# Chapter 1

# Theory

This chapter starts with an outline of the basic principles and concepts of the Standard Model of Particle Physics, the theoretical framework describing nature on the level of elementary particles. This is followed by an introduction to Supersymmetry, a promising class of theories aiming to solve some of the shortcomings of the Standard Model of Particle Physics (SM). By no means intended to be a full description, this chapter aims to highlight the important relations and consequences of the SM and Supersymmetry. A much more detailed introduction can be found in the references that the mathematical description in this chapter is based upon, namely Refs. [1, 2] for the SM and Refs. [3, 4] for Supersymmetry.

## 1.1 The Standard Model of Particle Physics

By the end of the 1920s, quantum mechanics and general relativity had been relatively well established and the consensus among physicists was that matter is composed of nuclear atoms consisting of electrons and protons. During the 1930s, a multitude of new experimental discoveries and theoretical puzzles excited physicists in three main currents of research: nuclear physics, cosmic rays and relativistic quantum mechanics [5]. Open questions at this time included e.g. the continuous spectrum of the $\beta$-decay, the nature of cosmic rays, or the negative energy states in Dirac's relativistic electron theory. As a result of these currents ultimately flowing together, the following decades saw elementary particle physics emerge as a new field of research.

Since these early times of particle physics, extraordinary progress has been made in describing nature at the subatomic scale. Today, a century later, the resulting theoretical framework, the SM, is the most fundamental, experimentally validated theory of nature known to mankind. It provides an extremely precise description of the interactions of elementary particles and—using the Large Electron Positron (LEP) collider—has been tested to an unprecedented level of accuracy up to the electroweak (EWK) scale. Given the remarkable success of the SM, it is not surprising that its history is paved with numerous awards for both experimental and theoretical work. In 1964, the Nobel prize was awarded to Feynman, Schwinger and Tomonoga for their fundamental work in quantum electrodynamics (QED), a quantum field theory allowing the precise calculation of fundamental processes like e.g. the anomalous magnetic moment of the electron that is known to a relative experimental uncertainty of $2.3 \times 10^{-10}$ [6]. In 1979,

**Table 1.1:** Names, electric charges and masses (rounded to three significant digits if known to that precision) of all observed fermions in the SM [7]. The symbols used in the following are indicated in parentheses after the particle names.

| | generation | particle | electric charge $[e]$ | mass |
|---|---|---|---|---|
| leptons | 1 | electron $(e)$ | $-1$ | 511 keV |
| | | electron neutrino $(\nu_e)$ | 0 | $<$ 1.1 eV |
| | 2 | muon $(\mu)$ | $-1$ | 106 MeV |
| | | muon neutrino $(\nu_\mu)$ | 0 | $<$ 0.19 MeV |
| | 3 | tau $(\tau)$ | $-1$ | 1.78 GeV |
| | | tau neutrino $(\nu_\tau)$ | 0 | $<$ 18.2 MeV |
| quarks | 1 | up $(u)$ | $\frac{2}{3}$ | 2.16 MeV |
| | | down $(d)$ | $-\frac{1}{3}$ | 4.67 MeV |
| | 2 | charm $(c)$ | $\frac{2}{3}$ | 1.27 GeV |
| | | strange $(s)$ | $-\frac{1}{3}$ | 93 MeV |
| | 3 | top $(t)$ | $\frac{2}{3}$ | 173 GeV |
| | | bottom $(b)$ | $-\frac{1}{3}$ | 4.18 GeV |

Glashow, Weinberg and Salam were awarded the Nobel prize for their work towards electroweak unification. The most prominent recent progress undoubtedly is the discovery of the Higgs boson, not only resulting in the Nobel prize being awarded to Englert and Higgs, but also completing the SM, roughly 50 years after the existence of the Higgs boson had been theorised.

### 1.1.1 Particle content of the SM

Apart from the experimentally non-vanishing neutrino masses, the SM successfully describes ordinary matter as well as their interactions, namely the electromagnetic, weak and strong interactions, leaving gravity as the only fundamental force not described within the SM. The particles in the SM are classified into two main categories, depending on their spin. Particles with half-integer spin follow Fermi-Dirac statistics and are called *fermions*. As they are subject to the Pauli exclusion principle, they make up ordinary matter. Particles with integer spin are called *bosons*, follow Bose-Einstein statistics and mediate the fundamental interactions between fermions.

Fermions are further divided into leptons and quarks, that each come in three generations with increasing masses†. The three electrically charged leptons are each associated to a corresponding neutral neutrino (more on this association in chapter section 1.1.2). While the SM assumes massless neutrinos, the observation of neutrino oscillations [8] implies the existence of at least two massive neutrinos. By extending the SM to allow non-vanishing neutrino masses, neutrino oscillations can be introduced through lepton generation mixing, described by the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [9]. Apart from an electric charge, the six quarks also carry a colour charge, of which three types exist: *red*, *green* and *blue* as well as their respective anti-colours. The mixing in the quark sector through the weak interaction can be described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [10, 11]. Finally, each fermion comes with its

† Neutrinos might not exist in a normal mass hierarchy but could also have an inverted mass hierarchy.

**Table 1.2:** Names, electric charges and masses (rounded to three significant digits if known to that precision) of all observed bosons in the SM [7]. The symbols used in the following are indicated in parentheses after the particle names.

| particle | spin | electric charge $[e]$ | mass |
|---|---|---|---|
| photon ($\gamma$) | 1 | 0 | 0 |
| gluon ($g$) | 1 | 0 | 0 |
| $W^{\pm}$ | 1 | $\pm 1$ | 80.4 GeV |
| $Z^0$ | 1 | 0 | 91.2 GeV |
| Higgs boson ($h$) | 0 | 0 | 125 GeV |

own anti-particle with same mass and spin, but inverted charge-like quantum numbers†. All fermions in the SM are listed in table 1.1.

The fundamental forces described by the SM are propagated by bosons with spin-1. The photon $\gamma$ couples to electrically charged particles and mediates the electromagnetic interaction. As the photon is massless, the electromagnetic force has infinite range. The strong force is mediated by gluons carrying one unit of colour and one unit of anti-colour. Due to colour-confinement, colour charged particles like quarks and gluons cannot exist as free particles and instead will always form colour-neutral bound states. Although nine gluon states would theoretically be possible, only eight of them are realised in nature—the colour-singlet state $\frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle)$ would result in long-range strong interactions, which have not been observed. Finally, the weak force is mediated by a total of three bosons, two charged $W^{\pm}$ bosons and a neutral $Z$ boson. The mediators of the weak force are massive, resulting in a finitely ranged interaction. They gain their masses through the Higgs mechanism (discussed in chapter section 1.1.2). All bosons known to the SM are listed in table 1.2.

### 1.1.2 The SM as a gauge theory

Formally, the SM is a collection of a special type of quantum field theories (QFTs), called gauge theories. QFT is the application of quantum mechanics to dynamical systems of fields—just as quantum mechanics is the quantisation of dynamical systems of particles—providing a uniform description of quantum mechanical particles and classical fields, while including special relativity.

In classical mechanics, the fundamental quantity is the action $S$, which is the time integral of the Lagrangian $L$, a functional characterising the state of a system of particles in terms of generalised coordinates $q_1, \ldots, q_n$. In field theory, the Lagrangian can be written as spatial integral of a Lagrangian density $\mathcal{L}(\phi_i, \partial_\mu \phi_i)$, which is a function of fields $\phi_i$ and their spacetime derivates $\partial_\mu \phi_i$. In the following, the Lagrangian density $\mathcal{L}$ will simply be referred to as the *Lagrangian*. The action can then be written as

$$S = \int L \, \mathrm{d}t = \int \mathcal{L}\left(\phi_i, \partial_\mu \phi_i\right) \mathrm{d}^4 x. \tag{1.1}$$

† The exact nature of anti-neutrinos is still an open question and ties into whether or not the neutrino mass matrix contains non-vanishing Majorana mass terms.

Using the principle of least action $\delta S = 0$, the equation of motions for each field are given by the Euler-Lagrange-equation,

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial \left( \partial_\mu \phi_i \right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0. \tag{1.2}$$

As opposed to the Hamiltonian formalism, the Lagrange formulation of field theory is especially well suited for the relativistic dynamics in particle physics, as it exhibits explicit Lorentz-invariance [2]. This is a direct consequence of the principle of least action, since boosted extrema in the action will still be extrema for Lorentz-invariant Lagrangians.

Symmetries are of central importance in the SM. As Emmy Noether has famously shown in 1918 for classical mechanics, every continuous symmetry of the action has a corresponding conservation law [12]. In the context of classical field theory, each generator of a continuous internal or spacetime symmetry transformation leads to a conserved current, and thus to a conserved charge. In QFTs, quantum versions of Noether's theorem, called Ward–Takahashi identities [13, 14] for Abelian theories and Slavnov–Taylor identities [15–17] for non-Abelian theories relate the conservation of quantum currents and charge-like quantum numbers to continuous symmetries of the Lagrangian.

From a theoretical point of view, the SM can be described by a non-Abelian Yang-Mills type [18] gauge theory based on the symmetry group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,$$

where $U(n)$ ($SU(n)$) describes (special) unitary groups, i.e. the Lie groups of $n \times n$ unitary matrices (with determinant 1, if special). $SU(3)_C$ generates quantum chromodynamics (QCD), describing the interaction of particles with colour charge $C$ through exchange of gluons, and $SU(2)_L \otimes U(1)_Y$ generates the electroweak interaction. Here, the subscript $Y$ represents the weak hypercharge, while the $L$ indicates that $SU(2)_L$ only couples to left-handed particles (right-handed antiparticles).

### Feyman diagrams

Transitioning from classical field theory to quantum field theory is typically done through either canonical quantisation or the usage of path integral formalism. As only the the simplest field theories can be solved analytically, i.e. those containing only free fields and no interactions, perturbation theory is used for calculating scattering cross sections and decay rates for any QFT containing interactions. Any transition matrix can then be written as a series expansion in the coupling constant, with each term represented by Feynman diagrams.

Using appropriate Feynman rules dictating the possible vertices (representing interactions between fields) and propagators (representing the propagation of fields), an infinite number of Feynman diagrams can be written down. Given the incoming and outgoing particles, all possible combinations of propagators and vertices that can be placed in between (i.e. all possible Feynman diagrams) represent the full perturbation series. Only the lowest order in the series is considered at leading order (LO), the next-lowest at next-to-leading order (NLO), and so on.

### Gauge principle

The gauge principle is fundamental to the SM and dictates that the existence of gauge fields is directly related to symmetries under local gauge transformations. QED, being the simplest gauge theory, can be taken to illustrate this important principle. The free Dirac Lagrangian for a single, non-interacting fermion with mass $m$ is given by

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi, \tag{1.3}$$

where $\psi$ is a four-component complex spinor field, $\bar{\psi} = \psi^\dagger \gamma^0$, and $\gamma^\mu$ with $\mu = 0, 1, 2, 3$ are the Dirac matrices with the usual anticommutation relations generating a matrix representation of the Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}\mathbb{1}_4. \tag{1.4}$$

It is worth noting that the free Dirac Lagrangian is invariant under a global $U(1)$ transformation

$$\psi \to e^{i\theta}\psi, \tag{1.5}$$

where the phase $\theta$ is spacetime independent and real-valued. In order to produce the physics of electromagnetism, the free Dirac Lagrangian however has to be invariant under *local* $U(1)$ phase transformations, which is not the case, as the transformed Lagrangian picks up an additional term from the spacetime derivative of the phase, $\partial_\mu\theta(x)$.

For the Dirac Lagrangian to become invariant under a local gauge transformation, a new vector field $A_\mu(x)$ has to be introduced and the partial derivative has to be replaced with the covariant derivative

$$\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu, \tag{1.6}$$

where $e$ can be identified with the elementary charge and represents the coupling of the fermion field to the gauge field $A_\mu$. The prescription of achieving local gauge invariance by replacing $\partial_\mu$ with $D_\mu$ is called *minimal coupling* and leads to a Lagrangian that is invariant under the transformations

$$\psi \to e^{i\theta(x)}\psi, \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta(x). \tag{1.7}$$

The modified Lagrangian now includes a term for interactions between the gauge field and the fermion field,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{int}} \\ &= \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - \left(e\bar{\psi}\gamma^\mu\psi\right)A_\mu,\end{aligned} \tag{1.8}$$

and is indeed invariant under a local phase transformation. Yet, it cannot be complete as it is still missing a term describing the kinematics of the free gauge field $A_\mu$. For a vector field, the kinetic term is described by the Proca Lagrangian

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu, \tag{1.9}$$

where $F^{\mu\nu} \equiv (\partial^\mu A^\nu - \partial^\nu A^\mu)$ is the field strength tensor that is invariant under the transformation in eq. (1.7). Since $A^\nu A_\nu$ is not invariant under the same transformation, the only way to keep the full Lagrangian invariant under a local phase transformation is by requiring $m_A = 0$, i.e. the introduced gauge field $A_\mu$ has to be massless, giving the Maxwell Lagrangian (ultimately generating the Maxwell equations),

$$\mathcal{L}_{\text{Maxwell}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{1.10}$$

This finally yields the full Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{QED}} &= \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}} \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu\right)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \left(e\bar{\psi}\gamma^\mu\psi\right)A_\mu,\end{aligned} \tag{1.11}$$

which can be identified to be the full Lagrangian of QED. The introduced gauge field $A_\mu$ is therefore nothing else than the electromagnetic potential with its associated massless particle, the photon. Thus, by applying the gauge principle on the free Dirac Lagrangian, i.e. forcing a global phase invariance to hold locally, a new massless gauge field has to be introduced, including interaction terms with the existing fields in the Lagrangian. In the case of the free Dirac Lagrangian, local gauge invariance produces all of QED.

As Yang and Mills have shown in 1954 [18], requiring a global phase invariance to hold locally is perfectly possible in the case of any continuous symmetry group. Considering a general non-Abelian symmetry group $G$, represented by a set of $n \times n$ unitary matrices $U(\alpha^1, \ldots, \alpha^N)$, parametrised by $N$ real parameters $\alpha^1, \ldots, \alpha^N$, then a gauge-invariant Lagrangian can be constructed with a similar prescription [1] as previously in the case of $U(1)$.

A total of $n$ fermion fields with mass $m$ are needed, arranged in an $n$-dimensional multiplet $\Psi = (\psi_1, \ldots, \psi_n)^T$. The free Lagrangian,

$$\mathcal{L}_{\text{free}} = \bar{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi, \tag{1.12}$$

is invariant under a global phase transformation of the form

$$\Psi(x) \to U(\alpha^1, \ldots, \alpha^N)\Psi(x). \tag{1.13}$$

Each element in the set of transformations $U$ can be written in terms of the group generators $T^a$ as

$$U(\alpha^1, \ldots, \alpha^N) = e^{i\alpha^a T^a}, \tag{1.14}$$

where the group indices $a = 1, \ldots, N$ are to be summed over. The group generators $T^a$ satisfy the commutation relations

$$[T^a, T^b] = if^{abc}T^c, \tag{1.15}$$

where $f^{abc}$ are the so-called structure constants quantifying the lack of commutativity between the generators. By convention, the basis for the generators $T^a$ is typically chosen such that $f^{abc}$ is completely anti-symmetric [1]. In order to make the Lagrangian invariant under local phase transformations, i.e. under transformations with a set of spacetime-dependent real parameters $\alpha^a(x)$, a vector field $\boldsymbol{W}_\mu$ together with a coupling constant $g$ have to be introduced through the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu - ig\boldsymbol{W}_\mu. \tag{1.16}$$

As $D_\mu$ acts on the $n$-dimensional multiplet $\Psi$, the introduced gauge field $\boldsymbol{W}_\mu$ has to be an $n \times n$ matrix and can thus be expanded in terms of the generators

$$\boldsymbol{W}_\mu(x) = T^a W^a_\mu(x), \tag{1.17}$$

thereby explicitly illustrating, that a total of $N$ gauge fields $W^a_\mu$ are introduced through the covariant derivative. Similar to QED above, the covariant derivative also introduces an interaction term of the form

$$\mathcal{L}_{\text{int}} = g\bar{\Psi}\gamma^\mu \boldsymbol{W}_\mu \Psi, \tag{1.18}$$

into the Lagrangian in eq. (1.12), coupling the gauge fields $W^a_\mu$ to the fermion multiplet. For infinitesimal $\alpha^a(x)$, the gauge fields gauge transform according to

$$W^a_\mu \to W^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} W^b_\mu \alpha^c, \tag{1.19}$$

where the term with $\alpha^a$ looks familiar from the $U(1)$ example and corresponds to the Abelian case, while the term with $f^{abc}$ introduces the non-Abelian structure into the theory [1]. The same non-Abelian structure is again clearly visible when introducing a kinetic term for the gauge fields into the Lagrangian

$$\mathcal{L}_W = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a}, \tag{1.20}$$

with the field-strength tensor now $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu$. As was already the case for QED, the above Lagrangian contains Abelian terms quadratic in $W$, describing the propagation of the free gauge fields. This time, the Lagrangian however also contains non-Abelian terms cubic and quartic in $W$, leading to self-interaction of the gauge fields.

#### Quantum chromodynamics

QCD, the gauge theory describing the strong interaction between quarks and gluons in the SM, is an example for a non-Abelian Yang-Mills theory. QCD is based on the gauge group $SU(3)_C$, with the subscript $C$ indicating that the quantum number associated with the symmetry group is the *colour*. Each quark is described by a triplet of fermion fields $q = (q_r, q_g, q_b)^T$, where the subscripts refer to the three different colours. The symmetry group $SU(3)$ has a total of $n^2 - 1 = 8$ generators, usually expressed in terms of the Gell-Mann matrices $\lambda^a$ [2]. The covariant derivative introducing the gauge fields $G^a_\mu$ acting on the quark triplets is then

$$D_\mu = \partial_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu, \tag{1.21}$$

with $g_s$ the coupling constant of the strong interaction, typically written as $\alpha_s = g_s^2/(4\pi)$ in analogy to the fine-structure constant in QED. Gauge invariance thus introduces a total of $N = 8$ gauge fields that can be identified with the eight gluons, leading to the full Lagrangian of QCD

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\gamma^\mu \partial_\mu - m_q)q - \sum_q -g_s \bar{q}\gamma^\mu \frac{\lambda^a}{2} q G^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}, \tag{1.22}$$

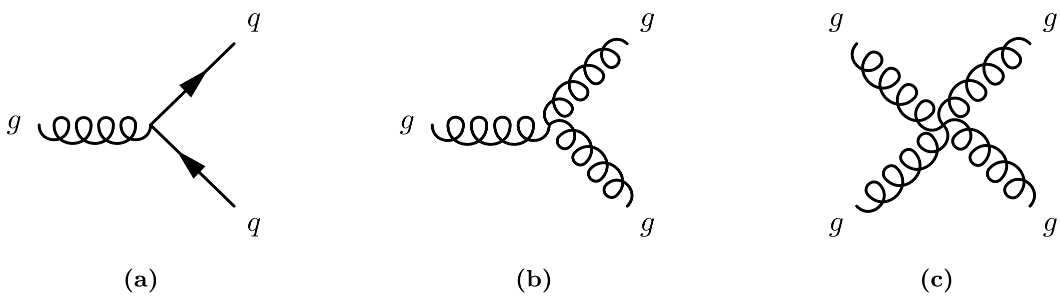


**Figure 1.1:** Possible vertices in QCD.

where $q = u, d, s, c, b, t$ and $G^a_{\mu\nu}$ are the gluon field strengths given by

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu. \tag{1.23}$$

As expected from the previous section, $\mathcal{L}_{\text{QCD}}$ contains terms that are cubic and quartic in the gluon fields, resulting in gluon self-interaction in the theory. All possible QCD interaction vertices involving gluons and quarks are shown in fig. 1.1. The gluon self-interaction leads to a number of phenomena unknown to Abelian theories, rendering the kinematics of QCD highly non-trivial.

In QCD, an effect similar to the electric charge screening in QED happens through quark-antiquark pairs, resulting in a screening of the colour charge. However, the existence of gluon loops in the gluon propagator due to gluon self-interaction creates an opposing *antiscreening* effect of colour charges. At short distances or large momentum scales, colour-charged particles essentially become free particles, a phenomenon called *asymptotic freedom*. In this regime, where $\alpha_s$ is sufficiently small, QCD processes can be calculated using perturbation theory. At large distances or small moment scales however, $\alpha_s$ becomes large and gluons interact very strongly with colour-charged particles, meaning that no free gluons or quarks can exist. This phenomenon is called *confinement* and implies that free quarks and gluons will be subject to *hadronisation*, i.e. form colourless bound states by combining with other quarks or gluons (that can be created from the vacuum). In a particle detector, hadronisation manifests itself as collimated showers of particles, called *jets*. At momentum scales where the strong coupling $\alpha_s$ becomes large ($\alpha_s \approx \mathcal{O}(1)$), QCD processes can no longer be calculated using perturbation theory and instead lattice QCD [19, 20] is used.

### Electroweak interaction

During the 1960s, Glashow, Weinberg and Salam [21–23] developed a unified theory of the electromagnetic and weak interactions, based on the $SU(2)_\text{L} \otimes U(1)_Y$ symmetry group. Known already experimentally from the Wu experiment [24] in 1956, weak interaction violates parity, i.e. the symmetry transformations have to act differently on the left-handed and right-handed fermion fields. The left- and right-handed components of a fermion field can be projected out using

$$\psi_\text{L} = \frac{1-\gamma^5}{2}\psi, \qquad \psi_\text{R} = \frac{1+\gamma^5}{2}\psi, \tag{1.24}$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. As the weak interaction only acts on left-handed fermions, they can be ordered as $SU(2)$ doublets

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix}u\\ d\end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix}c\\ s\end{pmatrix}_{\mathrm{L}}, \qquad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix}t\\ b\end{pmatrix}_{\mathrm{L}}. \tag{1.25}$$

The quantum number associated with $SU(2)$ symmetry transformations is called weak isospin $I$ with third component $I_3$. Fermion doublets have $I = 1/2$, with the upper component having $I_3 = 1/2$ and the lower component $I_3 = -1/2$. Right-handed fermion fields have $I = 0$, i.e. are singlet states in weak isospin space

$$e_{\mathrm{R}}, u_{\mathrm{R}}, d_{\mathrm{R}}, \qquad \mu_{\mathrm{R}}, c_{\mathrm{R}}, s_{\mathrm{R}}, \qquad \tau_{\mathrm{R}}, t_{\mathrm{R}}, b_{\mathrm{R}}, \tag{1.26}$$

and thus do not couple to the weak interaction. In the electroweak theory, neutrinos are assumed to be strictly massless, therefore no right-handed neutrino singlets exist.

The fermion doublets can be written in a free Lagrangian similar to eqs. (1.3) and (1.12),

$$\mathcal{L} = \bar{\psi}_{\mathrm{L}} i\gamma^\mu \partial_\mu \psi_{\mathrm{L}}, \tag{1.27}$$

with one crucial difference—the omission of the fermion masses. As $\bar{\psi}\psi = \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}$, mass terms would mix left- and right-handed terms and break gauge invariance. Section 1.1.2 will illustrate how fermion masses will instead be generated in the electroweak theory. For left-handed fermion fields, local $SU(2)_{\mathrm{L}}$ transformations can be written as

$$\psi_{\mathrm{L}} \to \exp\left(ig_2\alpha^a\frac{\sigma^a}{2}\right)\psi_{\mathrm{L}}, \tag{1.28}$$

where $g_2$ is the coupling constant, $\alpha^a$ (with $a = 1, 2, 3$) are real parameters and the Pauli matrices $\sigma^a$ are the generators of $SU(2)_{\mathrm{L}}$. By introducing the covariant derivative $D_\mu = \partial_\mu + ig_2\frac{\sigma^a}{2}W^a_\mu$ and including the usual kinetic term for the gauge fields, the Lagrangian becomes invariant under $SU(2)_{\mathrm{L}}$ transformations and reads

$$\mathcal{L} = \bar{\psi}_{\mathrm{L}} i\gamma^\mu D_\mu \psi_{\mathrm{L}} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu,a}, \tag{1.29}$$

with the gauge field strength tensors $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2\epsilon^{abc}W^b_\mu W^c_\nu$ where $\epsilon^{abc}$ are the structure constants. As previously in the case of QCD, the non-Abelian structure of the symmetry group causes self-interactions of the gauge fields.

In order to include electromagnetic interactions, the weak isospin group is extended with the $U(1)_Y$, corresponding to the multiplication of a phase factor $e^{i\alpha\frac{Y}{2}}$ to each of the preceding doublets and singlets. Here, $Y$ is the weak hypercharge as given by the Gell-Mann–Nishijima relation [25–27],

$$Q = I_3 + \frac{Y}{2}, \tag{1.30}$$

with $Q$ the electric charge. The electromagnetic group $U(1)_{\mathrm{em}}$ is then a subgroup of the combined electroweak gauge group [2].

By modifying the covariant derivative to include a $U(1)_Y$ gauge field and ensuring that $U(1)_Y$ acts the same on left-handed and right-handed fermions, it becomes $D_\mu = \partial_\mu + ig_2\frac{\sigma^a}{2}W^a_\mu + ig_1\frac{Y}{2}B_\mu$ for left-handed fermions and $D_\mu = \partial_\mu + ig_1\frac{Y}{2}B_\mu$ for right-handed fermions. The full electroweak Lagrangian then is

$$\begin{aligned}\mathcal{L}_{\text{electroweak}} =& \sum_j^6 \bar{\psi}^j_{\text{L}} i\gamma^\mu \left(\partial_\mu - ig_2\frac{\sigma^a}{2}W^a_\mu + ig_1\frac{Y}{2}B_\mu\right)\psi^j_{\text{L}} \\ &+ \sum_j^9 \bar{\psi}^j_{\text{R}} i\gamma^\mu \left(\partial_\mu + ig_1\frac{Y}{2}B_\mu\right)\psi^j_{\text{R}},\end{aligned} \tag{1.31}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, as usual.

### Spontaneous symmetry breaking

In the electroweak theory a total of three vector fields $W^a_\mu$ and one vector field $B_\mu$ are associated with the gauge groups $SU(2)_{\text{L}}$ and $U(1)_Y$, respectively. As has been shown explicitly through the example of QED in section 1.1.2, the gauge fields need to be massless for the resulting Lagrangian to be gauge invariant under the respective symmetry group. In addition, the electroweak symmetry group does not allow for fermion masses. Both gauge bosons of the weak interaction and the fermion are however manifestly massive, hence the electroweak symmetry has to be broken in the SM.

This spontaneous symmetry breaking is achieved through the Brout–Englert–Higgs mechanism [28–30]. In the SM, an isospin doublet of complex scalar fields, called Higgs doublet, is introduced

$$\Phi(x) = \begin{pmatrix}\phi^+(x)\\ \phi^0(x)\end{pmatrix}. \tag{1.32}$$

The Higgs doublet has hypercharge $Y = 1$, hence according to eq. (1.30), $\phi^+$ has electric charge $+1$ while $\phi^0$ is electrically neutral. With the covariant derivative introduced in section 1.1.2, the Higgs doublet gets an associated part in the SM Lagrangian,

$$\mathcal{L}_h = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi), \tag{1.33}$$

where $V(\Phi)$ is a gauge invariant potential

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2. \tag{1.34}$$

For positive and real parameters $\mu^2$ and $\lambda$, this potential has the form of a *Mexican hat* and an infinite number of minima for field configurations with $\Phi^\dagger\Phi = 2\mu^2/\lambda$. In the vacuum, i.e. in the ground state of the theory with minimal potential energy of the field, one of these minima is chosen such that the Higgs receives a vacuum expectation value (VEV)

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \qquad \text{with} \quad v = \frac{2\mu}{\sqrt{\lambda}} \approx 246\,\text{GeV}. \tag{1.35}$$

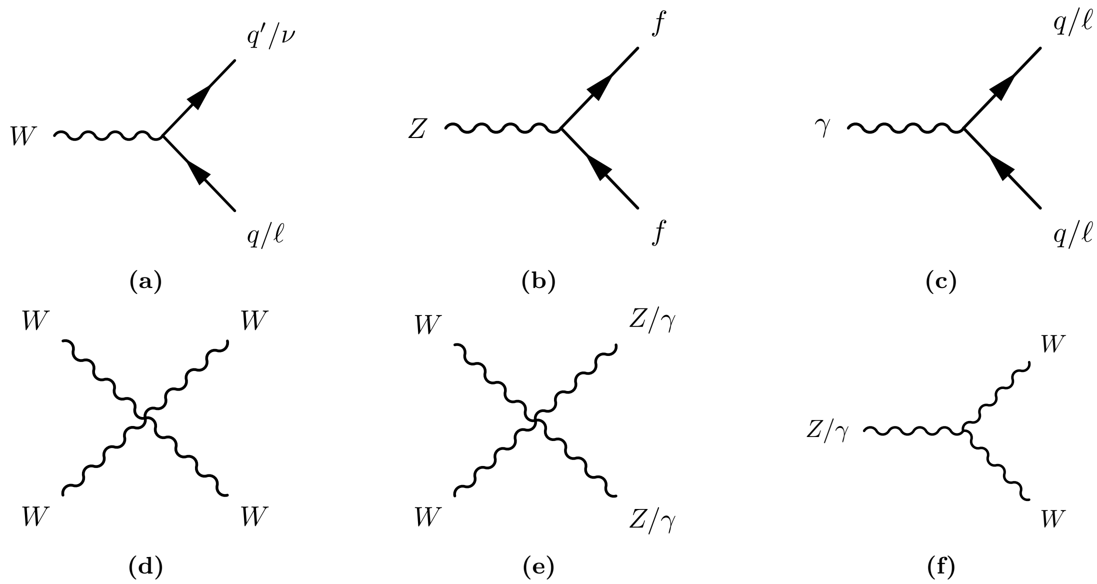


**Figure 1.2:** Possible vertices in the electroweak interaction.

This is neither invariant under a $SU(2)_{\mathrm{L}}$ transformation of the form $U = \exp(i\alpha^a \frac{\sigma^a}{2})$, nor under a $U(1)_Y$ transformation of the form $\exp(i\alpha\frac{Y}{2})$ [1]. Therefore, the electroweak gauge symmetry is spontaneously broken meaning that the Lagrangian has a symmetry that the vacuum does not have. It is worth noting that the $U(1)_{\mathrm{em}}$ gauge symmetry is not broken as the VEV of $\phi^+$ vanishes and $\phi^0$ is invariant under $U(1)_{\mathrm{em}}$.

The Higgs doublet can be expressed as excitations around the ground state

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + h(x) + i\chi(x) \end{pmatrix}, \tag{1.36}$$

where $h$, $\chi$, $\phi_1$ and $\phi_2$ are real-valued scalar fields with vanishing VEV. The Higgs potential can then be written as

$$V = \mu^2 h^2 + \frac{\mu^2}{v} h(h^2 + \chi^2 + \phi_1^2 + \phi_2^2) + \frac{\mu^2}{4v^2}(h^2 + \chi^2 + \phi_1^2 + \phi_2^2), \tag{1.37}$$

where only $h$ gets a mass term, thus describing an electrically neutral scalar particle with mass $m_h = \sqrt{2}\mu$. The remaining scalar fields remain massless, in accordance with the Nambu-Goldstone theorem [31, 32], stating that every spontaneously broken continuous symmetry generates a massless Goldstone boson. These bosons are unphysical and can be gauged away through a $SU(2)_{\mathrm{L}}$ transformation, such that the expansion around the vacuum from eq. (1.36) involves only the physical scalar $H(x)$,

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{1.38}$$

The gauge transformation bringing eq. (1.36) into the above form is called the *unitary gauge* [1].

In this gauge, the Higgs potential from eq. (1.34) has the form

$$V = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4, \tag{1.39}$$

containing cubic and quartic self-interactions of the Higgs field proportional to $m_h^2$. Inserting the excitation around the vacuum state in the kinetic term of $\mathcal{L}_\mathrm{h}$ yields mass terms for the vector bosons,

$$\mathcal{L}_h \propto \frac{v^2}{8} g_2^2 \left( W_\mu^1 W^{1,\mu} + W_\mu^2 W^{2,\mu} \right) + \frac{v^2}{8} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix}. \tag{1.40}$$

Instead of expressing the Lagrangian in terms of the fields $W_\mu^a$ and $B_\mu$ that make the original gauge invariance manifest, it can also be written in terms of the *physical* fields that correspond to the physical $W^\pm$, $Z$ and $\gamma$ bosons in the electroweak theory,

$$\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) && \text{with} \quad m_W = \frac{g_2}{2} v, \\
Z_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_\mu^3 - g_1 B_\mu) && \text{with} \quad m_Z = \frac{\sqrt{g_1^2 + g_2^2}}{2} v, \\
A_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 W_\mu^3 + g_2 B_\mu) && \text{with} \quad m_A = 0.
\end{aligned}$$

It is worth noting, that the massless photon field $A_\mu$ associated with the electromagnetic $U(1)_\mathrm{em}$ gauge symmetry is automatically recovered. All possible vertices between fermions and the physical electroweak gauge bosons are shown in fig. 1.2. The change of basis from $(W_\mu^3, B_\mu)$ to $(Z_\mu, A_\mu)$ [2] can also be written as a basis rotation with the weak mixing angle $\theta_W$,

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \qquad \text{with } \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{m_W}{m_Z}. \tag{1.41}$$

In the SM, not only the $W^\pm$ and $Z$ bosons but also fermions gain their masses through spontaneous breaking of the electroweak gauge symmetry. Fermion fields gain masses through gauge-invariant Yukawa interactions with the Higgs field. For one fermion generation, the respective Yukawa terms in the Lagrangian are

$$\mathcal{L}_\text{Yukawa,gen} = -\lambda_\ell \bar{L}_\mathrm{L} \Phi \ell_\mathrm{R} - \lambda_d \bar{Q}_\mathrm{L} \Phi d_\mathrm{R} - \lambda_u \bar{Q}_\mathrm{L} \Phi^\dagger u_\mathrm{R} + \text{h.c.}, \tag{1.42}$$

where $\lambda_f$ with $f = \ell, d, u$ are the dimensionless Yukawa couplings and $L_\mathrm{L} = (\nu_\mathrm{L}, \ell_\mathrm{L})^T$ and $Q_\mathrm{L} = (u_\mathrm{L}, d_\mathrm{L})^T$ are the left-handed lepton and quark doublets, respectively. The VEV of the Higgs field then gives rise to fermion mass terms in the Lagrangian, which, in the unitary gauge, yields for a single fermion generation

$$\mathcal{L}_\text{Yukawa,gen} = -\sum_{f=\ell,d,u} \left( m_f \bar{\psi}_f \psi_f + \frac{m_f}{v} H \bar{\psi}_f \psi_f \right) \qquad \text{with} \quad m_f = \frac{1}{\sqrt{2}} \lambda_f v. \tag{1.43}$$

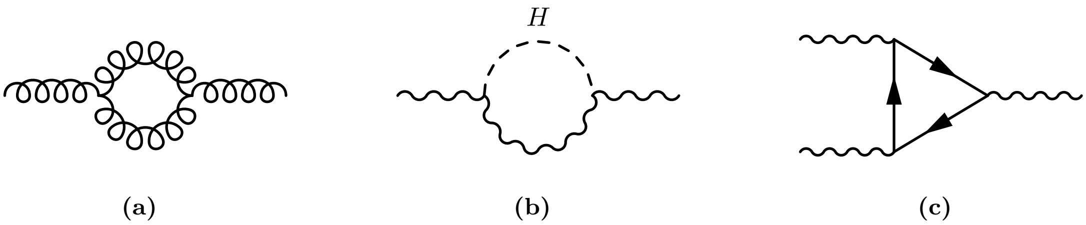


**Figure 1.3:** Examples of loop corrections to (a) the gluon propagator, (b) the $W$ or $Z$ propagator and (c) the cubic gauge boson vertex.

When introducing all three fermion generations, additional Yukawa terms mixing fermions of different generations appear in the Lagrangian [1]. The terms involving quark fields can be parametrised using the CKM matrix $V_{\mathrm{CKM}}$ [10, 11], quantifying the transition probability between quark generations. Since no right-handed neutrinos exist in the SM, no generation mixing in the lepton sector occurs and hence no neutrino mass terms are allowed in the SM. Neutrino oscillations have however been observed experimentally, thus at least one massive neutrino generation needs to exist. Their mixing can then be described with the PMNS matrix [9], allowing neutrinos to acquire mass e.g. through the see-saw mechanism [33].

### 1.1.3 Renormalisation and divergencies

At lowest order in the perturbative expansion, the momenta of the internal lines in the Feynman diagrams are fixed by the external particles. For higher orders where the diagrams involve loops, the momenta of the internal lines need to be integrated over as they are not fixed by energy-momentum conservation. Some examples of loop corrections to propagators and vertices are shown in fig. 1.3. As each vertex in the Feynman diagrams is associated with a coupling constant that is usually much smaller than 1 (apart from the non-perturbative regime of QCD), higher orders in the perturbative expansion contribute less and less to the total amplitude of the full expansion.

The momentum integrals in loop corrections however lead to *ultraviolet divergencies* for large momenta. In order to eliminate the divergencies, the integrals have to be *regularised*, e.g. by applying a cut-off scale $\Lambda$ or calculating the integrals in a number $D = 4 - \epsilon$ of dimensions where they converge. The potential divergencies are then absorbed in parameters of the Lagrangian, such as coupling constants and masses, after which the regulator is removed (e.g. $\epsilon \to 0$) again and a *renormalisation* procedure is applied, replacing the bare parameter values with the physical, measured values [1]. Renormalisation effectively absorbs the effects of quantum fluctuations acting on much smaller scales than the scale of the given problem in the parameters of the theory. As Veltmann and t'Hooft [34, 15] have shown, all Yang-Mills theories with massive gauge fields are renormalisable, making the SM as a whole a renormalisable theory.

## 1.2 Supersymmetry

Among the properties a quantum field theory might possess to make it more mathematically tractable, one specific higher symmetry reveals particularly far-reaching implications; a symmetry relating fermions and bosons, known as Supersymmetry (SUSY). The following section introduces the basic concepts of SUSY, a promising class of theories that could solve some of the shortcomings of the SM.

First, a motivation for the need of SUSY is given by highlighting some of the open questions of the SM. This is followed by an introduction to the mathematical description and phenomenological consequences of supersymmetric theories. The following sections are intended to highlight the most important concepts and relations, a much more complete and detailed introduction to SUSY can be found in Refs. [3, 4].

### 1.2.1 Shortcomings of the Standard Model

Although the SM is a remarkably successful theory able to predict and describe the interactions between elementary particles with unprecedented precision, there are still phenomena in nature that cannot be suitable understood within the theoretical framework of the SM. Those limitations and open questions are the reason for numerous searches looking for new physics beyond the Standard Model (BSM). Some of the aforementioned open questions are described in the following.

#### Dark Matter

The existence of dark matter (DM), i.e. non-luminous and non-absorbing matter is nowadays well established [7]. Some of the earliest hints for the existence of DM came from the observation that the rotation curves of luminous objects are not consistent with the expected velocities based on the gravitational attraction of the visible objects around them. Zwicky already postulated in 1933 the existence of DM [35] based on rotation curves of galaxies in the Coma cluster. In 1970, Rubin measured rotation curves of spiral galaxies [36], revealing again a significant disagreement with the theoretically expected curves given the visible matter in the galaxies. Based on Newtonian dynamics, the circular velocity of stars outside the bulge of galaxies is expected to fall off with increasing radius as $v(r) \propto 1/\sqrt{r}$ [37]. Rubin's observations however revealed that the velocities of stars outside the bulge stay approximately constant, strongly suggesting the existence of a non-luminous (or *dark*) matter halo around the galaxies. Surveys of galaxy clusters and observations of gravitational lensing effects e.g. in the bullet cluster [38] or the Abell 1689 cluster [39] have since then further consolidated the existence of large accumulations of non-luminous mass in the universe.

The anisotropies in cosmic microwave background (CMB), studied by the COBE [40, 41], WMAP [42, 43] and Planck missions [44] are well described by the Lambda Cold Dark Matter ($\Lambda$CDM) model [45], which includes a density for cold dark matter. Planck's latest results [46] suggest that the matter density of the universe is $\Omega_m = 0.3111 \pm 0.0056^\dagger$ and that ordinary baryonic matter only makes up $\sim 4.9\%$ of the universe's matter content, while DM accounts for the remaining $\sim 26.1\%$.

Candidates for non-baryonic DM need to satisfy certain conditions: they have to be stable on cosmological timescales (otherwise they would have decayed by now), they have to couple only very weakly to the electromagnet interaction (if at all, otherwise they would be luminous

† The remaining $\sim 69\%$ are taken up by *dark energy*, the nature of which is still an open question.

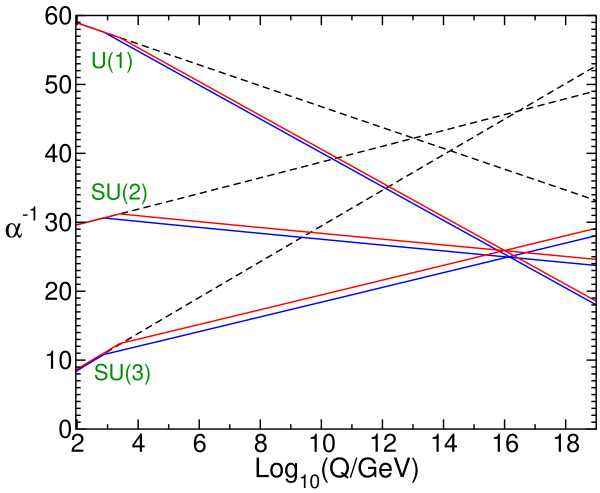


**Figure 1.4:** Evolution of the inverse coupling constants in the SM (dashed lines) and the MSSM (solid lines) in function of the energy scale $Q$. In the MSSM, the masses of the supersymmetric particles are treated as common threshold varied between 750 GeV and 2.5 TeV. Figure taken from Ref. [3].

matter) and they need to have the right relic density. Analyses of structure formations in the Universe have furthermore shown that most DM should have been *cold*, i.e. non-relativistic at the beginning of galaxy formation [37]. Candidates for DM particles are e.g. sterile neutrinos, axions, primordial black holes, or weakly interacting massive particles (WIMPs).

In the SM, the only DM candidate particle is the neutrino. Given the upper limits on the neutrino masses, an upper bound on their relic density can be computed, revealing that neutrinos are simply not abundant enough to be a dominant component of DM [37]. Many BSM theories naturally predict new WIMPs with masses in the GeV to TeV range. In many SUSY models with exact R-parity conservation (a quantity introduced in section 1.2.5), the lightest supersymmetric particle is neutral and stable and could be a good candidate for DM.

#### Unification of forces

Although the SM provides a good description of nature up to the energy scale probed with today's accelerators, some of its peculiar aspects hint to a more fundamental theory. A prominent example is the question why the electric charges of the electrons and the charges of the quarks of the protons and neutrons in the nuclei exactly cancel, making for electrically neutral atoms [1]. Or in other words: why are the charges of all observed particles simple multiples of the fundamental charge? And why are they quantised in the first place?

An explanation to many of these peculiarities comes naturally when describing the SM as a unified theory with a single non-Abelian gauge group, e.g. $SU(5)$ [47]. The larger symmetry group with a single coupling constant is then thought to be spontaneously broken at very high energy, such that the known SM interactions are recovered at the lower energies probed in today's experiments. In such a grand unified theory (GUT), the particles in the SM are arranged in anomaly-free$^\dagger$ irreducible representations of the gauge group, thereby e.g. naturally ensuring the fractional charges of quarks [2].

In the SM, the coupling constants run towards each other with increasing energy scale, but never exactly meet. In the MSSM with supersymmetric particles at the TeV scale the running

$^\dagger$ In the sense that loop corrections do not break symmetries the Lagrangian has.


(a) (b)

**Figure 1.5:** A massive fermion (a) and a hypothetical massive scalar particle (b) coupling to the Higgs boson.

couplings meet within their current uncertainties, hinting that a supersymmetric GUT could be a good candidate for describing physics at the unification scale. Figure 1.4 shows the running of the coupling constants in both the SM and the MSSM.

### The Hierarchy Problem

As the SM is a renormalisable gauge theory, finite results are obtained for all higher-order loop corrections, making the SM a theory that is in principle well-defined up to infinite energies. In renormalisation terms, this means that the cut-off scale $\Lambda$ is theoretically allowed to go to arbitrarily high values. It is however clear, that the SM cannot be a complete theory of nature and that at some unknown high-energy scale $\Lambda$, *new physics* has to appear. At the very least, a new theoretical framework becomes necessary at the Planck scale $M_P \approx 10^{18}$ GeV [3], where quantum gravitational effects can no longer be ignored.

The mass parameters of fermions and massive vector bosons are protected from large quantum corrections by chiral symmetry and gauge symmetry, respectively [48]. The mass parameter of the scalar Higgs field, on the other hand, receives loop corrections proportional at least to the scale at which new physics sets in. The coupling of the Higgs field to a fermion $f$ with mass $m_f$, depicted in fig. 1.5(a), yields a one-loop correction term to the Higgs square mass [3] given by

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda^2 + \dots \,. \tag{1.44}$$

Thus, in order to obtain the relatively low value of the Higgs mass in the order of $10^2$ GeV, the quantum corrections to the bare Higgs parameter have to be tuned in such a way that they almost cancel. Hence, if there is *any* scale of new physics even only several orders of magnitude higher than the electroweak scale, the resulting large quantum corrections to the Higgs mass immediately lead to a *fine-tuning* problem that is considered to be unnatural.

In SUSY, the Higgs mass is automatically protected from the large quantum corrections by the introduction of two complex scalar partners to each SM fermion. The quantum corrections from a hypothetical heavy complex scalar particle $S$ with mass $m_S$ as in fig. 1.5(b) yields a one-loop correction [3] given by

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 + 2m_S^2 \log\left(\Lambda/m_S\right) + \dots\right]. \tag{1.45}$$

Interestingly, the corrections in eq. (1.44) and eq. (1.45) enter with opposite signs. Thus, if $\lambda_S = |\lambda_f|^2$, then the large quantum corrections neatly cancel and no excessive fine-tuning is needed. The requirement $\lambda_S = |\lambda_f|^2$ means that the fermions and their supersymmetric bosonic partners would have same masses. Such particles would have been discovered long ago in particle physics experiments, meaning that SUSY must be a broken symmetry (see section 1.2.5 for a discussion on SUSY breaking) such that the supersymmetric particles acquire masses well above those of their SM partners.

#### Anomalous magnetic moment of the muon

One of the longest standing disagreements between experiment and theory in the SM is the anomalous magnetic moment of the muon [7]. The magnetic moment of the muon $\vec{\mu}_\mu$ is related to its intrinsic spin $\vec{S}$ through the gyromagnetic ratio $g_\mu$ by

$$\vec{\mu}_\mu = g_\mu \frac{q}{2m}\vec{S}. \tag{1.46}$$

For a structureless spin-1/2 particle with mass $m$ and charge $q = \pm e$, the gyromagnetic ratio is $g_\mu = 2$ [49]. Loop corrections coupling the muon spin to virtual fields cause small deviations, parameterised by the anomalous magnetic moment

$$a_\mu = \frac{1}{2}(g_\mu - 2). \tag{1.47}$$

The anomalous magnetic moment can be precisely measured as well as predicted within the SM, a comparison between experimental data and theoretical prediction thus directly tests the SM at quantum loop level and may hint to effects from new physics in case of discrepancies [50]. In the SM, the most dominant contribution to $a_\mu$ comes from QED corrections involving photon and fermion loops. An exemplary diagram is shown in fig. 1.6(a). Weak contributions involving the heavy $W^\pm$, $Z$ and Higgs particles are suppressed by their masses [7]. Although the contributions from QCD are relatively small, they give rise to the main theoretical uncertainties as they cannot be calculated from first principles [7].

The E821 experiment at Brookhaven National Lab (BNL) [49] has measured the anomalous magnetic moment of the muon and found a deviation from the SM expectation of

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(63)(48) \times 10^{-11}, \tag{1.48}$$

where the numbers in parentheses are the uncertainties from experiment and theory, respectively. This represents a deviation of $3.3\sigma$ [7] from the SM expectation.

In SUSY, additional Feynman diagrams exist involving the supersymmetric partners of the muon, the muon neutrino and the electroweak gauge bosons, and thus the measured deviation in $a_\mu$ can easily be accommodated in many supersymmetric models [51, 52]. Two exemplary lowest-order diagrams involving supersymmetric particles are shown in figs. 1.6(b) and 1.6(c).

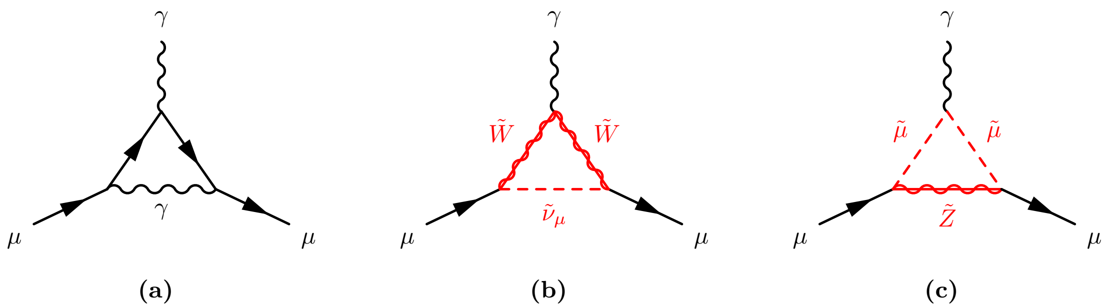


**Figure 1.6:** Electromagnetic (a) and supersymmetric (b), (c) contributions to $a_\mu$. Adapted from [50].

### 1.2.2 Supersymmetric Algebra

A generator of supersymmetric transformations is an anti-commuting spinor $Q$ that turns fermionic states $|f\rangle$ into bosonic states $|b\rangle$ and vice-versa.

$$Q\,|f\rangle = |b\rangle\,, \qquad\qquad Q\,|b\rangle = |f\rangle\,. \tag{1.49}$$

As spinors are complex objects, $Q^\dagger$ is also a symmetry operator. Both $Q$ and $Q^\dagger$ are necessarily fermionic and thus must carry half-integer spin, in the simplest case spin-1/2, meaning that SUSY must be a spacetime symmetry, i.e. a Poincaré symmetry. The Coleman–Mandula theorem [53] dictates that the symmetry group generating a consistent spacetime QFT must be the direct product of the internal symmetry group with the Poincaré group, which in principle rules out the possibility for SUSY. The Haag–Lopuszanski–Sohnius extension [54] however states that the only possible way of non-trivially combining internal and spacetime symmetry groups is to use a Lie superalgebra and fermionic spin-1/2 generators. Thus, in order to obey the Haag–Lopuszanski–Sohnius theorem and simultaneously allow for parity-violating interactions, the SUSY generators have to satisfy the following algebra of commutation and anti-commutation relations [4],

$$\begin{aligned}
\{Q, Q^\dagger\} &= 2\sigma_\mu P^\mu, \\
\{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0, \\
[P^\mu, Q] &= \left[P^\mu, Q^\dagger\right] = 0, \\
\{M^{\mu\nu}, Q\} &= \sigma^{\mu\nu} Q, \\
\{M^{\mu\nu}, Q^\dagger\} &= \bar{\sigma}^{\mu\nu} Q^\dagger,
\end{aligned} \tag{1.50}$$

where $P^\mu$ is the four-momentum generator of spacetime translations, $\sigma_\mu = (\mathbb{1}_2, \sigma_i)$, $\bar{\sigma}_\mu = (\mathbb{1}_2, -\sigma_i)$ with $i = 1, 2, 3$ and the Pauli matrices $\sigma_i$, and $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$ as well as $\bar{\sigma}^{\mu\nu} = \frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)$. This is the simplest version of SUSY, called $N = 1$ symmetry, as it introduces only one pair of generators. Supersymmetric theories with $N \geq 2$ pairs of generators also exist and generally have some theoretical advantages as e.g. fewer divergencies in the case of $N = 2$ or even no divergencies at all in the case of $N = 4$ [4]. SUSY models with $N \geq 2$ however do not allow for parity violation and thus fail to describe the physics of the SM, disqualifying them from a phenomenological point of view [4].

As both SUSY generators commute with spacetime translations (see eq. (1.50)), they also both commute with the squared mass operator $-P^2$. Consequently, particles related by the generators, called *superpartners*, must have equal eigenvalues under $-P^2$, i.e. they must have equal masses. Furthermore, the SUSY generators also commute with the gauge transformation generators, hence superpartners must have same electric charge, weak isospin and degrees of freedom in colour space [3].

### 1.2.3 Supermultiplets

The SM and SUSY particles are arranged in irreducible representations of the SUSY algebra, called *supermultiplets*, each containing both fermionic and bosonic states that are superpartners of each other. It can be shown that each supermultiplet has an equal number of fermion and boson degrees of freedom, $n_f = n_b$ [3].

The simplest supermultiplet $\Psi$ that can be constructed contains a single Weyl fermion $\psi$ and two real scalars, described by a single complex field $\phi$, called the *sfermion*. The Weyl fermion has two spin helicity states, hence $n_f = 2$, and the complex scalar field has two components with $n_b = 1$ each. An additional complex scalar field $F$, called *auxiliary field* and not corresponding to a physical particle, has to be introduced in order to allow the SUSY algebra to close off-shell (where the energy-momentum relation does not hold) [3]. The supermultiplet $\Psi$ thus reads

$$\Psi = (\phi, \psi, F). \tag{1.51}$$

Being a pure bookkeeping device, the auxiliary field does not propagate and can be eliminated on-shell with the equations of motion $F = F^* = 0$. This supermultiplet is called a *chiral* or *scalar* supermultiplet [3].

The next-simplest supermultiplet for which $n_f = n_b$ holds, is the *vector* or *gauge* supermultiplet $\Phi$ containing a spin-1 gauge boson $A_a^\mu$, where $a$ is the index of the gauge group. In order for the theory to be renormalisable, this gauge boson must be massless before spontaneous breaking of the symmetry. As a massless spin-1 boson has two helicity states, $n_b = 2$, the superpartner, called *gaugino*, must be a massless spin-1/2 Weyl fermion $\lambda_a$ with two helicity states such that $n_f = 2$ [3]. An auxiliary real bosonic field $D_a$ is needed in order to balance the degrees of freedom off-shell [4], completing the supermultiplet to be

$$\Phi = (\lambda_a, A_a^\mu, D_a). \tag{1.52}$$

Like the chiral auxiliary field, the gauge auxiliary field does not correspond to a physical particle and can be eliminated on-shell through its equations of motion [3].

### 1.2.4 Supersymmetric Lagrangian

The simplest supersymmetric model that can be shown to realise the superalgebra is the massless, non-interacting Wess–Zumino model [55, 3], given by

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} \\ &= \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi, \end{aligned} \tag{1.53}$$

with a massless complex scalar $\phi$ and a spin-1/2 fermion $\psi$, corresponding to a single chiral supermultiplet. As discussed in section 1.2.3, in order for this Lagrangian to satisfy the supersymmetry off-shell where the equations of motion cannot be used, an auxiliary complex scalar field $F$ has to be added. For a collection of $i$ chiral supermultiplets, the free Lagrangian thus reads

$$\begin{aligned}\mathcal{L}_{\text{free}} &= \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{aux}} \\ &= \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i,\end{aligned} \tag{1.54}$$

where the repeated indices $i$ are summed over. The auxiliary Lagrangian term $\mathcal{L}_{\text{aux}}$ implies the trivial equations of motion $F = F^* = 0$ which are needed to remove the auxiliary field in the on-shell case. The next step involves adding terms for non-gauge interactions for the chiral supermultiplets. Non-gauge interactions for chiral supermultiplets at most quadratic in the fermion fields can be achieved by introducing the term,

$$\mathcal{L}_{\text{int}} = \frac{1}{2} W^{ij}(\phi, \phi^*) \psi_i \psi_j + V(\phi, \phi^*) + c.c., \tag{1.55}$$

where $W^{ij}$ is a holomorphic† function of the complex scalar fields $\phi_i$ of the form [4]

$$W^{ij} = \frac{\partial W}{\partial \phi_i \partial \phi_j}. \tag{1.56}$$

Here, $W$ is called the *superpotential*. For the final Lagrangian to be renormalisable, the superpotential can at most be cubic [4], and can thus be written as

$$W = \frac{1}{2} m^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k, \tag{1.57}$$

where $y^{ij}$ are the Yukawa couplings between the scalar and the two fermions. Requiring $\mathcal{L}_{\text{int}}$ to be invariant under supersymmetry transformations further defines the potential $V$. The equations of motions of the auxiliary fields $F$ can be written as

$$F_i = \frac{\partial W(\phi)}{\partial \phi^i} = -W_i^*, \qquad F^{*i} = -\frac{\partial W(\phi)}{\partial \phi_i} = -W^i, \tag{1.58}$$

which thus yields for the potential $V = W_i^* W^i = F_i F^{*i}$. The full Lagrangian of the Wess-Zumino model [3] with general chiral interactions for $i$ chiral supermultiplets is then given by adding eq. (1.55) with eqs. (1.57) and (1.58) to the free Lagrangian in eq. (1.53). This yields,

$$\mathcal{L} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + \frac{1}{2} m^{ij} \psi_i \psi_j + \frac{1}{2} m^*_{ij} \psi^{\dagger i} \psi^{\dagger j} + \frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k + \frac{1}{2} y^*_{ijk} \phi^{*i} \psi^{\dagger j} \psi^{\dagger k} + V(\phi, \phi^*). \tag{1.59}$$

The Lagrangian in eq. (1.59) immediately reveals that, as expected by supersymmetry, the masses of the fermions and bosons in the same supermultiplet are identical. In order to incorporate gauge supermultiplets and consider the interactions between fermions and gauge bosons observed in the SM, the usual minimal coupling rule has to be applied, replacing $\partial_\mu$

† A holomorphic function is a complex-valued function in one or more complex variables that is complex differentiable in a neighbourhood for every point of its domain.

with $D_\mu$. This leads to equation of motions for the auxiliary fields $D^a$

$$D^a = -g(\phi^* T^a \phi), \tag{1.60}$$

where $T^a$ are the generators of the gauge group and $g$ is the coupling constant [3]. The potential then becomes

$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} \sum_a D^a D^a = W_i^* W^i + \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2, \tag{1.61}$$

where $a$ runs over the gauge groups that generally have differing gauge couplings [3, 4].

### 1.2.5 The Minimal Supersymmetric Standard Model

The MSSM is the simplest $N = 1$ supersymmetrisation of the SM in the sense that it introduces a minimal set of additional particles.

#### Particle content and interactions

The MSSM arranges all SM particles in one chiral (all the fermions and quarks) and one gauge (all spin-1 bosons) supermultiplet. As supersymmetric partners (*spartners*) have the same quantum numbers apart from spin, none of the SM particles can be spartners of each other. Thus, all spartners have to be new, unseen particles. Table 1.3 summaries the names, notations and spins of all spartners introduced in the MSSM. The naming convention is to prepend the names of the spartners of fermions with an 's' (e.g. *selectron*, *stop*, ...) and append '-ino' to the names of the spartners of the bosons (e.g. *Wino*, *Photino*, ...). Supersymmetric particles (*sparticles*) are generally denoted by adding a tilde to the symbol of SM particles (e.g. $\tilde{e}$, $\tilde{u}$, $\tilde{g}$).

An important detail to note is that right-handed and left-handed fermions get their own chiral supermultiplets and thus have distinct spartners, as otherwise the preference of the weak interaction for left-handed particles would be violated. For example, left-handed and right-handed quarks ($q_\mathrm{L}$, $q_\mathrm{R}$) get two different spartners ($\tilde{q}_\mathrm{L}$, $\tilde{q}_\mathrm{R}$), denoted with an index L and R. The index here refers to the handedness of the SM particle as scalar particles have only one helicity state. Additionally, the spartners of the left-handed and right-handed will mix to form physical mass eigenstates.

It is also worth asking why the spartners of SM particles are of lower spin in the first place, as e.g. spin-1 spartners of the SM fermions could also have been considered. The introduction of spin-1 bosons would entail the introduction of new gauge interactions, rendering the MSSM non-minimal [4]. Furthermore, introducing spartners with spin greater than 1 would make the resulting theory non-renormalisable [4].

In the MSSM, two Higgs doublets are needed in order to give masses to the up-type and down-type quarks via Yukawa couplings. A single Higgs field $h$ cannot be used for this as it would require Yukawa terms including the complex conjugate $h^*$, which is forbidden as the superpotential, being a holomorphic function of the fields, cannot depend on the complex conjugates of the same fields [4]. Additionally, the use of a single Higgs doublet would lead to gauge anomalies in the electroweak gauge symmetry [56]. Instead two complex Higgs doublets

**Table 1.3:** Particle content of the MSSM. The spin refers to the spin of the spartner. Adapted from [4].

| Particle | Spartner 0 | Spin |
|---|---|---|
| quarks $q$ | squarks $\tilde{q}$ | 0 |
| $\rightarrow$ top $t$ | stop $\tilde{t}$ | |
| $\rightarrow$ bottom $t$ | sbottom $\tilde{b}$ | |
| ... | | |
| leptons $\ell$ | sleptons $\tilde{\ell}$ | 0 |
| $\rightarrow$ electron $e$ | selectron $\tilde{e}$ | |
| $\rightarrow$ muon $\mu$ | smuon $\tilde{\mu}$ | |
| $\rightarrow$ tau $\tau$ | stau $\tilde{\tau}$ | |
| $\rightarrow$ neutrinos $\nu_\ell$ | stop $\tilde{\nu}_\ell$ | |
| gauge bosons | gauginos | 1/2 |
| $\rightarrow$ photon $\gamma$ | photino $\tilde{\gamma}$ | |
| $\rightarrow$ boson $Z$ | Zino $\tilde{Z}$ | |
| $\rightarrow$ boson $B$ | Bino $\tilde{B}$ | |
| $\rightarrow$ boson $W$ | Wino $\tilde{W}$ | |
| $\rightarrow$ gluon $g$ | gluino $\tilde{g}$ | |
| Higgs bosons $H_i^{\pm,0}$ | higgsinos $\tilde{H}_i^{\pm,0}$ | 1/2 |

with hypercharge $Y = \pm 1/2$ are used in the MSSM. The two Higgs doublets can be written as

$$H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}, \tag{1.62}$$

As illustrated in section 1.2.4 using the Wess–Zumino model, interactions are introduced using the superpotential. In the MSSM, the superpotential reads

$$W_{\text{MSSM}} = \bar{u}\boldsymbol{y_u}QH_u - \bar{d}\boldsymbol{y_d}QH_d - \bar{e}\boldsymbol{y_e}LH_d + \mu H_u H_d, \tag{1.63}$$

where $Q$ and $L$ correspond to the supermultiplets containing the left-handed quarks and leptons as well as their spartners, respectively. Likewise, $\bar{u}$, $\bar{d}$, $\bar{e}$ correspond to the supermultiplets containing the right-handed up-type quarks, down-type quarks and leptons as well as their spartners, respectively. The parameters $\boldsymbol{y_u}$, $\boldsymbol{y_d}$ and $\boldsymbol{y_e}$ are the $3 \times 3$ Yukawa coupling matrices. Except for the third generation, the Yukawa couplings are known to be relatively small [3] and are thus not of direct interest for the phenomenology of the theory. Phenomenologically more interesting are the supersymmetric gauge interactions that dominate the production and decay process of spartners in the MSSM [3]. The superpotential in eq. (1.63) illustrates again why two Higgs doublets are needed in the MSSM, since terms like $\bar{u}QH_d^*$ or $\bar{e}LH_u^*$ are not allowed due to the holomorphism of the superpotential. The term $\mu H_u H_d$ contains the *higgsino mass parameter* $\mu$ and is the supersymmetric version of the Higgs mass term in the SM Lagrangian.

### Soft supersymmetry breaking

As stated in section 1.2.2, all superpartners must have same quantum numbers apart from their spin. They especially also should have same masses, however such particles would have been discovered a long time ago and thus SUSY must be broken. Formally, SUSY should thus be an exact symmetry that is spontaneously broken because the Lagrangian has a symmetry under which the vacuum state is not invariant. However, if broken SUSY is still to provide a solution to the Hierarchy problem, i.e. cancel the quadratic divergencies in the loop corrections for the Higgs mass parameter, then the relations between the dimensionless couplings of the SM particles and their superpartners have to be maintained [3]. Hence, only symmetry breaking terms with positive mass dimension are allowed in the Lagrangian, especially also forbidding the presence of dimensionless SUSY-breaking couplings [3]. Such a breaking of SUSY is called *soft* breaking and can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}. \tag{1.64}$$

where, $\mathcal{L}_{\text{soft}}$ contains all the symmetry breaking terms while $\mathcal{L}_{\text{SUSY}}$ is the SUSY invariant Lagrangian with all the gauge and Yukawa interactions. In a softly broken SUSY, the loop corrections to the Higgs mass parameter depend quadratically on the largest mass scale associated with the soft terms ($m_{\text{soft}}$). As the fine-tuning problem reappears if $m_{\text{soft}}$ becomes too large, superpartners with masses not too far above the TeV scale are generally assumed [3].

A total of 105 new parameters with no counterpart in the SM are introduced through $\mathcal{L}_{\text{soft}}$ [3, 57]:

- gaugino mass parameters $M_1$, $M_2$ and $M_3$,
- trilinear scalar couplings, parametrised by $3 \times 3$ matrices in generation space $\boldsymbol{a_u}$, $\boldsymbol{a_d}$, $\boldsymbol{a_e}$, representing Higgs-squark-squark and Higgs-slepton-slepton interactions,
- Hermitian $3 \times 3$ matrices in generation space $\boldsymbol{m^2_Q}$, $\boldsymbol{m^2_{\bar{u}}}$, $\boldsymbol{m^2_{\bar{d}}}$, $\boldsymbol{m^2_L}$, $\boldsymbol{m^2_{\bar{e}}}$ that represent the sfermion masses,
- SUSY breaking parameters $m^2_{H_u}$, $m^2_{H_d}$ and $b$.

The sfermion mass matrices and the trilinear scalar couplings may introduce additional flavour mixing and CP violation, both of which are heavily constrained by experimental results. Flavour mixing in the lepton sector is for example constrained by an upper limit on $\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-12}$ [58]. Bounds on additional CP violation as well as squark mixing terms come from measurements of the electron and neutron electric moments and neutral meson systems[†] [**?** ]. Formally, in order to avoid these terms, SUSY breaking can be assumed to be *flavour-blind*, meaning that the mass matrices are approximately diagonal. The large Yukawa couplings for the third generation squarks and sfermions can then be achieved by assuming that the trilinear scalar couplings are proportional to the corresponding Yukawa coupling matrix [3].

As most of the parameters in the MSSM are related to soft SUSY breaking, it is not surprising that the phenomenology of the MSSM strongly depends on the exact breaking mechanism. The breaking is usually introduced to happen in a *hidden sector* and the effects of the breaking are then typically mediated by messenger particles from a messenger field to the *visible sector* containing all the particles of the MSSM. Since the hidden sector is assumed to be only coupled weakly or indirectly to the visible sector, the phenomenology mostly depends on the

† While it is theoretically possible to fine-tune the numerous phases in the MSSM such that cancelling contributions are generated, such possibilities will not be discussed in the following.

mechanism mediating the breaking. The two most popular mechanisms are *gravity-mediated* and *gauge-mediated* SUSY breaking.

Mediating SUSY breaking through gravity is an attractive approach, since all particles share gravitational interactions. This makes it easy to imagine gravitational effects to be the only connection between the hidden and the visible sectors. In such models SUSY breaking is mediated through effects of gravitational strength, suppressed by inverse powers of the Planck mass [7]. The mass of the gravitino—the spartner of the hypothetical mediator particle of gravity, called *graviton*—is typically of electroweak scale [59, 60]. Due to its couplings of gravitational strengths, it usually does not play a role in collider physics [7].

In gauge-mediated SUSY breaking (GMSB), additional messenger fields sharing gauge interactions with the MSSM fields are transmitting the breaking from the hidden to the visible sector. In such models, the gravitino is typically the LSP, as its mass ranges from a few eV to a few GeV, making it a candidate for DM [61].

### Mass spectrum

In the MSSM, electroweak symmetry breaking is generalised to the two Higgs doublets introduced in eq. (1.62). In total, the two doublets have eight degrees of freedom, three of which are used up to give masses to the $W^\pm$ and $Z$ bosons during the breaking of $SU(2)_\mathrm{L} \otimes U(1)_Y$ to $U(1)_\mathrm{em}$ (see section 1.1.2). Thus, five physical Higgs bosons appear in the MSSM; two neutral Higgs bosons even under CP transformation, called $h^0$ and $H^0$, one neutral Higgs boson odd under CP transformation, called $A^0$, and finally two charged Higgs bosons, called $H^\pm$. The two Higgs doublets $H_u$ and $H_d$ each get a VEV ($v_u$ and $v_d$, respectively) that are connected to the VEV $v$ of the SM Higgs field by

$$v_u^2 + v_d^2 = v^2. \tag{1.65}$$

Phenomenologically, the ratio of the two VEVs is usually considered, conventionally called $\tan\beta$,

$$\tan\beta = \frac{v_u}{v_d}. \tag{1.66}$$

Due to electroweak symmetry breaking, the gauginos and higgsinos are not mass eigenstates but mix to form states with definite mass, called *electroweakinos*:

- the two charged higgsinos mix with the two charged winos to form two charged mass eigenstates $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$, called *charginos*,
- the remaining neutral higgsinos mix with the photino, zino and bino to form four neutral mass eigenstates $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$, called *neutralinos*.

Both charginos and neutralinos are by convention labeled in ascending mass order. In the gauge-eigenstate basis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$, the neutralino mixing matrix reads [3]

$$\boldsymbol{M}_{\tilde{\chi}}^0 = \begin{pmatrix} M_1 & 0 & -g_1 v_d/\sqrt{2} & g_1 v_u/\sqrt{2} \\ 0 & M_2 & g_2 v_d/\sqrt{2} & -g_2 v_u/\sqrt{2} \\ -g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} & 0 & -\mu \\ g_1 v_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}, \tag{1.67}$$

where $M_1$ and $M_2$ stem directly from the soft SUSY breaking terms while the $-\mu$ terms are the higgsino mass terms. Entries with $g_1$ and $g_2$ come from Higgs-higgsino-gaugino couplings. The neutralino mixing matrix can be diagonalized to obtain the neutralino masses, which can be expressed in terms of the parameters $M_1$, $M_2$, $\mu$ and $\tan\beta$ [3]. As the exact forms of the mass expressions are relatively complicated [7], they are typically evaluated in limits where one of the mass parameters is significantly smaller than the other two. This is possible because $M_1$ and $M_2$ can be chosen to be real and positive through an appropriate phase redefinition of $\tilde{B}$ and $\tilde{W}$†. If neutralinos are dominated by the wino, bino or higgsino component, they are called wino-, bino- or higgsino-like, respectively, in the following.

The chargino mixing matrix can be written in a similar fashion. In the gauge-eigenstate $\psi^{\pm} = (\tilde{W}^{\pm}, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)$, it can be written as

$$\boldsymbol{M}_{\tilde{\chi}^{\pm}} = \begin{pmatrix} \mathbb{0}_2 & \boldsymbol{X}^T \\ \boldsymbol{X} & \mathbb{0}_2 \end{pmatrix} \qquad \text{with} \quad \boldsymbol{X} = \begin{pmatrix} M_2 & g_2 v_u \\ g_2 v_d & \mu \end{pmatrix}. \tag{1.68}$$

The masses of the charginos are then the eigenvalues of the doubly degenerate $4 \times 4$ matrix $\boldsymbol{M}^{\dagger}_{\tilde{\chi}^{\pm}} \boldsymbol{M}_{\tilde{\chi}^{\pm}}$ and can be expressed in terms of $M_2$, $\mu$ and $\sin 2\beta$ [3].

Squarks and sleptons also mix with each other. As in principle any scalars with same electric charge, colour charge and R-parity (see section 1.2.5) can mix with each other, the mass eigenstates of the sleptons and squarks should a priori be obtained through diagonalisation of three $6 \times 6$ mixing matrices (one for up-type squarks, one for down-type squarks and one for charged sleptons) and one $3 \times 3$ matrix (for sneutrinos). The assumption of flavour-blind soft SUSY breaking terms leads to most of the mixing angles being very small. As opposed to the first and second generation, the third generation sfermions have relatively large Yukawa couplings, therefore the superpartners of the left- and right-handed fermions mix to mass eigenstates $(\tilde{t}_1, \tilde{t}_2)$, $(\tilde{b}_1, \tilde{b}_2)$, $(\tilde{\tau}_1, \tilde{\tau}_2)$, again labeled in ascending mass order. The first and second generation sfermions, on the other hand, having very small Yukawa couplings, end up in nearly mass-degenerate, unmixed pairs.

The gluino, being the only colour octet fermion of the unbroken $SU(3)_C$ gauge group, cannot mix with another fermion and thus is a mass eigenstate with mass $m_{\tilde{g}} = |M_3|$ at tree level [3, 50].

### R-parity

The superpotential of the MSSM in principle allows additional gauge-invariant terms that are holomorphic in the chiral superfields but violate either lepton number (L) or baryon number (B). However, L- or B-violating processes have never been observed. Even worse, the L- and B-violating terms would cause a finite lifetime of the proton by allowing for it to decay e.g. via $p \to e^+\pi^0$, a process that is heavily constrained to have a lifetime longer than $1.6 \times 10^{34}$ years [62] as found by the Super-Kamiokande experiment.

In order to avoid these terms, a new symmetry, called *R-parity*, is introduced. R-parity is a multiplicatively conserved quantum number defined to be

$$P_R = (-1)^{3(B-L)+2s}, \tag{1.69}$$

† This makes the phase of $\mu$ in that convention a physical parameter that can no longer be rotated away through basis rotation.

where $s$ is the spin of the particle. Given this definition, all SM particles and the Higgs bosons have even R-parity ($P_R = +1$) while all spartners have odd R-parity ($P_R = -1$). Assuming R-parity to be exactly conserved at each vertex in the MSSM leads to a number of interesting phenomenological consequences:

- Sparticles are always produced in pairs.
- Heavier sparticles decay into lighter ones.
- The number of sparticles at each vertex must be even.
- The lightest supersymmetric particle (LSP) must be stable as it cannot decay any further without violating R-parity.

The nature of the LSP can be further constrained by cosmological observations [63]. If it were electrically charged or coupled to the strong interaction, it would have dissipated its energy and mixed with ordinary matter in the galactic disks where it would have formed anomalous heavy isotopes. Upper limits on such supersymmetric relics [64] thus heavily favour an electrically neutral and weakly interacting LSP. This excludes in particular the gluino as an LSP. Another possible LSP, the sneutrino, is ruled out by LEP and direct searches [65–67]. A gravitino LSP is especially attractive in gauge mediated theories.

Another promising option is a neutralino LSP. In large portions of the MSSM parameter space, a neutralino LSP produces a DM relic density that is compatible with the DM relic density measured by Planck [46, 64]. In the following, only R-parity conserving SUSY models with neutralino LSPs are considered.

### 1.2.6 The phenomenological MSSM

In addition to the 19 parameters of the SM, the MSSM adds a total of 105 additional parameters, too much to allow for a realistic exploration of the MSSM in a meaningful way. However, as already discussed in section 1.2.5, not all values of the 105 additional parameters lead to phenomenologically viable models. By requiring a set of phenomenological constraints, the 105 free parameters can be reduced to only 19 free parameters, spanning a model space called the phenomenological Minimal Supersymmetric Standard Model (pMSSM) [68, 69]. The free parameters in the pMSSM are listed in table 1.4.

The reduction of free parameters is obtained by applying the following constraints on the MSSM:

- No new source of CP violation, as discussed already in section 1.2.5, achieved by assuming all soft breaking parameters to be real.
- Minimal flavour violation, meaning that flavour-changing neutral currents (FCNCs), heavily constrained by experiment, are not allowed and the flavour physics is governed by the CKM matrix.
- First and second sfermion generations are mass-degenerate

**Table 1.4:** Parameters of the pMSSM.

| Parameter | Meaning |
|---|---|
| $\tan\beta$ | ratio of the Higgs doublet VEVs |
| $M_A$ | mass of the CP-odd Higgs boson |
| $\mu$ | Higgs-higgsino mass parameters |
| $M_1$, $M_2$, $M_3$ | wino, bino and gluino mass parameters |
| $m_{\tilde{q}}$, $m_{\tilde{u}_R}$, $m_{\tilde{d}_R}$, $m_{\tilde{\ell}}$, $m_{\tilde{e}_R}$ | first and second generation sfermion masses |
| $m_{\tilde{Q}}$, $m_{\tilde{t}_R}$, $m_{\tilde{b}_R}$, $m_{\tilde{L}}$, $m_{\tilde{\tau}_R}$ | third generation sfermion masses |
| $A_t$, $A_b$, $A_\tau$ | third generation trilinear couplings |

- The trilinear couplings and Yukawa couplings are negligible for the first and second sfermion generations.

The pMSSM does not make any assumptions on the physics above the TeV scale, and therefore does not assume a specific SUSY breaking mechanism. With its 19 free parameters, and the typical complexity of a search for SUSY, the pMSSM is still computationally extremely challenging to probe. Using appropriate approximations, the computational complexity can be simplified enough for exhaustive scans and comparisons to experimental data to become possible.

### 1.2.7 Simplified models

In searches for BSM physics at the Large Hadron Collider, it is common to use simplified models [70–72] as a way of reducing the available parameter space to a manageable level. Simplified models do not aim to represent complete supersymmetric models but are mostly defined by the empirical objects and kinematic variables used in the searches, typically allowing only a small number of sparticles to be involved in the decay chain (usually only two or three). Other sparticles are decoupled by setting their masses to be kinematically inaccessible at current collider experiments. The decay chains of the participating sparticles are determined by fixed branching ratios, often set to be 100%. Experimental bounds from non-observation of a given model are then usually presented in function of the physical masses of the sparticles involved in the decay chain. The model space spanned by the free parameters of the simplified model is typically called a *signal grid*, as each set of distinct mass parameter values, called *signal point*, occupies a single point in this space.

Simplified models have the inherent advantage that they circumvent the issue of having to search for SUSY in a vast parameter space where many of the parameters may only have small effects on observables. Their interpretation in terms of limits on individual SUSY production and decay topologies in function of sparticle masses is straightforward and very convenient. The hope is, that simplified models are a reasonable approximation of sizeable regions of parameter space of the more complete model they are embedded in [7]. The obvious downside is however, that the limits obtained in simplified models are not automatically a good approximation of the true underlying constraint on the respective model parameter when interpreted in more complete SUSY models. Often times, for example, the constraints set on sparticle masses in simplified models, significantly overestimate the true constraints obtained in more complex

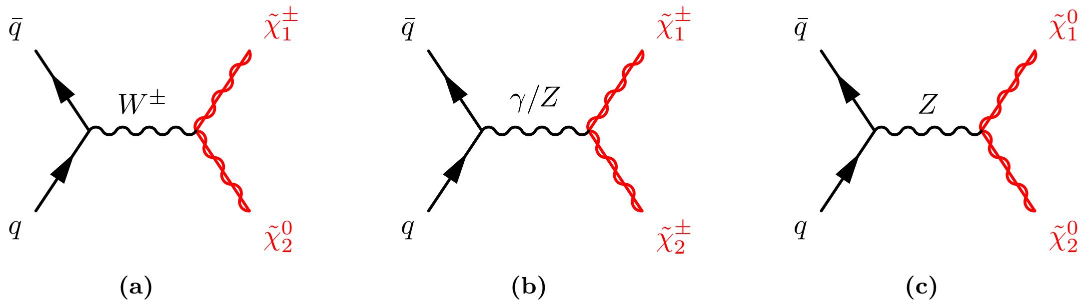


**Figure 1.7:** Dominant diagrams for production of electroweakino pairs at the Large Hadron Collider. Adapted from Ref. [3]

SUSY spectra, especially when the usual 100% branching fractions are assumed in the simplified models (see e.g. [73, 74]).

One way of circumventing these issues while sticking to the simplified model approach is to ensure that the limits obtained in different simplified models involving different production and decay mechanisms are combined into limits representing more complex SUSY spectra. In such an approach, the simplified model limits can be seen as building blocks for more complete and realistic SUSY models. Another possibility is to perform re-interpretations of SUSY searches—optimised for one ore more such simplified models—in more complete SUSY model spaces, like e.g. the pMSSM. This can not only demonstrate the sensitivity of existing SUSY searches beyond simplified models, but also potentially identify blind spots and model regions not covered by current searches. In addition, connections to (in)direct DM searches as well as SM measurements can be explored this way. Recent efforts in this direction include e.g. Refs. [73, 75, 76].

## 1.3 Search for electroweakinos

While both the ATLAS experiment [77] and CMS experiment [78] at the Large Hadron Collider at CERN set strong limits on the presence of gluinos and squarks at the TeV scale, the limits on electroweakinos are mostly still well below 1 TeV and thus offer ample space for SUSY to hide in. The reason for the relatively low limits on electroweakinos are the low cross-sections of electroweakino production, compared to those of squark and gluino production.

Apart from the electroweakino mass limits set by the current collider experiments, some additional limits from the LEP experiments are still relevant. Combining the results from all four LEP experiments leads to a general lower chargino mass limits of 103.5 GeV, except for corners of the phase space with low sneutrino mass [79]. For small mass splittings between the $\tilde{\chi}_1^\pm$ and the $\tilde{\chi}_1^0$, the lower limit is a little weaker with dedicated searches excluding charginos with $m(\tilde{\chi}_1^\pm) < 91.9$ GeV [79]. For the neutralino, a general lower limit on the lightest neutralino mass comes from limits on the invisible width of the $Z$ boson, excluding $m(\tilde{\chi}_1^0) < 45.5$ GeV$^\dagger$ [7].

† Depending on the coupling between the $Z$ boson and the lightest neutralino.

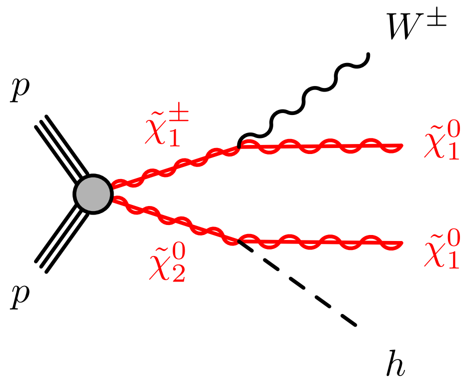


**Figure 1.8:** Diagram for $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ pair-production with subsequent decays into $\tilde{\chi}_1^{\pm} \to W^{\pm}\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0 \to h\tilde{\chi}_1^0$.

### 1.3.1 Production of electroweakinos at the Large Hadron Collider

If gluinos and squarks are heavier than a few TeV, i.e. too heavy to be within reach of the Large Hadron Collider, the direct production of electroweakinos might be the dominant production mode of SUSY. At hadron colliders, electroweakinos can be pair-produced directly via electroweak processes. The direct production of electroweakino pairs dominantly happens through electroweak gauge bosons from $s$-channel $q\bar{q}$ annihilation, as shown in fig. 1.7. Contributions from $t$-channels via squark exchange are typically of less importance [3].

### 1.3.2 Models used within this work

In SUSY scenarios where the sleptons and charged and pseudoscalar Higgs bosons are heavier than the charginos and neutralinos, a relatively pure wino lightest chargino decays predominantly through $\tilde{\chi}_1^{\pm} \to W^{\pm}\tilde{\chi}_1^0$, while the next-to-lightest neutralino decays via $\tilde{\chi}_2^0 \to Z/h\tilde{\chi}_1^0$. If, in addition, the higgsinos are much heavier than the wino, and the mass splitting between the two lightest neutralinos is larger than the Higgs boson mass, the decay $\tilde{\chi}_2^0 \to h\tilde{\chi}_1^0$ is the dominant decay mode of the $\tilde{\chi}_2^0$. In this case, both the $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ are wino-like and nearly mass-degenerate.

The main model used in the following is a simplified model considering direct production of a $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ pair where the lightest chargino decays via $\tilde{\chi}_1^{\pm} \to W^{\pm}\tilde{\chi}_1^0$ and the next-to-lightest neutralino decays via $\tilde{\chi}_2^0 \to h\tilde{\chi}_1^0$, each with 100% branching ratio. The lightest chargino $\tilde{\chi}_1^{\pm}$ and the next-to-lightest neutralino $\tilde{\chi}_2^0$ are assumed to be degenerate in mass and pure wino states, while the lightest neutralino $\tilde{\chi}_1^0$ is considered to be a pure bino lightest supersymmetric particle (LSP). The mass parameter hierarchy for this model thus is $|M_1| < |M_2| \ll |\mu|$.

The masses of $\tilde{\chi}_1^{\pm}/\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ are free parameters and are systematically varied, creating a two-dimensional signal grid to be scanned and compared to data. The Higgs boson mass is set to 125 GeV in accordance with the measured value [80, 81] and its branching ratios are the ones from the SM. Figure 1.8 shows an exemplary diagram for this simplified model.

In addition to the simplified model targeted by the SUSY search presented in the following, an additional class of models is considered in the second part of this work. These models are sampled directly from the pMSSM parameter space and are used to re-interpret the aforementioned SUSY search in the pMSSM. In accordance with the simplified model in fig. 1.8, the pMSSM models are sampled with a focus on electroweakinos, i.e. all electroweak parameters are set to be lighter than 2 TeV while first and second generation squarks are decoupled and set to have mass parameters of 10 TeV. Sleptons are also set to be decoupled with mass parameters fixed at 10 TeV. In order to yield a better rate of surviving models during the sampling without affecting the decays of the electroweakinos too much, third generation squark and gluino mass parameters are varied between 2–5 TeV and 1–5 TeV, respectively. No assumptions are made on the bino, wino or higgsino nature of either charginos or neutralinos. More details on the sampling of the pMSSM models are given in section 10.1.

# Abbreviations

**ΛCDM** Lambda Cold Dark Matter. 20

**BSM** beyond the Standard Model. 20, 21

**CKM** Cabibbo–Kobayashi–Maskawa. 8, 19, 32

**CMB** cosmic microwave background. 20

**DM** dark matter. 20, 21, 30, 32, 34

**EWK** electroweak. 7

**FCNC** flavour-changing neutral current. 32

**GUT** grand unified theory. 21, 22

**LEP** Large Electron Positron. 7, 32

**LSP** lightest supersymmetric particle. 35

**MSSM** Minimal Supersymmetric Standard Model. 21, 22, 27–32

**PMNS** Pontecorvo–Maki–Nakagawa–Sakata. 8, 19

**pMSSM** phenomenological Minimal Supersymmetric Standard Model. 32–36

**QCD** quantum chromodynamics. 10, 13–15, 19, 23

**QED** quantum electrodynamics. 7, 11–14, 16, 23

**QFT** quantum field theory. 9, 10, 24

**SM** Standard Model of Particle Physics. 7, 8, 10, 16, 19–23, 25, 27, 34, 35

**SUSY** Supersymmetry. 20, 23, 25

**VEV** vacuum expectation value. 16–18, 30, 33

**WIMP** weakly interacting massive particle. 21

# Bibliography


[1] I. C. Brock and T. Schorner-Sadenius, *Physics at the terascale*. Wiley, Weinheim, 2011. https://cds.cern.ch/record/1354959.

[2] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995. http://www.slac.stanford.edu/~mpeskin/QFT.html.

[3] S. P. Martin, "A Supersymmetry primer," arXiv:hep-ph/9709356 [hep-ph]. [Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[4] M. Bustamante, L. Cieri, and J. Ellis, "Beyond the Standard Model for Montaneros," in *5th CERN - Latin American School of High-Energy Physics*. 11, 2009. arXiv:0911.4409 [hep-ph].

[5] L. Brown, *The Birth of particle physics*. Cambridge University Press, Cambridge Cambridgeshire New York, 1986.

[6] P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA Recommended Values of the Fundamental Physical Constants: 2014," *Rev. Mod. Phys.* **88** no. 3, (2016) 035009, arXiv:1507.07956 [physics.atom-ph].

[7] P. D. Group, "Review of Particle Physics," *Progress of Theoretical and Experimental Physics* **2020** no. 8, (08, 2020) , https://academic.oup.com/ptep/article-pdf/2020/8/083C01/34673722/ptaa104.pdf. https://doi.org/10.1093/ptep/ptaa104. 083C01.

[8] **Super-Kamiokande** Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, arXiv:hep-ex/9807003 [hep-ex].

[9] Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880. [,34(1962)].

[10] N. Cabibbo, "Unitary symmetry and leptonic decays," *Phys. Rev. Lett.* **10** (Jun, 1963) 531–533. https://link.aps.org/doi/10.1103/PhysRevLett.10.531.

[11] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog. Theor. Phys.* **49** (1973) 652–657.

[12] E. Noether and M. A. Tavel, "Invariant variation problems," arXiv:physics/0503066.

[13] J. C. Ward, "An identity in quantum electrodynamics," *Phys. Rev.* **78** (Apr, 1950) 182–182. https://link.aps.org/doi/10.1103/PhysRev.78.182.

[14] Y. Takahashi, "On the generalized ward identity," *Il Nuovo Cimento (1955-1965)* **6** no. 2, (Aug, 1957) 371–375. https://doi.org/10.1007/BF02832514.

[15] G. 'tHooft, "Renormalization of massless yang-mills fields," *Nuclear Physics B* **33** no. 1, (1971) 173 – 199. http://www.sciencedirect.com/science/article/pii/0550321371903956.

[16] J. Taylor, "Ward identities and charge renormalization of the yang-mills field," *Nuclear Physics B* **33** no. 2, (1971) 436 – 444. http://www.sciencedirect.com/science/article/pii/0550321371902975.

[17] A. A. Slavnov, "Ward identities in gauge theories," *Theoretical and Mathematical Physics* **10** no. 2, (Feb, 1972) 99–104. https://doi.org/10.1007/BF01090719.

[18] C. N. Yang and R. L. Mills, "Conservation of isotopic spin and isotopic gauge invariance," *Phys. Rev.* **96** (Oct, 1954) 191–195. https://link.aps.org/doi/10.1103/PhysRev.96.191.

[19] K. G. Wilson, "Confinement of quarks," *Phys. Rev. D* **10** (Oct, 1974) 2445–2459. https://link.aps.org/doi/10.1103/PhysRevD.10.2445.

[20] T. DeGrand and C. DeTar, *Lattice Methods for Quantum Chromodynamics*. World Scientific, Singapore, 2006. https://cds.cern.ch/record/1055545.

[21] S. L. Glashow, "Partial-symmetries of weak interactions," *Nuclear Physics* **22** no. 4, (1961) 579 – 588. http://www.sciencedirect.com/science/article/pii/0029558261904692.

[22] S. Weinberg, "A model of leptons," *Phys. Rev. Lett.* **19** (Nov, 1967) 1264–1266. https://link.aps.org/doi/10.1103/PhysRevLett.19.1264.

[23] A. Salam and J. C. Ward, "Weak and electromagnetic interactions," *Il Nuovo Cimento (1955-1965)* **11** no. 4, (Feb, 1959) 568–577. https://doi.org/10.1007/BF02726525.

[24] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, "Experimental test of parity conservation in beta decay," *Phys. Rev.* **105** (Feb, 1957) 1413–1415. https://link.aps.org/doi/10.1103/PhysRev.105.1413.

[25] M. Gell-Mann, "The interpretation of the new particles as displaced charge multiplets," *Il Nuovo Cimento (1955-1965)* **4** no. 2, (Apr, 1956) 848–866. https://doi.org/10.1007/BF02748000.

[26] K. Nishijima, "Charge Independence Theory of V Particles*," *Progress of Theoretical Physics* **13** no. 3, (03, 1955) 285–304, `https://academic.oup.com/ptp/article-pdf/13/3/285/5425869/13-3-285.pdf`. https://doi.org/10.1143/PTP.13.285.

[27] T. Nakano and K. Nishijima, "Charge Independence for V-particles*," *Progress of Theoretical Physics* **10** no. 5, (11, 1953) 581–582, `https://academic.oup.com/ptp/article-pdf/10/5/581/5364926/10-5-581.pdf`. https://doi.org/10.1143/PTP.10.581.

[28] F. Englert and R. Brout, "Broken symmetry and the mass of gauge vector mesons," *Phys. Rev. Lett.* **13** (Aug, 1964) 321–323. https://link.aps.org/doi/10.1103/PhysRevLett.13.321.

[29] P. W. Higgs, "Broken symmetries and the masses of gauge bosons," *Phys. Rev. Lett.* **13** (Oct, 1964) 508–509. https://link.aps.org/doi/10.1103/PhysRevLett.13.508.

[30] P. W. Higgs, "Spontaneous symmetry breakdown without massless bosons," *Phys. Rev.* **145** (May, 1966) 1156–1163. https://link.aps.org/doi/10.1103/PhysRev.145.1156.

[31] Y. Nambu, "Quasiparticles and Gauge Invariance in the Theory of Superconductivity," *Phys. Rev.* **117** (1960) 648–663. [,132(1960)].

[32] J. Goldstone, "Field Theories with Superconductor Solutions," *Nuovo Cim.* **19** (1961) 154–164.

[33] V. Brdar, A. J. Helmboldt, S. Iwamoto, and K. Schmitz, "Type-I Seesaw as the Common Origin of Neutrino Mass, Baryon Asymmetry, and the Electroweak Scale," *Phys. Rev. D* **100** (2019) 075029, `arXiv:1905.12634 [hep-ph]`.

[34] G. 't Hooft and M. Veltman, "Regularization and renormalization of gauge fields," *Nuclear Physics B* **44** no. 1, (1972) 189 – 213. http://www.sciencedirect.com/science/article/pii/0550321372902799.

[35] F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln," *Helv. Phys. Acta* **6** (1933) 110–127. https://cds.cern.ch/record/437297.

[36] V. C. Rubin and W. K. Ford, Jr., "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions," *Astrophys. J.* **159** (1970) 379–403.

[37] G. Bertone, D. Hooper, and J. Silk, "Particle dark matter: Evidence, candidates and constraints," *Phys. Rept.* **405** (2005) 279–390, `arXiv:hep-ph/0404175`.

[38] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, "A direct empirical proof of the existence of dark matter," *Astrophys. J.* **648** (2006) L109–L113, `arXiv:astro-ph/0608407 [astro-ph]`.

[39] A. Taylor, S. Dye, T. J. Broadhurst, N. Benitez, and E. van Kampen, "Gravitational lens magnification and the mass of abell 1689," *Astrophys. J.* **501** (1998) 539, `arXiv:astro-ph/9801158`.

[40] C. Bennett *et al.*, "Four year COBE DMR cosmic microwave background observations: Maps and basic results," *Astrophys. J. Lett.* **464** (1996) L1–L4, `arXiv:astro-ph/9601067`.

[41] G. F. Smoot *et al.*, "Structure in the COBE Differential Microwave Radiometer First-Year Maps," *ApJS* **396** (September, 1992) L1.

[42] **WMAP** Collaboration, "Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results," *ApJS* **208** no. 2, (October, 2013) 20, `arXiv:1212.5225 [astro-ph.CO]`.

[43] **WMAP** Collaboration, "Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," *ApJS* **208** no. 2, (October, 2013) 19, `arXiv:1212.5226 [astro-ph.CO]`.

[44] **Planck** Collaboration, "Planck 2018 results. I. Overview and the cosmological legacy of Planck," *Astron. Astrophys.* **641** (2020) A1, `arXiv:1807.06205 [astro-ph.CO]`.

[45] A. Liddle, *An introduction to modern cosmology; 3rd ed.* Wiley, Chichester, Mar, 2015. https://cds.cern.ch/record/1976476.

[46] **Planck** Collaboration, "Planck 2018 results. VI. Cosmological parameters," *Astron. Astrophys.* **641** (2020) A6, `arXiv:1807.06209 [astro-ph.CO]`.

[47] H. Georgi and S. L. Glashow, "Unity of all elementary-particle forces," *Phys. Rev. Lett.* **32** (Feb, 1974) 438–441. https://link.aps.org/doi/10.1103/PhysRevLett.32.438.

[48] I. Aitchison, *Supersymmetry in Particle Physics. An Elementary Introduction*. Cambridge University Press, Cambridge, 2007.

[49] **Muon g-2** Collaboration, G. Bennett *et al.*, "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," *Phys. Rev. D* **73** (2006) 072003, `arXiv:hep-ex/0602035`.

[50] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events*. Cambridge University Press, 2006.

[51] A. Czarnecki and W. J. Marciano, "The Muon anomalous magnetic moment: A Harbinger for 'new physics'," *Phys. Rev. D* **64** (2001) 013014, `arXiv:hep-ph/0102122`.

[52] J. L. Feng and K. T. Matchev, "Supersymmetry and the anomalous magnetic moment of the muon," *Phys. Rev. Lett.* **86** (2001) 3480–3483, `arXiv:hep-ph/0102146`.

[53] S. Coleman and J. Mandula, "All possible symmetries of the $s$ matrix," *Phys. Rev.* **159** (Jul, 1967) 1251–1256. https://link.aps.org/doi/10.1103/PhysRev.159.1251.

[54] R. Haag, J. T. Lopuszanski, and M. Sohnius, "All Possible Generators of Supersymmetries of the s Matrix," *Nucl. Phys.* **B88** (1975) 257. [,257(1974)].

[55] J. Wess and B. Zumino, "Supergauge transformations in four dimensions," *Nuclear Physics B* **70** no. 1, (1974) 39 – 50. http://www.sciencedirect.com/science/article/pii/0550321374903551.

[56] H. Georgi and S. L. Glashow, "Gauge theories without anomalies," *Phys. Rev. D* **6** (Jul, 1972) 429–431. https://link.aps.org/doi/10.1103/PhysRevD.6.429.

[57] S. Dimopoulos and D. W. Sutter, "The Supersymmetric flavor problem," *Nucl. Phys. B* **452** (1995) 496–512, `arXiv:hep-ph/9504415`.

[58] **MEG** Collaboration, T. Mori, "Final Results of the MEG Experiment," *Nuovo Cim. C* **39** no. 4, (2017) 325, `arXiv:1606.08168 [hep-ex]`.

[59] H. P. Nilles, "Supersymmetry, Supergravity and Particle Physics," *Phys. Rept.* **110** (1984) 1–162.

[60] A. Lahanas and D. Nanopoulos, "The road to no-scale supergravity," *Physics Reports* **145** no. 1, (1987) 1 – 139. http://www.sciencedirect.com/science/article/pii/0370157387900342.

[61] J. L. Feng, A. Rajaraman, and F. Takayama, "Superweakly interacting massive particles," *Phys. Rev. Lett.* **91** (2003) 011302, `arXiv:hep-ph/0302215`.

[62] **Super-Kamiokande** Collaboration, K. Abe *et al.*, "Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in 0.31 megaton-years exposure of the Super-Kamiokande water Cherenkov detector," *Phys. Rev.* **D95** no. 1, (2017) 012004, `arXiv:1610.03597 [hep-ex]`.

[63] J. R. Ellis, "Beyond the standard model for hill walkers," in *1998 European School of High-Energy Physics*, pp. 133–196. 8, 1998. `arXiv:hep-ph/9812235`.

[64] J. R. Ellis, J. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, "Supersymmetric Relics from the Big Bang," *Nucl. Phys. B* **238** (1984) 453–476.

[65] D. O. Caldwell, R. M. Eisberg, D. M. Grumm, M. S. Witherell, B. Sadoulet, F. S. Goulding, and A. R. Smith, "Laboratory limits on galactic cold dark matter," *Phys. Rev. Lett.* **61** (Aug, 1988) 510–513. https://link.aps.org/doi/10.1103/PhysRevLett.61.510.

[66] M. Mori, M. M. Nojiri, K. S. Hirata, K. Kihara, Y. Oyama, A. Suzuki, K. Takahashi, M. Yamada, H. Takei, M. Koga, K. Miyano, H. Miyata, Y. Fukuda, T. Hayakawa, K. Inoue, T. Ishida, T. Kajita, Y. Koshio, M. Nakahata, K. Nakamura, A. Sakai, N. Sato, M. Shiozawa, J. Suzuki, Y. Suzuki, Y. Totsuka, M. Koshiba, K. Nishijima, T. Kajimura, T. Suda, A. T. Suzuki, T. Hara, Y. Nagashima, M. Takita, H. Yokoyama, A. Yoshimoto, K. Kaneyuki, Y. Takeuchi, T. Tanimori, S. Tasaka, and K. Nishikawa, "Search for neutralino dark matter heavier than the $w$ boson at kamiokande," *Phys. Rev. D* **48** (Dec, 1993) 5505–5518. https://link.aps.org/doi/10.1103/PhysRevD.48.5505.

[67] **CDMS** Collaboration, D. S. Akerib *et al.*, "Exclusion limits on the WIMP-nucleon cross section from the first run of the Cryogenic Dark Matter Search in the Soudan Underground Laboratory," *Phys. Rev. D* **72** (2005) 052009, `arXiv:astro-ph/0507190`.

[68] A. Djouadi, J.-L. Kneur, and G. Moultaka, "SuSpect: A Fortran code for the supersymmetric and Higgs particle spectrum in the MSSM," *Comput. Phys. Commun.* **176** (2007) 426–455, `arXiv:hep-ph/0211331`.

[69] C. F. Berger, J. S. Gainer, J. L. Hewett, and T. G. Rizzo, "Supersymmetry without prejudice," *Journal of High Energy Physics* **2009** no. 02, (Feb, 2009) 023–023. http://dx.doi.org/10.1088/1126-6708/2009/02/023.

[70] J. Alwall, P. Schuster, and N. Toro, "Simplified Models for a First Characterization of New Physics at the LHC," *Phys. Rev.* **D79** (2009) 075020, `arXiv:0810.3921 [hep-ph]`.

[71] **LHC New Physics Working Group** Collaboration, D. Alves, "Simplified Models for LHC New Physics Searches," *J. Phys.* **G39** (2012) 105005, `arXiv:1105.2838 [hep-ph]`.

[72] D. S. Alves, E. Izaguirre, and J. G. Wacker, "Where the Sidewalk Ends: Jets and Missing Energy Search Strategies for the 7 TeV LHC," *JHEP* **10** (2011) 012, `arXiv:1102.5338 [hep-ph]`.

[73] F. Ambrogi, S. Kraml, S. Kulkarni, U. Laa, A. Lessa, and W. Waltenberger, "On the coverage of the pMSSM by simplified model results," *Eur. Phys. J. C* **78** no. 3, (2018) 215, `arXiv:1707.09036 [hep-ph]`.

[74] O. Buchmueller and J. Marrouche, "Universal mass limits on gluino and third-generation squarks in the context of Natural-like SUSY spectra," *Int. J. Mod. Phys. A* **29** no. 06, (2014) 1450032, `arXiv:1304.2185 [hep-ph]`.

[75] **ATLAS** Collaboration, M. Aaboud *et al.*, "Dark matter interpretations of ATLAS searches for the electroweak production of supersymmetric particles in $\sqrt{s} = 8$ TeV proton-proton collisions," *JHEP* **09** (2016) 175, `arXiv:1608.00872 [hep-ex]`.

[76] **ATLAS** Collaboration, "Summary of the ATLAS experiment's sensitivity to supersymmetry after LHC Run 1 — interpreted in the phenomenological MSSM," *JHEP* **10** (2015) 134, `arXiv:1508.06608 [hep-ex]`.

[77] **ATLAS** Collaboration, "Mass reach of the atlas searches for supersymmetry." https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/ATL-PHYS-PUB-2020-020/fig_23.png, 2020.

[78] **CMS** Collaboration, "Summary plot moriond 2017." https://twiki.cern.ch/twiki/pub/CMSPublic/SUSYSummary2017/Moriond2017_BarPlot.pdf, 2017.

[79] L. S. W. Group, "Notes lepsusywg/02-04.1 and lepsusywg/01-03.1." http://lepsusy.web.cern.ch/lepsusy/, 2004. Accessed: 2021-02-11.

[80] **ATLAS** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys. Lett. B* **716** (2012) 1–29, `arXiv:1207.7214 [hep-ex]`.

[81] **CMS** Collaboration, S. Chatrchyan *et al.*, "Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC," *Phys. Lett. B* **716** (2012) 30–61, `arXiv:1207.7235 [hep-ex]`.

[82] CERN, "About cern." https://home.cern/about. Accessed: 2021-01-21.

[83] L. R. Evans and P. Bryant, "LHC Machine," *JINST* **3** (2008) S08001. 164 p. http://cds.cern.ch/record/1129806. This report is an abridged version of the LHC Design Report (CERN-2004-003).

[84] R. Scrivens, M. Kronberger, D. Küchler, J. Lettry, C. Mastrostefano, O. Midttun, M. O'Neil, H. Pereira, and C. Schmitzer, "Overview of the status and developments on primary ion sources at CERN*,". https://cds.cern.ch/record/1382102.

[85] E. Mobs, "The CERN accelerator complex - 2019. Complexe des accélérateurs du CERN - 2019,". https://cds.cern.ch/record/2684277. General Photo.

[86] **ATLAS** Collaboration, "The ATLAS Experiment at the CERN Large Hadron Collider," *JINST* **3** (2008) S08003.

[87] **CMS** Collaboration, S. Chatrchyan *et al.*, "The CMS Experiment at the CERN LHC," *JINST* **3** (2008) S08004.

[88] **ALICE** Collaboration, K. Aamodt *et al.*, "The ALICE experiment at the CERN LHC," *JINST* **3** (2008) S08002.

[89] **LHCb** Collaboration, J. Alves, A.Augusto *et al.*, "The LHCb Detector at the LHC," *JINST* **3** (2008) S08005.

[90] **TOTEM** Collaboration, G. Anelli *et al.*, "The TOTEM experiment at the CERN Large Hadron Collider," *JINST* **3** (2008) S08007.

[91] **LHCf** Collaboration, O. Adriani *et al.*, "Technical design report of the LHCf experiment: Measurement of photons and neutral pions in the very forward region of LHC,".

[92] **MoEDAL** Collaboration, J. Pinfold *et al.*, "Technical Design Report of the MoEDAL Experiment,".

[93] O. S. Bruning, P. Collier, P. Lebrun, S. Myers, R. Ostojic, J. Poole, and P. Proudlock, *LHC Design Report*. CERN Yellow Reports: Monographs. CERN, Geneva, 2004. https://cds.cern.ch/record/782076.