Fatgraphs of $M_{0,4}$

Automatically generated by FatGHoL 5.4
(See: `http://fatghol.googlecode.com/`)

2012-02-09

There are a total of 21 undecorated fatgraphs in the Kontsevich graph complex of $M_{0,4}$, originating 327 marked ones.

Contents

Notation

We denote $G_{m,j}$ the j-th graph in the set of undecorated fatgraphs with m edges; the symbol $G_{m,j}^{(k)}$ denotes the k-th inequivalent marking of $G_{m,j}$.

Fatgraph vertices are marked with lowercase latin letters "a", "b", "c", etc.; edges are marked with an arabic numeral starting from "1"; boundary cycles are denoted by lowercase greek letters "α", "β", etc.

Automorphisms are specified by their action on the set of vertices, edges, and boundary cycles: for each automorphism A_k, a table line lists how it permutes vertices, edges and boundary cycles relative to the identity morphism A_0. The automorphism table is printed only if the automorphism group is non-trivial.

Automorphisms that reverse the orientation of the unmarked fatgraph are indicated with a "†" symbol in the automorphism table; those that reverse the orientation of the marked fatgraphs are distinguished with a "‡" sign.

If a fatgraph is orientable, a "Markings" section lists all the inequivalent ways of assigning distinct numbers $\{0, \dots, n-1\}$ to the boundary cycles; this is of course a set of representatives for the orbits of $\mathfrak{S}_n$ under the action of $\mathrm{Aut}(G)$.

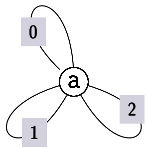
A separate section lists the differential of marked fatgraphs; graphs with null differential are omitted. If no marked fatgraph has a non-zero differential, the entire section is dropped.

Boundary cycles are specified using a "sequence of corners" notation: each corner is represented as ${}^pL^q$ where L is a latin letter indicating a vertex, and p, q are the attachment indices of the incoming and outgoing edges, respectively. Attachment indices match the Python representation of the vertex: e.g., if a=`Vertex([0,0,1])`, the two legs of edge 0 have attachment indices 0 and 1, and the boundary cycle enclosed by them is represented by the (single) corner ${}^0a^1$.

Fatgraphs with 3 edges / 1 vertex

There are 2 unmarked fatgraphs in this section, originating 40 marked fatgraphs (20 orientable, and 20 nonorientable).

The Fatgraph $G_{3,0}$ (8 orientable markings)




```
Fatgraph([
  Vertex([0, 0, 1, 1, 2, 2]),# a
])
```

Boundary cycles

$$\alpha = ({}^{0}a^{1})$$
$$\beta = ({}^{1}a^{2} \rightarrow {}^{3}a^{4} \rightarrow {}^{5}a^{0})$$
$$\gamma = ({}^{2}a^{3})$$
$$\delta = ({}^{4}a^{5})$$

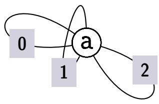
Automorphisms

A_0	a	0	1	2	α	β	γ	δ
A_1‡	a	1	2	0	γ	β	δ	α
A_2‡	a	2	0	1	δ	β	α	γ

Markings

	$G_{3,0}^{(0)}$	$G_{3,0}^{(1)}$	$G_{3,0}^{(2)}$	$G_{3,0}^{(3)}$	$G_{3,0}^{(4)}$	$G_{3,0}^{(5)}$	$G_{3,0}^{(6)}$	$G_{3,0}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

The Fatgraph $G_{3,1}$ (non-orientable, 12 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 0, 1, 2, 2]),# a
])
```

Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{0}a^{1})$$
$$\beta = ({}^{1}a^{2})$$
$$\gamma = ({}^{3}a^{4} \to {}^{5}a^{0})$$
$$\delta = ({}^{4}a^{5})$$

Automorphisms

A_0	a	0	1	2	α	β	γ	δ
A_1 †‡	a	2	1	0	γ	δ	α	β

Markings

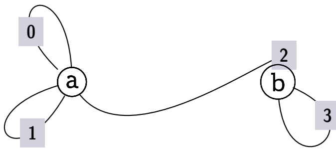
	$G^{(0)}_{3,1}$	$G^{(1)}_{3,1}$	$G^{(2)}_{3,1}$	$G^{(3)}_{3,1}$	$G^{(4)}_{3,1}$	$G^{(5)}_{3,1}$	$G^{(6)}_{3,1}$	$G^{(7)}_{3,1}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

	$G^{(8)}_{3,1}$	$G^{(9)}_{3,1}$	$G^{(10)}_{3,1}$	$G^{(11)}_{3,1}$
α	1	1	2	2
β	2	3	0	1
γ	3	2	3	3
δ	0	0	1	0

Fatgraphs with 4 edges / 2 vertices

There are 6 unmarked fatgraphs in this section, originating 198 marked fatgraphs (99 orientable, and 99 nonorientable).

The Fatgraph $G_{4,0}$ (24 orientable markings)




```
Fatgraph([
  Vertex([0, 0, 1, 1, 2]),# a
  Vertex([3, 3, 2]),      # b
])
```

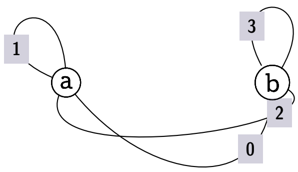
Boundary cycles

$$\alpha = ({}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^2b^0 \to {}^4a^0 \to {}^3a^4 \to {}^1b^2)$$
$$\gamma = ({}^2a^3)$$
$$\delta = ({}^0b^1)$$

Markings

Fatgraph $G_{4,0}$ only has the identity automorphism, so the marked fatgraphs $G_{4,0}^{(0)}$ to $G_{4,0}^{(24)}$ are formed by decorating boundary cycles of $G_{4,0}$ with all permutations of $(0,1,2,3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,1}$ (12 orientable markings)




```
Fatgraph([
  Vertex([1, 1, 2, 0]),# a
  Vertex([0, 2, 3, 3]),# b
])
```

Boundary cycles

$$\alpha = ({}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^3a^0 \to {}^3b^0 \to {}^1b^2)$$
$$\gamma = ({}^2a^3 \to {}^0b^1)$$
$$\delta = ({}^2b^3)$$

Automorphisms

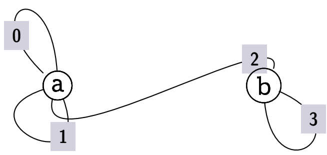
A_0	a	b	0	1	2	3	α	β	γ	δ
$A_1{}^\ddagger$	b	a	2	3	0	1	δ	β	γ	α

Markings

	$G_{4,1}^{(0)}$	$G_{4,1}^{(1)}$	$G_{4,1}^{(2)}$	$G_{4,1}^{(3)}$	$G_{4,1}^{(4)}$	$G_{4,1}^{(5)}$	$G_{4,1}^{(6)}$	$G_{4,1}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

	$G_{4,1}^{(8)}$	$G_{4,1}^{(9)}$	$G_{4,1}^{(10)}$	$G_{4,1}^{(11)}$
α	1	1	2	2
β	2	3	0	1
γ	0	0	1	0
δ	3	2	3	3

The Fatgraph $G_{4,2}$ (24 orientable markings)




```
Fatgraph([
  Vertex([0, 0, 1, 2, 1]),# a
  Vertex([3, 3, 2]),      # b
])
```

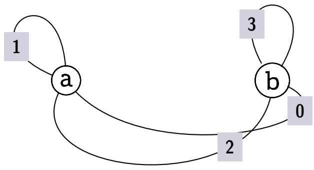
Boundary cycles

$$\alpha = ({}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^4a^0)$$
$$\gamma = ({}^2a^3 \to {}^2b^0 \to {}^3a^4 \to {}^1b^2)$$
$$\delta = ({}^0b^1)$$

Markings

Fatgraph $G_{4,2}$ only has the identity automorphism, so the marked fatgraphs $G_{4,2}^{(0)}$ to $G_{4,2}^{(24)}$ are formed by decorating boundary cycles of $G_{4,2}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,3}$ (non-orientable, 12 orientable markings)




```
Fatgraph([
  Vertex([1, 1, 2, 0]),# a
  Vertex([2, 0, 3, 3]),# b
])
```

Boundary cycles

$$\alpha = ({}^{0}a^{1})$$
$$\beta = ({}^{1}a^{2} \to {}^{3}a^{0} \to {}^{0}b^{1})$$
$$\gamma = ({}^{2}a^{3} \to {}^{3}b^{0} \to {}^{1}b^{2})$$
$$\delta = ({}^{2}b^{3})$$

Automorphisms

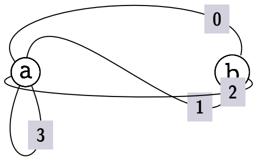
A_0	a	b	0	1	2	3	α	β	γ	δ
$A_1{}^{\dagger\ddagger}$	b	a	0	3	2	1	δ	γ	β	α

Markings

	$G_{4,3}^{(0)}$	$G_{4,3}^{(1)}$	$G_{4,3}^{(2)}$	$G_{4,3}^{(3)}$	$G_{4,3}^{(4)}$	$G_{4,3}^{(5)}$	$G_{4,3}^{(6)}$	$G_{4,3}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

	$G_{4,3}^{(8)}$	$G_{4,3}^{(9)}$	$G_{4,3}^{(10)}$	$G_{4,3}^{(11)}$
α	1	1	2	2
β	2	3	0	1
γ	0	0	1	0
δ	3	2	3	3

The Fatgraph $G_{4,4}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2, 3, 3]),# a
  Vertex([1, 2, 0]),      # b
])
```

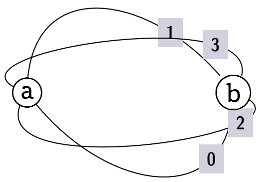
Boundary cycles

$$\alpha = ({}^2b^0 \to {}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^1b^2)$$
$$\gamma = ({}^2a^3 \to {}^0b^1 \to {}^4a^0)$$
$$\delta = ({}^3a^4)$$

Markings

Fatgraph $G_{4,4}$ only has the identity automorphism, so the marked fatgraphs $G_{4,4}^{(0)}$ to $G_{4,4}^{(24)}$ are formed by decorating boundary cycles of $G_{4,4}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,5}$ (non-orientable, 3 orientable markings)




```
Fatgraph([
  Vertex([1, 3, 2, 0]),# a
  Vertex([0, 2, 3, 1]),# b
])
```

Boundary cycles

$$\alpha = ({}^0a^1 \to {}^2b^3)$$
$$\beta = ({}^1a^2 \to {}^1b^2)$$
$$\gamma = ({}^2a^3 \to {}^0b^1)$$
$$\delta = ({}^3a^0 \to {}^3b^0)$$

Automorphisms

A_0	a	b	0	1	2	3	α	β	γ	δ

$A_1{}^{\dagger\ddagger}$	a	b	1	3	0	2	β	γ	δ	α
$A_2{}^{\ddagger}$	a	b	3	2	1	0	γ	δ	α	β
$A_3{}^{\dagger\ddagger}$	a	b	2	0	3	1	δ	α	β	γ
$A_4{}^{\ddagger}$	b	a	1	0	3	2	γ	β	α	δ
$A_5{}^{\dagger\ddagger}$	b	a	0	2	1	3	β	α	δ	γ
$A_6{}^{\ddagger}$	b	a	2	3	0	1	α	δ	γ	β
$A_7{}^{\dagger\ddagger}$	b	a	3	1	2	0	δ	γ	β	α

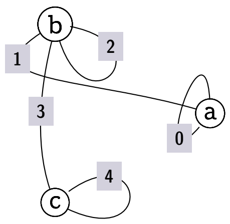
Markings

	$G_{4,5}^{(0)}$	$G_{4,5}^{(1)}$	$G_{4,5}^{(2)}$
α	0	0	0
β	1	1	2
γ	2	3	1
δ	3	2	3

Fatgraphs with 5 edges / 3 vertices

There are 7 unmarked fatgraphs in this section, originating 288 marked fatgraphs (144 orientable, and 144 nonorientable).

The Fatgraph $G_{5,0}$ (24 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 0]),   # a
  Vertex([1, 2, 2, 3]),# b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$
\begin{aligned}
\alpha &= ({}^{1}a^{2} \to {}^{0}a^{1} \to {}^{2}b^{3} \to {}^{3}b^{0} \to {}^{1}c^{2} \to {}^{2}c^{0} \to {}^{0}b^{1}) \\
\beta &= ({}^{2}a^{0}) \\
\gamma &= ({}^{1}b^{2}) \\
\delta &= ({}^{0}c^{1})
\end{aligned}
$$

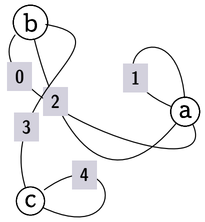
Markings

Fatgraph $G_{5,0}$ only has the identity automorphism, so the marked fatgraphs $G_{5,0}^{(0)}$ to $G_{5,0}^{(24)}$ are formed by decorating boundary cycles of $G_{5,0}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$
\begin{aligned}
D(G_{5,0}^{(0)}) &= +G_{4,0}^{(0)} \\
D(G_{5,0}^{(1)}) &= +G_{4,0}^{(1)} \\
D(G_{5,0}^{(2)}) &= +G_{4,0}^{(2)} \\
D(G_{5,0}^{(3)}) &= +G_{4,0}^{(3)} \\
D(G_{5,0}^{(4)}) &= +G_{4,0}^{(4)} \\
D(G_{5,0}^{(5)}) &= +G_{4,0}^{(5)} \\
D(G_{5,0}^{(8)}) &= +G_{4,0}^{(3)} \\
D(G_{5,0}^{(9)}) &= +G_{4,0}^{(2)} \\
D(G_{5,0}^{(10)}) &= +G_{4,0}^{(5)} \\
D(G_{5,0}^{(11)}) &= +G_{4,0}^{(4)} \\
D(G_{5,0}^{(14)}) &= +G_{4,0}^{(1)} \\
D(G_{5,0}^{(15)}) &= +G_{4,0}^{(0)} \\
D(G_{5,0}^{(16)}) &= +G_{4,0}^{(4)} \\
D(G_{5,0}^{(17)}) &= +G_{4,0}^{(5)} \\
D(G_{5,0}^{(20)}) &= +G_{4,0}^{(0)} \\
D(G_{5,0}^{(21)}) &= +G_{4,0}^{(1)} \\
D(G_{5,0}^{(22)}) &= +G_{4,0}^{(2)} \\
D(G_{5,0}^{(23)}) &= +G_{4,0}^{(3)}
\end{aligned}
$$

The Fatgraph $G_{5,1}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 1, 2, 0]),# a
  Vertex([0, 2, 3]),   # b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{0}a^{1})$$
$$\beta = ({}^{1}a^{2} \to {}^{2}c^{0} \to {}^{3}a^{0} \to {}^{1}c^{2} \to {}^{2}b^{0} \to {}^{1}b^{2})$$
$$\gamma = ({}^{2}a^{3} \to {}^{0}b^{1})$$
$$\delta = ({}^{0}c^{1})$$

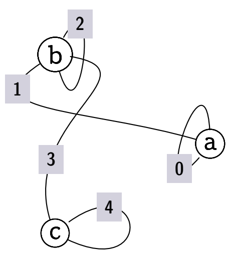
Markings

Fatgraph $G_{5,1}$ only has the identity automorphism, so the marked fatgraphs $G_{5,1}^{(0)}$ to $G_{5,1}^{(24)}$ are formed by decorating boundary cycles of $G_{5,1}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,1}^{(0)}) = +G_{4,0}^{(0)} + G_{4,0}^{(1)}$$
$$D(G_{5,1}^{(1)}) = +G_{4,0}^{(0)} + G_{4,0}^{(1)}$$
$$D(G_{5,1}^{(2)}) = +G_{4,0}^{(2)} + G_{4,0}^{(3)}$$
$$D(G_{5,1}^{(3)}) = +G_{4,0}^{(2)} + G_{4,0}^{(3)}$$
$$D(G_{5,1}^{(4)}) = +G_{4,0}^{(4)} + G_{4,0}^{(5)}$$
$$D(G_{5,1}^{(5)}) = +G_{4,0}^{(4)} + G_{4,0}^{(5)}$$
$$D(G_{5,1}^{(8)}) = +G_{4,0}^{(2)} + G_{4,0}^{(3)}$$
$$D(G_{5,1}^{(9)}) = +G_{4,0}^{(4)} + G_{4,0}^{(5)}$$
$$D(G_{5,1}^{(11)}) = +G_{4,0}^{(0)} + G_{4,0}^{(1)}$$

The Fatgraph $G_{5,2}$ (12 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 0]),   # a
  Vertex([1, 2, 3, 2]),# b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\begin{aligned}
\alpha &= ({}^{1}a^{2} \to {}^{3}b^{0} \to {}^{0}a^{1} \to {}^{0}b^{1}) \\
\beta &= ({}^{2}a^{0}) \\
\gamma &= ({}^{2}b^{3} \to {}^{2}c^{0} \to {}^{1}c^{2} \to {}^{1}b^{2}) \\
\delta &= ({}^{0}c^{1})
\end{aligned}$$

Automorphisms

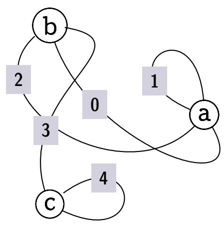
A_0	a	b	c	0	1	2	3	4	α	β	γ	δ
$A_1{}^{\ddagger}$	c	b	a	4	3	2	1	0	γ	δ	α	β

Markings

	$G_{5,2}^{(0)}$	$G_{5,2}^{(1)}$	$G_{5,2}^{(2)}$	$G_{5,2}^{(3)}$	$G_{5,2}^{(4)}$	$G_{5,2}^{(5)}$	$G_{5,2}^{(6)}$	$G_{5,2}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

	$G_{5,2}^{(8)}$	$G_{5,2}^{(9)}$	$G_{5,2}^{(10)}$	$G_{5,2}^{(11)}$
α	1	1	2	2
β	2	3	0	1
γ	3	2	3	3
δ	0	0	1	0

The Fatgraph $G_{5,3}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 1, 2, 0]),# a
  Vertex([2, 0, 3]),   # b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\begin{aligned}
\alpha &= ({}^{0}a^{1}) \\
\beta &= ({}^{1}a^{2} \to {}^{3}a^{0} \to {}^{0}b^{1}) \\
\gamma &= ({}^{2}a^{3} \to {}^{2}b^{0} \to {}^{2}c^{0} \to {}^{1}c^{2} \to {}^{1}b^{2}) \\
\delta &= ({}^{0}c^{1})
\end{aligned}$$

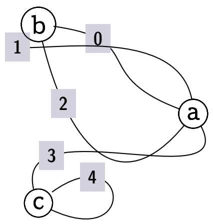
Markings

Fatgraph $G_{5,3}$ only has the identity automorphism, so the marked fatgraphs $G_{5,3}^{(0)}$ to $G_{5,3}^{(24)}$ are formed by decorating boundary cycles of $G_{5,3}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,3}^{(12)}) = +G_{4,0}^{(2)}$$
$$D(G_{5,3}^{(13)}) = +G_{4,0}^{(4)}$$
$$D(G_{5,3}^{(14)}) = +G_{4,0}^{(0)}$$
$$D(G_{5,3}^{(15)}) = +G_{4,0}^{(5)}$$
$$D(G_{5,3}^{(16)}) = +G_{4,0}^{(1)}$$
$$D(G_{5,3}^{(17)}) = +G_{4,0}^{(3)}$$
$$D(G_{5,3}^{(18)}) = +G_{4,0}^{(3)}$$
$$D(G_{5,3}^{(19)}) = +G_{4,0}^{(5)}$$
$$D(G_{5,3}^{(21)}) = +G_{4,0}^{(4)}$$
$$D(G_{5,3}^{(23)}) = +G_{4,0}^{(2)}$$

The Fatgraph $G_{5,4}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 2, 0]),   # b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{2}b^{0} \to {}^{0}a^{1})$$
$$\beta = ({}^{1}a^{2} \to {}^{1}b^{2})$$
$$\gamma = ({}^{2}a^{3} \to {}^{3}a^{0} \to {}^{0}b^{1} \to {}^{1}c^{2} \to {}^{2}c^{0})$$
$$\delta = ({}^{0}c^{1})$$

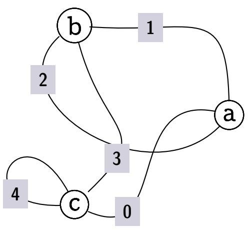
Markings

Fatgraph $G_{5,4}$ only has the identity automorphism, so the marked fatgraphs $G_{5,4}^{(0)}$ to $G_{5,4}^{(24)}$ are formed by decorating boundary cycles of $G_{5,4}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,4}^{(0)}) = +G_{4,0}^{(1)}$$
$$D(G_{5,4}^{(1)}) = +G_{4,0}^{(4)}$$
$$D(G_{5,4}^{(3)}) = +G_{4,0}^{(5)}$$
$$D(G_{5,4}^{(5)}) = +G_{4,0}^{(0)}$$
$$D(G_{5,4}^{(6)}) = +G_{4,0}^{(0)}$$
$$D(G_{5,4}^{(7)}) = +G_{4,0}^{(2)}$$
$$D(G_{5,4}^{(9)}) = +G_{4,0}^{(3)}$$
$$D(G_{5,4}^{(11)}) = +G_{4,0}^{(1)}$$

The Fatgraph $G_{5,5}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),   # a
  Vertex([2, 3, 1]),   # b
  Vertex([0, 3, 4, 4]),# c
])
```

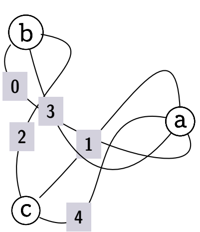
Boundary cycles

$$
\begin{aligned}
\alpha &= ({}^{0}a^{1} \to {}^{0}c^{1} \to {}^{1}b^{2}) \\
\beta &= ({}^{1}a^{2} \to {}^{3}c^{0} \to {}^{1}c^{2} \to {}^{0}b^{1}) \\
\gamma &= ({}^{2}a^{0} \to {}^{2}b^{0}) \\
\delta &= ({}^{2}c^{3})
\end{aligned}
$$

Markings

Fatgraph $G_{5,5}$ only has the identity automorphism, so the marked fatgraphs $G_{5,5}^{(0)}$ to $G_{5,5}^{(24)}$ are formed by decorating boundary cycles of $G_{5,5}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{5,6}$ (12 orientable markings)




```
Fatgraph([
  Vertex([1, 4, 3, 0]),# a
  Vertex([0, 3, 2]),   # b
  Vertex([4, 1, 2]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{0}a^{1} \to {}^{0}c^{1})$$
$$\beta = ({}^{1}a^{2} \to {}^{2}c^{0} \to {}^{1}b^{2})$$
$$\gamma = ({}^{2}a^{3} \to {}^{0}b^{1})$$
$$\delta = ({}^{3}a^{0} \to {}^{2}b^{0} \to {}^{1}c^{2})$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	α	β	γ	δ
A_1 ‡	a	c	b	4	3	2	1	0	γ	δ	α	β

Markings

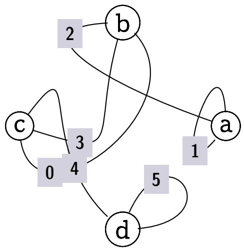
	$G_{5,6}^{(0)}$	$G_{5,6}^{(1)}$	$G_{5,6}^{(2)}$	$G_{5,6}^{(3)}$	$G_{5,6}^{(4)}$	$G_{5,6}^{(5)}$	$G_{5,6}^{(6)}$	$G_{5,6}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2

	$G_{5,6}^{(8)}$	$G_{5,6}^{(9)}$	$G_{5,6}^{(10)}$	$G_{5,6}^{(11)}$
α	1	1	2	2
β	2	3	0	1
γ	3	2	3	3
δ	0	0	1	0

Fatgraphs with 6 edges / 4 vertices

There are 6 unmarked fatgraphs in this section, originating 128 marked fatgraphs (64 orientable, and 64 nonorientable).

The Fatgraph $G_{6,0}$ (non-orientable, 12 orientable markings)




```
Fatgraph([
  Vertex([1, 2, 1]),# a
  Vertex([2, 3, 0]),# b
  Vertex([0, 3, 4]),# c
  Vertex([5, 5, 4]),# d
])
```

Boundary cycles

$$\alpha = ({}^{2}d^{0} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{2}c^{0} \to {}^{1}c^{2} \to {}^{2}b^{0} \to {}^{0}b^{1} \to {}^{1}d^{2})$$
$$\beta = ({}^{2}a^{0})$$
$$\gamma = ({}^{0}c^{1} \to {}^{1}b^{2})$$
$$\delta = ({}^{0}d^{1})$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β	γ	δ
$A_1{}^{\dagger\ddagger}$	d	c	b	a	3	5	4	0	2	1	α	δ	γ	β

Markings

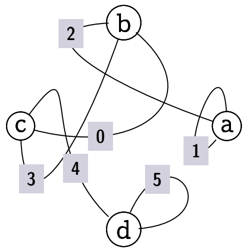
	$G_{6,0}^{(0)}$	$G_{6,0}^{(1)}$	$G_{6,0}^{(2)}$	$G_{6,0}^{(3)}$	$G_{6,0}^{(4)}$	$G_{6,0}^{(5)}$	$G_{6,0}^{(6)}$	$G_{6,0}^{(7)}$
α	0	0	0	1	1	1	2	2
β	1	1	2	0	0	2	0	0
γ	2	3	1	2	3	0	1	3
δ	3	2	3	3	2	3	3	1

	$G_{6,0}^{(8)}$	$G_{6,0}^{(9)}$	$G_{6,0}^{(10)}$	$G_{6,0}^{(11)}$
α	2	3	3	3
β	1	0	0	1
γ	0	1	2	0
δ	3	2	1	2

Differentials

$$
\begin{aligned}
D(G_{6,0}^{(0)}) &= -G_{5,0}^{(6)} \\
D(G_{6,0}^{(4)}) &= +G_{5,0}^{(6)} \\
D(G_{6,0}^{(6)}) &= -G_{5,0}^{(0)}
\end{aligned}
\qquad\qquad
\begin{aligned}
D(G_{6,0}^{(7)}) &= -G_{5,0}^{(1)} \\
D(G_{6,0}^{(8)}) &= +G_{5,0}^{(1)} \\
D(G_{6,0}^{(10)}) &= +G_{5,0}^{(0)}
\end{aligned}
$$

The Fatgraph $G_{6,1}$ (12 orientable markings)




```
Fatgraph([
  Vertex([1, 2, 1]),# a
  Vertex([2, 3, 0]),# b
  Vertex([3, 0, 4]),# c
  Vertex([5, 5, 4]),# d
])
```

Boundary cycles

$$
\begin{aligned}
\alpha &= ({}^1a^2 \to {}^2b^0 \to {}^0a^1 \to {}^0c^1 \to {}^0b^1) \\
\beta &= ({}^2a^0) \\
\gamma &= ({}^2d^0 \to {}^2c^0 \to {}^1d^2 \to {}^1c^2 \to {}^1b^2) \\
\delta &= ({}^0d^1)
\end{aligned}
$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β	γ	δ
$A_1{}^{\ddagger}$	d	c	b	a	0	5	4	3	2	1	γ	δ	α	β

Markings

	$G_{6,1}^{(0)}$	$G_{6,1}^{(1)}$	$G_{6,1}^{(2)}$	$G_{6,1}^{(3)}$	$G_{6,1}^{(4)}$	$G_{6,1}^{(5)}$	$G_{6,1}^{(6)}$	$G_{6,1}^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0

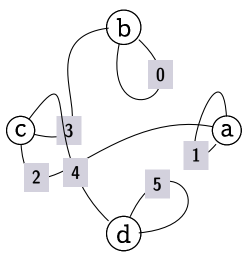
(continued.)

γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2
	$G^{(8)}_{6,1}$	$G^{(9)}_{6,1}$	$G^{(10)}_{6,1}$	$G^{(11)}_{6,1}$				
α	1	1	2	2				
β	2	3	0	1				
γ	3	2	3	3				
δ	0	0	1	0				

Differentials

$$D(G^{(0)}_{6,1}) = -G^{(2)}_{5,0}$$
$$D(G^{(1)}_{6,1}) = -G^{(3)}_{5,0}$$
$$D(G^{(2)}_{6,1}) = +G^{(3)}_{5,0}$$
$$D(G^{(4)}_{6,1}) = +G^{(2)}_{5,0}$$
$$D(G^{(6)}_{6,1}) = -G^{(4)}_{5,0}$$
$$D(G^{(7)}_{6,1}) = -G^{(5)}_{5,0}$$
$$D(G^{(8)}_{6,1}) = +G^{(5)}_{5,0}$$
$$D(G^{(10)}_{6,1}) = +G^{(4)}_{5,0}$$

The Fatgraph $G_{6,2}$ (8 orientable markings)




```
Fatgraph([
  Vertex([1, 2, 1]),# a
  Vertex([3, 0, 0]),# b
  Vertex([2, 3, 4]),# c
  Vertex([5, 5, 4]),# d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \to {}^0c^1 \to {}^1a^2 \to {}^0a^1 \to {}^0b^1 \to {}^1c^2 \to {}^2b^0 \to {}^2c^0 \to {}^1d^2)$$
$$\beta = ({}^2a^0)$$
$$\gamma = ({}^1b^2)$$
$$\delta = ({}^0d^1)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β	γ	δ
$A_1{}^{\ddagger}$	b	d	c	a	5	0	3	4	2	1	α	γ	δ	β
$A_2{}^{\ddagger}$	d	a	c	b	1	5	4	2	3	0	α	δ	β	γ

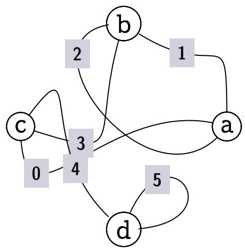
Markings

	$G_{6,2}^{(0)}$	$G_{6,2}^{(1)}$	$G_{6,2}^{(2)}$	$G_{6,2}^{(3)}$	$G_{6,2}^{(4)}$	$G_{6,2}^{(5)}$	$G_{6,2}^{(6)}$	$G_{6,2}^{(7)}$
α	0	0	1	1	2	2	3	3
β	1	1	0	0	0	0	0	0
γ	2	3	2	3	1	3	1	2
δ	3	2	3	2	3	1	2	1

Differentials

$$D(G_{6,2}^{(0)}) = +G_{5,0}^{(0)}$$
$$D(G_{6,2}^{(1)}) = +G_{5,0}^{(1)}$$
$$D(G_{6,2}^{(2)}) = +G_{5,0}^{(2)}$$
$$D(G_{6,2}^{(3)}) = +G_{5,0}^{(3)}$$
$$D(G_{6,2}^{(4)}) = +G_{5,0}^{(4)}$$
$$D(G_{6,2}^{(5)}) = +G_{5,0}^{(5)}$$
$$D(G_{6,2}^{(6)}) = +G_{5,0}^{(6)}$$

The Fatgraph $G_{6,3}$ (24 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 3, 1]),# b
  Vertex([0, 3, 4]),# c
  Vertex([5, 5, 4]),# d
])
```

Boundary cycles

$$\alpha = ({}^0a^1 \to {}^0c^1 \to {}^1b^2)$$
$$\beta = ({}^2d^0 \to {}^1a^2 \to {}^2c^0 \to {}^1c^2 \to {}^0b^1 \to {}^1d^2)$$
$$\gamma = ({}^2a^0 \to {}^2b^0)$$
$$\delta = ({}^0d^1)$$

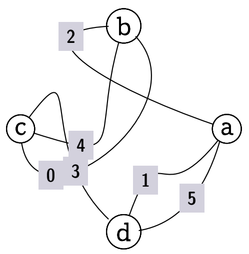
Markings

Fatgraph $G_{6,3}$ only has the identity automorphism, so the marked fatgraphs $G_{6,3}^{(0)}$ to $G_{6,3}^{(24)}$ are formed by decorating boundary cycles of $G_{6,3}$ with all permutations of $(0, 1, 2, 3)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{6,3}^{(0)}) = +G_{5,0}^{(2)}$$
$$D(G_{6,3}^{(2)}) = +G_{5,0}^{(4)}$$
$$D(G_{6,3}^{(4)}) = +G_{5,0}^{(6)}$$
$$D(G_{6,3}^{(6)}) = +G_{5,0}^{(0)}$$
$$D(G_{6,3}^{(8)}) = +G_{5,0}^{(5)}$$
$$D(G_{6,3}^{(12)}) = +G_{5,0}^{(1)}$$
$$D(G_{6,3}^{(14)}) = +G_{5,0}^{(3)}$$

The Fatgraph $G_{6,4}$ (non-orientable, 6 orientable markings)




```
Fatgraph([
  Vertex([5, 2, 1]),# a
  Vertex([2, 4, 0]),# b
  Vertex([0, 4, 3]),# c
  Vertex([5, 1, 3]),# d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \to {}^0a^1 \to {}^1c^2 \to {}^0b^1)$$
$$\beta = ({}^1a^2 \to {}^2b^0 \to {}^1d^2 \to {}^2c^0)$$
$$\gamma = ({}^2a^0 \to {}^0d^1)$$
$$\delta = ({}^0c^1 \to {}^1b^2)$$

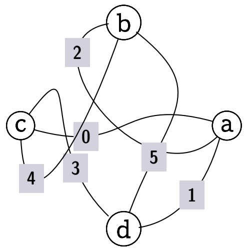
Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β	γ	δ
$A_1{}^{\ddagger}$	b	a	d	c	5	4	2	3	1	0	β	α	δ	γ
$A_2{}^{\dagger\ddagger}$	c	d	a	b	1	0	3	2	5	4	α	β	δ	γ
$A_3{}^{\dagger\ddagger}$	d	c	b	a	4	5	3	2	0	1	β	α	γ	δ

Markings

	$G_{6,4}^{(0)}$	$G_{6,4}^{(1)}$	$G_{6,4}^{(2)}$	$G_{6,4}^{(3)}$	$G_{6,4}^{(4)}$	$G_{6,4}^{(5)}$
α	0	0	0	1	1	2
β	1	2	3	2	3	3
γ	2	1	1	0	0	0
δ	3	3	2	3	2	1

The Fatgraph $G_{6,5}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 4, 5]),# b
  Vertex([4, 0, 3]),# c
  Vertex([1, 5, 3]),# d
])
```

Boundary cycles

$$\begin{aligned}\alpha &= ({}^2d^0 \to {}^0a^1 \to {}^1c^2)\\ \beta &= ({}^1a^2 \to {}^0c^1 \to {}^0b^1)\\ \gamma &= ({}^2a^0 \to {}^2b^0 \to {}^0d^1)\\ \delta &= ({}^2c^0 \to {}^1d^2 \to {}^1b^2)\end{aligned}$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β	γ	δ
$A_1{}^\ddagger$	a	d	b	c	2	0	1	4	5	3	β	γ	α	δ
$A_2{}^\ddagger$	a	c	d	b	1	2	0	5	3	4	γ	α	β	δ
$A_3{}^\ddagger$	b	d	c	a	4	2	5	0	3	1	β	δ	γ	α
$A_4{}^\ddagger$	b	a	d	c	5	4	2	3	1	0	δ	γ	β	α
$A_5{}^\ddagger$	b	c	a	d	2	5	4	1	0	3	γ	β	δ	α
$A_6{}^\ddagger$	c	d	a	b	0	4	3	2	1	5	β	α	δ	γ
$A_7{}^\ddagger$	c	b	d	a	3	0	4	1	5	2	α	δ	β	γ
$A_8{}^\ddagger$	c	a	b	d	4	3	0	5	2	1	δ	β	α	γ
$A_9{}^\ddagger$	d	c	b	a	5	1	3	2	4	0	γ	δ	α	β
$A_{10}{}^\ddagger$	d	a	c	b	3	5	1	4	0	2	δ	α	γ	β
$A_{11}{}^\ddagger$	d	b	a	c	1	3	5	0	2	4	α	γ	δ	β

Markings

	$G^{(0)}_{6,5}$	$G^{(1)}_{6,5}$
α	0	0
β	1	1
γ	2	3
δ	3	2

Markings of fatgraphs with trivial automorphisms

This appendix shows the numbering of marked fatgraphs when the base unmarked fatgraph G has only the trivial automorphism.

	$G^{(0)}$	$G^{(1)}$	$G^{(2)}$	$G^{(3)}$	$G^{(4)}$	$G^{(5)}$	$G^{(6)}$	$G^{(7)}$
α	0	0	0	0	0	0	1	1
β	1	1	2	2	3	3	0	0
γ	2	3	1	3	1	2	2	3
δ	3	2	3	1	2	1	3	2
	$G^{(8)}$	$G^{(9)}$	$G^{(10)}$	$G^{(11)}$	$G^{(12)}$	$G^{(13)}$	$G^{(14)}$	$G^{(15)}$
α	1	1	1	1	2	2	2	2
β	2	2	3	3	0	0	1	1
γ	0	3	0	2	1	3	0	3
δ	3	0	2	0	3	1	3	0
	$G^{(16)}$	$G^{(17)}$	$G^{(18)}$	$G^{(19)}$	$G^{(20)}$	$G^{(21)}$	$G^{(22)}$	$G^{(23)}$
α	2	2	3	3	3	3	3	3
β	3	3	0	0	1	1	2	2
γ	0	1	1	2	0	2	0	1
δ	1	0	2	1	2	0	1	0