

$$(1) S'_{ji} = \sum_{p,q} \alpha_{jp} \alpha_{iq} S_{pq} = \sum_{p,q} \alpha_{ip} \alpha_{jq} S_{qp} = \sum_{p,q} \alpha_{ip} \alpha_{jq} S_{pq} = S'_{ij}$$

$$(2) \bar{\Pi}'_{ij} = \sum_{p,q} \alpha_{ip} \alpha_{jq} \bar{\Pi}_{pq} = \sum_p \alpha_{ip} \left(\sum_q \bar{\Pi}_{pq} \alpha_{jq}^T \right) = \\ = \sum_p \alpha_{ip} (\bar{\Pi} \alpha^T)_{pj} = \alpha \bar{\Pi} \alpha^T$$

$$(3) \varepsilon'_{ij} = \alpha \varepsilon_{ij} \alpha^T = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \\ = \begin{pmatrix} \sin \theta \cos \theta - \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \\ -\sin^2 \theta - \cos^2 \theta & \sin \theta \cos \theta - \sin \theta \cos \theta \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \varepsilon_{ij}$$

(4) 1) Объект C имеет n^4 компонент

$$C'_{ijkl} = A'_{ij} B'_{kl} = \left(\sum_p \sum_q \alpha_{ip} \alpha_{jq} A_{pq} \right) \left(\sum_r \sum_s \alpha_{kr} \alpha_{ls} B_{rs} \right) = \\ = \sum_{p,q,r,s} \alpha_{ip} \alpha_{jq} \alpha_{kr} \alpha_{ls} (A_{pq} B_{rs}) = \sum_{p,q,r,s} \alpha_{ip} \alpha_{jq} \alpha_{kr} \alpha_{ls} C_{pqrs}$$

$$2) D'_{ic} = \sum_j C'_{ijjk} = \sum_j \sum_{p,q,r,s} \alpha_{ip} \alpha_{jq} \alpha_{jr} \alpha_{ks} C_{pqrs} = \sum_{p,s} \alpha_{ip} \alpha_{ks} \left[\sum_{q,r} \left(\sum_j \alpha_{qj}^T \alpha_{jr} \right) C_{pqrs} \right] = \\ = \sum_{p,s} \alpha_{ip} \alpha_{ks} \left(\sum_{q,r} \delta_{qr} C_{pqrs} \right) = \sum_{p,q} \alpha_{ip} \alpha_{eq} \left(\sum_j C_{pj dq} \right) = \sum_{p,q} \alpha_{ip} \alpha_{eq} D_{pq}$$

$$3) D' = \sum_i D'_{ic} = \sum_i \sum_{p,q} \alpha_{ip} \alpha_{iq} D_{pq} = \sum_{p,q} \left(\sum_i \alpha_{pi}^T \alpha_{iq} \right) D_{pq} = \\ = \sum_{p,q} \delta_{pq} D_{pq} = \sum_i D_{ii} = D$$

$$(5) D'_{ij} = \left(\frac{\partial^2 \varphi}{\partial x_i \partial x_j} \right)' = \left(\sum_p \alpha_{ip} \frac{\partial}{\partial x_p} \right) \left(\sum_q \alpha_{jq} \frac{\partial}{\partial x_q} \right) \varphi = \\ = \sum_{p,q} \alpha_{ip} \alpha_{jq} \frac{\partial^2 \varphi}{\partial x_p \partial x_q} = \sum_{p,q} \alpha_{ip} \alpha_{jq} D_{ij}$$

$$(6.) \quad \omega_k = \frac{1}{2} \sum_{i,j} A_{ij} \varepsilon_{ijk}$$

$$\begin{aligned} \omega'_k &= \frac{1}{2} \sum_{i,j} A'_{ij} \varepsilon'_{ijk} = \frac{1}{2} \sum_{i,j} \sum_{p,q,r,s,t} \alpha_{ip} \alpha_{jq} \alpha_{kr} \alpha_{ds} \alpha_{kt} A_{pq} \varepsilon_{rst} = \\ &= \frac{1}{2} \sum_{p,q,r,s,t} \left(\sum_i \alpha_{pi}^T \alpha_{ir} \right) \left(\sum_j \alpha_{qj}^T \alpha_{js} \right) \alpha_{kt} A_{pq} \varepsilon_{rst} = \\ &= \frac{1}{2} \sum_{p,q,r,s,t} \delta_{pq} \delta_{qs} \alpha_{kt} A_{pq} \varepsilon_{rst} = \frac{1}{2} \sum_{p,q} \sum_t \alpha_{kt} A_{pq} \varepsilon_{pqt} = \\ &= \sum_p \alpha_{kp} \left(\frac{1}{2} \sum_{i,j} A_{ij} \varepsilon_{ijp} \right). \end{aligned}$$

$$(7.) \quad \begin{array}{ll} 1) \quad \sum_j A_{ij} \delta_{jk} = A_{ik} & 2) \quad \sum_k \delta_{ik} \delta_{kj} = \delta_{ij} \\ \sum_i A_{ij} \delta_{ik} = A_{kj} & \sum_{i,k} \delta_{ik} \delta_{ik} = n, \text{ где } n - \text{размерность} \\ \sum_{i,j} A_{ij} \delta_{ij} = \sum_i A_{ii} & \sum_{i,k} \delta_{ik} \delta_{ki} = n \end{array}$$

$$(8.) \quad \delta_{il} \delta_{jlm} = \begin{cases} 1, & ijlm \in \{1111, 1212, 2121, 2222\} \\ 0, & \text{в остальных случаях} \end{cases}$$

$$\delta_{lm} \delta_{jle} = \begin{cases} 1, & ijlm \in \{1111, 1221, 2112, 2222\} \\ 0, & \text{в остальных случаях} \end{cases}$$

$$\delta_{il} \delta_{jlm} - \delta_{lm} \delta_{jle} = \begin{cases} 1, & ijlm \in \{1212, 2121\} \\ -1, & ijlm \in \{1221, 2112\} \\ 0, & \text{в остальных случаях} \end{cases} = \varepsilon_{ij} \varepsilon_{lm}$$

$$(9.) \quad 1) \quad \varepsilon_{ijk} \varepsilon_{lmn} = \delta_{il} \delta_{jm} \delta_{kn} - \delta_{il} \delta_{jn} \delta_{km} - \delta_{im} \delta_{jl} \delta_{kn} + \\ + \delta_{im} \delta_{jn} \delta_{kl} + \delta_{in} \delta_{jl} \delta_{km} - \delta_{in} \delta_{jm} \delta_{kl}$$

$$2) \quad \sum_k \varepsilon_{ijk} \varepsilon_{lmk} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

$$3) \quad \sum_{j,k} \varepsilon_{ijk} \varepsilon_{ejk} = 2 \delta_{il}$$

$$\begin{aligned}
 (10) \quad \sum_{i,j} A_{ij} S_{ij} &= \sum_{i,j} A'_{ij} S'_{ij} = \sum_{i,j} \sum_{p,q,r,s} \alpha_{ip} \alpha_{jq} \alpha_{ir} \alpha_{js} A_{pq} S_{rs} = \\
 &= \sum_{p,q,r,s} \left[\sum_{i,j} (\alpha_{pi}^T \alpha_{ir}) (\alpha_{qj}^T \alpha_{js}) \right] A_{pq} S_{rs} = \sum_{p,q,r,s} \delta_{pr} \delta_{qs} A_{pq} S_{rs} = \\
 &= \sum_{p,q} A_{pq} S_{pq} = - \sum_{p,q} A_{qp} S_{qp} = - \sum_{i,j} A_{ij} S_{ij} \Rightarrow \\
 &\Rightarrow \sum_{i,j} A_{ij} S_{ij} = + \left(\sum_{i,j} A_{ij} S_{ij} \right)' \equiv 0.
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad 1) \quad b'_i &= \sum_j \Pi'_{ij} a'_j = \sum_j \sum_{p,q,r} \alpha_{ip} \alpha_{jq} \alpha_{jr} \Pi_{pq} a_r = \\
 &= \sum_{p,q,r} \left(\sum_j \alpha_{jq}^T \alpha_{jr} \right) \alpha_{ip} \Pi_{pq} a_r = \sum_{p,q,r} \alpha_{ip} \delta_{qr} \Pi_{pq} a_r = \\
 &= \sum_{p,q} \alpha_{ip} \Pi_{pq} a_q = \sum_p \alpha_{ip} \left(\sum_q \Pi_{pq} a_q \right) = \sum_p \alpha_{ip} b_p
 \end{aligned}$$

$$\begin{aligned}
 2) \quad c'_i &= \sum_j \Pi'_{ij} a'_j = \sum_i \sum_{p,q,r} \alpha_{ip} \alpha_{jq} \alpha_{ir} \Pi_{pq} a_r = \\
 &= \sum_{p,q,r} \left(\sum_i \alpha_{pi}^T \alpha_{ir} \right) \alpha_{jq} \Pi_{pq} a_r = \sum_{p,q,r} \alpha_{jq} \delta_{pr} \Pi_{pq} a_r = \\
 &= \sum_{p,q} \alpha_{jq} \Pi_{pq} a_p = \sum_q \alpha_{jq} \left(\sum_i \Pi_{iq} a_i \right) = \sum_p \alpha_{jp} c_p
 \end{aligned}$$