(1.) $\oint_\Sigma \vec{E}\cdot d\vec{S} = 4\pi Q$

$E_r \cdot 2\pi r \cdot z = 4\pi\rho \cdot \pi r^2 \cdot z \implies 2\pi z\, d(rE_r) = 2\pi z \cdot 4\pi\rho \cdot r\, dr$

$\frac{1}{r}\frac{d}{dr}(rE_r) = \nabla\cdot\vec{E} \implies \frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right) = \Delta\varphi$

$\implies \Delta f = \frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)$

(2.) $\vec{H} = \nabla\times\vec{A} = \nabla\times\left(\frac{e}{c}\frac{\vec{v}}{R-\frac{\vec{v}\cdot\vec{R}}{c}}\right) = \frac{e\gamma^2}{cR^2}\left[\frac{R}{\gamma}\nabla\times\vec{v} - \nabla\left(R-\frac{\vec{v}\cdot\vec{R}}{c}\right)\times\vec{v}\right] =$

$= \frac{e\gamma^2}{cR^2}\left\{\frac{R}{\gamma}\nabla t'\times\dot{\vec{v}} - \left[\vec{n}\gamma\left(1-\frac{v^2}{c^2}\right) - \frac{\vec{v}}{c} + \gamma\frac{\vec{n}(\vec{R}\cdot\dot{\vec{v}})}{c^2}\right]\times\vec{v}\right\} =$

$= \vec{n}\times\frac{e\gamma^3}{R}\left\{\frac{1}{c^2}\left[\frac{\vec{n}\cdot\dot{\vec{v}}}{c}\vec{v} - \left(1-\frac{\vec{n}\cdot\vec{v}}{c}\right)\dot{\vec{v}}\right] - \frac{1}{R}\left(1-\frac{v^2}{c^2}\right)\frac{\vec{v}}{c}\right\} =$

$= \vec{n}\times\frac{e\gamma^3}{R}\left\{\frac{1}{c^2}\vec{n}\times\left[\left(\vec{n}-\frac{\vec{v}}{c}\right)\times\dot{\vec{v}}\right] + \frac{1}{R}\left(\vec{n}-\frac{\vec{v}}{c}\right)\left(1-\frac{v^2}{c^2}\right)\right\} = \vec{n}\times\vec{E}$

(3.) $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} = \frac{c}{4\pi}\vec{E}\times(\vec{n}\times\vec{E}) = \frac{c}{4\pi}\left[\vec{n}|\vec{E}|^2 - \vec{E}(\vec{n}\cdot\vec{E})\right]$

$\begin{cases} v \ll c \\ R\to\infty \end{cases} \implies \vec{E} \simeq \frac{e}{c^2R}\vec{n}\times(\vec{n}\times\dot{\vec{v}}) = \frac{e}{c^2R}\left[\vec{n}(\vec{n}\cdot\dot{\vec{v}}) - \dot{\vec{v}}\right]$

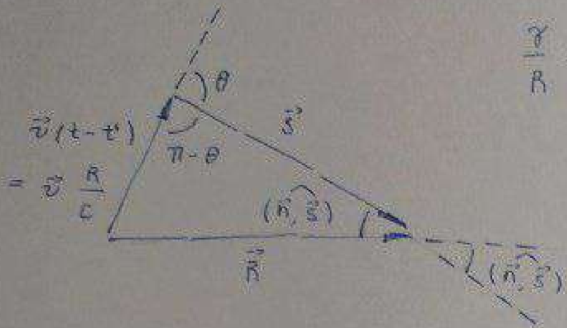
$\implies \begin{cases} |\vec{E}|^2 = \frac{e^2}{c^4R^2}\left[|\dot{\vec{v}}|^2 - (\vec{n}\cdot\dot{\vec{v}})^2\right] \\ \vec{n}\cdot\vec{E} \equiv 0 \end{cases} \implies \vec{S} = \frac{e^2}{c^3}\frac{1}{4\pi R^2}\left[|\dot{\vec{v}}|^2 - (\vec{n}\cdot\dot{\vec{v}})^2\right]\vec{n}$

(4.) $\dot{\vec{v}} = \begin{pmatrix} \pm\frac{4v_0}{T} \\ 0 \\ 0 \end{pmatrix}$ $\quad \begin{cases} v \ll c \\ r\to\infty \end{cases} \implies \vec{S} \simeq \frac{e^2}{c^3}\frac{1}{4\pi r^2}\left(\frac{4v_0}{T}\right)^2(1-\cos^2\varphi\sin^2\theta)\,\vec{n}$

$\langle P\rangle_T = \frac{e^2}{c^3}\frac{1}{4\pi}\left(\frac{4v_0}{T}\right)^2\int_0^\pi\int_0^{2\pi}(1-\cos^2\varphi\sin^2\theta)\sin\theta\, d\varphi\, d\theta = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{4v_0}{T}\right)^2$

(5.) $\vec v = \overrightarrow{const} \Longrightarrow \vec E = \frac{e\gamma^3}{R^2}\left(\vec n - \frac{\vec v}{c}\right)\left(1-\frac{v^2}{c^2}\right)\Big|_{t=t'}$

$$\vec s = \vec r - \vec r_0(t) = \vec r - \vec r_0(t') - \vec v(t-t') = \vec R - \vec v \frac{R}{c} = R\left(\vec n - \frac{\vec v}{c}\right)$$



$$\frac{\gamma}{R} = \frac{1}{R\left(1-\frac{\vec n\cdot\vec v}{c}\right)} = \frac{1}{\vec n \cdot R\left(\vec n - \frac{\vec v}{c}\right)} = \frac{1}{\vec n\cdot\vec s} = \frac{1}{s\cos(\widehat{\vec n,\vec s})}$$

$$\frac{v\frac{R}{c}}{\sin(\widehat{\vec n,\vec s})} = \frac{R}{\sin(\pi-\theta)} \Longrightarrow \sin(\widehat{\vec n,\vec s}) = \frac{v}{c}\sin\theta$$

$$\Longrightarrow \cos(\widehat{\vec n,\vec s}) = \sqrt{1-\frac{v^2}{c^2}\sin^2\theta}$$

$$\vec E = \frac{e\gamma^3}{R^2}\left(\vec n-\frac{\vec v}{c}\right)\left(1-\frac{v^2}{c^2}\right) = e\left(\frac{\gamma}{R}\right)^3 \vec s\left(1-\frac{v^2}{c^2}\right) = \frac{e\vec s}{s^3}\,\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

(6.) $\nabla\cdot\vec A + \frac{1}{c}\partial_t\varphi = \nabla\cdot\left(\frac{e}{c}\frac{\vec v}{R-\frac{\vec v\cdot\vec R}{c}}\right) + \frac{1}{c}\partial_t\left(\frac{e}{R-\frac{\vec v\cdot\vec R}{c}}\right) =$

$$= \frac{e\gamma^2}{cR^2}\left\{\left[\frac{R}{\gamma}\nabla\cdot\vec v - \nabla\left(R-\frac{\vec v\cdot\vec R}{c}\right)\cdot\vec v\right] - \partial_t\left(R-\frac{\vec v\cdot\vec R}{c}\right)\right\} =$$

$$= \frac{e\gamma^2}{cR^2}\left\{\frac{R}{\gamma}\nabla t'\cdot\dot{\vec v} - \left[\vec n\gamma\left(1-\frac{v^2}{c^2}\right) - \frac{\vec v}{c} + \gamma\frac{\vec n(\vec R\cdot\dot{\vec v})}{c^2}\right]\cdot\vec v + \right.$$

$$\left. + (\vec n\cdot\vec v)\gamma - \gamma\frac{v^2}{c} + \gamma\frac{\vec R\cdot\dot{\vec v}}{c}\right\} =$$

$$= \frac{e\gamma^3}{R}\left\{\frac{\vec n\cdot\dot{\vec v}}{c^2}\left(1-\frac{\vec n\cdot\vec v}{c}\right)(1-1) + \frac{1}{R}\frac{v^2}{c^2}\frac{\vec n\cdot\vec v}{c}(1-1)\right\} \equiv 0$$

(7.)

$$\vec{r} = \begin{pmatrix} r\cos\omega t \\ r\sin\omega t \\ 0 \end{pmatrix} \Longrightarrow \dot{\vec{v}} = \begin{pmatrix} -\omega^2 r\cos\omega t \\ -\omega^2 r\sin\omega t \\ 0 \end{pmatrix}$$

$$\begin{cases} v_0 \ll c \\ R \to \infty \end{cases} \Longrightarrow \vec{E} \simeq \frac{e}{c^2 R}\vec{n}\times(\vec{n}\times\dot{\vec{v}}) = \frac{e}{c^2 R}(-\omega^2 r)\begin{pmatrix} \cos(\omega t'-\varphi)\cos\varphi\sin^2\theta - \cos\omega t' \\ \cos(\omega t'-\varphi)\sin\varphi\sin^2\theta - \sin\omega t' \\ \cos(\omega t'-\varphi)\sin\theta\cos\theta \end{pmatrix}$$

$$\vec{H} = \vec{n}\times\vec{E} = \frac{e}{c^2 R}(-\omega^2 r)\begin{pmatrix} \sin\omega t'\cos\theta \\ -\cos\omega t'\cos\theta \\ -\sin(\omega t'-\varphi)\sin\theta \end{pmatrix}$$

$$\begin{cases} H_\varphi = \vec{H}\cdot\hat{\varphi} = \dfrac{e}{c^2 R}\omega^2 r\cos\theta\cos(\omega t'-\varphi) \\ H_\theta = \vec{H}\cdot\hat{\theta} = \dfrac{e}{c^2 R}(-\omega^2 r)\sin(\omega t'-\varphi) \end{cases} \Longrightarrow$$

$$\Longrightarrow \begin{cases} \cos\theta = 0 \Longrightarrow \theta = \dfrac{\pi}{2} \Longrightarrow \text{поляризация – линейная} \\ \cos\theta = \pm 1 \Longrightarrow \theta \in \{0, \pi\} \Longrightarrow \text{поляризация – круговая} \end{cases}$$