Cyclicity in multivariate time series
and applications to functional MRI data

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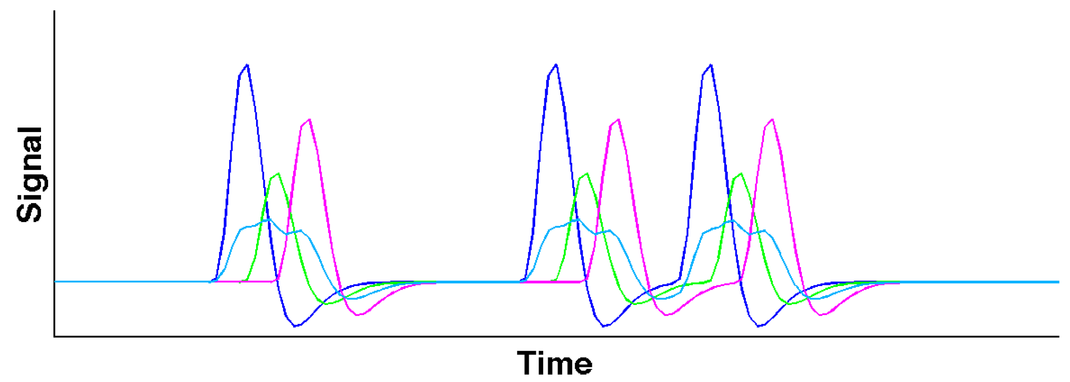
A challenge in time series, and in particular fMRI, data analysis is the absence of a natural time scale at which to consider the data. We aim to develop a tool to extract information which is invariant with respect to reparameterization of the time coordinate.

Cyclic but not periodic



Cyclic but not periodic: *No period* P *can be found such that shifting the process along the time axis by* P *leaves it invariant.*

Examples include cardiac cycles, patterns in gait, population dynamics in closed ecosystems, and business cycles.

Iterated integrals

What are the functions of trajectories that do not depend on how one traverses them?

Iterated integrals of order k: *the vector space generated by the functionals*

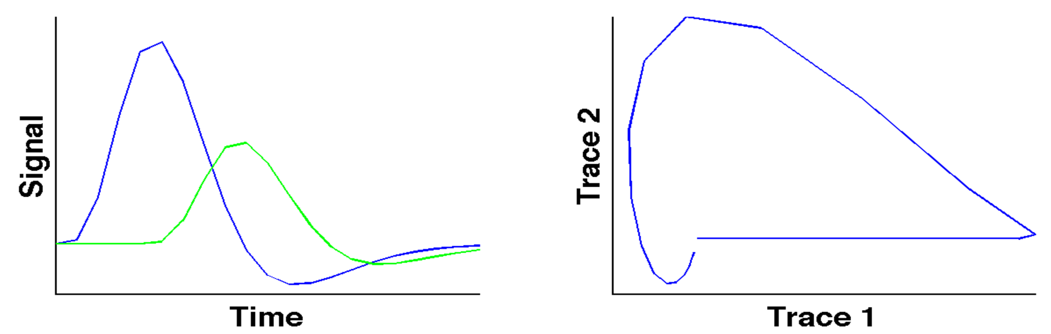
$$I(\gamma) := \sum_{1\leq j\leq d} \int_{t_s}^{t_f} I_k(\gamma_{t_s,t})\, d\gamma_j(t)$$

where I_k *are the iterated integrals of order* $< k$.

The iterated integrals of order 2 are spanned by the functionals

$$I_{k,l} = \int_{t_s}^{t_f} \gamma_k(t)\, d\gamma_l(t),$$

which is the algebraic area of the projections of γ onto coordinate 2-planes.



Chain of offsets model (COOM)

Assume processes being monitored track the same underlying function ϕ

$$f_k(t) = a_k\phi(t-\alpha_k) \qquad \gamma_k(t) = a_k\phi(R(\tau)-\alpha_k) \qquad \text{for } k = \{1,\dots,n\}.$$

Can we discover the cyclic order of the functions f_k, or, equivalently, α_k by recovering from the (perhaps noisy version) of the trajectory, γ, the sampled lead matrix?

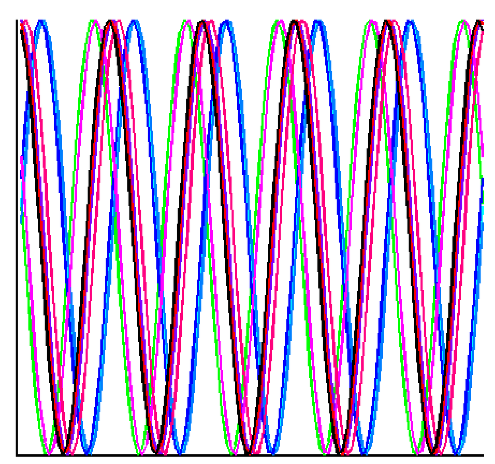
Lead matrix lemma

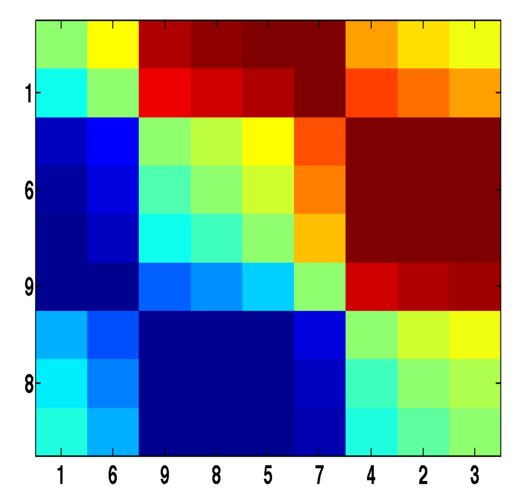
The lead matrix over period A^p is given by

$$A(\phi) = 2\pi a_k a_l\left(\sum_{m\geq 1} m|c_m|^2 \sin\left(m(a_k - a_l)\right)\right)$$

Artificial data

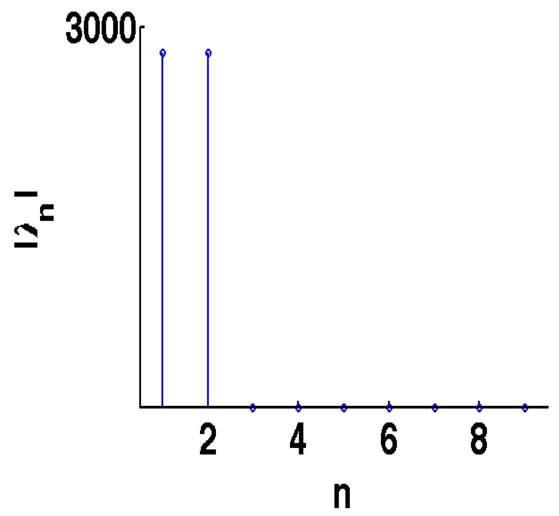
Set of $n = 9$ noiseless traces of the form $\sin(t-\alpha_k)$, for random shift α_k in $\{1,\dots,n\}$.

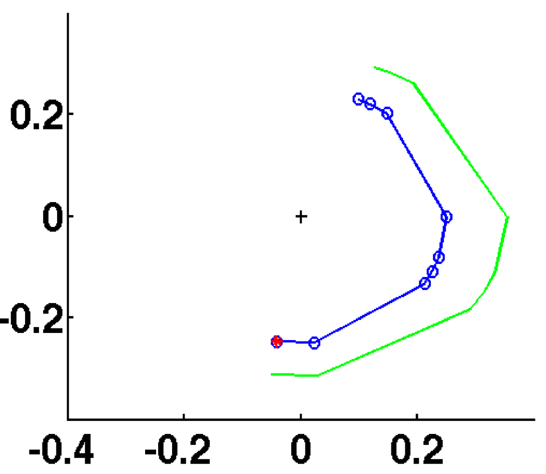




We can approximate A^p by the rank 2 matrix PAP, where P is spanned by the first eigenvector pair.

The ratio λ_1/λ_3 indicates how well the lead matrix can be described by the noiseless harmonic model (for which the ratio is $+\infty$)

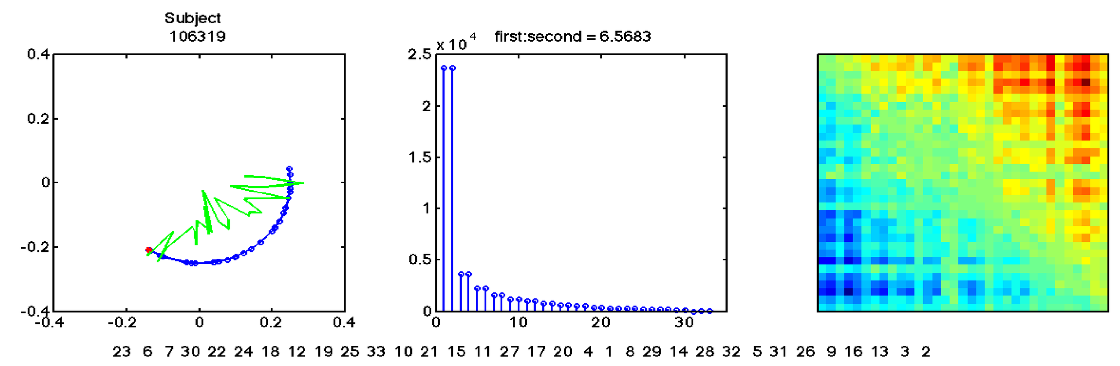


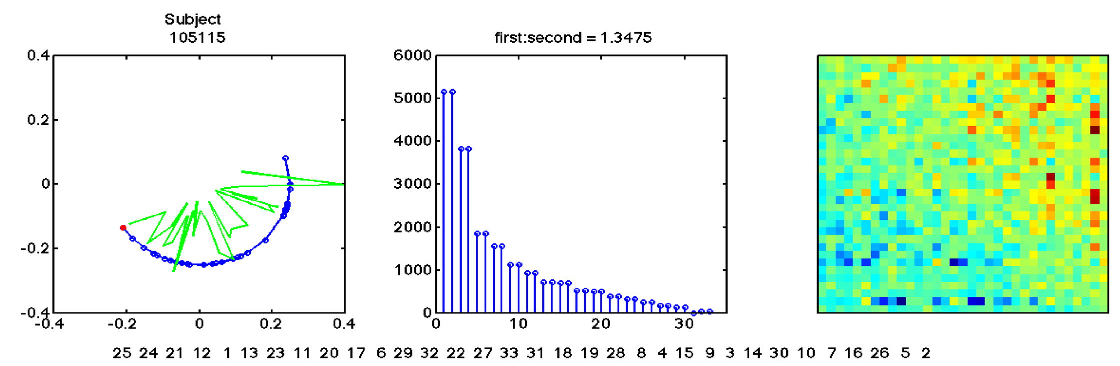


The angular arguments of the components of the first eigenvector of A^p are sorted to obtain the desired cyclic order.

[Baryshnikov and Schlafly, 2016]

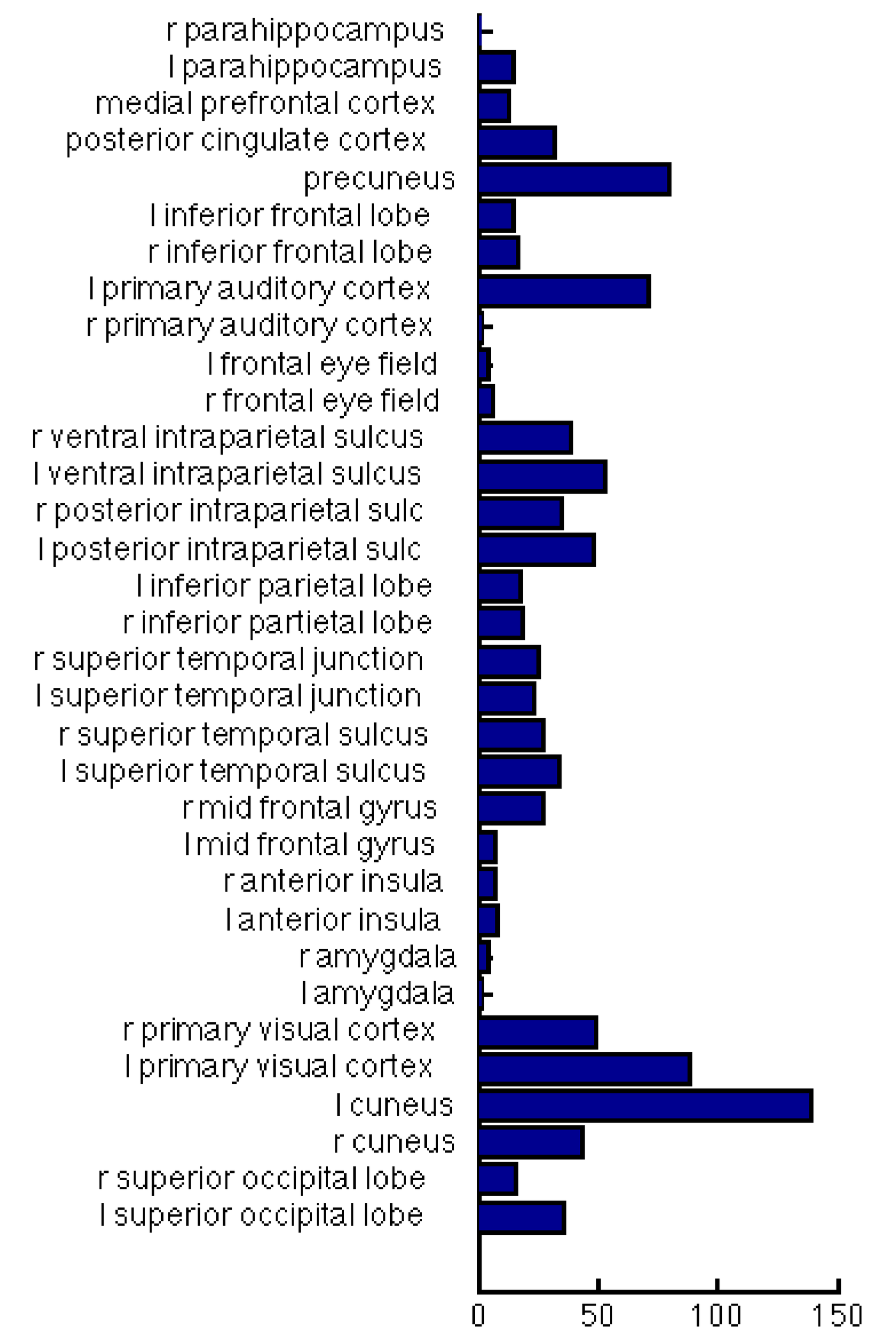
Analysis of HCP data





Above: Results for two HCP subjects with varying levels of clarity in the cyclic analysis.

Below: Regions with high phase magnitudes across all subjects.



References :

Baryshnikov, Y. and Schlafly, E. (2016).
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