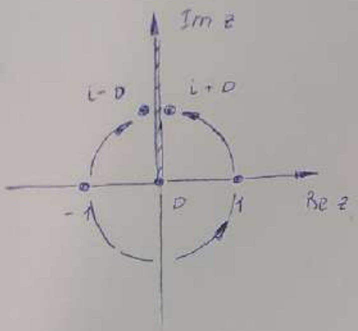
1.

а) $\varphi(z) = \sqrt[3]{z}$, $z \in [0, i\infty]$, $\varphi(-1) = e^{i\frac{\pi}{3}}$

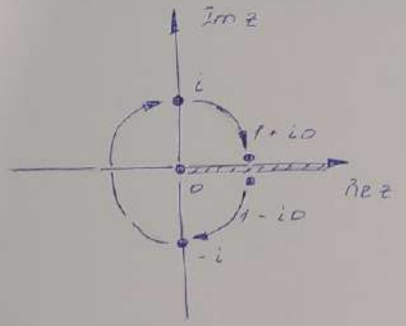


$$\varphi(1) = e^{i\frac{\pi}{3}} \sqrt[3]{\frac{1}{1}}\, e^{\frac{i}{3}\pi} = e^{i\frac{2\pi}{3}}$$

$$\varphi(i+0) = e^{i\frac{\pi}{3}} \sqrt[3]{\frac{1}{1}}\, e^{\frac{i}{3}\frac{3\pi}{2}} = e^{i\frac{5\pi}{6}}$$

$$\varphi(i-0) = e^{i\frac{\pi}{3}} \sqrt[3]{\frac{1}{1}}\, e^{-\frac{i}{3}\frac{\pi}{2}} = e^{i\frac{\pi}{6}}$$

б) $\varphi(z) = \ln z$, $z \in [0, +\infty]$, $\varphi(1-i0) = 0$

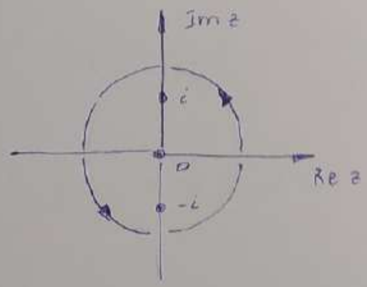


$$\varphi(1+i0) = 0 + \ln\frac{1}{1} - i\,2\pi = -i\,2\pi$$

$$\varphi(i) = 0 + \ln\frac{1}{1} - i\,\frac{3\pi}{2} = -i\,\frac{3\pi}{2}$$

$$\varphi(-i) = 0 + \ln\frac{1}{1} - i\,\frac{\pi}{2} = -i\,\frac{\pi}{2}$$

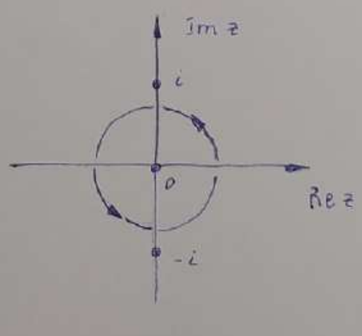
3.



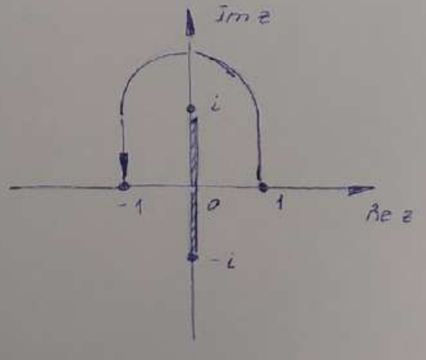
$$I_1 = \oint\limits_{|z|>1} \sqrt{1+z^2}\, dz = -2\pi i \operatorname*{Res}_{z=\infty} \sqrt{1+z^2} =$$

$$= 2\pi i \operatorname*{Res}_{z=0} \frac{\sqrt{1+z^2}}{z^3} = 2\pi i \operatorname*{Res}_{z=0} \frac{\left(\ldots + \frac{1}{2}z^2 - \ldots\right)}{z^3} =$$

$$= 2\pi i \operatorname*{Res}_{z=0} \left(\ldots + \frac{1}{2}\,\frac{1}{z} + \ldots\right) = \pi i$$



$$I_2 = \oint\limits_{|z|<1} \sqrt{1+z^2}\, dz = 0$$



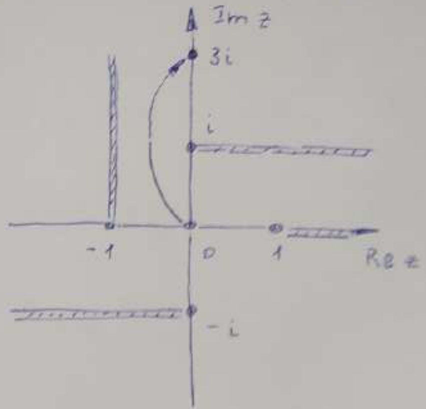
$f(z) = \sqrt{1+z^2}$ $\quad f(1) = \sqrt{2}$ (согл-но усл-ию)

$$f(-1) = \sqrt{2}\,\sqrt{\frac{2}{2}}\, e^{\frac{i}{2}\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)} = -\sqrt{2}$$

$\Longrightarrow$ следует выбрать разрез а)

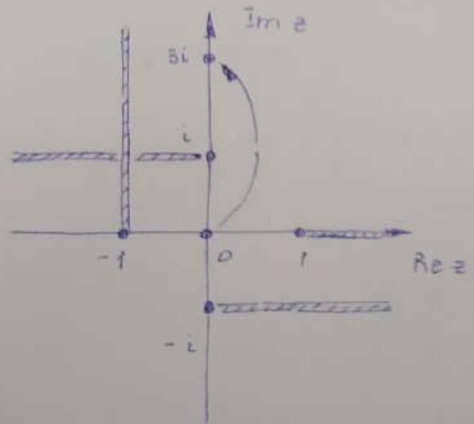
4. $\varphi(z) = \sqrt[3]{1+z^2}$, $\varphi(0) = 1$

1)



$$\varphi(3i) = 1 \cdot \sqrt[3]{\frac{8}{1}} \cdot e^{\frac{i}{3}(-\pi+0)} = 2e^{-i\frac{\pi}{3}}$$

2)



$$\varphi(3i) = 1 \cdot \sqrt[3]{\frac{8}{1}} \, e^{\frac{i}{3}(\pi+0)} = 2e^{i\frac{\pi}{3}}$$

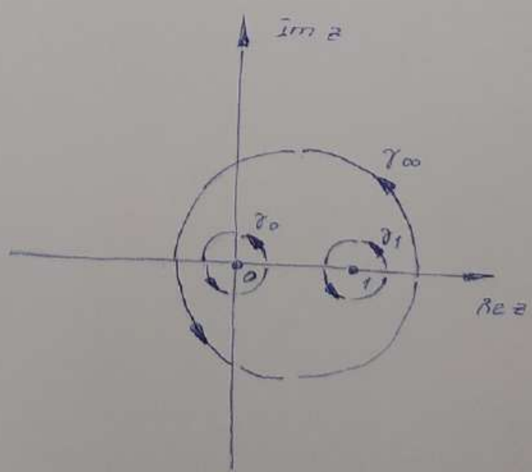
(6.) $f(z) = z^a (z-1)^b$

$$\begin{cases} \Delta_{\gamma_0} \arg z + \Delta_{\gamma_0} \arg(z-1) = 2\pi + 0 = 2\pi \\ \Delta_{\gamma_1} \arg z + \Delta_{\gamma_1} \arg(z-1) = 0 + 2\pi = 2\pi \\ \Delta_{\gamma_\infty} \arg z + \Delta_{\gamma_\infty} \arg(z-1) = 2\pi + 2\pi = 4\pi \end{cases} \Rightarrow N(1,1) = 0$$

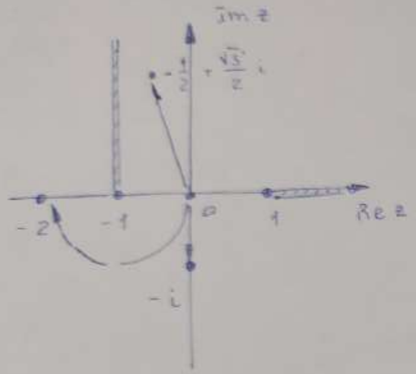
$$\begin{cases} \Delta_{\gamma_0} \arg z + \frac{1}{2} \Delta_{\gamma_0} \arg(z-1) = 2\pi + 0 = 2\pi \\ \Delta_{\gamma_1} \arg z + \frac{1}{2} \Delta_{\gamma_1} \arg(z-1) = 0 + \pi = \pi \\ \Delta_{\gamma_\infty} \arg z + \frac{1}{2} \Delta_{\gamma_\infty} \arg(z-1) = 2\pi + \pi = 3\pi \end{cases} \Rightarrow N\left(1, \frac{1}{2}\right) = 2$$

$$\begin{cases} \frac{1}{2} \Delta_{\gamma_0} \arg z + \frac{1}{3} \Delta_{\gamma_0} \arg(z-1) = \pi + 0 = \pi \\ \frac{1}{2} \Delta_{\gamma_1} \arg z + \frac{1}{3} \Delta_{\gamma_1} \arg(z-1) = 0 + \frac{2\pi}{3} = \frac{2\pi}{3} \\ \frac{1}{2} \Delta_{\gamma_\infty} \arg z + \frac{1}{3} \Delta_{\gamma_\infty} \arg(z-1) = \pi + \frac{2\pi}{3} = \frac{5\pi}{3} \end{cases} \Rightarrow N\left(\frac{1}{2}, \frac{1}{3}\right) = 3$$

$$\begin{cases} \frac{2}{3} \Delta_{\gamma_0} \arg z + \frac{1}{3} \Delta_{\gamma_0} \arg(z-1) = \frac{4\pi}{3} + 0 = \frac{4\pi}{3} \\ \frac{2}{3} \Delta_{\gamma_1} \arg z + \frac{1}{3} \Delta_{\gamma_1} \arg(z-1) = 0 + \frac{2\pi}{3} = \frac{2\pi}{3} \\ \frac{2}{3} \Delta_{\gamma_\infty} \arg z + \frac{1}{3} \Delta_{\gamma_\infty} \arg(z-1) = \frac{4\pi}{3} + \frac{2\pi}{3} = 2\pi \end{cases} \Rightarrow N\left(\frac{2}{3}, \frac{1}{3}\right) = 2$$



(2.) $\varphi(z) = \ln(1 - z^2)$, $\varphi(0) = -2\pi i$



1) $\varphi(-2) = -2\pi i + \ln \frac{3}{1} + i(0 - \pi) = \ln 3 - 3\pi i$

2) $\varphi(-i) = -2\pi i + \ln \frac{2}{1} + i\left(\frac{\pi}{4} - \frac{\pi}{4}\right) = \ln 2 - 2\pi i$

3) $\varphi\left(\frac{-1 + \sqrt{3}\, i}{2}\right) = -2\pi i + \ln \frac{\sqrt{3}}{1} + i\left(-\operatorname{atan} \frac{1}{\sqrt{3}} + \operatorname{atan} \sqrt{3}\right) =$

$= \frac{1}{2} \ln 3 - \frac{11\pi}{6} i$