}\gamma^\mu\psi\right)A_\mu, \end{aligned} \tag{1.8}$$

and is indeed invariant under a local phase transformation. Yet, it still cannot be complete as it is missing a term describing the kinematics of the free gauge field A_μ. For a vector field, the kinetic term is described by the Proca Lagrangian

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu, \tag{1.9}$$

where $F^{\mu\nu} \equiv (\partial^\mu A^\nu - \partial^\nu A^\mu)$ is the field strength tensor that is invariant under the transformation in eq. (1.7). Since $A^\nu A_\nu$ is not invariant under the same transformation, the only way to keep the full Lagrangian invariant under a local phase transformation is by requiring $m_A = 0$, i.e. the introduced gauge field A_μ has to be massless, giving the Maxwell Lagrangian (ultimately generating the Maxwell equations)

$$\mathcal{L}_{\text{Maxwell}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{1.10}$$

This finally yields the full Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{QED}} &= \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}} \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu\right)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \left(e\bar{\psi}\gamma^\mu\psi\right)A_\mu \end{aligned} \tag{1.11}$$

which can be identified to be the full Lagrangian of QED. The introduced gauge field A_μ is therefore nothing else but the electromagnetic potential with its associated massless particle, the photon. Thus, by applying the gauge principle on the free Dirac Lagrangian, i.e. forcing a global phase invariance to hold locally, a new massless gauge field including interaction terms

[†] The prescription of achieving local gauge invariance by replacing ∂_μ with D_μ is called *minimal coupling*.

with the existing fields in the Lagrangian has to be introduced. In the case of the free Dirac Lagrangian, local gauge invariance produces all of quantum electrodynamics.

As Yang and Mills have shown in 1954 [16], requiring a global phase invariance to hold locally is perfectly possible in the case of any continuous symmetry group. Considering a general non-Abelian symmetry group G, represented by a set of $n \times n$ unitary matrices $U(\alpha^1, \ldots, \alpha^N)$, parametrised by N real parameters $\alpha^1, \ldots, \alpha^N$, then a gauge-invariant Lagrangian can be constructed with a similar prescription [1] as previously in the case of $U(1)$.

A total of n fermion fields with mass m are needed, arranged in an n-dimensional multiplet $\Psi = (\psi_1, \ldots, \psi_n)^T$. The free Lagrangian

$$\mathcal{L}_{\text{free}} = \bar{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi, \tag{1.12}$$

is invariant under a global phase transformation

$$\Psi(x) \to U(\alpha^1, \ldots, \alpha^N)\Psi(x), \tag{1.13}$$

Each element in the set of transformations U can be written in terms of the group generators T^a

$$U(\alpha^1, \ldots, \alpha^N) = e^{i\alpha^a T^a}, \tag{1.14}$$

where the group indices $a = 1, \ldots, N$ are to be summed over. The group generators T^a satisfy the commutation relations

$$[T^a, T^b] = if^{abc}T^c, \tag{1.15}$$

with f^{abc} the so-called structure constants quantifying the lack of commutativity between the generators. By convention, the basis for the generators T^a is typically chosen such that f^{abc} is completely anti-symmetric.

In order to make the Lagrangian invariant under local phase transformations, i.e. under transformations with a set of spacetime-dependent real parameters $\alpha^a(x)$ a vector field $\boldsymbol{W}_\mu$ together with a coupling constant g have to be introduced through the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu - ig\boldsymbol{W}_\mu. \tag{1.16}$$

As D_μ acts on the n-dimensional multiplet Ψ, the introduced gauge field $\boldsymbol{W}_\mu$ has to be an $n \times n$ matrix and can thus be expanded in terms of the generators

$$\boldsymbol{W}_\mu(x) = T^a W^a_\mu(x), \tag{1.17}$$

explicitly illustrating, that a total of N gauge fields W^a_μ are introduced through the covariant derivative. Similar to QED above, the covariant derivative also introduces an interaction term of the form

$$\mathcal{L}_{\text{int}} = g\bar{\Psi}\gamma^\mu \boldsymbol{W}_\mu \Psi, \tag{1.18}$$

in the Lagrangian in eq. (1.12), coupling the gauge fields W^a_μ to the fermion fields. For infinitesimal $\alpha^a(x)$, the gauge fields gauge transform according to

$$W^a_\mu \to W^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc}W^b_\mu \alpha^c, \tag{1.19}$$

where the term with α^a looks familiar from the $U(1)$ example and corresponds to the Abelian case, while the term with f^{abc} introduces the non-Abelian structure into the theory. The non-Abelian structure is again clearly visible when introducing a kinetic term for the gauge fields into the Lagrangian

$$\mathcal{L}_{\mathrm{W}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a}, \tag{1.20}$$

with the field-strength tensor now $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu$. As was already the case for QED, the above Lagrangian contains Abelian terms quadratic in W, describing the propagation of the free gauge fields. This time, the Lagrangian however also contains non-Abelian terms cubic and quartic in W, leading to self-interaction of the gauge fields.

Quantum chromodynamics

Quantum chromodynamics (QCD), the gauge theory describing the strong interaction between quarks and gluons in the SM, is an example for a non-Abelian Yang-Mills theory. QCD is based on the gauge group $SU(3)_C$, with the subscript C indicating that the quantum number associated with the symmetry group is the *colour*. Each quark is described by a triplet of fermion fields $q = (q_r, q_g, q_b)^T$, where the subscripts refer to the three different colours. The symmetry group $SU(3)$ has a total of $n^2 - 1 = 8$ generators, usually expressed in terms of the Gell-Mann matrices λ^a. The covariant derivative introducing the gauge fields G^a_μ acting on the quark triplets is then

$$D_\mu = \partial_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu, \tag{1.21}$$

with g_s the coupling constant of the strong interaction, that is typically written as $\alpha_s = g_s^2/(4\pi)$ in analogy to the fine-structure constant in QED. Gauge invariance thus introduces a total of $N = 8$ gauge fields that can be identified with the eight gluons, leading to the full Lagrangian of QCD

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(i\gamma^\mu \partial_\mu - m_q) q - \sum_q -g_s \bar{q} \gamma^\mu \frac{\lambda^a}{2} q G^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}, \tag{1.22}$$

where $q = u, d, s, c, b, t$ and $G^a_{\mu\nu}$ are the gluon field strengths given by

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu. \tag{1.23}$$

As expected from the previous section, $\mathcal{L}_{\mathrm{QCD}}$ contains terms that are cubic and quartic in the gluon fields, resulting in gluon self-interaction in the theory. The gluon self-interactions lead to a number of phenomena unknown to Abelian theories, rendering the kinematics of QCD highly non-trivial.

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In QCD, a similar effect to the electric charge screening in QED happens through quark-antiquark pairs, resulting in a screening of the colour charge. However, the existence of gluon loops in the gluon propagator due to gluon self-interaction creates an opposing *antiscreening* effect of colour charges. At short distances or large momentum scales, colour-charged particles essentially become free particles, a phenomenon that is called *asymptotic freedom*. In this regime, where α_s is sufficiently small, QCD processes can be calculated using perturbation theory. At large distances or small moment scales however, α_s becomes large and gluons interact very strongly with colour-charged particles, meaning that no free gluons or quarks can exist. This phenomenon is called *confinement* and implies that free quarks and gluons will be subject to *hadronisation*, i.e. form colourless bound states by combining with other quarks or gluons (that can be created from the vacuum). In a particle detector, hadronisation manifests itself as collimated showers of particles, called *jets*.

Add QCD vertices

At momentum scales where the strong coupling α_s becomes large ($\alpha_s \approx \mathcal{O}(1)$), QCD processes can no longer be calculated using perturbation theory and instead lattice QCD [17, 18] is used.

Electroweak interaction

add gluon loops

During the 1960s, Glashow, Weinberg and Salam [19–21] developed a unified theory of the electromagnetic and weak interactions, based on the $SU(2)_\mathrm{L} \otimes U(1)_Y$ symmetry group. Known already experimentally from the Wu experiment'[22] in 1956, weak interaction violates parity, i.e. the symmetry transformations have to act differently on the left-handed and right-handed fermion fields. The left- and right-handed components of a fermion field can be projected out using

$$\psi_\mathrm{L} = \frac{1-\gamma^5}{2}\psi, \qquad \psi_\mathrm{R} = \frac{1+\gamma^5}{2}\psi, \tag{1.24}$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. As the weak interaction only acts on left-handed fermions, they can be ordered as $SU(2)$ doublets

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_\mathrm{L}, \quad \begin{pmatrix}u \\ d\end{pmatrix}_\mathrm{L}, \qquad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_\mathrm{L}, \quad \begin{pmatrix}c \\ s\end{pmatrix}_\mathrm{L}, \qquad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_\mathrm{L}, \quad \begin{pmatrix}t \\ b\end{pmatrix}_\mathrm{L}. \tag{1.25}$$

The quantum number associated with $SU(2)$ symmetry transformations is called weak isospin I with third component I_3. Fermion doublets have $I = 1/2$, with the upper component having $I_3 = 1/2$ and the lower component $I_3 = -1/2$. Right-handed fermion fields have $I = 0$, i.e. are singlet states in weak isospin space

$$e_\mathrm{R}, u_\mathrm{R}, d_\mathrm{R}, \qquad \mu_\mathrm{R}, c_\mathrm{R}, s_\mathrm{R}, \qquad \tau_\mathrm{R}, t_\mathrm{R}, b_\mathrm{R}, \tag{1.26}$$

and thus do not couple to the weak interaction. In the electroweak theory, neutrinos are assumed to be strictly massless, therefore no right-handed neutrino singlets exist.

The fermion doublets can be written in a free Lagrangian similar to eqs. (1.3) and (1.12)

$$\mathcal{L} = \bar{\psi}_\mathrm{L} i\gamma^\mu \partial_\mu \psi_\mathrm{L}, \tag{1.27}$$

with one crucial difference—the omission of the fermion masses. As $\bar{\psi}\psi = \bar{\psi}_\mathrm{L}\psi_\mathrm{R} + \bar{\psi}_\mathrm{R}\psi_\mathrm{L}$, mass terms would mix left- and right-handed terms and break gauge invariance. Section 1.1.2 will illustrate how fermion masses will instead be generated in the electroweak theory. For left-handed fermion fields, local $SU(2)_\mathrm{L}$ transformations can be written as

$$\psi_\mathrm{L} \to \exp\left(ig_2\alpha^a \frac{\sigma^a}{2}\right)\psi_\mathrm{L}, \tag{1.28}$$

where g_2 is the coupling constant, α^a with $a = 1, 2, 3$ are real parameters and the Pauli matrices σ^a are the generators of $SU(2)_\mathrm{L}$. By introducing the covariant derivative $D_\mu = \partial_\mu + ig_2\frac{\sigma^a}{2}W^a_\mu$ and including the usual kinetic term for the gauge fields, the Lagrangian becomes invariant under $SU(2)_\mathrm{L}$ transformations and reads

$$\mathcal{L} = \bar{\psi}_\mathrm{L} i\gamma^\mu D_\mu \psi_\mathrm{L} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a}, \tag{1.29}$$

with the gauge field strength tensors $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu$ where ϵ^{abc} are the structure constants. As previously in the case of QCD, the non-Abelian structure of the symmetry group causes self-interactions of the gauge fields.

In order to include electromagnetic interactions, the weak isospin group is extended with the $U(1)_Y$, corresponding to the multiplication of a phase factor $e^{i\alpha\frac{Y}{2}}$ to each of the preceding doublets and singlets. Here, Y is the weak hypercharge as given by the Gell-Mann–Nishijima relation [23–25]

$$Q = I_3 + \frac{Y}{2}, \tag{1.30}$$

with Q the electric charge. The electromagnetic group $U(1)_\mathrm{em}$ as a subgroup of the combined electroweak gauge group.

By modifying the covariant derivative to include a $U(1)_Y$ gauge field and ensuring that $U(1)_Y$ acts the same on left- and on right-handed fermions it becomes $D_\mu = \partial_\mu + ig_2 \frac{\sigma^a}{2} W^a_\mu + ig_1 \frac{Y}{2} B_\mu$ for left-handed fermions and $D_\mu = \partial_\mu + ig_1 \frac{Y}{2} B_\mu$ for right-handed fermions. Then the full electroweak Lagrangian reads

$$\begin{aligned} \mathcal{L}_\mathrm{electroweak} = &\sum_j^6 \bar{\psi}^j_\mathrm{L} i\gamma^\mu \left(\partial_\mu - ig_2 \frac{\sigma^a}{2} W^a_\mu + ig_1 \frac{Y}{2} B_\mu\right) \psi^j_\mathrm{L} \\ &+ \sum_j^9 \bar{\psi}^j_\mathrm{R} i\gamma^\mu \left(\partial_\mu + ig_1 \frac{Y}{2} B_\mu\right) \psi^j_\mathrm{R} \end{aligned} \tag{1.31}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

Spontaneous symmetry breaking

In the electroweak theory a total of three vector fields W^a_μ and one vector field B_μ are associated with the gauge groups $SU(2)_\mathrm{L}$ and $U(1)_Y$, respectively. As has been shown explicitly through the example of QED in section 1.1.2, the gauge fields need to be massless for the resulting Lagrangian to be gauge invariant under the respective symmetry group. In addition, the electroweak symmetry group does not allow for fermion masses. Both gauge bosons of the weak interaction and the fermion are however manifestly massive, hence the electroweak symmetry has to be broken in the SM.

This spontaneous symmetry breaking is achieved through the Brout-Englert-Higgs mechanism [26–28]. In the SM, an isospin doublet of complex scalar fields, called Higgs doublet, is introduced

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}. \tag{1.32}$$

The Higgs doublet has hypercharge $Y = 1$, hence according to eq. (1.30), ϕ^+ has electric charge $+1$ while ϕ^0 is electrically neutral. With the covariant derivative introduced in section 1.1.2, the

Higgs doublet gets an associated part in the SM Lagrangian reading

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \tag{1.33}$$

where $V(\Phi)$ is a gauge invariant potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2. \tag{1.34}$$

For positive and real parameters μ^2 and λ, this potential has the form of a *Mexican hat* and an infinite number of minima for field configurations with $\Phi^\dagger \Phi = 2\mu^2/\lambda$. In the vacuum, i.e. in the ground state of the theory with minimal potential energy of the field, one of these minima is chosen such that the Higgs receives a vacuum expectation value (VEV)

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{with} \quad v = \frac{2\mu}{\sqrt{\lambda}}. \tag{1.35}$$

This is neither invariant under a $SU(2)_{\mathrm{L}}$ transformation of the form $U = \exp(i\alpha^a \frac{\sigma^a}{2})$, nor under a $U(1)_Y$ transformation of the form $\exp(i\alpha \frac{Y}{2})$, therefore the electroweak gauge symmetry is spontaneously broken; the Lagrangian has a symmetry that the vacuum does not have. It is worth noting that the $U(1)_{\mathrm{em}}$ gauge symmetry is not broken as the VEV of ϕ^+ vanishes and ϕ^0 is invariant under $U(1)_{\mathrm{em}}$.

The Higgs doublet can be expressed as excitations around the ground state

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + H(x) + i\chi(x) \end{pmatrix}, \tag{1.36}$$

where H, χ, ϕ_1 and ϕ_2 are real scalar fields with vanishing VEV. The Higgs potential can then be written as

$$V = \mu^2 H^2 + \frac{\mu^2}{v} H(H^2 + \chi^2 + \phi_1^2 + \phi_2^2) + \frac{\mu^2}{4v^2}(H^2 + \chi^2 + \phi_1^2 + \phi_2^2), \tag{1.37}$$

where only H gets a mass term, thus describing an electrically neutral scalar particle with mass $m_H = \sqrt{2}\mu$. The remaining scalar fields remain massless, in accordance with the Nambu-Goldstone theorem [29, 30], stating that every spontaneously broken continuous symmetry generates a massless Goldstone boson. These bosons are unphysical and can be gauged away through a $SU(2)_{\mathrm{L}}$ transformation, such that the expansion around the vacuum from eq. (1.36) involves only the physical scalar $H(x)$

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}. \tag{1.38}$$

The gauge transformation bringing eq. (1.36) into the above form is called the *unitary gauge*. In this gauge, the Higgs potential from eq. (1.34) has the form

$$V = \frac{m_H^2}{2} H^2 + \frac{m_H^2}{2v} H^3 + \frac{m_H^2}{8v^2} H^4, \tag{1.39}$$

containing cubic and quartic self-interactions of the Higgs field proportional to m_H^2. Inserting the excitation around the vacuum state in the kinetic term of the $\mathcal{L}_\mathrm{H}$ yields mass terms for the vector bosons

$$\mathcal{L}_\mathrm{H} \propto \frac{v^2}{8} g_2^2 \left(W_\mu^1 W^{1,\mu} + W_\mu^2 W^{2,\mu} \right) + \frac{v^2}{8} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix}, \tag{1.40}$$

Instead of expressing the Lagrangian in terms of the fields W_μ^a and B_μ that make the original gauge invariance manifest, it can also be written in terms of the *physical* fields that correspond to the physical $W^\pm$, Z and γ bosons in the electroweak theory

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2) && \text{with} \quad m_W = \frac{g_2}{2} v, \\ Z_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2 W_\mu^3 - g_1 B_\mu) && \text{with} \quad m_Z = \frac{\sqrt{g_1^2 + g_2^2}}{2} v, \\ Z_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1 W_\mu^3 + g_2 B_\mu) && \text{with} \quad m_A = 0. \end{aligned}$$

The change of basis from (W_μ^3, B_μ) to (Z_μ, A_μ) [2] can also be written as a basis rotation with the weak mixing angle θ_W

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \qquad \text{with } \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{m_W}{m_Z}. \tag{1.41}$$

In the SM, not only the $W^\pm$ and Z bosons but also fermions gain their masses through spontaneous breaking of the electroweak gauge symmetry. Fermion fields gain masses through gauge-invariant Yukawa interactions with the Higgs field. For one fermion generation, the respective Yukawa terms in the Lagrangian are

$$\mathcal{L}_\text{Yukawa,gen} = -\lambda_\ell \bar{L}_\mathrm{L} \Phi \ell_\mathrm{R} - \lambda_d \bar{Q}_\mathrm{L} \Phi d_\mathrm{R} - \lambda_u \bar{Q}_\mathrm{L} \Phi^\dagger u_\mathrm{R} + \text{h.c.}, \tag{1.42}$$

where λ_f with $f = \ell, d, u$ are the dimensionless Yukawa couplings and $L_\mathrm{L} = (\nu_\mathrm{L}, \ell_\mathrm{L})^T$ and $Q_\mathrm{L} = (u_\mathrm{L}, d_\mathrm{L})^T$ are the left-handed lepton and quark doublets, respectively. The VEV of the Higgs field then gives rise to fermion mass terms in the Lagrangian, which, in the unitary gauge, reads for a single fermion generation

$$\mathcal{L}_\text{Yukawa,gen} = -\sum_{f=\ell,d,u} \left(m_f \bar{\psi}_f \psi_f + \frac{m_f}{v} H \bar{\psi}_f \psi_f \right) \qquad \text{with} \quad m_f = \frac{1}{\sqrt{2}} \lambda_f v. \tag{1.43}$$

When introducing all three fermion generations, additional Yukawa terms mixing fermions of different generations appear in the Lagrangian. The terms involving quark fields can be parametrised using the CKM matrix V_CKM [8, 9], quantifying the probability of . Since no right-handed neutrinos exist in the SM, no generation mixing in the lepton sector occurs. Neutrino oscillations have however been observed experimentally, thus at least one massive neutrino generation needs to exist. Their mixing can then be described with the Pontecorvo-

Maki-Nakagawa-Sakata (PMNS) matrix [7], allowing neutrinos to acquire mass e.g. through the see-saw mechanism [31].

1.1.3 Renormalisation and divergencies

1.2 Supersymmetry

Chapter 2

The LHC and ATLAS

Chapter 3

Data and Monte Carlo Simulation

3.1 Data

Chapter 4

Statistical data analysis

Chapter 5

Analysis

Chapter 6

Summary

Here be dragons/

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Symbols

Acronyms / Abbreviations

CKM	Cabibbo-Kobayashi-Maskawa, page 6
LEP	Large Electron Positron Collider, page 5
PMNS	Pontecorvo–Maki–Nakagawa–Sakata, page 6
QCD	Quantum Chromodynamics, page 7
QED	Quantum Electrodynamics, page 5
QFT	Quantum Field Theory, page 7
SM	Standard Model of Particle Physics, page 5
VEV	Vacuum expectation value, page 13

Appendix A

A.1 N-1 plots for cut-scan results

Appendix B

B.1 Scatter plots comparing truth and reco yields in the SRs

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Selbstständigkeitserklärung

Hiermit erkläre ich, die vorliegende Arbeit mit dem Titel

WIP: Work in Progress Title

WIP: Work in Progress Title

selbständig verfasst zu haben und keine anderen als die in der Arbeit angegebenen Quellen und Hilfsmittel benutzt zu haben.

Eric Schanet

München, den 01. Mai 2021