3D surface: mortality and kin competition vs fitness

First, I generated lots of kernels -- so far, still using a nonparametric approach. For now, I've reduced the number of distance bins to 6 (0: staying home, 1: distance 1, ... , 5: distance 5).

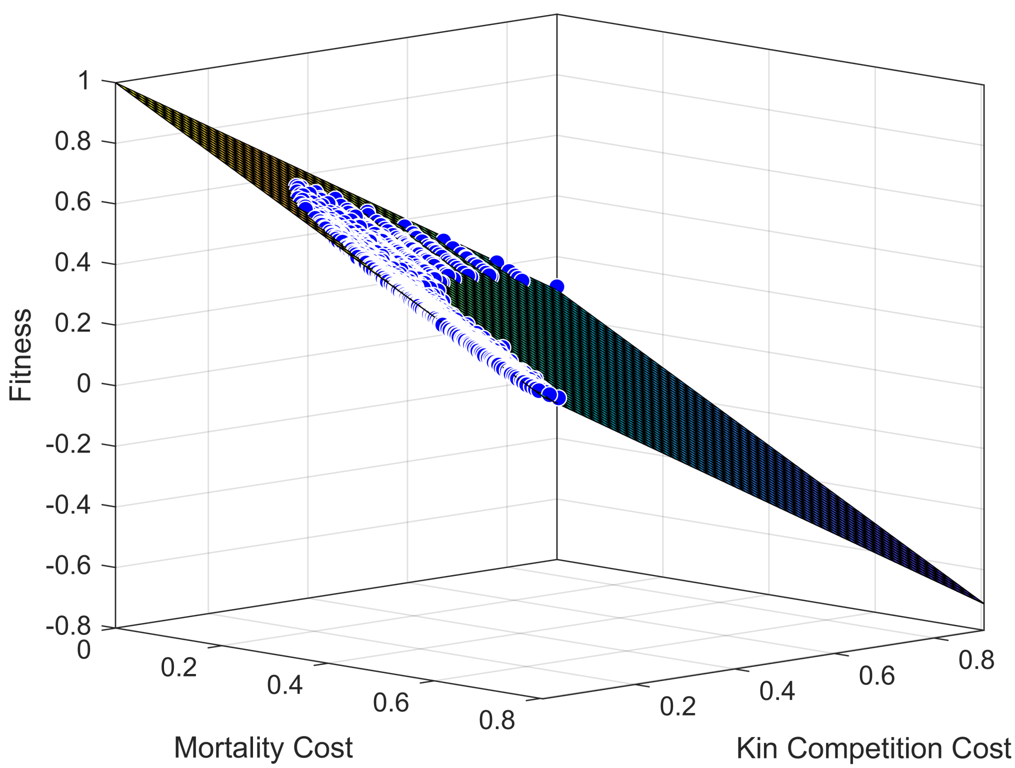
To generate the kernels, I divided the probability mass of 1 into twelfths. Every possible way to distribute those twelfths among the 6 bins is one kernel.

e.g. [12/12, 0, 0, 0, 0, 0] or [11/12, 1/12, 0, 0, 0, 0] or [1/12, 1/12, 1/12, 1/12, 1/12, 7/12], etc. In total, that's 6188 kernels.

For each kernel, I simulated one generation of dispersal and used it to calculate:

- mortality: the proportion of larvae that did not reach suitable habitat
- kin competition: the number of siblings a larva encounters, divided by its total number of siblings. (Note: this is a slight change from previously, when I used the total number of competitors as the denominator. The two quantities give similar results, because the habitat is symmetrical and saturated, but I think total number of siblings is more relevant to kin competition, and it's less sensitive to the population-level reproductive rate.)
- fitness: (1 - mortality) * (1 - kin competition)

On this plot, each kernel is represented by a blue dot. A surface is fit through them.



The fitness surface is almost a flat plane. Based on the definitions, we know that fitness = 1 - mortality cost - kin competition cost + ϵ.

$\epsilon = \sum_d v_d m_d k_d$, where v_d is the mass of the displacement kernel at distance d, m_d is the mortality cost at distance d, and k is the kin competition cost at distance d.

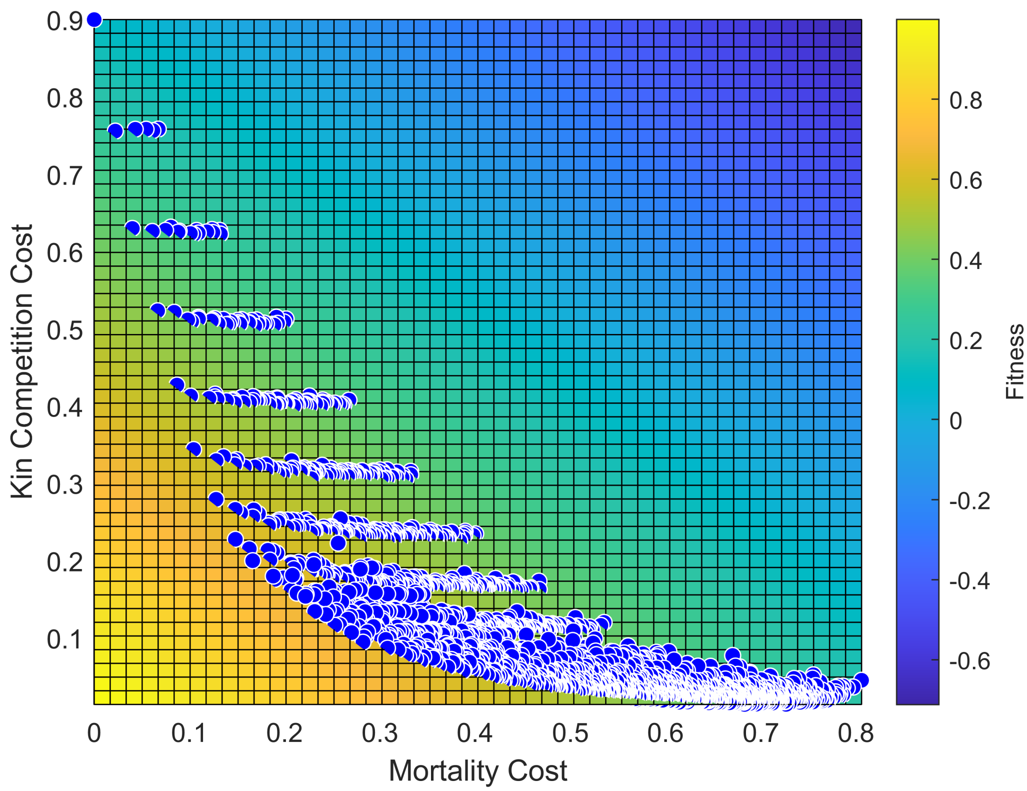
ϵ is small relative to the other quantities, so the plane shown above is a pretty good fit through the points and is roughly described by: fitness = 1 - mortality - kin competition.

```
Linear model Poly11:
f(x,y) = p00 + p10*x + p01*y
Coefficients (with 95% confidence bounds):
  p00 =       1.014  (1.012, 1.015)
  p10 =      -0.995  (-0.9969, -0.9931)
  p01 =      -1.027  (-1.03, -1.024)
```

[Digression: how does the term ϵ change across the surface?]

2D plot: mortality vs kin competition

Since the fitness surface is flat, instead of looking at it in 3-d, let's look at the mortality cost vs kin competition cost plane.

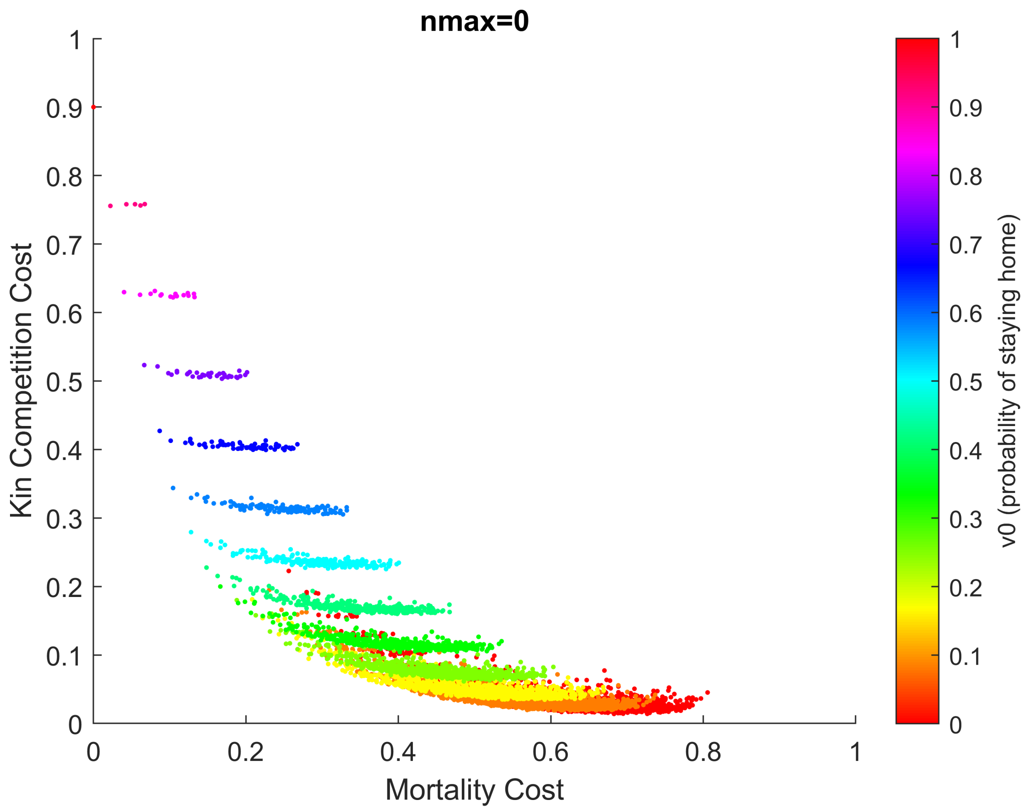


There are only a limited number of combinations of mortality and kin competition that are possible. This constraint is set by the habitat and navigation ability.

Dissecting features of the mortality-kin competition plot

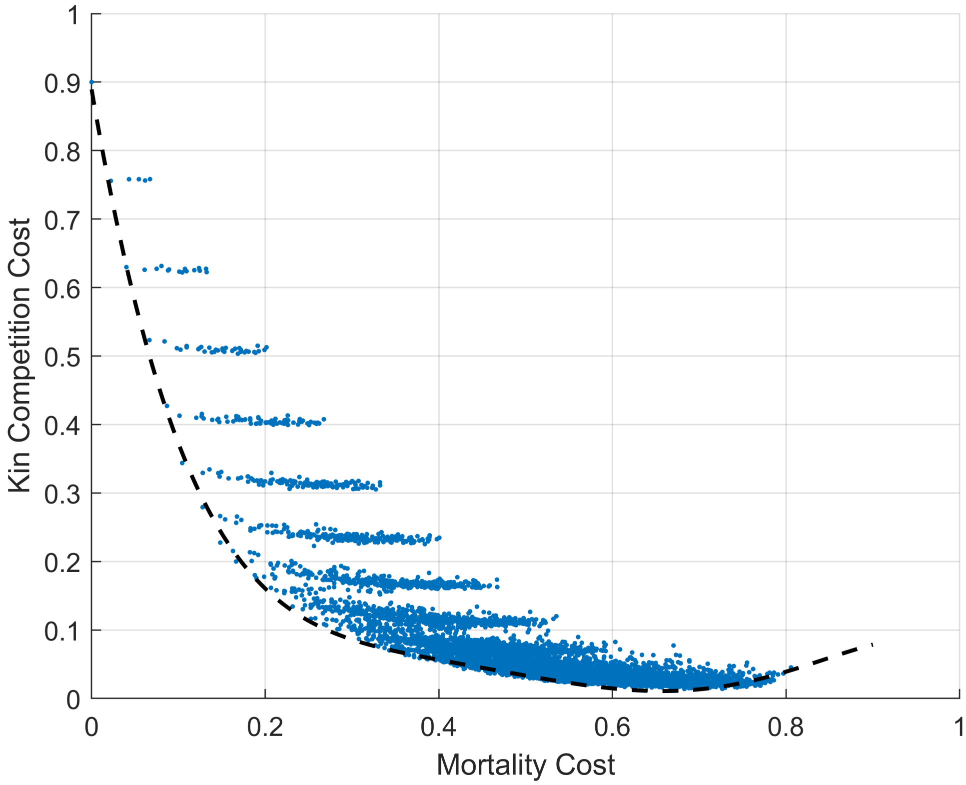
We can get an idea of what sorts of kernels lead to what points on the plot by looking at some patterns. Here, the color of the points corresponds to the mass in the first bin of the kernel (i.e., staying home).

e.g., the pink strip of dots around kin competition = 0.6 corresponds to all kernels with a 10/12 probability of staying home. The other 3/12 can be distributed among the remaining 5 bins in a variety of ways: ([10/12, 1/12, 1/12, 1/12, 0, 0]) or ([10/12, 0, 1/12, 1/12, 1/12, 0]) or ([10/12, 0, 0, 0, 0, 3/12]), etc. In any of these cases, since there are only 10 offspring per parent, two siblings that leave home are unlikely to land in the same patch. So kin competition costs are roughly the same in all cases. Mortality costs, though, are greater for kernels with the 3/12 allocated to longer distances.



Constraints

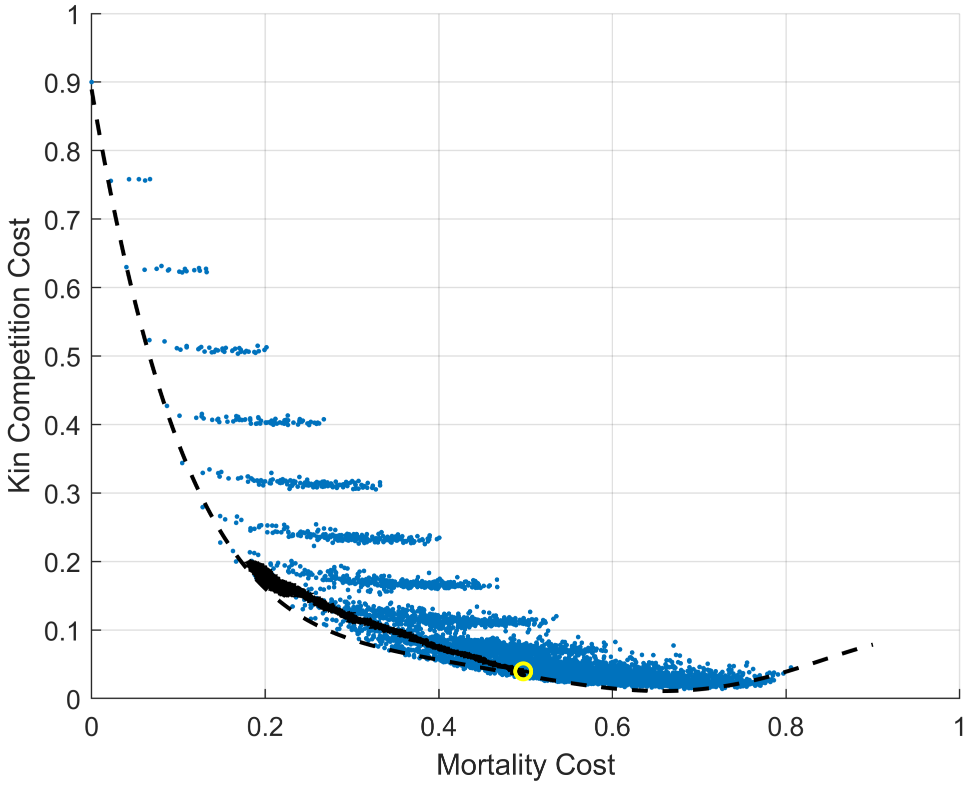
What portions of the plane can be covered, given this seascape and navigation ability? We can roughly draw a line around the lower boundary. This line represents points where neither (mortality or kin competition) cost can be decreased without increasing the other.



I think our sample of kernels gives a good idea of where the boundaries of the shape should be. Additional kernels would fill in space between, but I don't think they'd push the outline at all. [Give some evidence for this.]

Add a trajectory

Simulate kernel evolution, starting with a uniform kernel, for 100 000 generations. Calculate mortality and kin competition costs at each generation. Thin by 100 generations, and plot trajectory.



Kin Competition Cost
Mortality Cost
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1