Chapter 3

Statistical data analysis

Statistical models are used in order to quantify the correspondence between theoretical predictions and the experimental observations in searches for Supersymmetry (SUSY). This chapter introduces the statistical concepts, methods and formulae used for statistical inference in this work. A frequentist approach to statistics is employed, interpreting probabilities as the frequencies of the outcomes of repeatable experiments that may either be real, based on computer simulations, or mathematical abstraction [7]. The ensuing description largely follows Refs. [145, 146].

3.1 The likelihood function

In measurements in high energy physics (HEP), a *statistical model* $f(\boldsymbol{x}|\boldsymbol{\phi})$ is a parametric family of probability density functions (pdfs) describing the probability of observing data $\boldsymbol{x}$ given a set of model parameters $\boldsymbol{\phi}^{\dagger}$. The latter typically describe parameters of the physical theory or unknown detector effects. The *likelihood function* $L(\boldsymbol{\phi})$ is then numerically equivalent to $f(\boldsymbol{x}|\boldsymbol{\phi})$ with $\boldsymbol{x}$ fixed. As opposed to the pdf $f(\boldsymbol{x})$ describing the value of f as a function of $\boldsymbol{x}$ given a fixed set of parameters $\boldsymbol{\phi}$, the likelihood refers to the value of f as a function of $\boldsymbol{\phi}$ given a fixed value of $\boldsymbol{x}^{\S}$.

Searches for beyond the Standard Model (BSM) physics are typically centred around the measurement of several disjoint binned distributions, called *channels c*, each associated with different event selection criteria (as opposed to different scattering processes) yielding observed event counts $\boldsymbol{n}$. In such counting experiments where each event is independently drawn from the same underlying distribution, each bin is fundamentally described by a Poisson term. The Poisson probability to observe n events with a expectation of ν events, $\mathrm{Pois}(n|\nu)$, is given by

$$\mathrm{Pois}(n|\nu) = \frac{\nu^n}{n!}e^{-\nu}. \tag{3.1}$$

[†]   Sets of parameters (as opposed to single parameters) will henceforth be written using bold face.
[§]   This important difference is why the likelihood is written here as $L(\boldsymbol{\phi})$ instead of the equally common $L(\boldsymbol{x}|\boldsymbol{\phi})$.

The expectation ν_{cb} in each channel c and bin b is a sum over the set of physics processes considered, called *samples* in the following,

$$\nu_{cb} = \sum_{s \in \text{samples}} \nu_{csb}(\boldsymbol{\eta}, \boldsymbol{\chi}), \tag{3.2}$$

where ν_{csb} is the expected sample rate in a given bin of a given channel. The sample-wise rates are in general a function of the the model parameters $\boldsymbol{\phi}$ that can either be *free parameters* $\boldsymbol{\eta}$ or *constrained parameters* $\boldsymbol{\chi}$. Possible modifications of the expected sample rates due to model parameters are considered to be either multiplicative or additive changes to the nominal estimate ν_{csb}^0 of the form

$$\nu_{csb}(\boldsymbol{\eta}, \boldsymbol{\chi}) = \left(\prod_i f_{csb}^i(\boldsymbol{\eta}, \boldsymbol{\chi}) \right) \left(\nu_{csb}^0 + \sum_j \Delta_{csb}^j(\boldsymbol{\eta}, \boldsymbol{\chi}) \right). \tag{3.3}$$

Free parameters directly determined by the Poisson terms for the data observations are called *normalisation factors*. The constrained parameters represent the systematic uncertainties considered in the model, which—in frequentist statistics—have fixed but unknown true values. The degree to which they cause a deviation of the expected event rates from the nominal event rates is limited through *constraint terms* $c_{\boldsymbol{\chi}}(a_{\boldsymbol{\chi}}|\boldsymbol{\chi})$ that can be viewed as *auxiliary measurements* with global observed data $\boldsymbol{a}$.

For a given observation $\boldsymbol{x} = (\boldsymbol{n}, \boldsymbol{a})$ of observed events $\boldsymbol{n}$ and auxiliary data $\boldsymbol{a}$, the likelihood then reads

$$L(\boldsymbol{\eta}, \boldsymbol{\chi}) = \prod_{c \in \text{channels}} \prod_{b \in \text{bins}_c} \text{Pois}(n_{cb}|\nu_{cb}(\boldsymbol{\eta}, \boldsymbol{\chi})) \prod_{\chi \in \boldsymbol{\chi}} c_{\chi}(a_{\chi}|\chi), \tag{3.4}$$

where, given a certain integrated luminosity, n_{cb} and ν_{cb} refer to the corresponding observed and expected rate of events, respectively [147]. Most of the systematic uncertainties are so-called *interpolation parameters* $\boldsymbol{\alpha}$ representing either normalisation uncertainties or correlated shape uncertainties. Their constraint terms $c_{\boldsymbol{\alpha}}(a_{\boldsymbol{\alpha}}|\boldsymbol{\alpha})$ are parametrised by a Gaussian with mean $a = 0|\alpha$ and variance $\sigma = 1$, with $\alpha = 0$ representing the nominal value. The *up* and *down* variations are then given by $\alpha = \pm 1$, thereby representing $\pm 1\sigma$ variations. The impact of any given value of the parameter on the event rates is subsequently evaluated through polynomial interpolation and exponential extrapolation, a method that avoids discontinuous first and second derivatives at $\alpha = 0$ and ensures positive values for the predicted event rates [148].

Sample rates derived directly from theory calculations (i.e. Monte Carlo (MC) simulation), are scaled to the integrated luminosity corresponding to the observed data. As discussed in section 2.1.2, the integrated luminosity is itself a measurement that is subject to uncertainties, requiring an additional constraint term in the likelihood. It is parametrised by a Gaussian with mean corresponding to the nominal integrated luminosity measurement and variance equal to the integrated luminosity measurement uncertainty. Uncertainties arising from the finite size of the MC datasets often used to derive estimated event rates are modelled by bin-wise scale factors γ_b. The constraint terms are Gaussian distributions with central value equal to unity and variances calculated from the individual uncertainties of the samples defined in the respective channel.

As the event rate in a given bin can depend on multiple parameters, and likewise, a single parameter can affect the expected event rate in multiple bins across various channels, correlations between the model parameters $\boldsymbol{\phi}$ can occur.

The above prescription for building binned likelihoods is called the HISTFACTORY template [148]. In this work, two independent implementations of the HISTFACTORY template are used. The implementation predominantly adopted in part II uses ROOFIT [149] and ROOSTATS [150] for model parameter estimation and hypothesis tests (using `Minuit` [151] and implemented within the ROOT framework [152, 153]), and HISTFITTER [154] as interface for steering theses processes and bookkeeping the results. The second implementation, mostly used in part III, makes use of `pyhf` [155, 156], a pure-`python` implementation of HISTFACTORY that is independent from the ROOT environment. The `pyhf` implementation of HISTFACTORY relies on NUMPY [157] and uses computational graph libraries like PYTORCH [158], TENSORFLOW [159] and JAX [160] to significantly speed up the parameter estimation process by leveraging the computational advantages of tensor algebra and automatic differentiation.

Apart from separating the model parameter set into free and constrained parameters $\boldsymbol{\phi} = (\boldsymbol{\eta}, \boldsymbol{\chi})$, a separate partition $\boldsymbol{\phi} = (\boldsymbol{\psi}, \boldsymbol{\theta})$ is frequently used in the context of hypothesis testing. Here, $\boldsymbol{\psi}$ are so-called *parameters of interests* of the model for which hypothesis tests are performed, and $\boldsymbol{\theta}$ are *nuisance parameters* that are not of immediate interest but need to be accounted for to correctly model the data. In the search presented in this work, the only Parameter of Interest (POI) is the *signal strength* parameter μ, representing the ratio of the signal process cross section to its reference cross section as expected from theory. The expected event rate ν_i in each bin i can then be parametrised through

$$\nu_b = \mu S_b + B_b, \tag{3.5}$$

where S_b and B_b are the bin-wise expected signal and background rates, respectively. Fixing $\mu = 0$ thus yields an expected event rate containing only Standard Model of Particle Physics (SM) processes, which is why this is also called a *background-only* configuration. Setting $\mu = 1$ represents a *signal-plus-background* description at nominal signal cross section. Scanning multiple values of μ allows to set limits on the visible cross sections of the signal models considered in the search.

3.2 Parameter estimation

Given a likelihood $L(\boldsymbol{\phi})$ for a fixed set of observations $\boldsymbol{x}$, a measurement can be understood as a parameter estimation. In general, an estimator $\hat{\phi}$ is a function of the observed data used to estimate the true value of the model parameter ϕ.

In particle physics, the most commonly used estimator is the maximum likelihood estimator (MLE). The MLEs for the model parameters $\hat{\boldsymbol{\phi}}$ are defined to be the parameter values that maximise $L(\boldsymbol{\phi})$, or equivalently, maximise $\ln L(\boldsymbol{\phi})$ and minimise $-\ln L(\boldsymbol{\phi})$. The logarithm of the likelihood is used for computational reasons, as it not only reduces the computational complexity by avoiding exponentials and products, but also avoids potential problems arising from finite floating point precision. As the logarithm is a monotonically increasing function, $\ln L(\boldsymbol{\phi})$ has maxima at the same parameter values as $L(\boldsymbol{\phi})$. The negative logarithm of the

likelihood is chosen in order to stick to the convention of optimisers in statistical packages typically minimising the result of a *loss function.*

The MLEs $\hat{\boldsymbol{\phi}}$ can thus be found by solving the system of equations

$$\frac{\partial \ln L}{\partial \phi_i} = 0, \tag{3.6}$$

where the index i runs over all parameters. Due to the complexity of the likelihood, a solution typically needs to be found numerically using minimisation algorithms. In the following, the parameter estimation is referred to as a *fit* of the model to data, and the maximum likelihood estimates of the parameters are consequently called *best-fit values.*

3.3 Statistical tests

In addition to estimating the values of model parameters, searches for SUSY are naturally interested in claiming discovery (or alternatively exclusion) of hypothesised signal models. In the frequentist approach, this can be formulated in terms of hypothesis tests, evaluating a *null hypothesis H_0* against an *alternative hypothesis H_1*, with the goal of rejecting the null hypothesis. For discovering a new signal process, H_0 is defined to describe only known SM processes and thus called *background-only* hypothesis. The alternative hypothesis H_1 is then the *signal-plus-background* hypothesis describing both SM background processes as well as the signal process considered. When excluding a signal model the signal-plus-background hypothesis takes over the role of H_0 and is tested against the background-only hypothesis.

The degree of agreement of observed data with a certain hypothesis H is quantified by calculating a *p*-value, representing the probability of finding data of greater or more extreme incompatibility under assumption of H. The hypothesis can then be considered as excluded if its observed *p*-value is below a specified threshold. It is common to convert the *p*-value into a *significance Z*, defined in such a way that a Gaussian distributed observable with measured value Z standard deviations above its mean gives a one-sided upper tail probability equal to p. This yields the expression

$$Z = \Phi^{-1}(1-p), \tag{3.7}$$

where Φ^{-1} is the quantile of the standard Gaussian. In HEP, discovery of a signal then conventionally requires a significance of at least $Z = 5$, while exclusion of a signal hypothesis at 95% confidence level requires a *p*-value of 0.05, i.e. $Z = 1.64$ [146].

The *p*-values are calculated using a *test statistic* that parameterises the compatibility between the hypothesis and data in a single value. At the LHC experiments, the test statistics used for hypothesis testing are based on the *profile likelihood ratio $\lambda(\mu)$*, defined to be

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})}, \tag{3.8}$$

where the conditional maximum likelihood estimators (CMLEs) $\hat{\hat{\boldsymbol{\theta}}}$ are the values of $\boldsymbol{\theta}$ that maximise the likelihood with μ fixed. The distribution of the profile likelihood ratio depends explicitly on μ, and implicitly on $\boldsymbol{x} = (\boldsymbol{n}, \boldsymbol{a})$, but is asymptotically (i.e. in the limit of a large

number of events) independent of the nuisance parameters $\boldsymbol{\theta}^\dagger$ in the case where the tested value of μ is the true value μ' [145]. The asymptotic independence from $\boldsymbol{\theta}$ follows from Wilks' theorem [161] and is one of the main motivations for using the profile likelihood ratio, as it avoids the problem of having to compute p-values for all possible values of $\boldsymbol{\theta}$. A generalisation to tested values of μ not corresponding to the true value μ' can be derived using Wald's theorem [162], allowing to obtain the distribution $f(\lambda(\mu)|\mu', \boldsymbol{\theta})$. The profile likelihood ratio takes values between 0 and 1, with $\lambda(\mu) = 1$ corresponding to cases where the tested value of μ is in good agreement with the observed data. In eq. (3.8), the nuisance parameters result in a broadening of the profile likelihood distribution, reflecting the loss of information about μ due to systematic uncertainties [146].

As the rate of signal processes considered in the following is in general non-negative, an estimator for μ should satisfy $\hat{\mu} \geq 0$. In order to avoid the formal complications of having a boundary at $\mu = 0$, it is convenient to consider an effective estimator $\hat{\mu}$ that is allowed to become negative, provided that the respective Poisson terms for $\mu S_b + B_b$ remain positive. By imposing a constraint equivalent to requiring $\mu \geq 0$ on the test statistic itself, it is possible to avoid the formal problems of having a boundary at $\mu = 0$. This leads to the alternative definition of the profile likelihood as

$$\tilde{\lambda}(\mu) = \begin{cases} \frac{L(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(\hat{\mu},\hat{\boldsymbol{\theta}})}, & \hat{\mu} \geq 0, \\ \frac{L(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(0,\hat{\hat{\boldsymbol{\theta}}}(0))}, & \hat{\mu} < 0, \end{cases} \tag{3.9}$$

where $\hat{\hat{\boldsymbol{\theta}}}(0)$ and $\hat{\hat{\boldsymbol{\theta}}}(\mu)$ are the CMLEs of $\boldsymbol{\theta}$ given a signal strength parameter of 0 and μ, respectively.

Discovery

For the important special case where $\mu = 0$ is tested in a model with $\mu \geq 0$, i.e. discovery of a non-negative signal (rejection of the background-only hypothesis), the profile likelihood $\tilde{\lambda}(\mu)$ is used to build the test statistic

$$q_0 = -2 \ln \tilde{\lambda}(0) = \begin{cases} -2 \ln \lambda(0), & \hat{\mu} \geq 0, \\ 0, & \hat{\mu} < 0. \end{cases} \tag{3.10}$$

This definition ensures that the background-only hypothesis is not rejected due to a downward fluctuation in data, causing $\hat{\mu} < 0$. In case more events are seen in data than expected based on the background-only hypothesis, eq. (3.10) produces increasingly large values of q_0, corresponding to an increasing incompatibility between data and the background-only hypothesis. The p-value quantifying the disagreement between the $\mu = 0$ hypothesis and data can then be computed using

$$p_0 = \int_{q_{0,\mathrm{obs}}}^{\infty} f(q_0|0) \, \mathrm{d}q, \tag{3.11}$$

with $q_{0,\mathrm{obs}}$ the observed value of the test statistic q_0 in data and $f(q_0|0)$ the pdf of q_0 under assumption of the background-only hypothesis. In the asymptotic limit [146] with a single POI,

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$\dagger$ Eliminated by choosing specific values of the nuisance parameters for a given $\boldsymbol{x}$ and μ, often referred to as *profiling*.

the test statistic q_0 can be written as

$$q_0 = \begin{cases} \hat{\mu}^2/\sigma^2, & \hat{\mu} \geq 0, \\ 0, & \hat{\mu} < 0, \end{cases} \tag{3.12}$$

where $\hat{\mu}$ has a Gaussian distribution with mean μ' and variance σ^2. In the case of $\mu' = 0$, the pdf of q_0 has the form of a half χ^2 distribution with one degree of freedom, and its cumulative distribution is $F(q_0|0) = \Phi(\sqrt{q_0})$ [145]. Using eq. (3.7), the p-value obtained with eq. (3.11) can be expressed with the significance Z_0 as

$$Z_0 = \sqrt{q_0}. \tag{3.13}$$

Exclusion and upper limits

If the background-only ($\mu = 0$) hypothesis cannot be rejected, the hypotheses can be switched around and instead the signal-plus-background ($\mu = 1$) hypothesis can be tested against a hypothesis where signal events are produced at a rate smaller than μ""
For excluding the signal-plus-background hypothesis and setting upper limits on the signal strength μ, the test statistic is defined as

$$\tilde{q}_\mu = \begin{cases} -2\ln\tilde{\lambda}(\mu), & \hat{\mu} \leq \mu, \\ 0, & \hat{\mu} > \mu \end{cases} = \begin{cases} -2\ln\frac{L(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(\hat{\mu},\hat{\boldsymbol{\theta}})}, & \hat{\mu} \geq 0, \\ -2\ln\frac{L(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(0,\hat{\hat{\boldsymbol{\theta}}}(0))}, & 0 \leq \hat{\mu} \leq \mu, \\ 0 & \hat{\mu} > \mu. \end{cases} \tag{3.14}$$

Setting $\tilde{q}_\mu = 0$ in the case where $\hat{\mu} > \mu$ ensures that an overfluctuation of data is not considered as evidence against the signal hypothesis. This is opposed to the definition of q_0, where an underfluctuation of data ($\hat{\mu} < \mu$) is not regarded to be evidence against the background-only hypothesis. The p-value, quantifying the level of agreement between data and the tested value of μ is then given by

$$p_\mu = \int_{\tilde{q}_{\mu,\text{obs}}}^{\infty} f(\tilde{q}_\mu|\mu)\,\mathrm{d}\tilde{q}_\mu, \tag{3.15}$$

where, as before, $\tilde{q}_{\mu,\text{obs}}$ is the observed value of the test statistic in data and $f(\tilde{q}_\mu|\mu)$ is the pdf of $\tilde{q}_\mu$ given the hypothesis μ. In the asymptotic limit [146], the test statistic $\tilde{q}_\mu$ can be written as

$$\tilde{q}_\mu = \begin{cases} \mu^2\sigma^2 - 2\mu\hat{\mu}/\sigma^2, & \hat{\mu} \geq 0, \\ (\mu - \hat{\mu})^2\sigma^2, & 0 \leq \hat{\mu} \leq \mu, \\ 0 & \hat{\mu} > \mu, \end{cases} \tag{3.16}$$

which yields for the significance Z_μ the expression

$$Z_\mu = \begin{cases} \sqrt{\tilde{q}_\mu}, & 0 < \tilde{q}_\mu \leq \mu^2/\sigma^2 \\ \frac{\tilde{q}_\mu + \mu^2/\sigma^2}{2\mu/\sigma}, & \tilde{q}_\mu > \mu^2/\sigma^2. \end{cases} \tag{3.17}$$
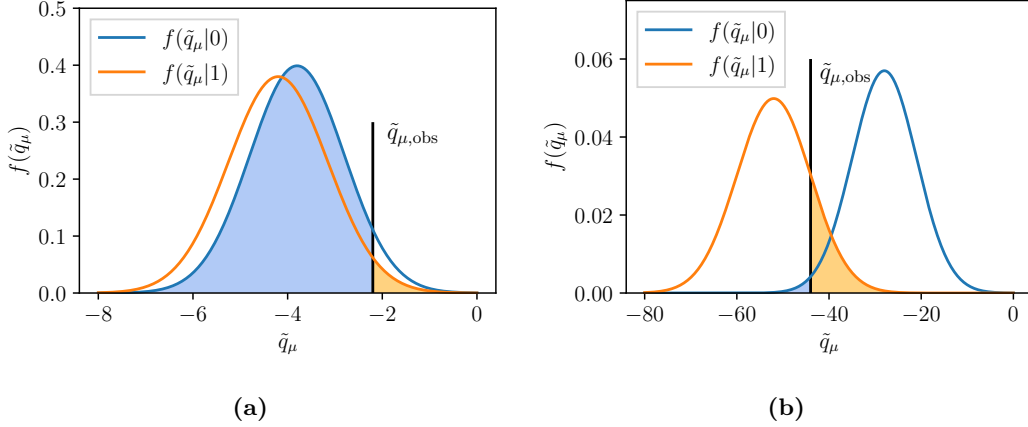
Figure 3.1: Distribution of the pdfs of the signal plus background (in orange) and background-only (in blue) models. The coloured areas represent the p_{s+b} and p_b values, respectively. Figure (a) shows a case where both pdfs are close together, while figure (a) shows a case where both a well separated. Figures created by the author but based on Ref. [163].

3.4   CL$_s$ approach

In the CL$_{s+b}$ method, a signal-plus-background model is excluded if $p_{s+b} < \alpha$, where α is defined by the desired confidence level, typically CL $= 1 - \alpha = 95\%$, and p_{s+b} can be calculated using the test statistic $\tilde{q}_\mu$ (with $\mu = 1$) introduced in eq. (3.14). If the experiment has very low sensitivity to a specific signal-plus-background model, e.g. because the production cross section is too low, the distribution of the test statistic of the signal-plus-background model will be very close to that of the background-only model. In case of an underfluctuation in data, the $\mu = 1$ model can then be falsely excluded, even though no sensitivity is expected. Figure 3.1 illustrates this with a simple example. In fact, the exclusion of models to which the experiment has no sensitivity has a probability of at least α [163].

This problem can be remedied by adopting the CL$_s$ method [164], altering the threshold for excluding a model in a way to avoid exclusion of models to which the experiment has very low sensitivity. The CL$_s$ value is defined as

$$\mathrm{CL}_s = \frac{p_{s+b}}{1 - p_b}, \tag{3.18}$$

where p_b is the p-value of the background-only hypothesis. If the distributions of the test statistics for the signal-plus-background and the background-only models are close to each other (as seen in fig. 3.1(a)) a small value of p_{s+b} due to an underfluctuation in data will entail a large value of p_b. Consequently, in the calculation of the CL$_s$ value, p_{s+b} will be penalised by $1 - p_b$ (that will be close to 0), resulting in CL$_s > p_{s+b}$, preventing the exclusion of the signal-plus-background model. Conversely, in the case where the two test statistics are well-separated (see fig. 3.1(b)) and $p_{s+b} < \alpha$, then p_b will also be small and thus CL$_s$ will be close to p_{s+b} obtained by the frequentist approach.

3.5   Asimov dataset

Searches for BSM physics are not only interested in the significance obtained using the dataset measured by the experiment, but also in the expected (or median) significance obtained for rejecting different values of μ. For example, for rejecting the $\mu = 1$ hypothesis, searches are interested in the expected significance obtained assuming data generated according to the $\mu = 0$ hypothesis.

The expected experimental sensitivity can be determined using an artificial dataset called the *Asimov dataset*, defined such that MLEs of all parameters determined using Asimov data correspond to the true parameter values. This is achieved by constructing a dataset where the number of events in each bin is exactly equal to the expected event rate in that bin. Using Asimov data, the Asimov likelihood L_{A} as well as the profile likelihood λ_{A} can be evaluated and thus a median significance can be determiend. Non-integer values for data are not an issue as factorial terms from the Poisson probabilities are canceled in the profile likelihood and can thus be dropped altogether.

3.6   Sensitivity estimation

When designing search regions for an analysis, it is necessary to achieve an optimal signal-to-background separation power. A significance metric is needed in order to quantify the separation power and have a metric to optimise for. In the following, the expected discovery significance introduced in Ref. [165] is used. As the full statistical model is in general not yet known when designing the search regions, appropriate assumptions have to be made. In a *cut-and-count* selection where only the total number of events after a selection are relevant (and not e.g. their distribution), the significance is determined by the total number of signal events S, the total number of background events B and the uncertainty on the expected number of background events ΔB. This can be modelled as a so-called *on/off problem*[†] [165, 166], where the cut-and-count experiment uses two bins, a signal region (SR) enriched in signal events, and a control region (CR) containing only background events. In the background-only hypothesis, the parameter $\tau = n_{\mathrm{CR}}/n_{\mathrm{SR}}$ then denotes the ratio between the event rate in the CR, n_{CR}, and the event rate in the SR, n_{SR}.

If τ is known, then the likelihood of this simple configuration can be written in terms of the expected background event rate

$$L(\mu, B) = \mathrm{Pois}(n_{\mathrm{SR}}|\mu S + B) \cdot \mathrm{Pois}(n_{\mathrm{CR}}|\tau B), \tag{3.19}$$

where μ is again the signal strength parameter. The relative background uncertainty can thus be treated as coming from a Poisson-distributed auxiliary measurement containing only background

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[†]   The *on/off* nomenclature originates from gamma ray astronomy where *on* refers to the telescope pointing on-source (measuring signal and background photons), while *off* refers to it pointing off-source (measuring only background photons). This problem is an exact analog to the HEP problem herein, where the off-source measurement typically corresponds to some sideband measurement.

(i.e. in the CR) with corresponding uncertainty $\sqrt{\tau B}$, leading to the approximation

$$\tau = \frac{B}{\Delta B^2}. \tag{3.20}$$

As n_{SR} and n_{CR} are each drawn from a Poisson probability with unknown means ν_{SR} and ν_{CR}, the background-only hypothesis corresponds exactly to the case where the ratio of Poisson means $\lambda = \nu_{\mathrm{CR}}/\nu_{\mathrm{SR}}$ is equal to τ [165]. The two Poisson terms can then be written as the product of a single Poisson term with mean $n_{\mathrm{tot}} = n_{\mathrm{SR}} + n_{\mathrm{CR}}$ and the binomial probability of picking n_{SR} events out of n_{tot} with probability $\rho = \nu_{\mathrm{SR}}/\nu_{\mathrm{tot}} = 1/(1 + \lambda)$. The likelihood can thus be written as

$$\begin{aligned} L(\mu, B) &= \mathrm{Pois}(n_{\mathrm{tot}}|\lambda_{\mathrm{tot}}) \cdot B(n_{\mathrm{SR}}|\rho, n_{\mathrm{tot}}) \\ &= \frac{e^{-\lambda_{\mathrm{tot}}}\lambda_{\mathrm{tot}}^{n_{\mathrm{tot}}}}{n_{\mathrm{tot}}!} \cdot \binom{n_{\mathrm{tot}}}{n_{\mathrm{SR}}}\rho_{\mathrm{tot}}^{\lambda}(1 - \rho)^{n_{\mathrm{tot}} - n_{\mathrm{SR}}}. \end{aligned} \tag{3.21}$$

Since the background-only hypothesis cannot only be expressed as $\mu = 0$ but also as $\nu_{\mathrm{SR}} = \nu_B$, $\lambda = \tau$, and especially also as $\rho = 1/(1 + \tau)$ [165], its p-value can be calculated using the well-know frequentist binomial test,

$$p_{\mathrm{B}} = \sum_{j=n_{\mathrm{SR}}}^{n_{\mathrm{tot}}} B(j|n_{\mathrm{tot}}, \rho). \tag{3.22}$$

The significance corresponding to p_{B} can be derived using eq. (3.7) and is computable in a numerically fast way using the incomplete beta function. The algorithm used for calculating Z_{B} in this work is implemented in the `RooStats::NumberCountingUtils` methods in ROOT.

Bibliography

[1] I. C. Brock and T. Schorner-Sadenius, *Physics at the terascale*. Wiley, Weinheim, 2011.
https://cds.cern.ch/record/1354959.

[2] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*.
Addison-Wesley, Reading, USA, 1995.
http://www.slac.stanford.edu/~mpeskin/QFT.html.

[3] S. P. Martin, "A Supersymmetry primer," `arXiv:hep-ph/9709356 [hep-ph]`. [Adv. Ser.
Direct. High Energy Phys.18,1(1998)].

[4] M. Bustamante, L. Cieri, and J. Ellis, "Beyond the Standard Model for Montaneros," in
5th CERN - Latin American School of High-Energy Physics. 11, 2009. `arXiv:0911.4409
[hep-ph]`.

[5] L. Brown, *The Birth of particle physics*. Cambridge University Press, Cambridge
Cambridgeshire New York, 1986.

[6] P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA Recommended Values of the
Fundamental Physical Constants: 2014," *Rev. Mod. Phys.* **88** no. 3, (2016) 035009,
`arXiv:1507.07956 [physics.atom-ph]`.

[7] P. D. Group, "Review of
Particle Physics," *Progress of Theoretical and Experimental Physics* **2020** no. 8, (08, 2020) ,
`https://academic.oup.com/ptep/article-pdf/2020/8/083C01/34673722/ptaa104.pdf`.
https://doi.org/10.1093/ptep/ptaa104. 083C01.

[8] **Super-Kamiokande** Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of
atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, `arXiv:hep-ex/9807003
[hep-ex]`.

[9] Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary
particles," *Prog. Theor. Phys.* **28** (1962) 870–880. [,34(1962)].

[10] N. Cabibbo, "Unitary symmetry and leptonic decays," *Phys. Rev. Lett.* **10** (Jun, 1963)
531–533. https://link.aps.org/doi/10.1103/PhysRevLett.10.531.

[11] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak
Interaction," *Prog. Theor. Phys.* **49** (1973) 652–657.

[12] E. Noether and M. A. Tavel, "Invariant variation problems," `arXiv:physics/0503066`.

[13] J. C. Ward, "An identity in quantum electrodynamics," *Phys. Rev.* **78** (Apr, 1950)
182–182. https://link.aps.org/doi/10.1103/PhysRev.78.182.

[14] Y. Takahashi, "On the generalized ward identity," *Il Nuovo Cimento (1955-1965)* **6** no. 2, (Aug, 1957) 371–375. https://doi.org/10.1007/BF02832514.

[15] G. 'tHooft, "Renormalization of massless yang-mills fields," *Nuclear Physics B* **33** no. 1, (1971) 173 – 199. http://www.sciencedirect.com/science/article/pii/0550321371903956.

[16] J. Taylor, "Ward identities and charge renormalization of the yang-mills field," *Nuclear Physics B* **33** no. 2, (1971) 436 – 444. http://www.sciencedirect.com/science/article/pii/0550321371902975.

[17] A. A. Slavnov, "Ward identities in gauge theories," *Theoretical and Mathematical Physics* **10** no. 2, (Feb, 1972) 99–104. https://doi.org/10.1007/BF01090719.

[18] C. N. Yang and R. L. Mills, "Conservation of isotopic spin and isotopic gauge invariance," *Phys. Rev.* **96** (Oct, 1954) 191–195. https://link.aps.org/doi/10.1103/PhysRev.96.191.

[19] K. G. Wilson, "Confinement of quarks," *Phys. Rev. D* **10** (Oct, 1974) 2445–2459. https://link.aps.org/doi/10.1103/PhysRevD.10.2445.

[20] T. DeGrand and C. DeTar, *Lattice Methods for Quantum Chromodynamics*. World Scientific, Singapore, 2006. https://cds.cern.ch/record/1055545.

[21] S. L. Glashow, "Partial-symmetries of weak interactions," *Nuclear Physics* **22** no. 4, (1961) 579 – 588. http://www.sciencedirect.com/science/article/pii/0029558261904692.

[22] S. Weinberg, "A model of leptons," *Phys. Rev. Lett.* **19** (Nov, 1967) 1264–1266. https://link.aps.org/doi/10.1103/PhysRevLett.19.1264.

[23] A. Salam and J. C. Ward, "Weak and electromagnetic interactions," *Il Nuovo Cimento (1955-1965)* **11** no. 4, (Feb, 1959) 568–577. https://doi.org/10.1007/BF02726525.

[24] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, "Experimental test of parity conservation in beta decay," *Phys. Rev.* **105** (Feb, 1957) 1413–1415. https://link.aps.org/doi/10.1103/PhysRev.105.1413.

[25] M. Gell-Mann, "The interpretation of the new particles as displaced charge multiplets," *Il Nuovo Cimento (1955-1965)* **4** no. 2, (Apr, 1956) 848–866. https://doi.org/10.1007/BF02748000.

[26] K. Nishijima, "Charge Independence Theory of V Particles*," *Progress of Theoretical Physics* **13** no. 3, (03, 1955) 285–304, https://academic.oup.com/ptp/article-pdf/13/3/285/5425869/13-3-285.pdf. https://doi.org/10.1143/PTP.13.285.

[27] T. Nakano and K. Nishijima, "Charge Independence for V-particles*," *Progress of Theoretical Physics* **10** no. 5, (11, 1953) 581–582, https://academic.oup.com/ptp/article-pdf/10/5/581/5364926/10-5-581.pdf. https://doi.org/10.1143/PTP.10.581.

[28] F. Englert and R. Brout, "Broken symmetry and the mass of gauge vector mesons," *Phys. Rev. Lett.* **13** (Aug, 1964) 321–323. https://link.aps.org/doi/10.1103/PhysRevLett.13.321.

[29] P. W. Higgs, "Broken symmetries and the masses of gauge bosons," *Phys. Rev. Lett.* **13** (Oct, 1964) 508–509. https://link.aps.org/doi/10.1103/PhysRevLett.13.508.

[30] P. W. Higgs, "Spontaneous symmetry breakdown without massless bosons," *Phys. Rev.* **145** (May, 1966) 1156–1163. https://link.aps.org/doi/10.1103/PhysRev.145.1156.

[31] Y. Nambu, "Quasiparticles and Gauge Invariance in the Theory of Superconductivity," *Phys. Rev.* **117** (1960) 648–663. [,132(1960)].

[32] J. Goldstone, "Field Theories with Superconductor Solutions," *Nuovo Cim.* **19** (1961) 154–164.

[33] V. Brdar, A. J. Helmboldt, S. Iwamoto, and K. Schmitz, "Type-I Seesaw as the Common Origin of Neutrino Mass, Baryon Asymmetry, and the Electroweak Scale," *Phys. Rev. D* **100** (2019) 075029, arXiv:1905.12634 [hep-ph].

[34] G. 't Hooft and M. Veltman, "Regularization and renormalization of gauge fields," *Nuclear Physics B* **44** no. 1, (1972) 189 – 213. http://www.sciencedirect.com/science/article/pii/0550321372902799.

[35] F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln," *Helv. Phys. Acta* **6** (1933) 110–127. https://cds.cern.ch/record/437297.

[36] V. C. Rubin and W. K. Ford, Jr., "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions," *Astrophys. J.* **159** (1970) 379–403.

[37] G. Bertone, D. Hooper, and J. Silk, "Particle dark matter: Evidence, candidates and constraints," *Phys. Rept.* **405** (2005) 279–390, arXiv:hep-ph/0404175.

[38] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, "A direct empirical proof of the existence of dark matter," *Astrophys. J.* **648** (2006) L109–L113, arXiv:astro-ph/0608407 [astro-ph].

[39] A. Taylor, S. Dye, T. J. Broadhurst, N. Benitez, and E. van Kampen, "Gravitational lens magnification and the mass of abell 1689," *Astrophys. J.* **501** (1998) 539, arXiv:astro-ph/9801158.

[40] C. Bennett *et al.*, "Four year COBE DMR cosmic microwave background observations: Maps and basic results," *Astrophys. J. Lett.* **464** (1996) L1–L4, arXiv:astro-ph/9601067.

[41] G. F. Smoot *et al.*, "Structure in the COBE Differential Microwave Radiometer First-Year Maps," *ApJS* **396** (September, 1992) L1.

[42] **WMAP** Collaboration, "Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results," *ApJS* **208** no. 2, (October, 2013) 20, arXiv:1212.5225 [astro-ph.CO].

[43] **WMAP** Collaboration, "Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," *ApJS* **208** no. 2, (October, 2013) 19, arXiv:1212.5226 [astro-ph.CO].

[44] **Planck** Collaboration, "Planck 2018 results. I. Overview and the cosmological legacy of Planck," *Astron. Astrophys.* **641** (2020) A1, arXiv:1807.06205 [astro-ph.CO].

[45] A. Liddle, *An introduction to modern cosmology; 3rd ed.* Wiley, Chichester, Mar, 2015. https://cds.cern.ch/record/1976476.

[46] **Planck** Collaboration, "Planck 2018 results. VI. Cosmological parameters," *Astron. Astrophys.* **641** (2020) A6, arXiv:1807.06209 [astro-ph.CO].

[47] H. Georgi and S. L. Glashow, "Unity of all elementary-particle forces," *Phys. Rev. Lett.* **32** (Feb, 1974) 438–441. https://link.aps.org/doi/10.1103/PhysRevLett.32.438.

[48] I. Aitchison, *Supersymmetry in Particle Physics. An Elementary Introduction.* Cambridge University Press, Cambridge, 2007.

[49] **Muon g-2** Collaboration, G. Bennett *et al.*, "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," *Phys. Rev. D* **73** (2006) 072003, `arXiv:hep-ex/0602035`.

[50] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events.* Cambridge University Press, 2006.

[51] A. Czarnecki and W. J. Marciano, "The Muon anomalous magnetic moment: A Harbinger for 'new physics'," *Phys. Rev. D* **64** (2001) 013014, `arXiv:hep-ph/0102122`.

[52] J. L. Feng and K. T. Matchev, "Supersymmetry and the anomalous magnetic moment of the muon," *Phys. Rev. Lett.* **86** (2001) 3480–3483, `arXiv:hep-ph/0102146`.

[53] S. Coleman and J. Mandula, "All possible symmetries of the s matrix," *Phys. Rev.* **159** (Jul, 1967) 1251–1256. https://link.aps.org/doi/10.1103/PhysRev.159.1251.

[54] R. Haag, J. T. Lopuszanski, and M. Sohnius, "All Possible Generators of Supersymmetries of the s Matrix," *Nucl. Phys.* **B88** (1975) 257. [,257(1974)].

[55] J. Wess and B. Zumino, "Supergauge transformations in four dimensions," *Nuclear Physics B* **70** no. 1, (1974) 39 – 50. http://www.sciencedirect.com/science/article/pii/0550321374903551.

[56] H. Georgi and S. L. Glashow, "Gauge theories without anomalies," *Phys. Rev. D* **6** (Jul, 1972) 429–431. https://link.aps.org/doi/10.1103/PhysRevD.6.429.

[57] S. Dimopoulos and D. W. Sutter, "The Supersymmetric flavor problem," *Nucl. Phys. B* **452** (1995) 496–512, `arXiv:hep-ph/9504415`.

[58] **MEG** Collaboration, T. Mori, "Final Results of the MEG Experiment," *Nuovo Cim. C* **39** no. 4, (2017) 325, `arXiv:1606.08168 [hep-ex]`.

[59] H. P. Nilles, "Supersymmetry, Supergravity and Particle Physics," *Phys. Rept.* **110** (1984) 1–162.

[60] A. Lahanas and D. Nanopoulos, "The road to no-scale supergravity," *Physics Reports* **145** no. 1, (1987) 1 – 139. http://www.sciencedirect.com/science/article/pii/0370157387900342.

[61] J. L. Feng, A. Rajaraman, and F. Takayama, "Superweakly interacting massive particles," *Phys. Rev. Lett.* **91** (2003) 011302, `arXiv:hep-ph/0302215`.

[62] **Super-Kamiokande** Collaboration, K. Abe *et al.*, "Search for proton decay via $p \rightarrow e^+ \pi^0$ and $p \rightarrow \mu^+ \pi^0$ in 0.31 megaton-years exposure of the Super-Kamiokande water Cherenkov detector," *Phys. Rev.* **D95** no. 1, (2017) 012004, `arXiv:1610.03597 [hep-ex]`.

[63] J. R. Ellis, "Beyond the standard model for hill walkers," in *1998 European School of High-Energy Physics*, pp. 133–196. 8, 1998. `arXiv:hep-ph/9812235`.

[64] J. R. Ellis, J. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, "Supersymmetric Relics from the Big Bang," *Nucl. Phys. B* **238** (1984) 453–476.

[65] D. O. Caldwell, R. M. Eisberg, D. M. Grumm, M. S. Witherell, B. Sadoulet, F. S. Goulding, and A. R. Smith, "Laboratory limits on galactic cold dark matter," *Phys. Rev. Lett.* **61** (Aug, 1988) 510–513. https://link.aps.org/doi/10.1103/PhysRevLett.61.510.

[66] M. Mori, M. M. Nojiri, K. S. Hirata, K. Kihara, Y. Oyama, A. Suzuki, K. Takahashi, M. Yamada, H. Takei, M. Koga, K. Miyano, H. Miyata, Y. Fukuda, T. Hayakawa, K. Inoue, T. Ishida, T. Kajita, Y. Koshio, M. Nakahata, K. Nakamura, A. Sakai, N. Sato, M. Shiozawa, J. Suzuki, Y. Suzuki, Y. Totsuka, M. Koshiba, K. Nishijima, T. Kajimura, T. Suda, A. T. Suzuki, T. Hara, Y. Nagashima, M. Takita, H. Yokoyama, A. Yoshimoto, K. Kaneyuki, Y. Takeuchi, T. Tanimori, S. Tasaka, and K. Nishikawa, "Search for neutralino dark matter heavier than the w boson at kamiokande," *Phys. Rev. D* **48** (Dec, 1993) 5505–5518. https://link.aps.org/doi/10.1103/PhysRevD.48.5505.

[67] **CDMS** Collaboration, D. S. Akerib *et al.*, "Exclusion limits on the WIMP-nucleon cross section from the first run of the Cryogenic Dark Matter Search in the Soudan Underground Laboratory," *Phys. Rev. D* **72** (2005) 052009, arXiv:astro-ph/0507190.

[68] A. Djouadi, J.-L. Kneur, and G. Moultaka, "SuSpect: A Fortran code for the supersymmetric and Higgs particle spectrum in the MSSM," *Comput. Phys. Commun.* **176** (2007) 426–455, arXiv:hep-ph/0211331.

[69] C. F. Berger, J. S. Gainer, J. L. Hewett, and T. G. Rizzo, "Supersymmetry without prejudice," *Journal of High Energy Physics* **2009** no. 02, (Feb, 2009) 023–023. http://dx.doi.org/10.1088/1126-6708/2009/02/023.

[70] J. Alwall, P. Schuster, and N. Toro, "Simplified Models for a First Characterization of New Physics at the LHC," *Phys. Rev.* **D79** (2009) 075020, arXiv:0810.3921 [hep-ph].

[71] **LHC New Physics Working Group** Collaboration, D. Alves, "Simplified Models for LHC New Physics Searches," *J. Phys.* **G39** (2012) 105005, arXiv:1105.2838 [hep-ph].

[72] D. S. Alves, E. Izaguirre, and J. G. Wacker, "Where the Sidewalk Ends: Jets and Missing Energy Search Strategies for the 7 TeV LHC," *JHEP* **10** (2011) 012, arXiv:1102.5338 [hep-ph].

[73] F. Ambrogi, S. Kraml, S. Kulkarni, U. Laa, A. Lessa, and W. Waltenberger, "On the coverage of the pMSSM by simplified model results," *Eur. Phys. J. C* **78** no. 3, (2018) 215, arXiv:1707.09036 [hep-ph].

[74] O. Buchmueller and J. Marrouche, "Universal mass limits on gluino and third-generation squarks in the context of Natural-like SUSY spectra," *Int. J. Mod. Phys. A* **29** no. 06, (2014) 1450032, arXiv:1304.2185 [hep-ph].

[75] **ATLAS** Collaboration, M. Aaboud *et al.*, "Dark matter interpretations of ATLAS searches for the electroweak production of supersymmetric particles in $\sqrt{s} = 8$ TeV proton-proton collisions," *JHEP* **09** (2016) 175, arXiv:1608.00872 [hep-ex].

[76] **ATLAS** Collaboration, "Summary of the ATLAS experiment's sensitivity to supersymmetry after LHC Run 1 — interpreted in the phenomenological MSSM," *JHEP* **10** (2015) 134, arXiv:1508.06608 [hep-ex].

[77] **ATLAS** Collaboration, "Mass reach of the atlas searches for supersymmetry."
      https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/
      ATL-PHYS-PUB-2020-020/fig__23.png, 2020.

[78] **CMS** Collaboration, "Summary plot moriond 2017." https://twiki.cern.ch/twiki/pub/
      CMSPublic/SUSYSummary2017/Moriond2017__BarPlot.pdf, 2017.

[79] L. S. W. Group, "Notes lepsusywg/02-04.1 and lepsusywg/01-03.1."
      http://lepsusy.web.cern.ch/lepsusy/, 2004. Accessed: 2021-02-11.

[80] **ATLAS** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for
      the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys. Lett. B*
      716 (2012) 1–29, `arXiv:1207.7214 [hep-ex]`.

[81] **CMS** Collaboration, S. Chatrchyan *et al.*, "Observation of a New Boson at a Mass of
      125 GeV with the CMS Experiment at the LHC," *Phys. Lett. B* **716** (2012) 30–61,
      `arXiv:1207.7235 [hep-ex]`.

[82] CERN, "About cern." https://home.cern/about. Accessed: 2021-01-21.

[83] CERN, "CERN Annual report 2019," tech. rep., CERN, Geneva, 2020.
      https://cds.cern.ch/record/2723123.

[84] O. S. Bruning, P. Collier, P. Lebrun, S. Myers, R. Ostojic, J. Poole, and P. Proudlock,
      LHC Design Report. CERN Yellow Reports: Monographs. CERN, Geneva, 2004.
      https://cds.cern.ch/record/782076.

[85] M. Blewett and N. Vogt-Nilsen, "Proceedings of the 8th international conference on
      high-energy accelerators, cern 1971. conference held at geneva, 20–24 september 1971.,"
      tech. rep., 1971, 1971.

[86] L. R. Evans and P. Bryant, "LHC Machine," *JINST* **3** (2008) S08001. 164 p.
      http://cds.cern.ch/record/1129806. This report is an abridged version of the LHC Design
      Report (CERN-2004-003).

[87] R. Scrivens, M. Kronberger, D. Küchler, J. Lettry, C. Mastrostefano, O. Midttun,
      M. O'Neil, H. Pereira, and C. Schmitzer, "Overview of the status and developments on
      primary ion sources at CERN*,". https://cds.cern.ch/record/1382102.

[88] M. Vretenar, J. Vollaire, R. Scrivens, C. Rossi, F. Roncarolo, S. Ramberger, U. Raich,
      B. Puccio, D. Nisbet, R. Mompo, S. Mathot, C. Martin, L. A. Lopez-Hernandez,
      A. Lombardi, J. Lettry, J. B. Lallement, I. Kozsar, J. Hansen, F. Gerigk, A. Funken, J. F.
      Fuchs, N. Dos Santos, M. Calviani, M. Buzio, O. Brunner, Y. Body, P. Baudrenghien,
      J. Bauche, and T. Zickler, *Linac4 design report*, vol. 6 of *CERN Yellow Reports:
      Monographs*. CERN, Geneva, 2020. https://cds.cern.ch/record/2736208.

[89] E. Mobs, "The CERN accelerator complex - 2019. Complexe des accélérateurs du CERN
      - 2019,". https://cds.cern.ch/record/2684277. General Photo.

[90] **ATLAS** Collaboration, "The ATLAS Experiment at the CERN Large Hadron Collider,"
      JINST **3** (2008) S08003.

[91] **CMS** Collaboration, S. Chatrchyan *et al.*, "The CMS Experiment at the CERN LHC,"
      JINST **3** (2008) S08004.

[92] **ALICE** Collaboration, K. Aamodt *et al.*, "The ALICE experiment at the CERN LHC," *JINST* **3** (2008) S08002.

[93] **LHCb** Collaboration, J. Alves, A.Augusto *et al.*, "The LHCb Detector at the LHC," *JINST* **3** (2008) S08005.

[94] **TOTEM** Collaboration, G. Anelli *et al.*, "The TOTEM experiment at the CERN Large Hadron Collider," *JINST* **3** (2008) S08007.

[95] **LHCf** Collaboration, O. Adriani *et al.*, "Technical design report of the LHCf experiment: Measurement of photons and neutral pions in the very forward region of LHC,".

[96] **MoEDAL** Collaboration, J. Pinfold *et al.*, "Technical Design Report of the MoEDAL Experiment,".

[97] **ATLAS** Collaboration, "ATLAS Public Results - Luminosity Public Results Run 2,". https://twiki.cern.ch/twiki/bin/view/AtlasPublic/LuminosityPublicResultsRun2. Accessed: 2021-01-17.

[98] **ATLAS** Collaboration, Z. Marshall, "Simulation of Pile-up in the ATLAS Experiment," *J. Phys. Conf. Ser.* **513** (2014) 022024.

[99] "First beam in the LHC - accelerating science,". https://home.cern/news/news/accelerators/record-luminosity-well-done-lhc. Accessed: 2021-01-10.

[100] **ATLAS Collaboration** Collaboration, "Luminosity determination in *pp* collisions at $\sqrt{s} = 13$ TeV using the ATLAS detector at the LHC," Tech. Rep. ATLAS-CONF-2019-021, CERN, Geneva, Jun, 2019. https://cds.cern.ch/record/2677054.

[101] **ATLAS** Collaboration, M. Aaboud *et al.*, "Luminosity determination in pp collisions at $\sqrt{s} = 8$ TeV using the ATLAS detector at the LHC," *Eur. Phys. J. C* **76** no. 12, (2016) 653, arXiv:1608.03953 [hep-ex].

[102] G. Avoni, M. Bruschi, G. Cabras, D. Caforio, N. Dehghanian, A. Floderus, B. Giacobbe, F. Giannuzzi, F. Giorgi, P. Grafström, V. Hedberg, F. L. Manghi, S. Meneghini, J. Pinfold, E. Richards, C. Sbarra, N. S. Cesari, A. Sbrizzi, R. Soluk, G. Ucchielli, S. Valentinetti, O. Viazlo, M. Villa, C. Vittori, R. Vuillermet, and A. Zoccoli, "The new LUCID-2 detector for luminosity measurement and monitoring in ATLAS," *Journal of Instrumentation* **13** no. 07, (Jul, 2018) P07017–P07017. https://doi.org/10.1088/1748-0221/13/07/p07017.

[103] S. van der Meer, "Calibration of the effective beam height in the ISR," Tech. Rep. CERN-ISR-PO-68-31. ISR-PO-68-31, CERN, Geneva, 1968. https://cds.cern.ch/record/296752.

[104] P. Grafström and W. Kozanecki, "Luminosity determination at proton colliders," *Progress in Particle and Nuclear Physics* **81** (2015) 97 – 148. http://www.sciencedirect.com/science/article/pii/S0146641014000878.

[105] "New schedule for CERN's accelerators and experiments,". https://home.cern/news/press-release/cern/first-beam-lhc-accelerating-science. Accessed: 2021-01-10.

[106] **ATLAS** Collaboration, G. Aad *et al.*, "Luminosity Determination in *pp* Collisions at $\sqrt{s} = 7$ TeV Using the ATLAS Detector at the LHC," *Eur. Phys. J. C* **71** (2011) 1630, `arXiv:1101.2185 [hep-ex]`.

[107] **ATLAS Collaboration** Collaboration, G. Aad *et al.*, "Improved luminosity determination in pp collisions at $\sqrt{s} = 7$ TeV using the ATLAS detector at the LHC. Improved luminosity determination in pp collisions at sqrt(s) = 7 TeV using the ATLAS detector at the LHC," *Eur. Phys. J. C* **73** no. CERN-PH-EP-2013-026. CERN-PH-EP-2013-026, (Feb, 2013) 2518. 27 p. https://cds.cern.ch/record/1517411. Comments: 26 pages plus author list (39 pages total), 17 figures, 9 tables, submitted to EPJC, All figures are available at <a href=.

[108] "Record luminosity: well done LHC,". https://home.cern/news/news/accelerators/new-schedule-cerns-accelerators-and-experiments. Accessed: 2021-01-10.

[109] A. G., B. A. I., B. O., F. P., L. M., R. L., and T. L., *High-Luminosity Large Hadron Collider (HL-LHC): Technical Design Report V. 0.1*. CERN Yellow Reports: Monographs. CERN, Geneva, 2017. https://cds.cern.ch/record/2284929.

[110] J. Pequenao, "Computer generated image of the whole ATLAS detector." Mar, 2008.

[111] **ATLAS** Collaboration, "ATLAS: Detector and physics performance technical design report. Volume 1,".

[112] J. Pequenao, "Computer generated image of the ATLAS inner detector." Mar, 2008.

[113] **ATLAS Collaboration** Collaboration, K. Potamianos, "The upgraded Pixel detector and the commissioning of the Inner Detector tracking of the ATLAS experiment for Run-2 at the Large Hadron Collider," Tech. Rep. ATL-PHYS-PROC-2016-104, CERN, Geneva, Aug, 2016. https://cds.cern.ch/record/2209070. 15 pages, EPS-HEP 2015 Proceedings.

[114] **ATLAS IBL** Collaboration, B. Abbott *et al.*, "Production and Integration of the ATLAS Insertable B-Layer," *JINST* **13** no. 05, (2018) T05008, `arXiv:1803.00844 [physics.ins-det]`.

[115] **ATLAS** Collaboration, "ATLAS Insertable B-Layer Technical Design Report," Tech. Rep. CERN-LHCC-2010-013. ATLAS-TDR-19, Sep, 2010. http://cds.cern.ch/record/1291633.

[116] **ATLAS** Collaboration, G. Aad *et al.*, "ATLAS b-jet identification performance and efficiency measurement with $t\bar{t}$ events in pp collisions at $\sqrt{s} = 13$ TeV," *Eur. Phys. J. C* **79** no. 11, (2019) 970, `arXiv:1907.05120 [hep-ex]`.

[117] ATLAS Collaboration, "Particle Identification Performance of the ATLAS Transition Radiation Tracker." ATLAS-CONF-2011-128, 2011. https://cds.cern.ch/record/1383793.

[118] J. Pequenao, "Computer Generated image of the ATLAS calorimeter." Mar, 2008.

[119] J. Pequenao, "Computer generated image of the ATLAS Muons subsystem." Mar, 2008.

[120] S. Lee, M. Livan, and R. Wigmans, "Dual-Readout Calorimetry," *Rev. Mod. Phys.* **90** no. arXiv:1712.05494. 2, (Dec, 2017) 025002. 40 p. https://cds.cern.ch/record/2637852. 44 pages, 53 figures, accepted for publication in Review of Modern Physics.

[121] M. Leite, "Performance of the ATLAS Zero Degree Calorimeter," Tech. Rep.
ATL-FWD-PROC-2013-001, CERN, Geneva, Nov, 2013.
https://cds.cern.ch/record/1628749.

[122] S. Abdel Khalek *et al.*, "The ALFA Roman Pot Detectors of ATLAS," *JINST* **11** no. 11,
(2016) P11013, `arXiv:1609.00249 [physics.ins-det]`.

[123] U. Amaldi, G. Cocconi, A. Diddens, R. Dobinson, J. Dorenbosch, W. Duinker,
D. Gustavson, J. Meyer, K. Potter, A. Wetherell, A. Baroncelli, and C. Bosio, "The real
part of the forward proton proton scattering amplitude measured at the cern intersecting
storage rings," *Physics Letters B* **66** no. 4, (1977) 390 – 394.
http://www.sciencedirect.com/science/article/pii/0370269377900223.

[124] L. Adamczyk, E. Banaś, A. Brandt, M. Bruschi, S. Grinstein, J. Lange, M. Rijssenbeek,
P. Sicho, R. Staszewski, T. Sykora, M. Trzebiński, J. Chwastowski, and K. Korcyl,
"Technical Design Report for the ATLAS Forward Proton Detector," Tech. Rep.
CERN-LHCC-2015-009. ATLAS-TDR-024, May, 2015.
https://cds.cern.ch/record/2017378.

[125] **ATLAS** Collaboration, A. R. Martínez, "The Run-2 ATLAS Trigger System," *J. Phys.
Conf. Ser.* **762** no. 1, (2016) 012003.

[126] **ATLAS Collaboration** Collaboration, *ATLAS level-1 trigger: Technical Design Report.*
Technical Design Report ATLAS. CERN, Geneva, 1998.
https://cds.cern.ch/record/381429.

[127] **ATLAS** Collaboration, G. Aad *et al.*, "Operation of the ATLAS trigger system in Run
2," *JINST* **15** no. 10, (2020) P10004, `arXiv:2007.12539 [physics.ins-det]`.

[128] **ATLAS Collaboration** Collaboration, P. Jenni, M. Nessi, M. Nordberg, and K. Smith,
ATLAS high-level trigger, data-acquisition and controls: Technical Design Report.
Technical Design Report ATLAS. CERN, Geneva, 2003.
https://cds.cern.ch/record/616089.

[129] **ATLAS** Collaboration, G. Aad *et al.*, "The ATLAS Simulation Infrastructure," *Eur.
Phys. J. C* **70** (2010) 823–874, `arXiv:1005.4568 [physics.ins-det]`.

[130] T. Gleisberg, S. Hoeche, F. Krauss, M. Schonherr, S. Schumann, F. Siegert, and
J. Winter, "Event generation with SHERPA 1.1," *JHEP* **02** (2009) 007,
`arXiv:0811.4622 [hep-ph]`.

[131] A. Buckley *et al.*, "General-purpose event generators for LHC physics," *Phys. Rept.* **504**
(2011) 145–233, `arXiv:1101.2599 [hep-ph]`.

[132] V. N. Gribov and L. N. Lipatov, "Deep inelastic e p scattering in perturbation theory,"
Sov. J. Nucl. Phys. **15** (1972) 438–450.

[133] J. Blumlein, T. Doyle, F. Hautmann, M. Klein, and A. Vogt, "Structure functions in deep
inelastic scattering at HERA," in *Workshop on Future Physics at HERA (To be followed
by meetings 7-9 Feb and 30-31 May 1996 at DESY).* 9, 1996. `arXiv:hep-ph/9609425`.

[134] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr,
and G. Watt, "LHAPDF6: parton density access in the LHC precision era," *Eur. Phys. J.
C* **75** (2015) 132, `arXiv:1412.7420 [hep-ph]`.

[135] M. Bengtsson and T. Sjostrand, "Coherent Parton Showers Versus Matrix Elements: Implications of PETRA - PEP Data," *Phys. Lett. B* **185** (1987) 435.

[136] S. Catani, F. Krauss, R. Kuhn, and B. R. Webber, "QCD matrix elements + parton showers," *JHEP* **11** (2001) 063, `arXiv:hep-ph/0109231`.

[137] L. Lonnblad, "Correcting the color dipole cascade model with fixed order matrix elements," *JHEP* **05** (2002) 046, `arXiv:hep-ph/0112284`.

[138] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand, "Parton Fragmentation and String Dynamics," *Phys. Rept.* **97** (1983) 31–145.

[139] B. Andersson, *The Lund Model*. Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology. Cambridge University Press, 1998.

[140] D. Amati and G. Veneziano, "Preconfinement as a Property of Perturbative QCD," *Phys. Lett. B* **83** (1979) 87–92.

[141] D. Yennie, S. Frautschi, and H. Suura, "The infrared divergence phenomena and high-energy processes," *Annals of Physics* **13** no. 3, (1961) 379–452. https://www.sciencedirect.com/science/article/pii/0003491661901518.

[142] M. Dobbs and J. B. Hansen, "The HepMC C++ Monte Carlo event record for High Energy Physics," *Comput. Phys. Commun.* **134** (2001) 41–46.

[143] **GEANT4** Collaboration, S. Agostinelli *et al.*, "GEANT4: A Simulation toolkit," *Nucl. Instrum. Meth.* **A506** (2003) 250–303.

[144] **ATLAS Collaboration** Collaboration, "The new Fast Calorimeter Simulation in ATLAS," Tech. Rep. ATL-SOFT-PUB-2018-002, CERN, Geneva, Jul, 2018. https://cds.cern.ch/record/2630434.

[145] K. Cranmer, "Practical Statistics for the LHC," in *2011 European School of High-Energy Physics*, pp. 267–308. 2014. `arXiv:1503.07622 [physics.data-an]`.

[146] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, "Asymptotic formulae for likelihood-based tests of new physics," *Eur. Phys. J.* **C71** (2011) 1554, `arXiv:1007.1727 [physics.data-an]`. [Erratum: Eur. Phys. J.C73,2501(2013)].

[147] ATLAS Collaboration, "Reproduction searches for new physics with the ATLAS experiment through publication of full statistical likelihoods." ATL-PHYS-PUB-2019-029, 2019. https://cds.cern.ch/record/2684863.

[148] **ROOT Collaboration** Collaboration, K. Cranmer, G. Lewis, L. Moneta, A. Shibata, and W. Verkerke, "HistFactory: A tool for creating statistical models for use with RooFit and RooStats," Tech. Rep. CERN-OPEN-2012-016, New York U., New York, Jan, 2012. https://cds.cern.ch/record/1456844.

[149] W. Verkerke and D. P. Kirkby, "The RooFit toolkit for data modeling," *eConf* **C0303241** (2003) MOLT007, `arXiv:physics/0306116 [physics]`. [,186(2003)].

[150] L. Moneta, K. Belasco, K. S. Cranmer, S. Kreiss, A. Lazzaro, D. Piparo, G. Schott, W. Verkerke, and M. Wolf, "The RooStats Project," *PoS* **ACAT2010** (2010) 057, `arXiv:1009.1003 [physics.data-an]`.

[151] F. James and M. Roos, "MINUIT: a system for function minimization and analysis of the parameter errors and corrections," *Comput. Phys. Commun.* **10** no. CERN-DD-75-20, (Jul, 1975) 343–367. 38 p. https://cds.cern.ch/record/310399.

[152] R. Brun and F. Rademakers, "ROOT: An object oriented data analysis framework," *Nucl. Instrum. Meth.* **A389** (1997) 81–86.

[153] I. Antcheva *et al.*, "Root — a c++ framework for petabyte data storage, statistical analysis and visualization," *Computer Physics Communications* **182** no. 6, (2011) 1384 – 1385. http://www.sciencedirect.com/science/article/pii/S0010465511000701.

[154] M. Baak, G. J. Besjes, D. Côte, A. Koutsman, J. Lorenz, and D. Short, "HistFitter software framework for statistical data analysis," *Eur. Phys. J.* **C75** (2015) 153, arXiv:1410.1280 [hep-ex].

[155] L. Heinrich, M. Feickert, G. Stark, and K. Cranmer, "pyhf: pure-python implementation of histfactory statistical models," *Journal of Open Source Software* **6** no. 58, (2021) 2823. https://doi.org/10.21105/joss.02823.

[156] L. Heinrich, M. Feickert, and G. Stark, "pyhf: v0.6.0." https://github.com/scikit-hep/pyhf.

[157] C. R. Harris, K. J. Millman, S. J. van der Walt, R. Gommers, P. Virtanen, D. Cournapeau, E. Wieser, J. Taylor, S. Berg, N. J. Smith, R. Kern, M. Picus, S. Hoyer, M. H. van Kerkwijk, M. Brett, A. Haldane, J. F. del R'ıo, M. Wiebe, P. Peterson, P. G'erard-Marchant, K. Sheppard, T. Reddy, W. Weckesser, H. Abbasi, C. Gohlke, and T. E. Oliphant, "Array programming with NumPy," *Nature* **585** no. 7825, (Sept., 2020) 357–362. https://doi.org/10.1038/s41586-020-2649-2.

[158] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan, T. Killeen, Z. Lin, N. Gimelshein, L. Antiga, A. Desmaison, A. Kopf, E. Yang, Z. DeVito, M. Raison, A. Tejani, S. Chilamkurthy, B. Steiner, L. Fang, J. Bai, and S. Chintala, "Pytorch: An imperative style, high-performance deep learning library," in *Advances in Neural Information Processing Systems 32*, H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alché-Buc, E. Fox, and R. Garnett, eds., pp. 8024–8035. Curran Associates, Inc., 2019. http://papers.neurips.cc/paper/9015-pytorch-an-imperative-style-high-performance-deep-learning-library.pdf.

[159] M. Abadi, A. Agarwal, P. Barham, E. Brevdo, Z. Chen, C. Citro, G. S. Corrado, A. Davis, J. Dean, M. Devin, S. Ghemawat, I. Goodfellow, A. Harp, G. Irving, M. Isard, Y. Jia, R. Jozefowicz, L. Kaiser, M. Kudlur, J. Levenberg, D. Mané, R. Monga, S. Moore, D. Murray, C. Olah, M. Schuster, J. Shlens, B. Steiner, I. Sutskever, K. Talwar, P. Tucker, V. Vanhoucke, V. Vasudevan, F. Viégas, O. Vinyals, P. Warden, M. Wattenberg, M. Wicke, Y. Yu, and X. Zheng, "TensorFlow: Large-scale machine learning on heterogeneous systems," 2015. https://www.tensorflow.org/. Software available from tensorflow.org.

[160] J. Bradbury, R. Frostig, P. Hawkins, M. J. Johnson, C. Leary, D. Maclaurin, and S. Wanderman-Milne, "JAX: composable transformations of Python+NumPy programs," 2018. http://github.com/google/jax.

[161] S. S. Wilks, "The large-sample distribution of the likelihood ratio for testing composite hypotheses," *Ann. Math. Statist.* **9** no. 1, (03, 1938) 60–62. https://doi.org/10.1214/aoms/1177732360.

[162] A. Wald, "Tests of statistical hypotheses concerning several parameters when the number of observations is large," *Transactions of the American Mathematical Society* **54** no. 3, (1943) 426–482. https://doi.org/10.1090/S0002-9947-1943-0012401-3.

[163] G. Cowan, "Statistics for Searches at the LHC," in *69th Scottish Universities Summer School in Physics: LHC Physics*, pp. 321–355. 7, 2013. arXiv:1307.2487 [hep-ex].

[164] A. L. Read, "Presentation of search results: the CL_S technique," *J. Phys. G* **28** (2002) 2693.

[165] R. D. Cousins, J. T. Linnemann, and J. Tucker, "Evaluation of three methods for calculating statistical significance when incorporating a systematic uncertainty into a test of the background-only hypothesis for a Poisson process," *Nucl. Instrum. Meth. A* **595** no. 2, (2008) 480, arXiv:physics/0702156 [physics.data-an].

[166] K. CRANMER, "Statistical challenges for searches for new physics at the lhc," *Statistical Problems in Particle Physics, Astrophysics and Cosmology* (May, 2006) . http://dx.doi.org/10.1142/9781860948985_0026.

[167] ATLAS Collaboration, "Search for direct pair production of a chargino and a neutralino decaying to the 125 GeV Higgs boson in $\sqrt{s} = 8$ TeV pp collisions with the ATLAS detector," *Eur. Phys. J. C* **75** (2015) 208, arXiv:1501.07110 [hep-ex].

[168] ATLAS Collaboration, "Search for chargino and neutralino production in final states with a Higgs boson and missing transverse momentum at $\sqrt{s} = 13$ TeV with the ATLAS detector," *Phys. Rev. D* **100** (2019) 012006, arXiv:1812.09432 [hep-ex].

[169] CMS Collaboration, "Search for electroweak production of charginos and neutralinos in WH events in proton–proton collisions at $\sqrt{s} = 13$ TeV," *JHEP* **11** (2017) 029, arXiv:1706.09933 [hep-ex].

[170] ATLAS Collaboration, "Search for direct production of electroweakinos in final states with one lepton, missing transverse momentum and a Higgs boson decaying into two b-jets in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector," *Eur. Phys. J. C* **80** (2020) 691, arXiv:1909.09226 [hep-ex].

[171] ATLAS Collaboration, "Improvements in $t\bar{t}$ modelling using NLO+PS Monte Carlo generators for Run 2." ATL-PHYS-PUB-2018-009, 2018. https://cds.cern.ch/record/2630327.

[172] ATLAS Collaboration, "Modelling of the $t\bar{t}H$ and $t\bar{t}V(V = W, Z)$ processes for $\sqrt{s} = 13$ TeV ATLAS analyses." ATL-PHYS-PUB-2016-005, 2016. https://cds.cern.ch/record/2120826.

[173] ATLAS Collaboration, "ATLAS simulation of boson plus jets processes in Run 2." ATL-PHYS-PUB-2017-006, 2017. https://cds.cern.ch/record/2261937.

[174] ATLAS Collaboration, "Multi-Boson Simulation for 13 TeV ATLAS Analyses." ATL-PHYS-PUB-2017-005, 2017. https://cds.cern.ch/record/2261933.

[175] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, "The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations," *JHEP* **07** (2014) 079, arXiv:1405.0301 [hep-ph].