Exercise 1

$f(x) = x^3 + 3x - 1$ [0,1]

Iterations

1. a = 0 b = 1 c = $\frac{1}{2}$ $f(\frac{1}{2}) = \frac{5}{8}$ → b = $\frac{1}{2}$
2. a = 0 b = $\frac{1}{2}$ c = $\frac{1}{4}$ $f(\frac{1}{4}) = \frac{-15}{64}$ → a = $\frac{1}{4}$
3. a = $\frac{1}{4}$ b = $\frac{1}{2}$ c = $\frac{3}{8}$ $f(\frac{3}{8}) = \frac{91}{512}$ → b = $\frac{3}{8}$
4. a = $\frac{1}{4}$ b = $\frac{3}{8}$ c = $\frac{5}{16}$ $f(\frac{5}{16}) = \frac{-131}{4096}$ → a = $\frac{5}{16}$
5. a = $\frac{5}{16}$ b = $\frac{3}{8}$ c = $\frac{11}{32}$ $f(\frac{11}{32}) = \frac{2355}{32768}$ → b = $\frac{11}{32}$
6. a = $\frac{5}{16}$ b = $\frac{11}{32}$ c = $\frac{21}{64}$ $f(\frac{21}{64}) = \frac{5165}{262144}$ → b = $\frac{21}{64}$
7. a = $\frac{5}{16}$ b = $\frac{21}{64}$ c = $\frac{41}{128}$ $f(\frac{41}{128}) = \frac{-12999}{2097152}$ → a = $\frac{41}{128}$
8. a = $\frac{41}{128}$ b = $\frac{21}{64}$ c = $\frac{83}{256}$ $f(\frac{83}{256}) = 0.00673741102219$ → b = $\frac{83}{256}$
9. a = $\frac{41}{128}$ b = $\frac{83}{256}$ c = $\frac{165}{512}$ $f(\frac{165}{512}) = 0.000265814363956$ → b = $\frac{165}{512}$
10. a = $\frac{41}{128}$ b = $\frac{165}{512}$ c = $\frac{329}{1024}$ $f(\frac{329}{1024}) = -0.00296721514314$ → a = $\frac{329}{1024}$

Max iterations of 10: c = $\frac{329}{1024}$ so the zero is approximately at x = $\frac{329}{1024}$

g(x)

$x^3 - 2\sin x$, on [0.5,2]

1. a = 0.5 b = 2 c = $\frac{5}{4}$ $f(\frac{2.5}{2}) = 0.0551557612888$ → b = $\frac{5}{4}$
2. a = 0.5 b = $\frac{5}{4}$ c = $\frac{7}{8}$ $f(\frac{7}{8}) = -0.865165129472$ → a = $\frac{7}{8}$
3. a = $\frac{7}{8}$ b = $\frac{5}{4}$ c = $\frac{17}{16}$ $f(\frac{17}{16}) = -0.547686979709$ → a = $\frac{17}{16}$
4. a = $\frac{17}{16}$ b = $\frac{5}{4}$ c = $\frac{37}{32}$ $f(\frac{37}{32}) = -0.284791400798$ → a = $\frac{37}{32}$

5. a = $\frac{37}{32}$ b = $\frac{5}{4}$ c = $\frac{77}{64}$ f($\frac{77}{64}$) = -0.124798615509 → a = $\frac{77}{64}$

7. a = $\frac{77}{64}$ b = $\frac{5}{4}$ c = $\frac{157}{128}$ f($\frac{157}{128}$) = -0.0373598065251 → a = $\frac{157}{128}$

8. a = $\frac{157}{128}$ b = $\frac{5}{4}$ c = $\frac{317}{256}$ f($\frac{317}{256}$) = 0.00825801590073 → b = $\frac{317}{256}$

9. a = $\frac{157}{128}$ b = $\frac{317}{256}$ c = $\frac{631}{512}$ f($\frac{631}{512}$) = -0.0147102162427 → a = $\frac{631}{512}$

10. a = $\frac{631}{512}$ b = $\frac{317}{256}$ c = $\frac{1265}{1024}$ f($\frac{1265}{1024}$) = -0.00326601417057 → a = $\frac{1265}{1024}$

After 10 iterations: c = $\frac{1265}{1024}$ so the zero is approximately at x = $\frac{1265}{1024}$

<u>h(x)</u>

x + 10 - x cosh(50/x) a = 120 b = 130

Bisection method can not be applied to this function with the starting values [120,130].

cos(h(50/x)) is limited from -1 to 1. Assuming the maximum value of 1 which will be the minimum value of h(x) if x > 0, the rewritten equation would be x + 10 – x(1). As long as x > 0, then the function will never reach 0.

120 and 130 are both > 0 so h(120) and h(130) are both positive as well which does not satisfy the requirement of f(a)f(b) < 0.

Therefore, bisection method cannot be done on h(x).

Exercise 2

$f(x) = x^3 + 2x^2 + 10x - 20 \qquad x_0 = 2$

$f'(x) = 3x^2 + 4x + 10$

Iterations

1.

$f(2) = 16 \qquad f'(2) = 30 \qquad \frac{f}{f'} = \frac{16}{30} = \frac{8}{15}$

$x_1 = 2 - \frac{8}{15} = \frac{22}{15}$

2.

$f(\frac{22}{15}) = \frac{7168}{3375} \qquad f'(\frac{22}{15}) = \frac{558}{25} \qquad \frac{f}{f'} = \frac{3584}{37665}$

$x_2 = \frac{22}{15} - \frac{3584}{37665} = \frac{51658}{37665}$

3.

$f(\frac{51658}{37665}) = 0.0570866419043 \qquad f'(\frac{51658}{37665}) = 21.1291836673 \qquad \frac{f}{f'} = \frac{0.0570866419043}{21.1291836673}$

$x_3 = \frac{51658}{37665} - \frac{0.0570866419043}{21.1291836673} = 1.36881022263$

4.

$f(1.36881022263) = 0.0000446143247927 \quad f'(1.36881022263) = 21.0961651672$

$\frac{f}{f'} = \frac{0.0000446143247927}{10.0001784633}$

$x_4 = 1.36881022263 - \frac{0.0000446143247927}{21.0961651672} = 1.368808107822667050897$

4.

$f(1.36880810783) = 1.8200552176 \times 10^{-10} \quad f'(1.36880810783) = 21.0961393395$

$\frac{f}{f'} = \frac{1.8200552176 \times 10\text{^}-10}{21.0961393395}$

$x_4 = 1.36880810783 - \frac{1.8200552176 \times 10\text{^}-10}{21.0961393395} = 1.368808107821372567 33$

5.

$f(1.36880810782137256733) = -7.1054273576 \times 10^{-15}$

$f'(1.36880810782137256733) = 21.0961393394$

$\frac{f}{f'} = \frac{-7.1054273576 \times 10^{\wedge}-15}{21.0961393394}$

$x_4 = 1.36880810782137256733 - \frac{-7.1054273576 \times 10^{\wedge}-15}{21.0961393394} = 1.3688081078213729$

Change between of x_4 and x_3 is 0.000000000000000332 which is very small so the zero is near x_4 or $x = 1.3688081078213729$.

Exercise 3

$f(x) = x^3 + 2x^2 + 10x - 20$ $x_0 = 2$ $x_1 = 1$

Iteration

1. $f(x_0) = f(2) = 16$ $f(x_1) = f(1) = -7$

$$x_2 = x_1 - \frac{(x1 - x0)f(x1)}{f(x1) - f(x0)} = 1 - \frac{(1-2)(-7)}{-7-16} = \frac{30}{23}$$

2. $f(x_1) = f(1) = -7$ $f(x_2) = f(\frac{30}{23}) = \frac{-16240}{12167}$

$$x_3 = x_2 - \frac{(x2 - x1)f(x2)}{f(x2) - f(x1)} = \frac{30}{23} - \frac{(\frac{30}{23} - 1)(\frac{-16240}{12167})}{\frac{-16240}{12167} - -7} = \frac{13550}{9847}$$

3. $f(x_2) = f(\frac{30}{23}) = \frac{-16240}{12167}$ $f(x_3) = f(\frac{13550}{9847}) = 0.153173294867$

$$x_4 = x_3 - \frac{(x3 - x2)f(x3)}{f(x3) - f(x2)} = \frac{13550}{9847} - \frac{(\frac{13550}{9847} - \frac{30}{23})f(\frac{13550}{9847})}{f\left(\frac{13550}{9847}\right) - f(\frac{30}{23})} = \underline{1.36867195353}$$

4. $f(x_3) = f(\frac{13550}{9847}) = 0.153173294867$ $f(x_4) = f(1.3686\ldots) = -0.00287221670414$

$$x_5 = x_4 - \frac{(x4 - x3)f(x4)}{f(x4) - f(x3)} = 1.36867195.. - \frac{(1.36867\ldots - \frac{13550}{9847})(-0.0028722..)}{-0.00287221670 - 0.153173294} = \underline{1.36880782253}$$

5.

$f(x_4) = f(1.36867\ldots) = -0.00287221670414$

$f(x_5) = f(1.36880\ldots) = -0.00000601854605264$

$$x_6 = x_5 - \frac{(x5 - x4)f(x5)}{f(x5) - f(x4)} = 1.36880782253 - \frac{(1.3688\ldots - 1.3686\ldots) - 0.00000601854605264}{-0.00000601854605264 - -0.00287221670414} =$$

$x_6 = \underline{1.36880810783}$

6.

$f(x_5) = f(1.36880782253) = -0.00000601854605264$

$f(x_6) = f(1.36880810783...) = 1.8200552176 \times 10^{-10}$

$x_7 = x_6 - \frac{(x6 - x5)f(x6)}{f(x6) - f(x5)} = 1.36880810783 - \frac{(1.368808... - 1.368807...) - (1.8200552176 \text{ x } 10\text{^}-10)}{(1.8200552176 \text{ x } 10\text{^}-10) - -0.00000601854605264} =$

$x_6 = \underline{1.36880810782}$

Change between x5 and x6: 0.00000000001

The change between the two points is getting very small so the zero must be around

x = <u>1.36880810782.</u>