(1.) $S'_{ji} = \sum_{p,q} \alpha_{jp}\alpha_{iq} S_{pq} = \sum_{p,q} \alpha_{ip}\alpha_{jq} S_{qp} = \sum_{p,q} \alpha_{ip}\alpha_{jq} S_{pq} = S'_{ij}$

(2.) $\Pi_{ij} = \sum_{p,q} \alpha_{ip}\alpha_{jq}\Pi_{pq} = \sum_{p} \alpha_{ip}\left(\sum_{q} \Pi_{pq}\alpha^{T}_{qj}\right) =$

$= \sum_{p} \alpha_{ip}(\Pi\alpha^{T})_{pj} = \alpha\Pi\alpha^{T}$

(3.)

$$\varepsilon'_{ij} = \alpha\varepsilon_{ij}\alpha^{T} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} =$$

$$= \begin{pmatrix} \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \\ -\sin^2\theta - \cos^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \varepsilon_{ij}$$

(4.) 1) Объект C имеет n^4 компонент

$$C'_{ijkl} = A'_{ij}B'_{kl} = \left(\sum_{p}\sum_{q} \alpha_{ip}\alpha_{iq}A_{pq}\right)\left(\sum_{r}\sum_{s} \alpha_{kr}\alpha_{ls}B_{rs}\right) =$$

$$= \sum_{p,q,r,s} \alpha_{ip}\alpha_{jq}\alpha_{kr}\alpha_{ls}(A_{pq}B_{rs}) = \sum_{p,q,r,s} \alpha_{ip}\alpha_{jq}\alpha_{kr}\alpha_{ls}C_{pqrs}$$

2) $D'_{il} = \sum_{j} C'_{ijjl} = \sum_{j}\sum_{p,q,r,s} \alpha_{ip}\alpha_{jq}\alpha_{jr}\alpha_{ls}C_{pqrs} = \sum_{p,s} \alpha_{ip}\alpha_{ls}\left[\sum_{q,r}\left(\sum_{j} \alpha^{T}_{qj}\alpha_{jr}\right)C_{pqrs}\right.$

$= \sum_{p,s} \alpha_{ip}\alpha_{ls}\left(\sum_{q,r} \delta_{qr}C_{pqrs}\right) = \sum_{p,q} \alpha_{ip}\alpha_{lq}\left(\sum_{j} C_{pjjq}\right) = \sum_{p,q} \alpha_{ip}\alpha_{lq}D_{pq}$

3) $D' = \sum_{i} D'_{ii} = \sum_{i}\sum_{p,q} \alpha_{ip}\alpha_{iq}D_{pq} = \sum_{p,q}\left(\sum_{i} \alpha^{T}_{pi}\alpha_{iq}\right)D_{pq} =$

$= \sum_{p,q} \delta_{pq}D_{pq} = \sum_{i} D_{ii} = D$

(5.) $D'_{ij} = \left(\dfrac{\partial^2\varphi}{\partial x_i\,\partial x_j}\right)' = \left(\sum_{p} \alpha_{ip}\dfrac{\partial}{\partial x_p}\right)\left(\sum_{q} \alpha_{jq}\dfrac{\partial}{\partial x_q}\right)\varphi =$

$= \sum_{p,q} \alpha_{ip}\alpha_{jq}\dfrac{\partial^2\varphi}{\partial x_p\,\partial x_q} = \sum_{p,q} \alpha_{ip}\alpha_{jq}D_{ij}$

(6.) $\omega_k = \frac{1}{2} \sum_{i,j} A_{ij} \varepsilon_{ijk}$

$$\omega'_k = \frac{1}{2} \sum_{i,j} A'_{ij} \varepsilon'_{ijk} = \frac{1}{2} \sum_{i,j} \sum_{p,q,r,s,t} \alpha_{ip}\alpha_{jq}\alpha_{kr}\alpha_{js}\alpha_{kt} A_{pq} \varepsilon_{rst} =$$

$$= \frac{1}{2} \sum_{p,q,r,s,t} \Big(\sum_i \alpha^T_{pi} \alpha_{ir} \Big) \Big(\sum_j \alpha^T_{qj} \alpha_{js} \Big) \alpha_{kt} A_{pq} \varepsilon_{rst} =$$

$$= \frac{1}{2} \sum_{p,q,r,s,t} \delta_{pq} \delta_{qs} \alpha_{kt} A_{pq} \varepsilon_{rst} = \frac{1}{2} \sum_{p,q} \sum_t \alpha_{kt} A_{pq} \varepsilon_{pqt} =$$

$$= \sum_p \alpha_{kp} \Big(\frac{1}{2} \sum_{i,j} A_{ij} \varepsilon_{ijp} \Big).$$

(7.) 1) $\sum_j A_{ij} \delta_{jk} = A_{ik}$

$\sum_i A_{ij} \delta_{ik} = A_{kj}$

$\sum_{i,j} A_{ij} \delta_{ij} = \sum_i A_{ii}$

2) $\sum_k \delta_{ik} \delta_{kj} = \delta_{ij}$

$\sum_{i,k} \delta_{ik} \delta_{ik} = n$, где n - размерность

$\sum_{i,k} \delta_{ik} \delta_{ki} = n$

(8.)

$$\delta_{il}\delta_{jm} = \begin{cases} 1, & ijlm \in \{1111, 1212, 2121, 2222\} \\ 0, & \text{в остальных случаях} \end{cases}$$

$$\delta_{im}\delta_{jl} = \begin{cases} 1, & ijlm \in \{1111, 1221, 2112, 2222\} \\ 0, & \text{в остальных случаях} \end{cases}$$

$$\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = \begin{cases} 1, & ijlm \in \{1212, 2121\} \\ -1, & ijlm \in \{1221, 2112\} \\ 0, & \text{в остальных случаях} \end{cases} = \varepsilon_{ij}\varepsilon_{lm}$$

(9.) 1) $\varepsilon_{ijk}\varepsilon_{lmn} = \delta_{il}\delta_{jm}\delta_{kn} - \delta_{il}\delta_{jn}\delta_{km} - \delta_{im}\delta_{jl}\delta_{kn} +$
$+ \delta_{im}\delta_{jn}\delta_{kl} + \delta_{in}\delta_{jl}\delta_{km} - \delta_{in}\delta_{jm}\delta_{kl}$

2) $\sum_k \varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

3) $\sum_{j,k} \varepsilon_{ijk}\varepsilon_{ljk} = 2\delta_{il}$

(10.) $\sum_{i,j} A_{ij} S_{ij} = \sum_{i,j} A'_{ij} S'_{ij} = \sum_{i,j} \sum_{p,q,r,s} \alpha_{ip} \alpha_{jq} \alpha_{ir} \alpha_{js} A_{pq} S_{rs} =$

$= \sum_{p,q,r,s} \left[\sum_{i,j} (\alpha^T_{pi} \alpha_{ir})(\alpha^T_{qj} \alpha_{js}) \right] A_{pq} S_{rs} = \sum_{p,q,r,s} \delta_{pr} \delta_{qs} A_{pq} S_{rs} =$

$= \sum_{p,q} A_{pq} S_{pq} = - \sum_{p,q} A_{qp} S_{qp} = - \sum_{i,j} A_{ij} S_{ij} \Longrightarrow$

$\Longrightarrow \sum_{i,j} A_{ij} S_{ij} = + \left(\sum_{i,j} A_{ij} S_{ij} \right)' \equiv 0.$