Stochastic Optimization of Floating-point Programs with Tunable Precision

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Overview

- **Floating-point kernels:** Nearly ubiquitous in high-performance computing

- **Getting the best performance:** Requires full exploitation of the dark corners of an instruction set and how it interacts with machine resources

- **But we can do even better:** Give up on full precision for applications that don't need it

Overview

- **Stochastic Optimization:** Automated method for generating high-performance kernels that trade a reduction in precision for improvements in code size and runtime

Goals and Related Work

- **Mixed fixed / floating-point kernels:** Build on previous work [schkufza 13, sharma 13]
- **Improved approximation:** Give user finer-grained control [lam 13, rubio-gonzalez 13]
- **Assembly level**: Direct optimization, no further trusted optimization kernel [sidiroglou-douskos 11]
- **Optimize kernels:** Improve end-to-end performance automatically [chilimbi 10, zou 12]

Example

expert →

```
vmovddup   %xmm0, %xmm0
vmulpd     (%rdi), %xmm0, %xmm2
vroundpd   $0, %xmm2, %xmm2
vmulpd     0x10(%rdi), %xmm2, %xmm1
vcvtpd2dqx %xmm2, %xmm3
vmulpd     0x20(%rdi), %xmm2, %xmm2
vaddpd     %xmm1, %xmm0, %xmm1
vmovapd    0x30(%rdi), %xmm0
vpaddd     0x40(%rdi), %xmm3, %xmm3
vpslld     $20, %xmm3, %xmm3
vpshufd    $114, %xmm3, %xmm3
vaddpd     %xmm2, %xmm1, %xmm1
vmulpd     0x50(%rdi), %xmm1, %xmm2
vaddpd     0x60(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0x70(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0x80(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0x90(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xa0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xb0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xc0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xd0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xe0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     0xf0(%rdi), %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm2
vaddpd     %xmm0, %xmm2, %xmm2
vmulpd     %xmm1, %xmm2, %xmm1
vaddpd     %xmm0, %xmm1, %xmm0
vmulpd     %xmm3, %xmm0, %xmm0
retq
```

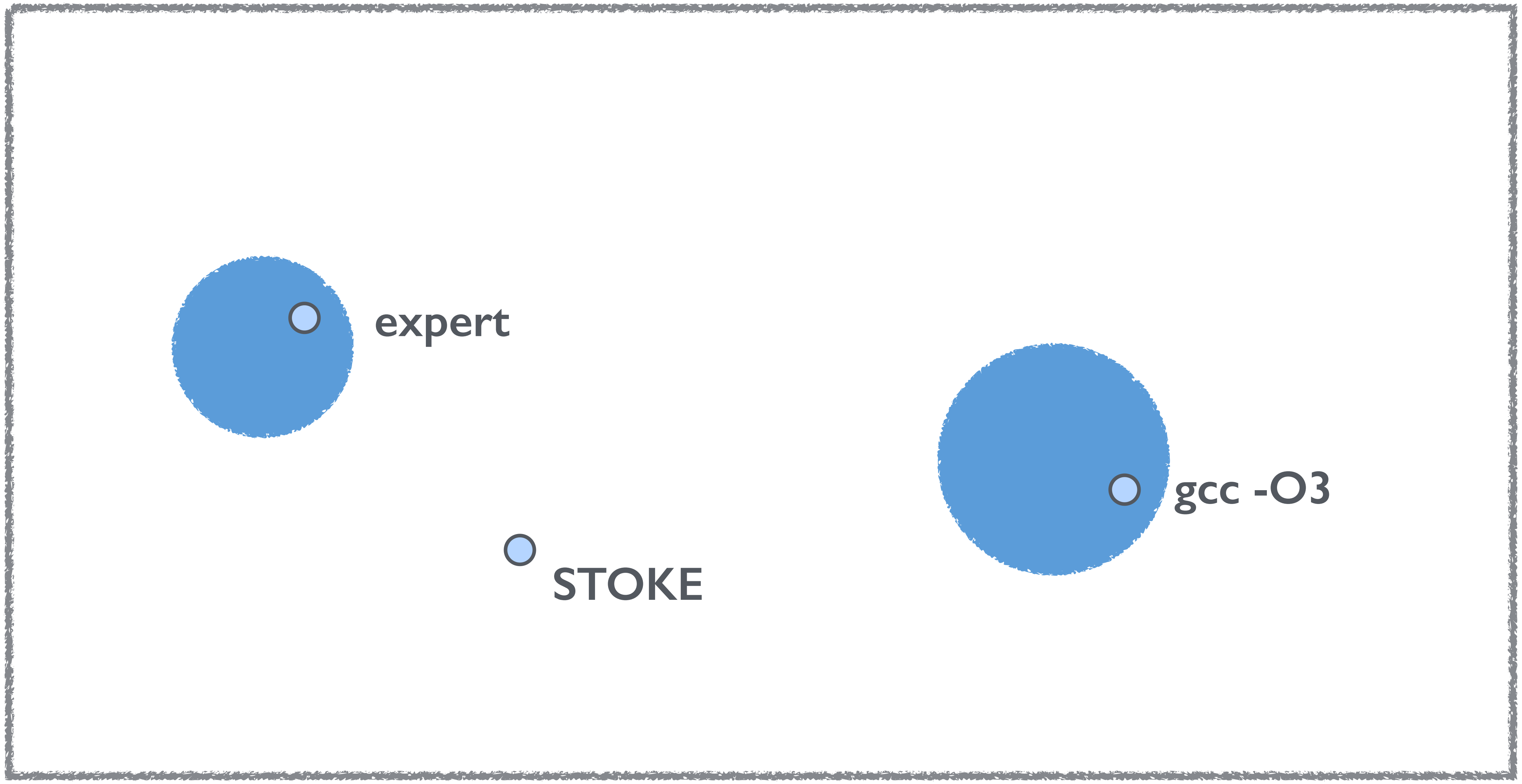
STOKE →

```
vmulpd      (%rdi), %xmm0, %xmm2
vroundpd    $0xfffffffffffffffe, %xmm2, %xmm2
vcvttpd2dq  %xmm2, %xmm3
vmulpd      0x10(%rdi), %xmm2, %xmm1
vlddqu      0x90(%rdi), %xmm2
vaddpd      %xmm1, %xmm0, %xmm1
vmulpd      %xmm1, %xmm2, %xmm2
vpaddw      0x40(%rdi), %xmm3, %xmm3
vmovapd     0x30(%rdi), %xmm0
vpsllq      $0x14, %xmm3, %xmm3
vaddpd      0xa0(%rdi), %xmm2, %xmm2
vmulpd      %xmm1, %xmm2, %xmm2
vaddpd      0xb0(%rdi), %xmm2, %xmm2
vmulsd      %xmm1, %xmm2, %xmm2
vaddpd      0xc0(%rdi), %xmm2, %xmm2
vmulpd      %xmm1, %xmm2, %xmm2
vaddpd      0xd0(%rdi), %xmm2, %xmm2
vmulpd      %xmm1, %xmm2, %xmm2
vaddpd      0xe0(%rdi), %xmm2, %xmm2
vmulsd      %xmm1, %xmm2, %xmm2
vaddpd      0xf0(%rdi), %xmm2, %xmm2
vmulsd      %xmm1, %xmm2, %xmm2
vaddsd      %xmm0, %xmm2, %xmm2
vpshufd     $0x3, %xmm3, %xmm3
vmulpd      %xmm1, %xmm2, %xmm1
vaddsd      %xmm0, %xmm1, %xmm0
vmulpd      %xmm3, %xmm0, %xmm0
retq
```

Optimization Notes

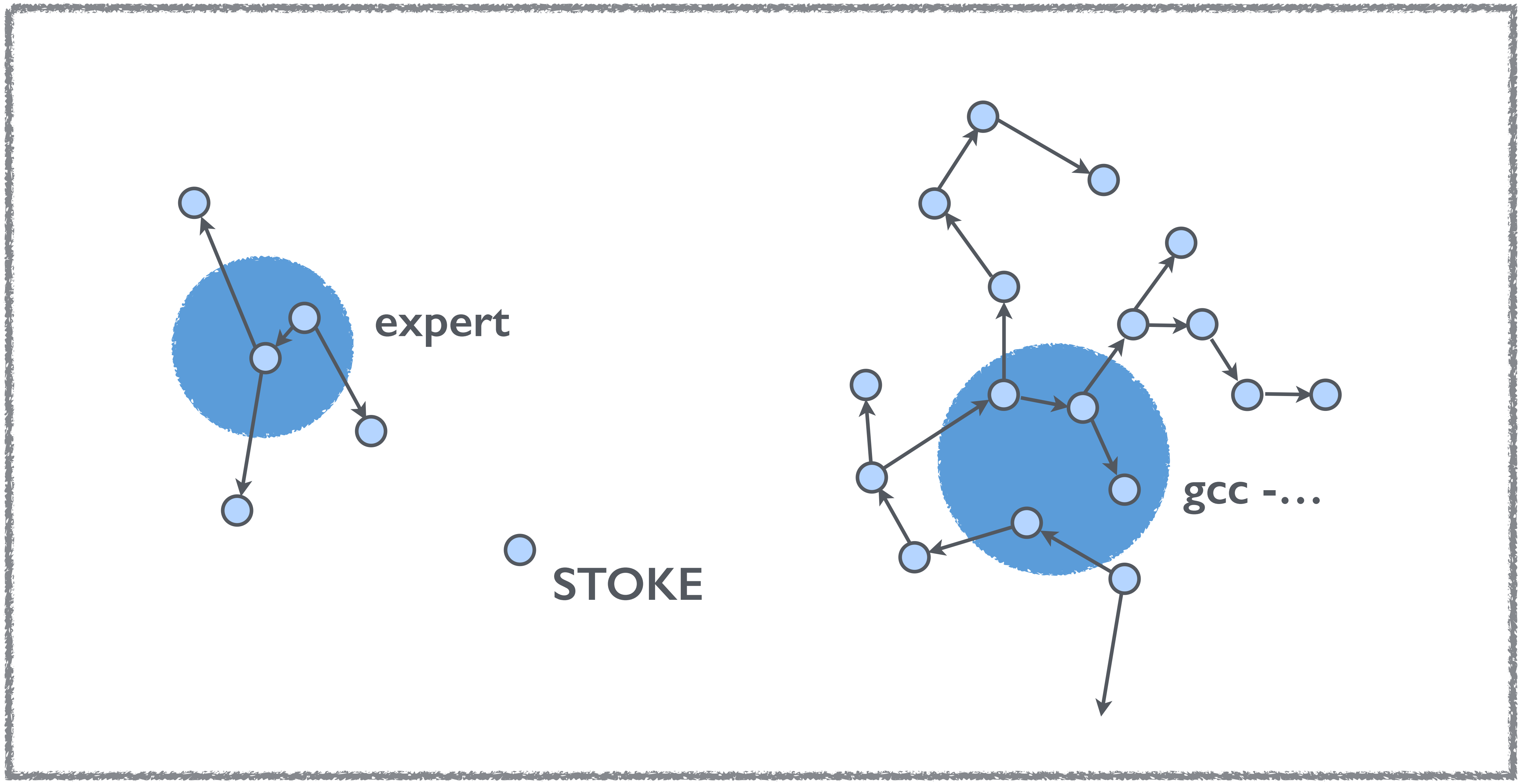
- **Smaller kernel:** 38 LOC reduced to 28 LOC
- **Performance improvement:** 57% kernel speedup, produces a 27% overall task speedup
- **Highly specialized:** Obeys application-specific error bound requirements for all inputs between -3.0 and 0

Intuition



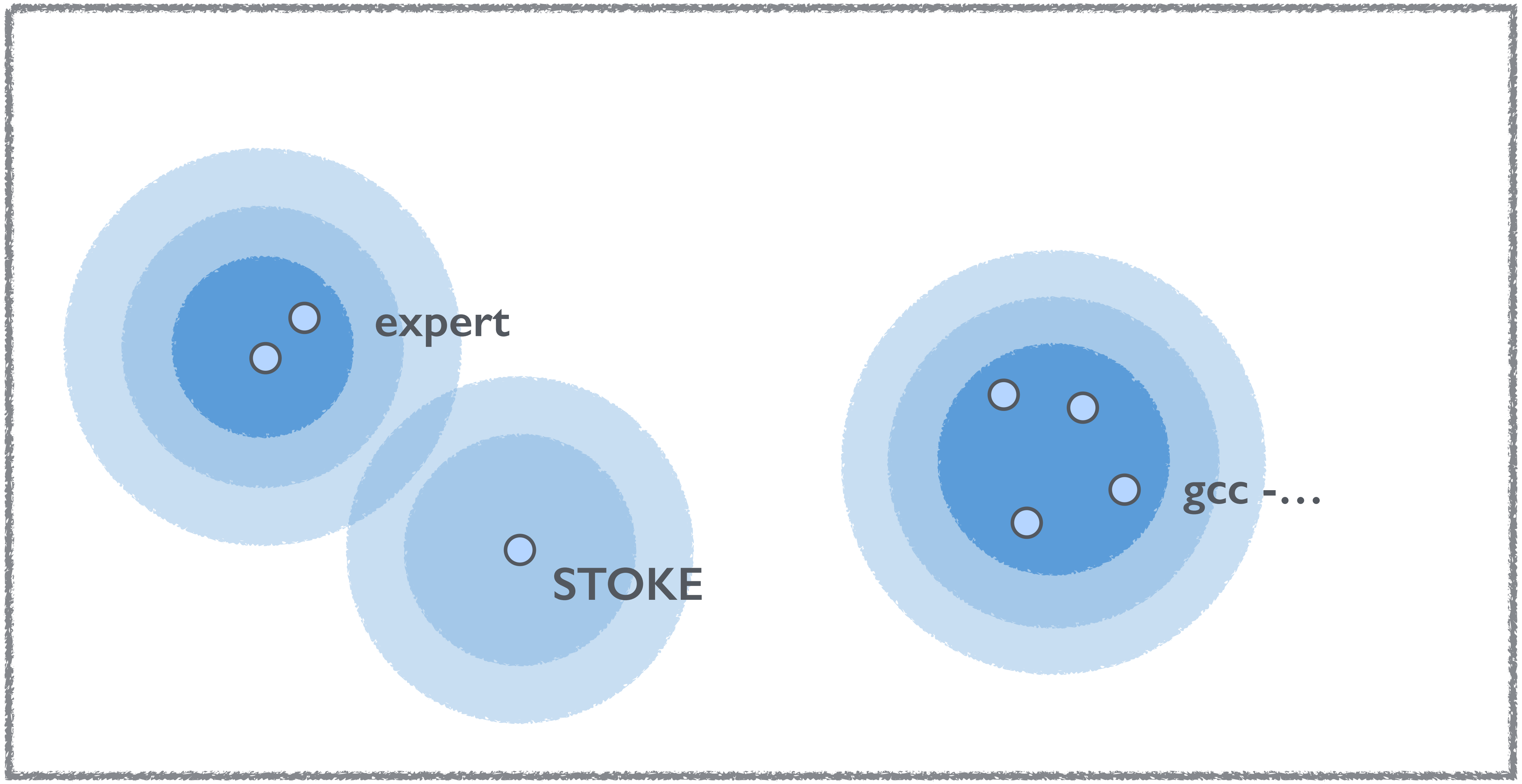
Abstract space of programs (*rewrites*): Blue regions contains points that are bit-wise equivalent to the *target*

Intuition



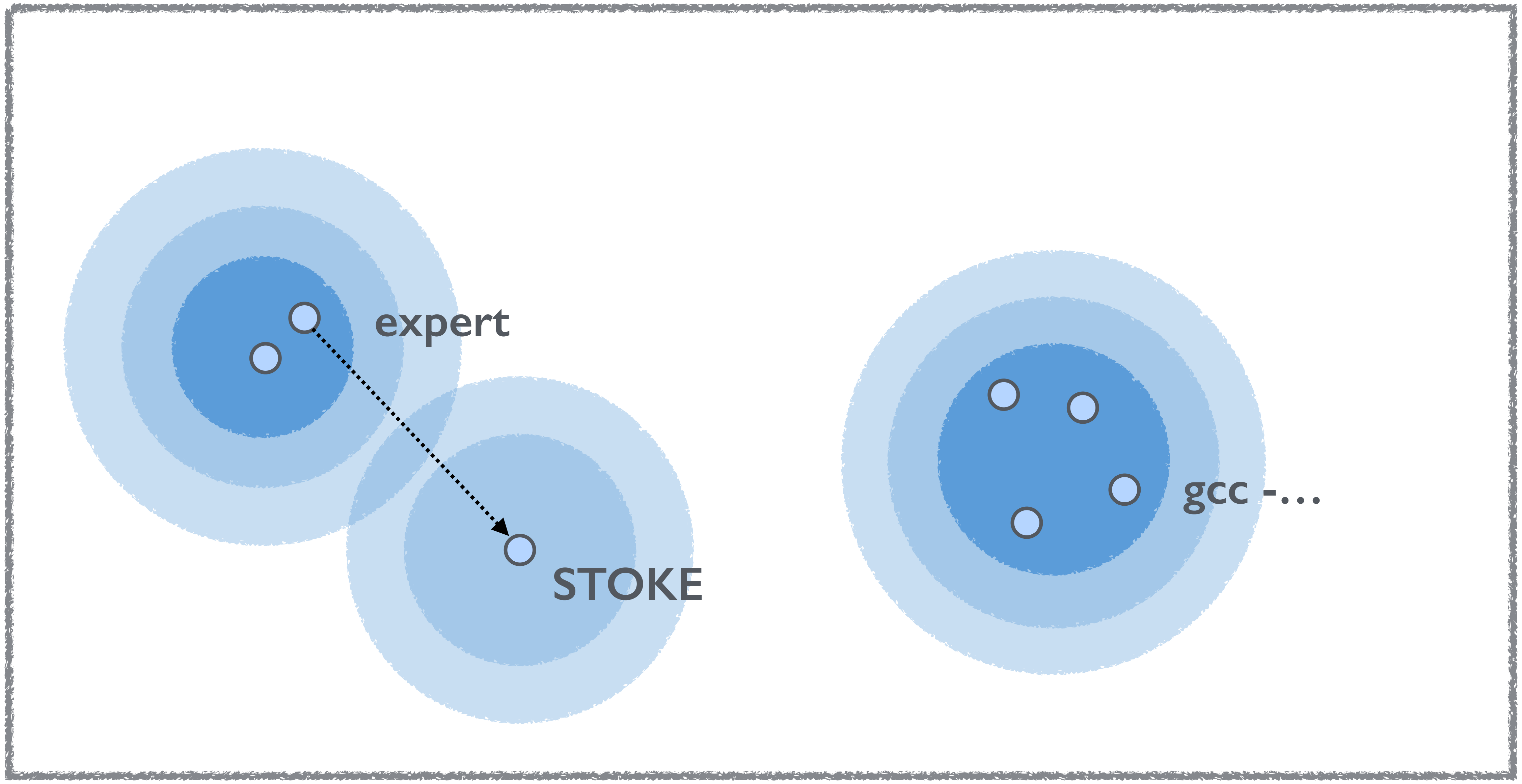
Few opportunities: Floating-point instruction set semantics complicate optimization

Intuition



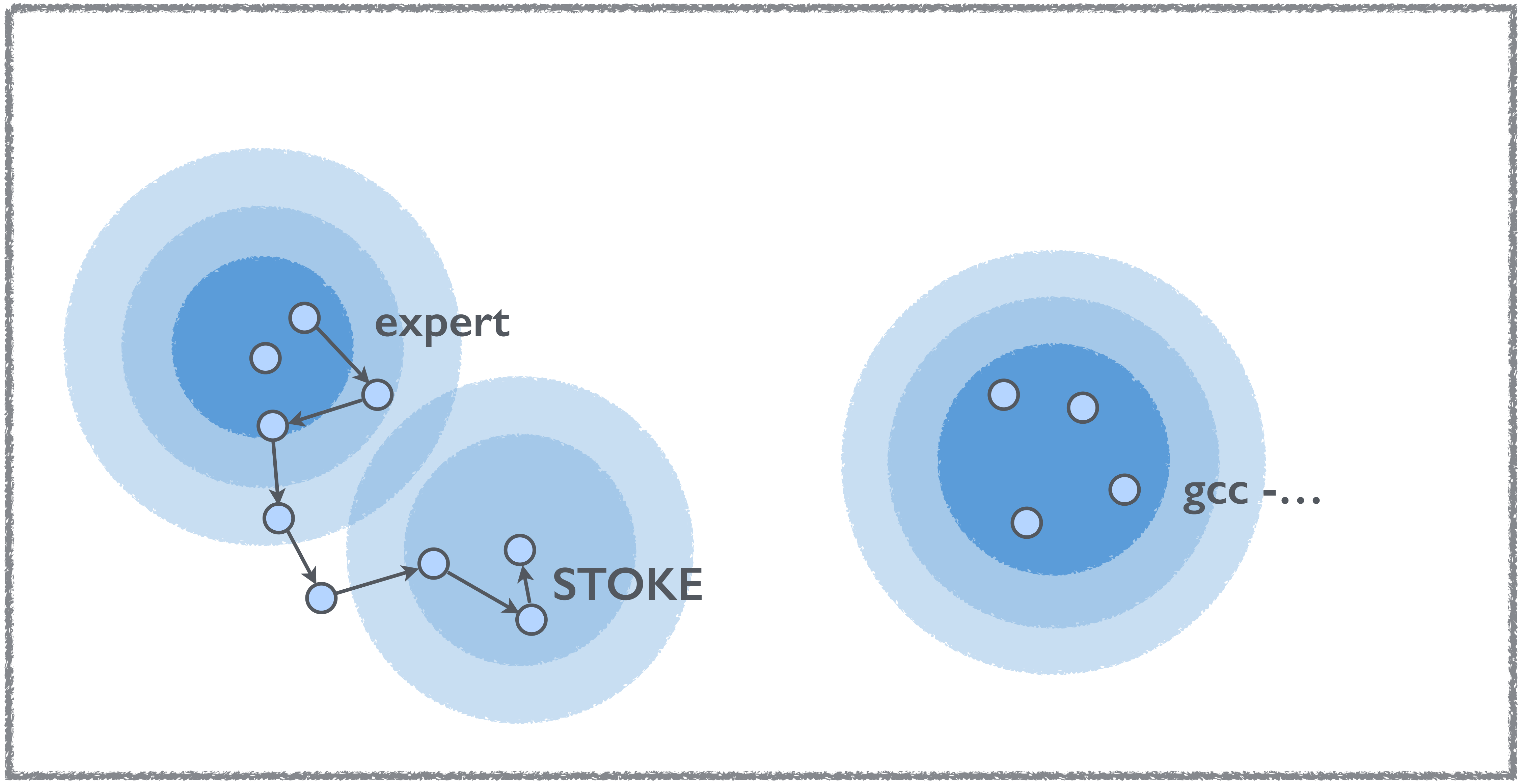
Relax correctness: Most applications don't require full precision results for all possible inputs

Intuition



New opportunities: Gain access to high performance optimizations that were previous inaccessible

Intuition



Non-trivial task: Semantics preserving transformations ill-suited to this task; prefer stochastic search [schkufza 13]

What’s Required

1. **Search Procedure:** Markov Chain Monte Carlo Sampling

2. **Cost Function:** Formal encoding of rewrite quality to guide search; should balance competing constraints of precision and performance

MCMC Sampling

- **Widely used:** For many domains, the only known tractable solution method for high dimensional irregular search spaces [andrieu 03][chenney 00]

- **Guarantees:** Draws samples in proportion to their value; higher value points are sampled more frequently

- **No claim of convergence rate:** Works well in practice for the benchmarks that we consider

MCMC Sampling

Algorithm:

1. Select an initial program
2. Repeat (**millions** to **billions** of times)
   A. Propose a random change and evaluate cost
   B. If (decreased) { accept }
   C. If (increased) { maybe accept anyway }

Transformations

original

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

Transformations

insert

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
imulq rsi, rdx
...
```

original

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movl ecx, ecx
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```

insert

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
imulq rsi, rdx
...
```

delete

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

Transformations

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movl ecx, ecx
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insert

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shrq 32, rsi
andl ff, r9d
movq rcx, rax
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imulq r9, rax
imulq rsi, rdx
...
```

delete

...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
~~movq rcx, rax~~
movl edx, edx
imulq r9, rax
...

instruction

```
...
movl ecx, ecx
shrq 32, rsi
salq 16, rcx
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

Transformations

original

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

insert

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
imulq rsi, rdx
...
```

opcode

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
subl edx, edx
imulq r9, rax
...
```

delete

...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
~~movq rcx, rax~~
movl edx, edx
imulq r9, rax
...

instruction

```
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movl ecx, ecx
shrq 32, rsi
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movl ecx, ecx
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instruction

```
...
movl ecx, ecx
shrq 32, rsi
salq 16, rcx
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

operand

```
...
movl ecx, ecx
shrq 32, rcx
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

Transformations

original

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

insert

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
imulq rsi, rdx
...
```

opcode

```
...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
movq rcx, rax
subl edx, edx
imulq r9, rax
...
```

delete

...
movl ecx, ecx
shrq 32, rsi
andl ff, r9d
~~movq rcx, rax~~
movl edx, edx
imulq r9, rax
...

instruction

```
...
movl ecx, ecx
shrq 32, rsi
salq 16, rcx
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

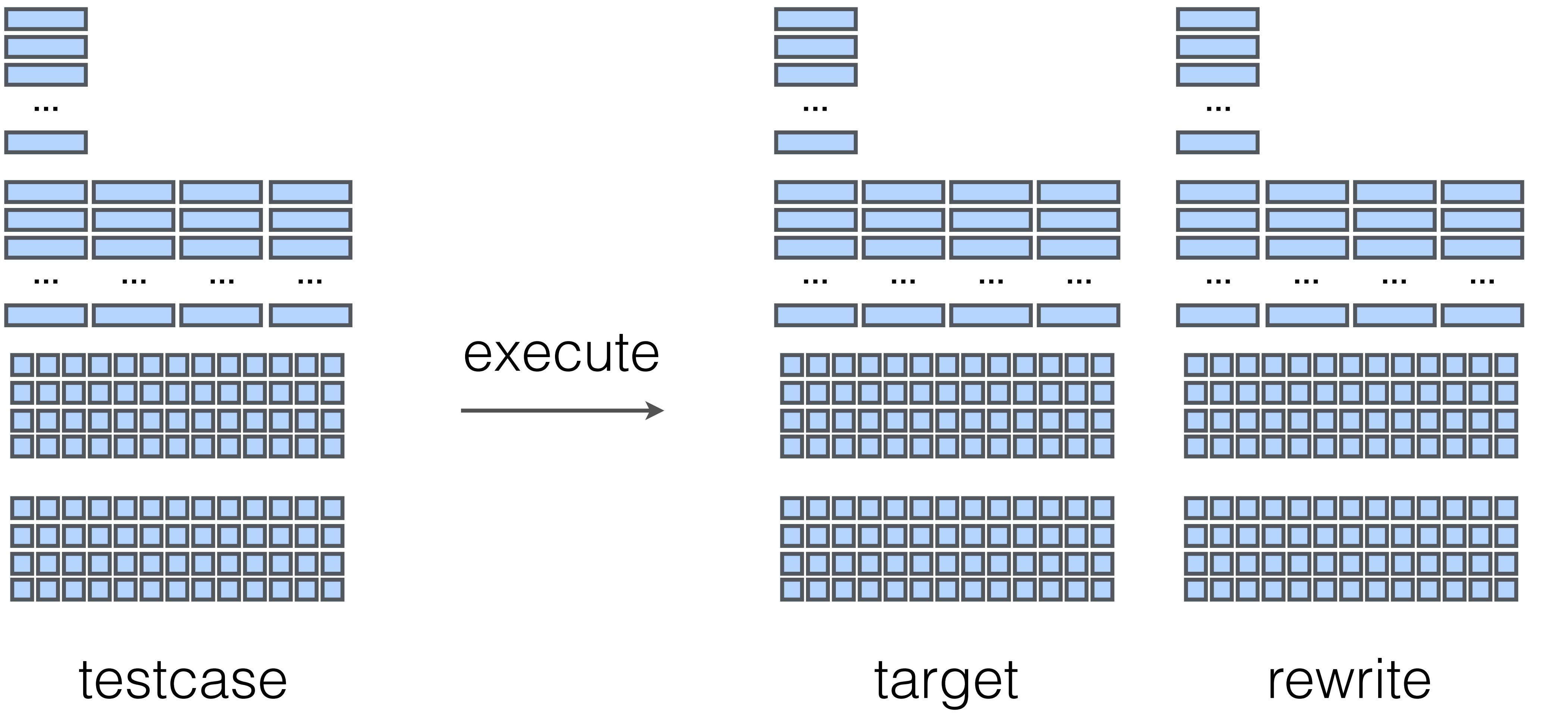
swap

```
...
movl ecx, ecx
movl edx, edx
andl ff, r9d
movq rcx, rax
shrq 32, rsi
imulq r9, rax
...
```

operand

```
...
movl ecx, ecx
shrq 32, rcx
andl ff, r9d
movq rcx, rax
movl edx, edx
imulq r9, rax
...
```

Precision Function

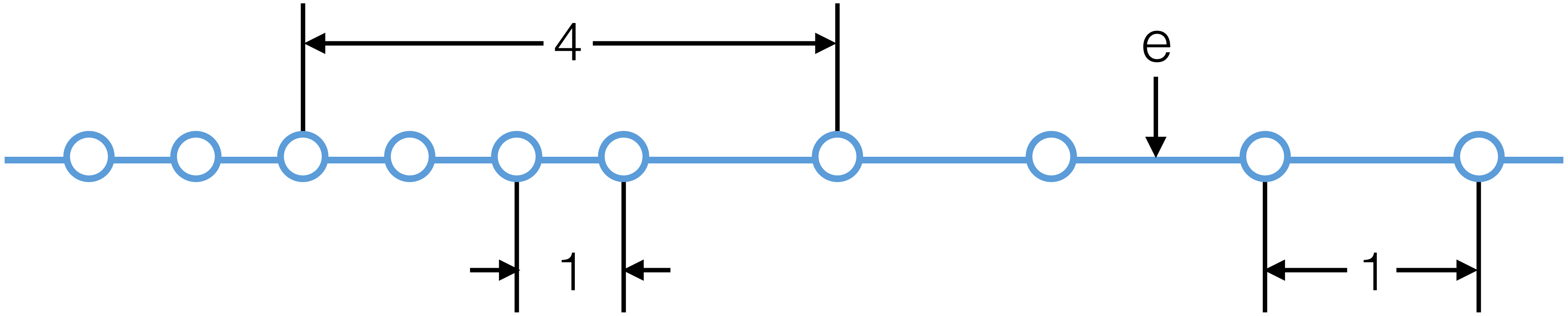


Comparison: Execute target and rewrite on identical state and compare live outputs

Precision Function

- **Uncertainty in Last Place:** Measures the distance between a real number and the closest representable floating-point value

- **Widely Used:** Most scientific applications measure precision in terms of ULPs; 0.5 is the gold standard but very expensive to obtain, most settle for 1 to 2



Verification

- **Guarantees:** How do we know that an optimization is “precise enough” for all possible inputs?
- **Decision Procedures:** Bit-blasting techniques don’t scale beyond a few lines of code; can’t handle mixed fixed- and floating-point codes [darulova 14]
- **Abstract Interpretation:** Can’t prove even bitwise equality in many common cases; can’t handle mixed fixed- and floating-point codes [haller 12]
- **Bottom Line:** No standard techniques can do this

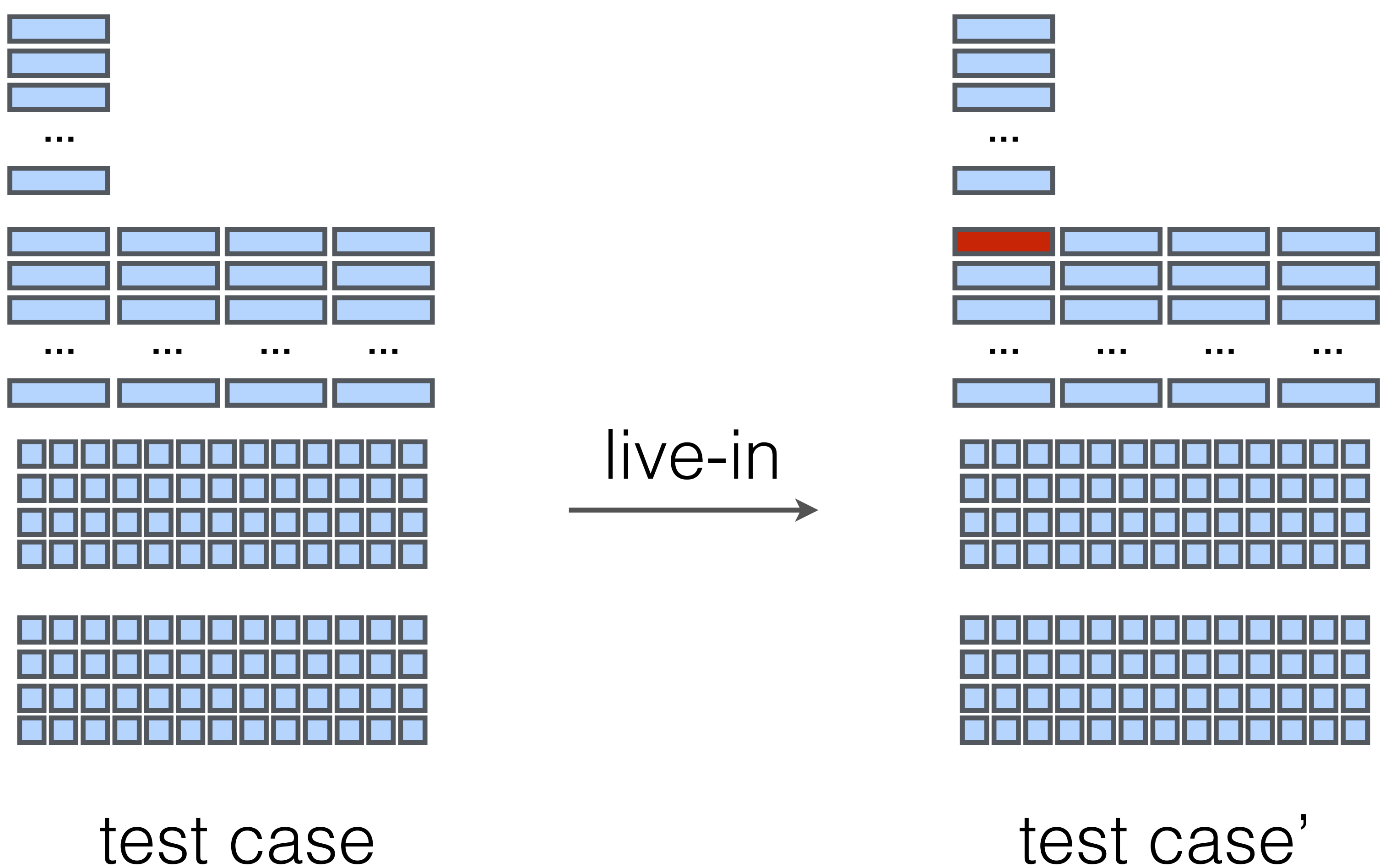
Validation

- **Relaxed problem statement:** Claim with high confidence that there is no input that will cause an error in excess of maximum user bound
- **Error function:** Run original and optimized code on identical inputs and measure ULP Error

  error(x) = **ULP**(**eval**(target, x), **eval**(rewrite, x))

- **Goal:** Show max error is below user-defined bound

Validation

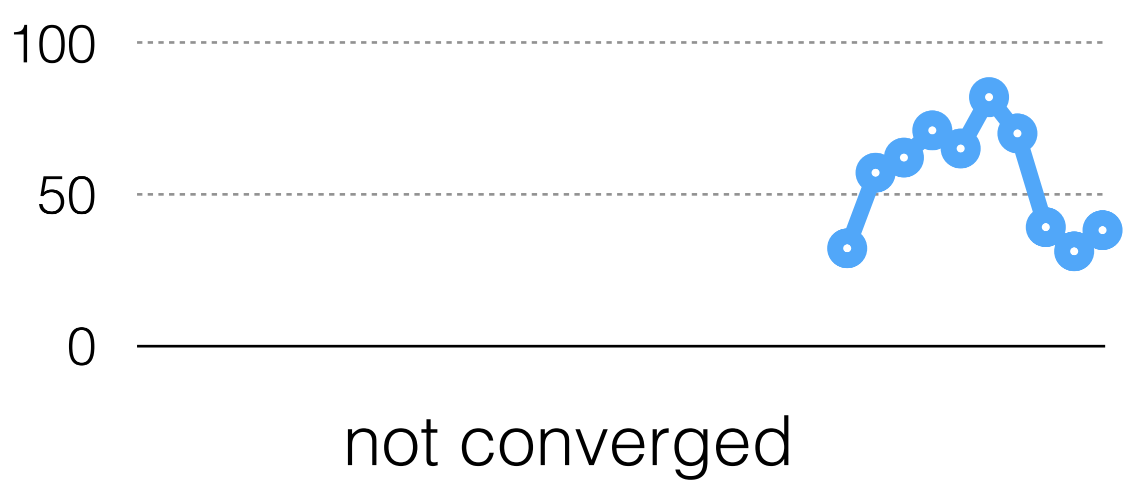


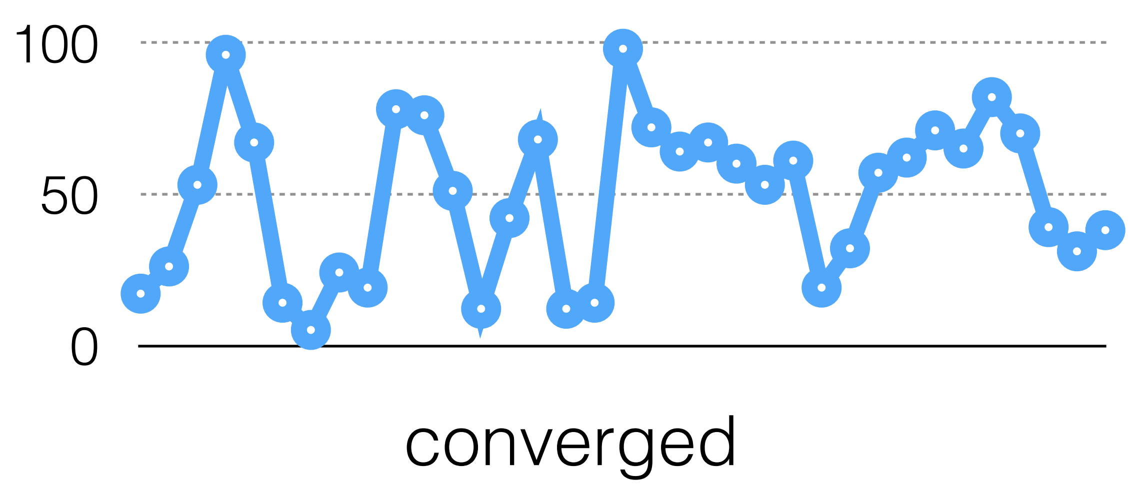
Search: Use MCMC sampling to search for test case inputs that maximize the error function; just one transform

Termination

- **Mixing Tests:** Statistical tests [geweke 92] for producing a high-confidence guarantee that search has sampled uniformly across the domain of a function

- **Therefore:** High confidence that search has discovered all local maxima implies high confidence that it has discovered the global maximum

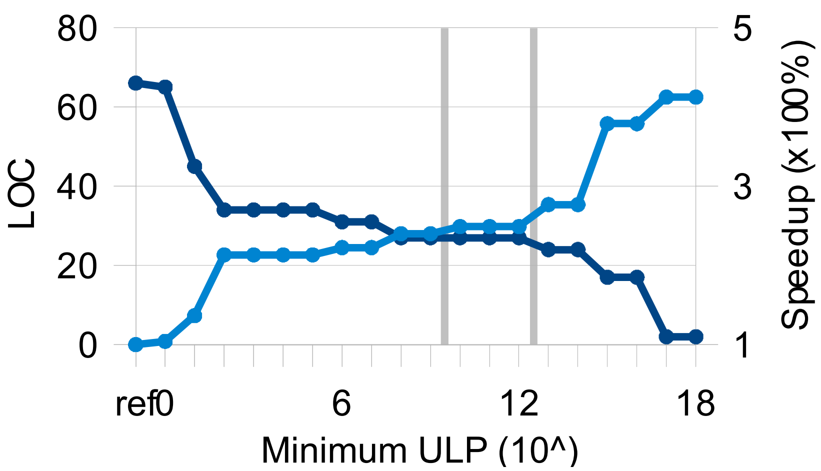


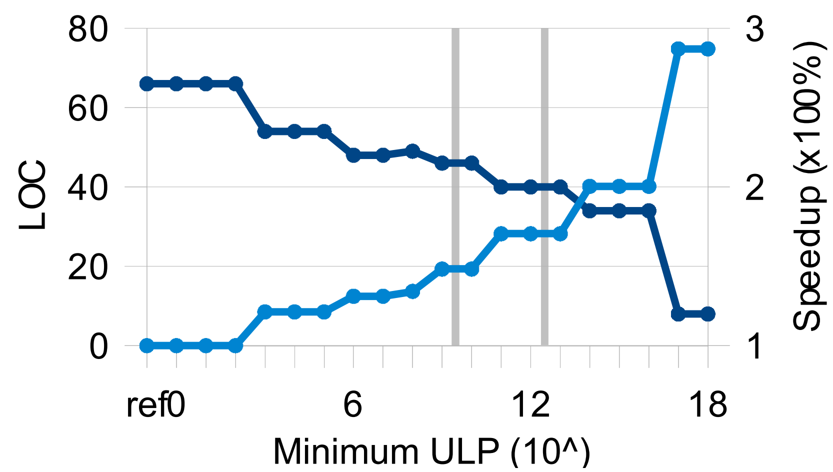


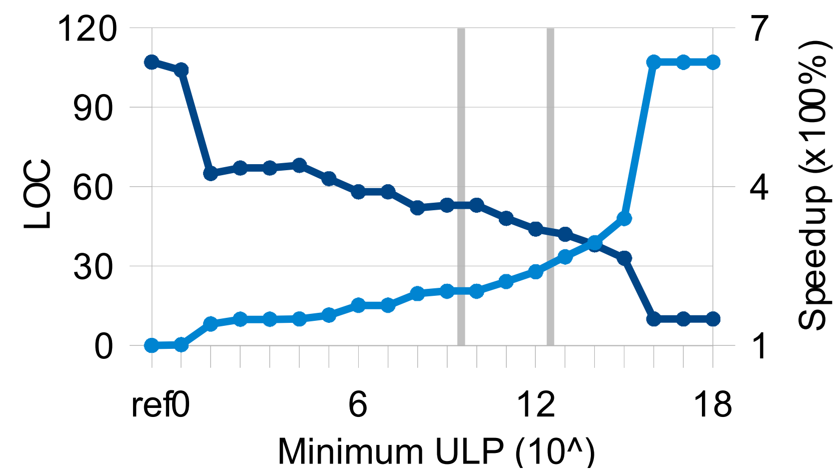
Evaluation

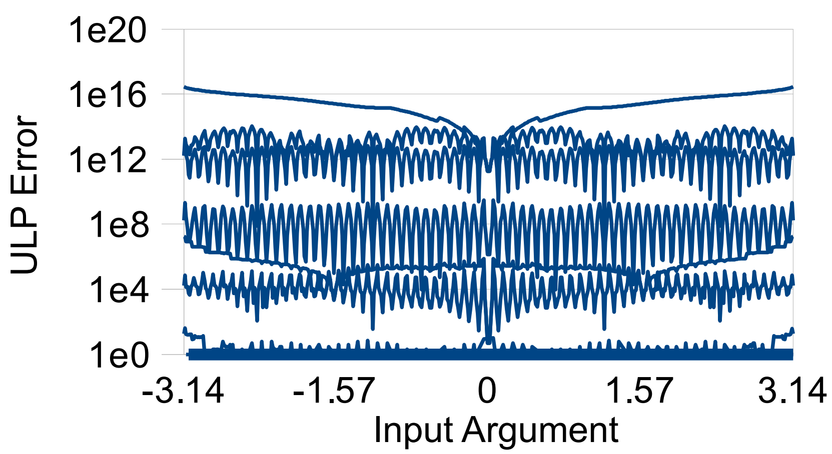
- **Numeric simulation:** S3D, 3-dimensional direct numerical solver for HCCI combustion
- **C library:** libimf, Intel's hand-written implementation of the C numerics library math.h
- **Computer graphics:** A ray tracer

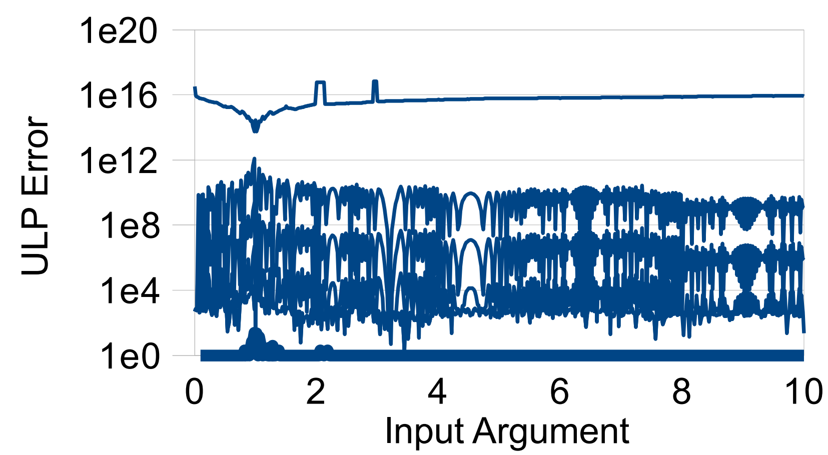
libimf

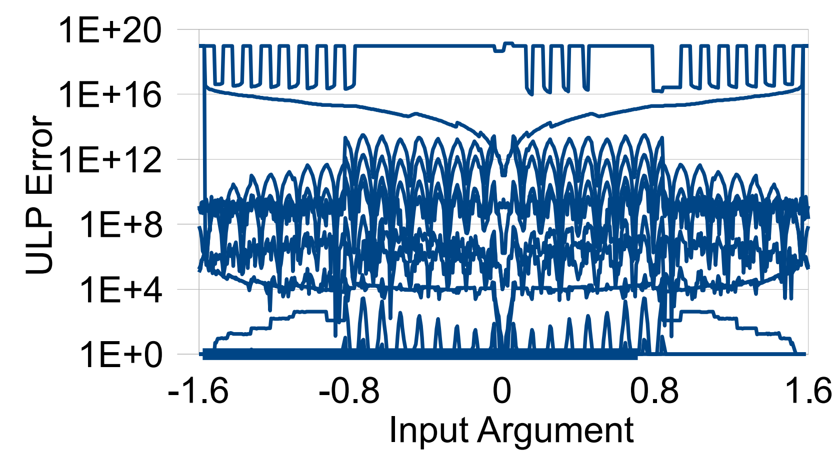












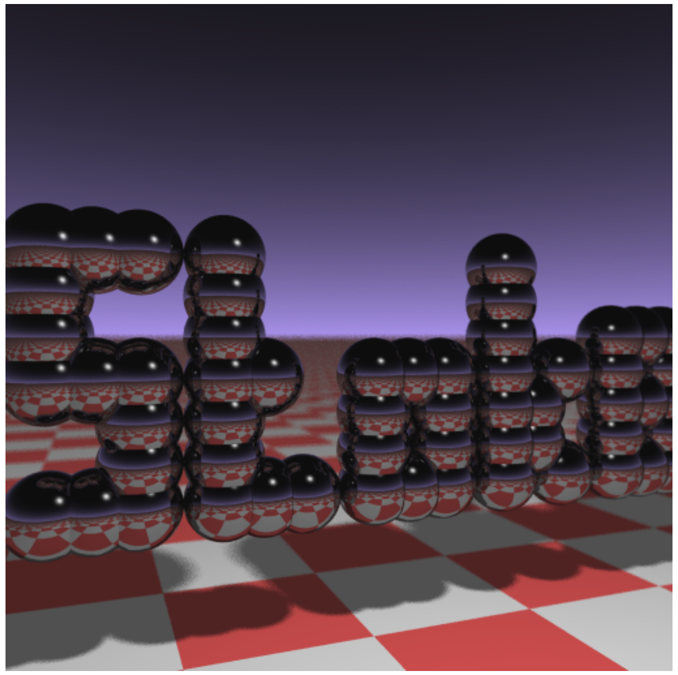
sin()

log()

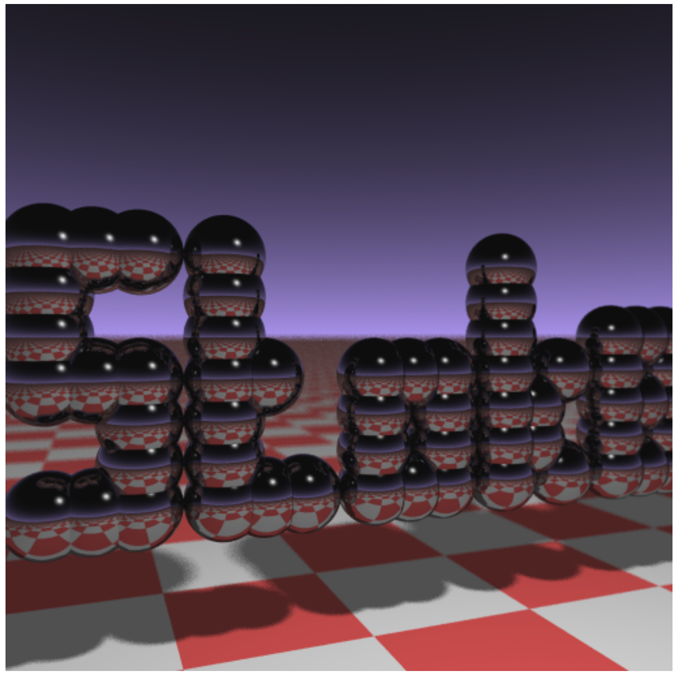
tan()

Ray Tracer

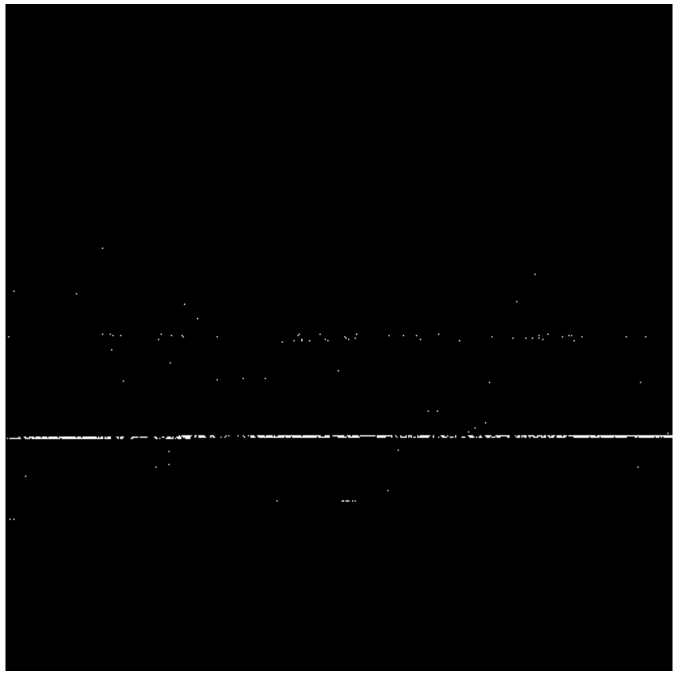
- **Bit-wise correct:** 30% speedup optimizing vector kernels
- **Depth of field blur:** Random perturbations made to viewing camera angle; 6% speedup by relaxing precision requirements



bit-wise correct



relaxed precision

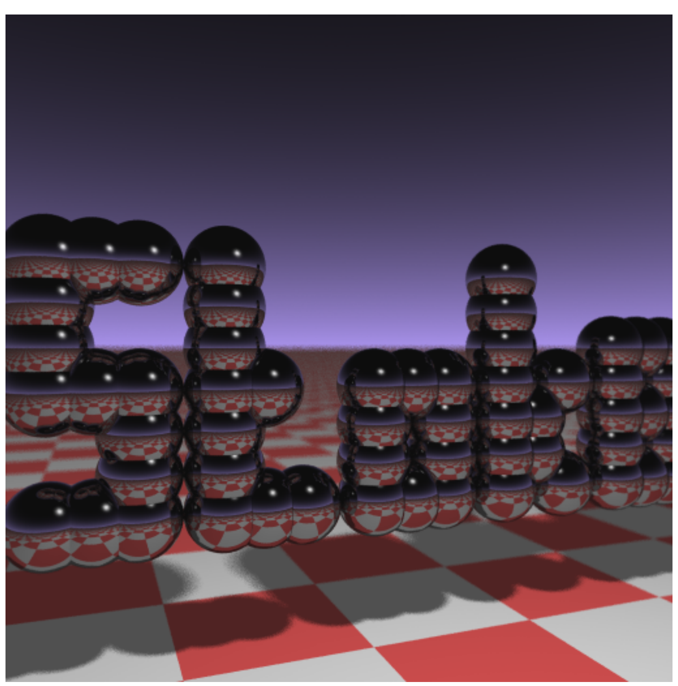


error pixels (white)

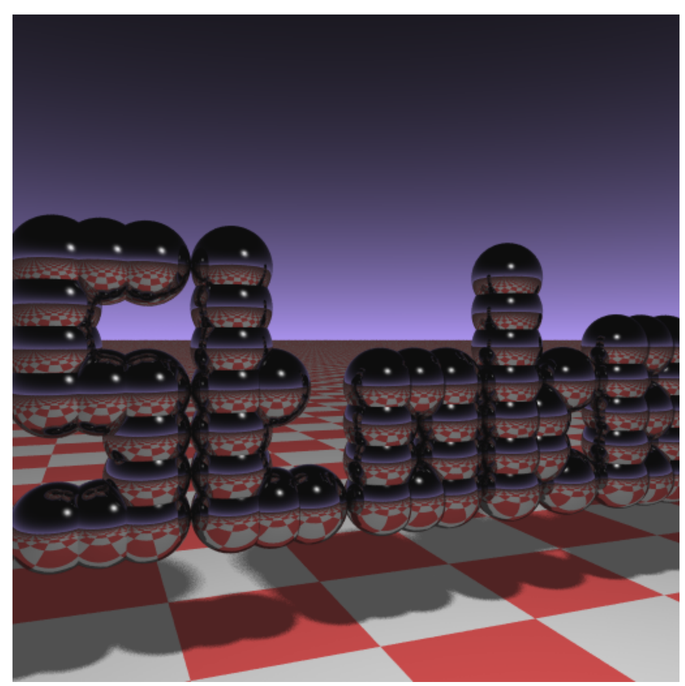
Overfitting

- **Over-relaxation:** If minimum tolerable error exceeds the variance of random perturbations, they are removed altogether

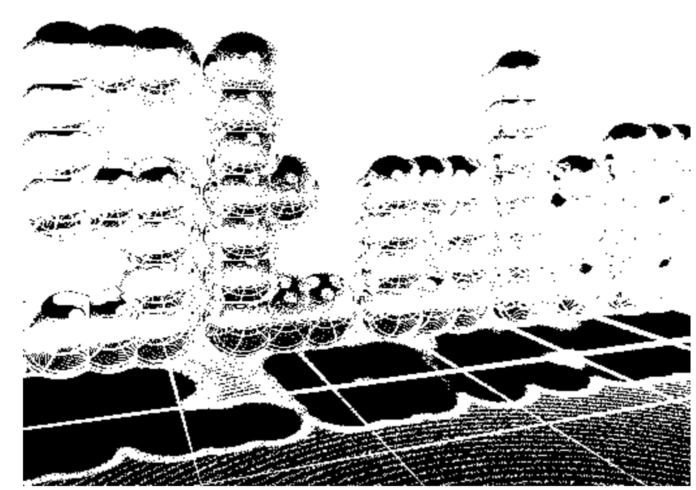
- **Faster still:** But depth of field blur has been removed



bit-wise correct



over-relaxed precision



error pixels (white)

Summary

- **Micro-optimization:** For many interesting application domains, once data movement is orchestrated correctly, even a single instruction can make a difference

- **New approach:** Use random search to experiment with imprecise intermediate optimizations

- **New opportunities for optimization:** Identify applications that can tolerate a loss of precision and produce code that is specialized to that domain

- **Download:** www.github.com/eschkufz/stoke-release