

Fatgraphs of $M_{2,1}$

Automatically generated by FatGHoL 5.4
(See: <http://fatghol.googlecode.com/>)

2012-02-09

There are a total of 160 undecorated fatgraphs in the Kontsevich graph complex of $M_{2,1}$, originating 160 marked ones.

Contents

Notation	2
Fatgraphs with 4 edges / 1 vertex	3
Fatgraphs with 5 edges / 2 vertices	4
Fatgraphs with 6 edges / 3 vertices	13
Fatgraphs with 7 edges / 4 vertices	36
Fatgraphs with 8 edges / 5 vertices	65
Fatgraphs with 9 edges / 6 vertices	81

Notation

We denote $G_{m,j}$ the j -th graph in the set of undecorated fatgraphs with m edges; the symbol $G_{m,j}^{(k)}$ denotes the k -th inequivalent marking of $G_{m,j}$.

Fatgraph vertices are marked with lowercase latin letters “a”, “b”, “c”, etc.; edges are marked with an arabic numeral starting from “1”; boundary cycles are denoted by lowercase greek letters “ α ”, “ β ”, etc.

Automorphisms are specified by their action on the set of vertices, edges, and boundary cycles: for each automorphism A_k , a table line lists how it permutes vertices, edges and boundary cycles relative to the identity morphism A_0 . The automorphism table is printed only if the automorphism group is non-trivial.

Automorphisms that reverse the orientation of the unmarked fatgraph are indicated with a “†” symbol in the automorphism table; those that reverse the orientation of the marked fatgraphs are distinguished with a “‡” sign.

If a fatgraph is orientable, a “Markings” section lists all the inequivalent ways of assigning distinct numbers $\{0, \dots, n-1\}$ to the boundary cycles; this is of course a set of representatives for the orbits of \mathfrak{S}_n under the action of $\text{Aut}(G)$.

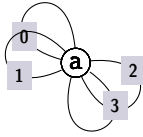
A separate section lists the differential of marked fatgraphs; graphs with null differential are omitted. If no marked fatgraph has a non-zero differential, the entire section is dropped.

Boundary cycles are specified using a “sequence of corners” notation: each corner is represented as ${}^pL^q$ where L is a latin letter indicating a vertex, and p, q are the attachment indices of the incoming and outgoing edges, respectively. Attachment indices match the Python representation of the vertex: e.g., if `a=Vertex([0,0,1])`, the two legs of edge 0 have attachment indices 0 and 1, and the boundary cycle enclosed by them is represented by the (single) corner ${}^0a^1$.

Fatgraphs with 4 edges / 1 vertex

There are 4 unmarked fatgraphs in this section, originating 4 marked fatgraphs (3 orientable, and 1 nonorientable).

The Fatgraph $G_{4,0}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 0, 1, 3, 2, 3, 2]),# a
])
```

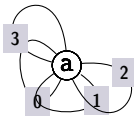
Boundary cycles

$$\alpha = ({}^7a^0 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^6a^7 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^5a^6 \rightarrow {}^4a^5)$$

Automorphisms

A_0	a	0	1	2	3	α
A_1	a	3	2	1	0	α

The Fatgraph $G_{4,1}$ (1 orientable marking)

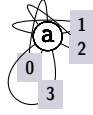


```
Fatgraph([
  Vertex([3, 0, 3, 1, 0, 2, 1, 2]),# a
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3a^4 \rightarrow {}^6a^7 \rightarrow {}^7a^0 \rightarrow {}^5a^6)$$

The Fatgraph $G_{4,2}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 2, 1, 3, 0, 3, 2, 1]),# a
])
```

Boundary cycles

$$\alpha = ({}^7a^0 \rightarrow {}^6a^7 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^4a^5 \rightarrow {}^5a^6)$$

The Fatgraph $G_{4,3}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 3, 2, 1, 0, 3, 2, 1]),# a
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^5a^6 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^7a^0 \rightarrow {}^6a^7)$$

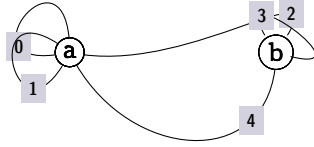
Automorphisms

A_0	a	0	1	2	3	α
A_1^\dagger	a	3	0	1	2	α
A_2	a	2	3	0	1	α
A_3^\dagger	a	1	2	3	0	α
A_4	a	0	1	2	3	α
A_5^\dagger	a	3	0	1	2	α
A_6	a	2	3	0	1	α
A_7^\dagger	a	1	2	3	0	α

Fatgraphs with 5 edges / 2 vertices

There are 21 unmarked fatgraphs in this section, originating 21 marked fatgraphs (20 orientable, and 1 nonorientable).

The Fatgraph $G_{5,0}$ (1 orientable marking)

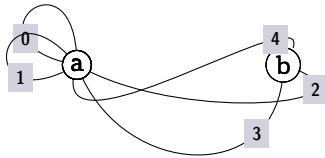


```
Fatgraph([
  Vertex([0, 1, 0, 1, 4, 2]),# a
  Vertex([4, 3, 2, 3]),      # b
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^2b^3 \rightarrow ^1a^2 \rightarrow ^3b^0 \rightarrow ^5a^0 \rightarrow ^4a^5 \rightarrow ^0b^1 \rightarrow ^1b^2)$$

The Fatgraph $G_{5,1}$ (1 orientable marking)

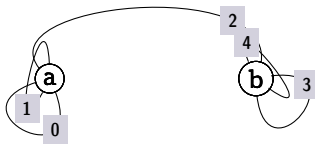


```
Fatgraph([
  Vertex([0, 1, 0, 1, 4, 3, 2]),# a
  Vertex([3, 2, 4]),            # b
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^5a^6 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^2a^3 \rightarrow ^4a^5 \rightarrow ^2b^0 \rightarrow ^0b^1 \rightarrow ^6a^0)$$

The Fatgraph $G_{5,2}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 2, 0, 1, 0]),# a
  Vertex([3, 4, 3, 2, 4]),# b
])
```

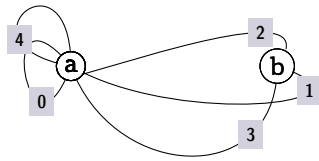
Boundary cycles

$$\alpha = (^4b^0 \rightarrow ^4a^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^2a^3 \rightarrow ^3b^4 \rightarrow ^3a^4 \rightarrow ^2b^3 \rightarrow ^1b^2)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	b	a	4	3	2	1	0	α

The Fatgraph $G_{5,3}$ (1 orientable marking)

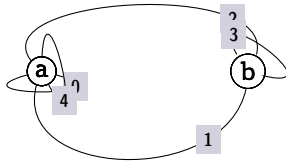


```
Fatgraph([
  Vertex([4, 0, 4, 2, 0, 3, 1]),# a
  Vertex([3, 1, 2]),           # b
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^5a^6 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^6a^0)$$

The Fatgraph $G_{5,4}$ (1 orientable marking)

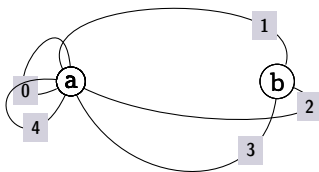


```
Fatgraph([
  Vertex([4, 2, 0, 1, 4, 0]),# a
  Vertex([1, 3, 2, 3]),      # b
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^5a^0 \rightarrow {}^3a^4 \rightarrow {}^2b^3 \rightarrow {}^0b^1)$$

The Fatgraph $G_{5,5}$ (1 orientable marking)

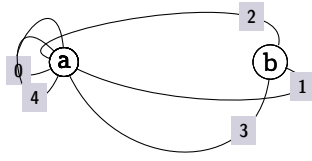


```
Fatgraph([
  Vertex([0, 1, 4, 0, 4, 3, 2]),# a
  Vertex([3, 2, 1]),           # b
])
```

Boundary cycles

$$\alpha = (^4a^5 \rightarrow ^2a^3 \rightarrow ^5a^6 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^1a^2 \rightarrow ^3a^4 \rightarrow ^2b^0 \rightarrow ^0b^1 \rightarrow ^6a^0)$$

The Fatgraph $G_{5,6}$ (1 orientable marking)

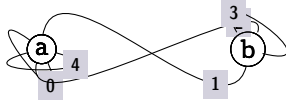


```
Fatgraph([
  Vertex([0, 4, 2, 0, 4, 3, 1]), # a
  Vertex([3, 1, 2]),             # b
])
```

Boundary cycles

$$\alpha = (^4a^5 \rightarrow ^2a^3 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^5a^6 \rightarrow ^3a^4 \rightarrow ^2b^0 \rightarrow ^1b^2 \rightarrow ^6a^0)$$

The Fatgraph $G_{5,7}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 4, 2, 0, 4]), # a
  Vertex([1, 3, 2, 3]),       # b
])
```

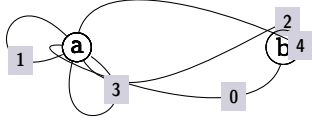
Boundary cycles

$$\alpha = (^4a^5 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^1a^2 \rightarrow ^3b^0 \rightarrow ^5a^0 \rightarrow ^3a^4 \rightarrow ^2b^3 \rightarrow ^0b^1)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1^\dagger	a	b	0	2	1	3	4	α

The Fatgraph $G_{5,8}$ (1 orientable marking)

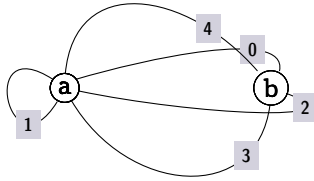


```
Fatgraph([
  Vertex([4, 1, 0, 1, 3, 2, 3]),# a
  Vertex([0, 4, 2]),             # b
])
```

Boundary cycles

$$\alpha = ({}^3a^4 \rightarrow {}^5a^6 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^6a^0)$$

The Fatgraph $G_{5,9}$ (1 orientable marking)

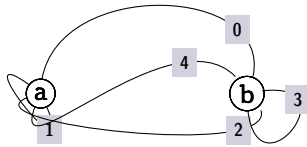


```
Fatgraph([
  Vertex([4, 1, 0, 1, 3, 2]),# a
  Vertex([3, 2, 0, 4]),      # b
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^5a^0 \rightarrow {}^4a^5 \rightarrow {}^1b^2)$$

The Fatgraph $G_{5,10}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 4, 1]),# a
  Vertex([3, 2, 3, 0, 4]),# b
])
```

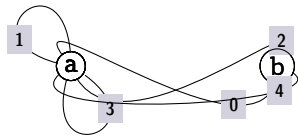
Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^4 \rightarrow {}^4b^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	b	a	2	3	0	1	4	α

The Fatgraph $G_{5,11}$ (1 orientable marking)

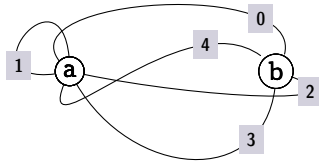


```
Fatgraph([
  Vertex([1, 0, 1, 4, 3, 2, 3]),# a
  Vertex([0, 4, 2]),           # b
])
```

Boundary cycles

$$\alpha = ({}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^5a^6 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^6a^0)$$

The Fatgraph $G_{5,12}$ (1 orientable marking)

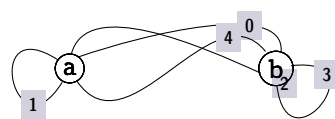


```
Fatgraph([
  Vertex([1, 0, 1, 4, 3, 2]),# a
  Vertex([3, 2, 0, 4]),      # b
])
```

Boundary cycles

$$\alpha = ({}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^5a^0 \rightarrow {}^4a^5 \rightarrow {}^2b^3 \rightarrow {}^1b^2)$$

The Fatgraph $G_{5,13}$ (1 orientable marking)

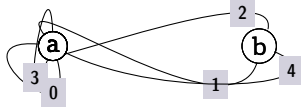


```
Fatgraph([
  Vertex([2, 1, 0, 1, 4]),# a
  Vertex([3, 2, 3, 0, 4]),# b
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^2b^3 \rightarrow ^1a^2 \rightarrow ^3b^4 \rightarrow ^4b^0 \rightarrow ^4a^0 \rightarrow ^0b^1 \rightarrow ^1b^2)$$

The Fatgraph $G_{5,14}$ (1 orientable marking)

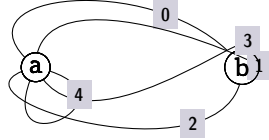


```
Fatgraph([
  Vertex([3, 1, 0, 2, 3, 0, 4]),# a
  Vertex([1, 4, 2]),             # b
])
```

Boundary cycles

$$\alpha = (^4a^5 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^1a^2 \rightarrow ^3a^4 \rightarrow ^2b^0 \rightarrow ^1b^2 \rightarrow ^6a^0 \rightarrow ^5a^6)$$

The Fatgraph $G_{5,15}$ (1 orientable marking)

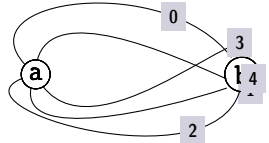


```
Fatgraph([
  Vertex([1, 0, 2, 4, 3, 4]),# a
  Vertex([2, 1, 3, 0]),      # b
])
```

Boundary cycles

$$\alpha = (^4a^5 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^1a^2 \rightarrow ^3b^0 \rightarrow ^5a^0 \rightarrow ^3a^4 \rightarrow ^2b^3 \rightarrow ^0b^1)$$

The Fatgraph $G_{5,16}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 4, 3]),# a
  Vertex([2, 1, 4, 3, 0]),# b
])
```

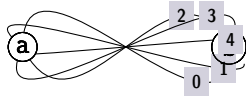
Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^4b^0 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^4 \rightarrow {}^3a^4 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	b	a	4	1	3	2	0	α

The Fatgraph $G_{5,17}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 4, 3, 2]),# a
  Vertex([0, 1, 4, 3, 2]),# b
])
```

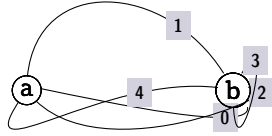
Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^3b^4 \rightarrow {}^4b^0 \rightarrow {}^4a^0 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	a	b	1	4	0	2	3	α
A_2	a	b	4	3	1	0	2	α
A_3	a	b	3	2	4	1	0	α
A_4	a	b	2	0	3	4	1	α
A_5	b	a	0	1	2	3	4	α
A_6	b	a	1	4	0	2	3	α
A_7	b	a	4	3	1	0	2	α
A_8	b	a	3	2	4	1	0	α
A_9	b	a	2	0	3	4	1	α

The Fatgraph $G_{5,18}$ (1 orientable marking)

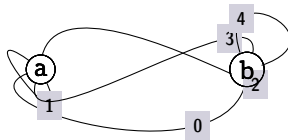


```
Fatgraph([
  Vertex([1, 0, 4, 2]),      # a
  Vertex([3, 0, 2, 3, 1, 4]),# b
])
```

Boundary cycles

$$\alpha = ({}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2b^3 \rightarrow {}^2a^3 \rightarrow {}^3b^4 \rightarrow {}^4b^5 \rightarrow {}^0b^1 \rightarrow {}^5b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{5,19}$ (1 orientable marking)



```
Fatgraph([
  Vertex([2, 1, 0, 3, 1]),# a
  Vertex([0, 2, 4, 3, 4]),# b
])
```

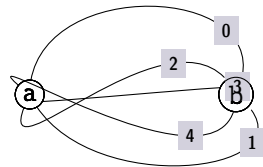
Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^4b^0 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^4 \rightarrow {}^3a^4 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	b	a	0	4	3	2	1	α

The Fatgraph $G_{5,20}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 4, 3, 2, 1]),# a
  Vertex([4, 1, 3, 0, 2]),# b
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^4 \rightarrow {}^4b^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

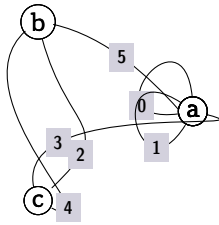
Automorphisms

A_0	a	b	0	1	2	3	4	α
A_1	a	b	4	0	1	2	3	α
A_2	a	b	3	4	0	1	2	α
A_3	a	b	2	3	4	0	1	α
A_4	a	b	1	2	3	4	0	α

Fatgraphs with 6 edges / 3 vertices

There are 45 unmarked fatgraphs in this section, originating 45 marked fatgraphs (39 orientable, and 6 nonorientable).

The Fatgraph $G_{6,0}$ (1 orientable marking)

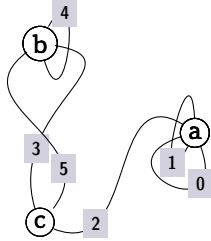


```
Fatgraph([
  Vertex([0, 1, 0, 1, 5, 3]), # a
  Vertex([4, 2, 5]),          # b
  Vertex([4, 2, 3]),          # c
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^5a^0 \rightarrow {}^0c^1 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,1}$ (1 orientable marking)

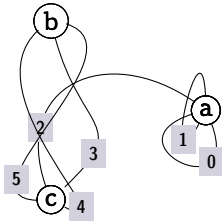


```
Fatgraph([
  Vertex([1, 2, 0, 1, 0]),# a
  Vertex([5, 4, 3, 4]),    # b
  Vertex([2, 5, 3]),       # c
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2b^3 \rightarrow ^2a^3 \rightarrow ^3b^0 \rightarrow ^1c^2 \rightarrow ^2c^0 \rightarrow ^0b^1 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,2}$ (1 orientable marking)

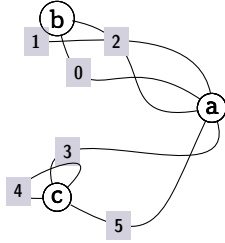


```
Fatgraph([
  Vertex([1, 2, 0, 1, 0]),# a
  Vertex([4, 3, 5]),      # b
  Vertex([4, 3, 2, 5]),   # c
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^1c^2 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^1a^2 \rightarrow ^3c^0 \rightarrow ^2c^3 \rightarrow ^0b^1 \rightarrow ^2b^0)$$

The Fatgraph $G_{6,3}$ (1 orientable marking)

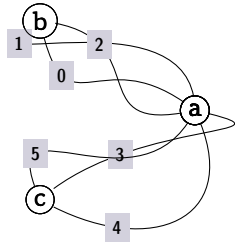


```
Fatgraph([
  Vertex([1, 0, 2, 5, 3]),# a
  Vertex([1, 0, 2]),      # b
  Vertex([5, 4, 3, 4]),   # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3c^0 \rightarrow {}^2c^3 \rightarrow {}^4a^0 \rightarrow {}^0b^1 \rightarrow {}^2b^0)$$

The Fatgraph $G_{6,4}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2, 5, 4, 3]),# a
  Vertex([1, 0, 2]),      # b
  Vertex([4, 3, 5]),      # c
])
```

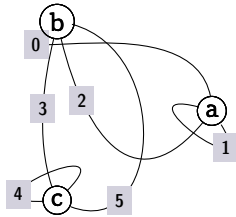
Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1a^2 \rightarrow {}^5a^0 \rightarrow {}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1^\dagger	a	c	b	4	5	3	2	0	1	α

The Fatgraph $G_{6,5}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([0, 2, 5, 3]),# b
  Vertex([5, 4, 3, 4]),# c
])
```

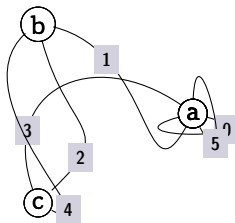
Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^2b^3 \rightarrow {}^3c^0 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1^\dagger	c	b	a	5	4	3	2	1	0	α

The Fatgraph $G_{6,6}$ (1 orientable marking)

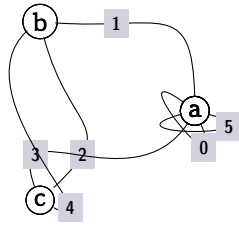


```
Fatgraph([
  Vertex([5, 3, 0, 1, 5, 0]),# a
  Vertex([4, 2, 1]),# b
  Vertex([4, 2, 3]),# c
])
```

Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^5a^0 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,7}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 5, 3, 0, 5]), # a
  Vertex([4, 2, 1]),          # b
  Vertex([4, 2, 3]),          # c
])
```

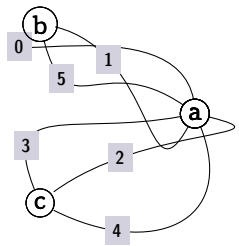
Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^5a^0 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1^\dagger	a	c	b	0	3	2	1	4	5	α

The Fatgraph $G_{6,8}$ (1 orientable marking)

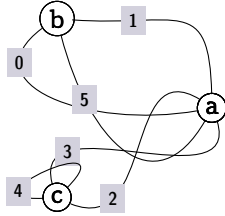


```
Fatgraph([
  Vertex([0, 5, 3, 1, 4, 2]), # a
  Vertex([0, 5, 1]),          # b
  Vertex([4, 2, 3]),          # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^5a^0 \rightarrow {}^2c^0 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,9}$ (1 orientable marking)

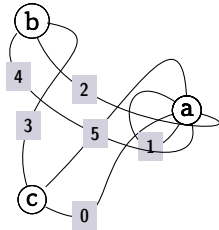


```
Fatgraph([
  Vertex([1, 2, 0, 5, 3]),# a
  Vertex([0, 5, 1]),      # b
  Vertex([2, 4, 3, 4]),   # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^3c^0 \rightarrow {}^2c^3 \rightarrow {}^4a^0 \rightarrow {}^2b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,10}$ (1 orientable marking)

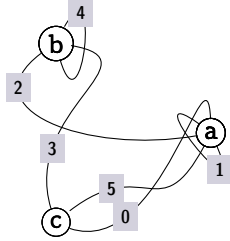


```
Fatgraph([
  Vertex([5, 1, 0, 1, 4, 2]),# a
  Vertex([4, 2, 3]),         # b
  Vertex([0, 5, 3]),         # c
])
```

Boundary cycles

$$\alpha = ({}^3a^4 \rightarrow {}^5a^0 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,11}$ (1 orientable marking)

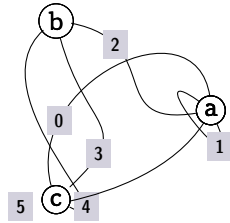


```
Fatgraph([
  Vertex([0, 1, 2, 5, 1]),# a
  Vertex([2, 4, 3, 4]),    # b
  Vertex([0, 5, 3]),       # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^4a^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,12}$ (1 orientable marking)

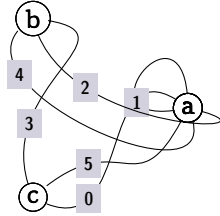


```
Fatgraph([
  Vertex([0, 1, 2, 5, 1]),# a
  Vertex([4, 3, 2]),      # b
  Vertex([4, 3, 0, 5]),   # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^3c^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^4a^0 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^2b^0)$$

The Fatgraph $G_{6,13}$ (1 orientable marking)

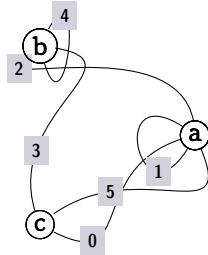


```
Fatgraph([
  Vertex([1, 0, 1, 5, 4, 2]),# a
  Vertex([4, 2, 3]),          # b
  Vertex([0, 5, 3]),          # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^5a^0 \rightarrow {}^2c^0 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,14}$ (1 orientable marking)

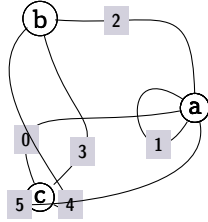


```
Fatgraph([
  Vertex([2, 1, 0, 1, 5]),# a
  Vertex([2, 4, 3, 4]),   # b
  Vertex([0, 5, 3]),      # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^4a^0 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^1b^2 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,15}$ (1 orientable marking)

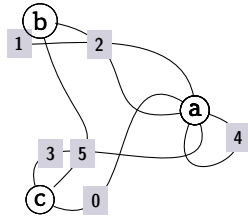


```
Fatgraph([
  Vertex([2, 1, 0, 1, 5]),# a
  Vertex([4, 3, 2]),      # b
  Vertex([4, 3, 0, 5]),   # c
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^3c^0 \rightarrow ^2a^3 \rightarrow ^0c^1 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^1a^2 \rightarrow ^1c^2 \rightarrow ^2c^3 \rightarrow ^4a^0 \rightarrow ^2b^0 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,16}$ (1 orientable marking)

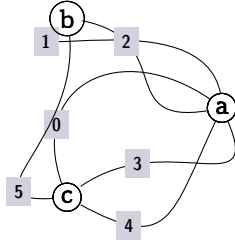


```
Fatgraph([
  Vertex([1, 0, 2, 4, 3, 4]),# a
  Vertex([1, 5, 2]),          # b
  Vertex([0, 5, 3]),          # c
])
```

Boundary cycles

$$\alpha = (^1c^2 \rightarrow ^4a^5 \rightarrow ^0c^1 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^1a^2 \rightarrow ^5a^0 \rightarrow ^2c^0 \rightarrow ^3a^4 \rightarrow ^2b^0 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,17}$ (1 orientable marking)

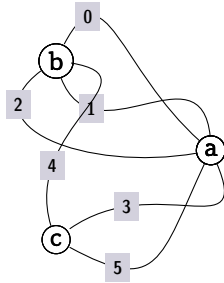


```
Fatgraph([
  Vertex([1, 0, 2, 4, 3]),# a
  Vertex([1, 5, 2]),      # b
  Vertex([4, 3, 0, 5]),   # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^3c^0 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^4a^0 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^2b^0)$$

The Fatgraph $G_{6,18}$ (1 orientable marking)

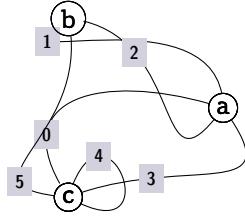


```
Fatgraph([
  Vertex([1, 0, 2, 5, 3]),# a
  Vertex([2, 1, 4, 0]),   # b
  Vertex([5, 3, 4]),      # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,19}$ (1 orientable marking)

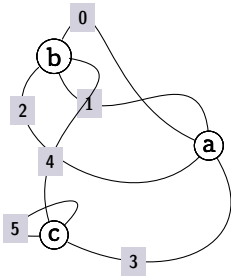


```
Fatgraph([
  Vertex([1, 0, 2, 3]), # a
  Vertex([1, 5, 2]),    # b
  Vertex([4, 3, 4, 0, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^4c^0 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3a^0 \rightarrow {}^2b^0 \rightarrow {}^0c^1 \rightarrow {}^3c^4 \rightarrow {}^0b^1 \rightarrow {}^2c^3)$$

The Fatgraph $G_{6,20}$ (1 orientable marking)

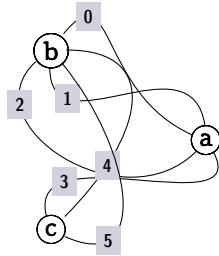


```
Fatgraph([
  Vertex([1, 0, 2, 3]), # a
  Vertex([2, 1, 4, 0]), # b
  Vertex([3, 5, 4, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3a^0 \rightarrow {}^3b^0 \rightarrow {}^3c^0 \rightarrow {}^2c^3 \rightarrow {}^2b^3 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,21}$ (1 orientable marking)

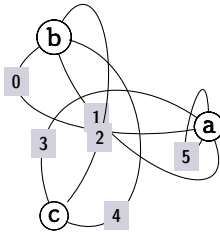


```
Fatgraph([
  Vertex([1, 0, 2, 3]), # a
  Vertex([2, 1, 5, 4, 0]), # b
  Vertex([5, 4, 3]), # c
])
```

Boundary cycles

$$\alpha = (^4b^0 \rightarrow ^0c^1 \rightarrow ^2a^3 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2b^3 \rightarrow ^3a^0 \rightarrow ^1c^2 \rightarrow ^3b^4 \rightarrow ^2c^0 \rightarrow ^0b^1 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,22}$ (1 orientable marking)

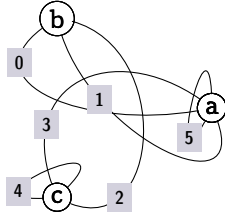


```
Fatgraph([
  Vertex([5, 3, 0, 5, 1]), # a
  Vertex([0, 1, 4, 2]), # b
  Vertex([4, 2, 3]), # c
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^2a^3 \rightarrow ^3b^0 \rightarrow ^1c^2 \rightarrow ^2c^0 \rightarrow ^2b^3 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,23}$ (1 orientable marking)

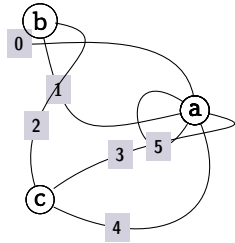


```
Fatgraph([
  Vertex([5, 3, 0, 5, 1]),# a
  Vertex([0, 1, 2]),      # b
  Vertex([2, 4, 3, 4]),   # c
])
```

Boundary cycles

$$\alpha = (^3a^4 \rightarrow ^0c^1 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^3c^0 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^1c^2 \rightarrow ^2c^3 \rightarrow ^0b^1 \rightarrow ^2b^0 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,24}$ (1 orientable marking)

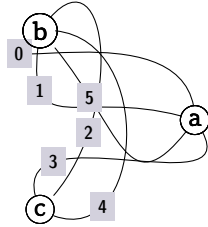


```
Fatgraph([
  Vertex([0, 5, 1, 5, 4, 3]),# a
  Vertex([0, 1, 2]),          # b
  Vertex([4, 3, 2]),          # c
])
```

Boundary cycles

$$\alpha = (^1c^2 \rightarrow ^3a^4 \rightarrow ^0c^1 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^1a^2 \rightarrow ^5a^0 \rightarrow ^2c^0 \rightarrow ^4a^5 \rightarrow ^2b^0 \rightarrow ^1b^2)$$

The Fatgraph $G_{6,25}$ (1 orientable marking)

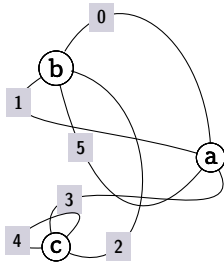


```
Fatgraph([
  Vertex([0, 1, 5, 3]), # a
  Vertex([0, 1, 5, 4, 2]), # b
  Vertex([4, 2, 3]), # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^4b^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^1c^2 \rightarrow {}^3b^4 \rightarrow {}^1b^2 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,26}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 1, 5, 3]), # a
  Vertex([1, 5, 2, 0]), # b
  Vertex([2, 4, 3, 4]), # c
])
```

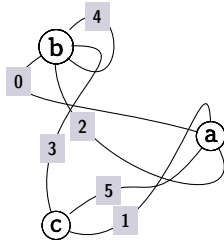
Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^0b^1 \rightarrow {}^3a^0 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^2c^3 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1^\dagger	b	a	c	0	1	3	2	4	5	α

The Fatgraph $G_{6,27}$ (1 orientable marking)

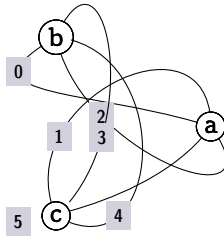


```
Fatgraph([
  Vertex([1, 0, 5, 2]), # a
  Vertex([0, 2, 4, 3, 4]), # b
  Vertex([1, 5, 3]), # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^4b^0 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^3b^4 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,28}$ (1 orientable marking)

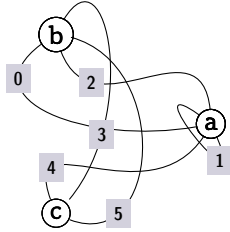


```
Fatgraph([
  Vertex([1, 0, 5, 2]), # a
  Vertex([0, 2, 4, 3]), # b
  Vertex([4, 3, 1, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^2b^3 \rightarrow {}^3c^0 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,29}$ (1 orientable marking)

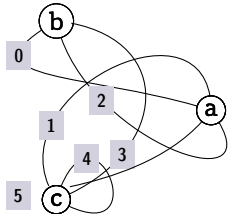


```
Fatgraph([
  Vertex([2, 1, 0, 4, 1]),# a
  Vertex([0, 2, 5, 3]),   # b
  Vertex([5, 3, 4]),      # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,30}$ (1 orientable marking)

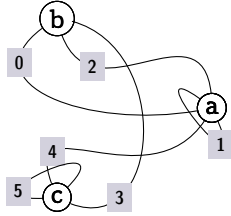


```
Fatgraph([
  Vertex([1, 0, 5, 2]),   # a
  Vertex([0, 2, 3]),      # b
  Vertex([4, 3, 4, 1, 5]),# c
])
```

Boundary cycles

$$\alpha = ({}^4c^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^1c^2 \rightarrow {}^2c^3 \rightarrow {}^3c^4 \rightarrow {}^0b^1 \rightarrow {}^2b^0)$$

The Fatgraph $G_{6,31}$ (1 orientable marking)

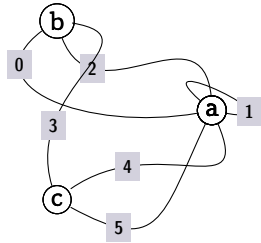


```
Fatgraph([
  Vertex([2, 1, 0, 4, 1]),# a
  Vertex([0, 2, 3]),      # b
  Vertex([3, 5, 4, 5]),   # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3c^0 \rightarrow {}^2b^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1 \rightarrow {}^2c^3)$$

The Fatgraph $G_{6,32}$ (1 orientable marking)

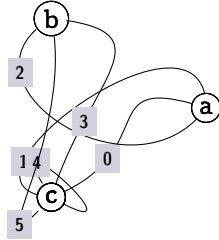


```
Fatgraph([
  Vertex([2, 1, 0, 5, 4, 1]),# a
  Vertex([0, 2, 3]),      # b
  Vertex([5, 4, 3]),      # c
])
```

Boundary cycles

$$\alpha = ({}^3a^4 \rightarrow {}^5a^0 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1a^2 \rightarrow {}^1c^2 \rightarrow {}^4a^5 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,33}$ (1 orientable marking)

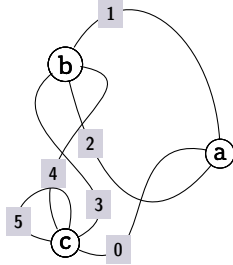


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 5, 3]),      # b
  Vertex([4, 0, 3, 4, 1, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^5c^0 \rightarrow {}^4c^5 \rightarrow {}^3c^4 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^2c^3)$$

The Fatgraph $G_{6,34}$ (1 orientable marking)

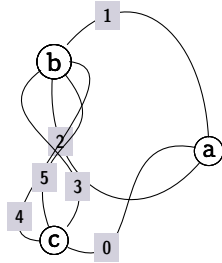


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([3, 2, 4, 1]),   # b
  Vertex([0, 3, 5, 4, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^4c^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^3c^4 \rightarrow {}^1b^2 \rightarrow {}^2c^3)$$

The Fatgraph $G_{6,35}$ (1 orientable marking)

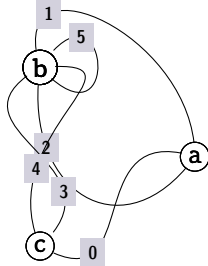


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([3, 2, 5, 4, 1]),# b
  Vertex([0, 3, 5, 4]),   # c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^4b^0 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^0b^1 \rightarrow {}^1c^2 \rightarrow {}^3b^4 \rightarrow {}^2c^3 \rightarrow {}^2b^3 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,36}$ (1 orientable marking)

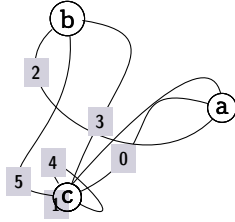


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([3, 2, 5, 4, 5, 1]),# b
  Vertex([0, 3, 4]),      # c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^4b^5 \rightarrow {}^1c^2 \rightarrow {}^3b^4 \rightarrow {}^2b^3 \rightarrow {}^5b^0 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,37}$ (1 orientable marking)

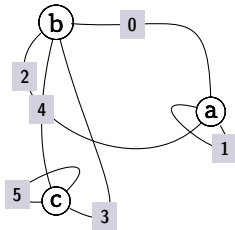


```
Fatgraph([
  Vertex([1, 0, 2]),          # a
  Vertex([2, 5, 3]),          # b
  Vertex([4, 0, 3, 4, 5, 1]), # c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^0c^1 \rightarrow {}^5c^0 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^1c^2 \rightarrow {}^4c^5 \rightarrow {}^2c^3 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^3c^4)$$

The Fatgraph $G_{6,38}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]), # a
  Vertex([2, 3, 0, 4]), # b
  Vertex([3, 5, 4, 5]), # c
])
```

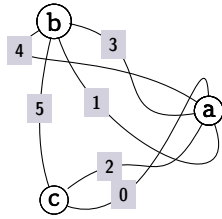
Boundary cycles

$$\alpha = ({}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^2b^3 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1	a	b	c	2	1	0	4	3	5	α
A_2	c	b	a	3	5	4	2	0	1	α
A_3	c	b	a	4	5	3	0	2	1	α

The Fatgraph $G_{6,39}$ (1 orientable marking)

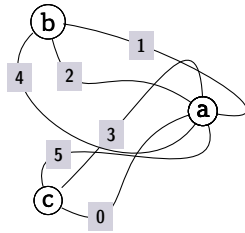


```
Fatgraph([
  Vertex([0, 4, 3, 2, 1]),# a
  Vertex([4, 1, 3, 5]),   # b
  Vertex([0, 2, 5]),      # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1)$$

The Fatgraph $G_{6,40}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([3, 2, 0, 4, 5, 1]),# a
  Vertex([4, 2, 1]),         # b
  Vertex([0, 3, 5]),         # c
])
```

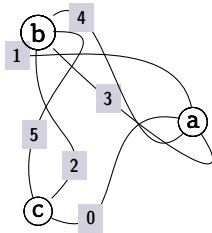
Boundary cycles

$$\alpha = ({}^4a^5 \rightarrow {}^5a^0 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^1c^2 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1	a	b	c	3	4	1	5	2	0	α
A_2	a	b	c	5	2	4	0	1	3	α
A_3^\dagger	a	c	b	2	5	3	1	0	4	α
A_4^\dagger	a	c	b	1	0	5	4	3	2	α
A_5^\dagger	a	c	b	4	3	0	2	5	1	α

The Fatgraph $G_{6,41}$ (1 orientable marking)

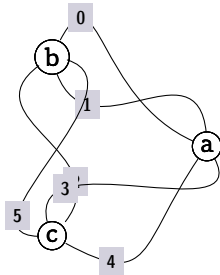


```
Fatgraph([
  Vertex([1, 0, 4, 3]), # a
  Vertex([1, 2, 3, 5, 4]), # b
  Vertex([0, 2, 5]), # c
])
```

Boundary cycles

$$\alpha = ({}^2b^3 \rightarrow {}^4b^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^1c^2 \rightarrow {}^3b^4 \rightarrow {}^1b^2)$$

The Fatgraph $G_{6,42}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 4, 3]), # a
  Vertex([2, 1, 5, 0]), # b
  Vertex([4, 2, 3, 5]), # c
])
```

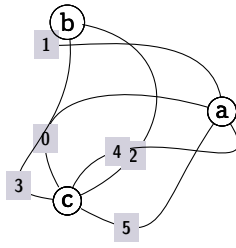
Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^3c^0 \rightarrow {}^2c^3 \rightarrow {}^2b^3 \rightarrow {}^0b^1)$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1^\dagger	a	c	b	3	4	5	0	1	2	α

The Fatgraph $G_{6,43}$ (1 orientable marking)

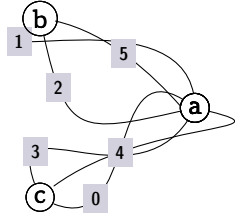


```
Fatgraph([
  Vertex([1, 0, 5, 4]),    # a
  Vertex([1, 3, 2]),       # b
  Vertex([5, 2, 4, 0, 3]), # c
])
```

Boundary cycles

$$\alpha = ({}^4c^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^2b^0 \rightarrow {}^1c^2 \rightarrow {}^3c^4 \rightarrow {}^1b^2 \rightarrow {}^2c^3)$$

The Fatgraph $G_{6,44}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3, 5, 4]), # a
  Vertex([1, 2, 5]),          # b
  Vertex([0, 4, 3]),          # c
])
```

Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^4a^5 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1a^2 \rightarrow {}^5a^0 \rightarrow {}^3a^4 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1b^2)$$

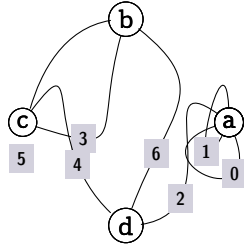
Automorphisms

A_0	a	b	c	0	1	2	3	4	5	α
A_1	a	b	c	3	2	5	4	0	1	α
A_2	a	b	c	4	5	1	0	3	2	α

Fatgraphs with 7 edges / 4 vertices

There are 52 unmarked fatgraphs in this section, originating 52 marked fatgraphs (43 orientable, and 9 nonorientable).

The Fatgraph $G_{7,0}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 2, 0, 1, 0]),# a
  Vertex([5, 3, 6]),      # b
  Vertex([5, 3, 4]),      # c
  Vertex([2, 6, 4]),      # d
])
```

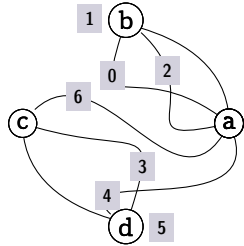
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^3a^4 \rightarrow ^1c^2 \rightarrow ^4a^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^2a^3 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Differentials

$$D(G_{7,0}^{(0)}) = +2G_{6,0}^{(0)}$$

The Fatgraph $G_{7,1}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 6, 4]),# a
  Vertex([1, 0, 2]),      # b
  Vertex([5, 3, 6]),      # c
  Vertex([5, 3, 4]),      # d
])
```

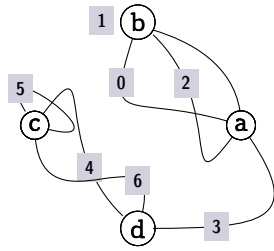
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^2c^0 \rightarrow ^1a^2 \rightarrow ^1c^2 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^1b^2 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Differentials

$$D(G_{7,1}^{(0)}) = +G_{6,0}^{(0)}$$

The Fatgraph $G_{7,2}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 0, 2]),  # b
  Vertex([6, 5, 4, 5]),# c
  Vertex([3, 6, 4]),  # d
])
```

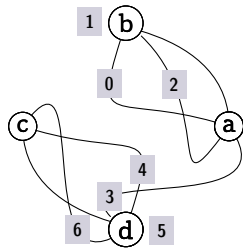
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3a^0 \rightarrow {}^2b^0 \rightarrow {}^3c^0 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,2}^{(0)}) = +G_{6,0}^{(0)}$$

The Fatgraph $G_{7,3}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 0, 2]),  # b
  Vertex([5, 4, 6]),  # c
  Vertex([5, 4, 3, 6]),# d
])
```

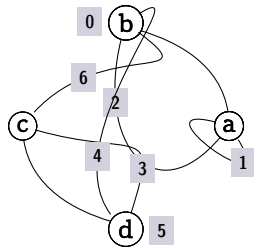
Boundary cycles

$$\alpha = (^3d^0 \rightarrow ^0c^1 \rightarrow ^2a^3 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2c^0 \rightarrow ^3a^0 \rightarrow ^1c^2 \rightarrow ^0d^1 \rightarrow ^2d^3 \rightarrow ^2b^0 \rightarrow ^1b^2 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	d	c	b	a	5	6	4	3	2	0	1	α

The Fatgraph $G_{7,4}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([0, 2, 6, 4]),# b
  Vertex([5, 3, 6]),   # c
  Vertex([5, 3, 4]),   # d
])
```

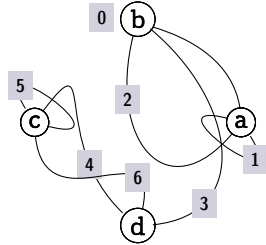
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2b^3 \rightarrow ^0c^1 \rightarrow ^3a^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^2a^3 \rightarrow ^3b^0 \rightarrow ^1c^2 \rightarrow ^2c^0 \rightarrow ^0d^1 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Differentials

$$D(G_{7,4}^{(0)}) = +G_{6,1}^{(2)}$$

The Fatgraph $G_{7,5}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([0, 2, 3]),  # b
  Vertex([6, 5, 4, 5]),# c
  Vertex([3, 6, 4]),  # d
])
```

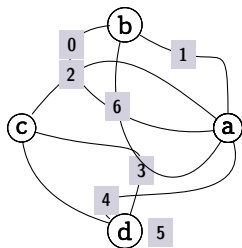
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^3c^0 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^2b^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	c	d	a	b	6	5	4	3	2	1	0	α

The Fatgraph $G_{7,6}$ (1 orientable marking)

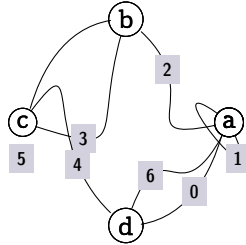


```
Fatgraph([
  Vertex([1, 2, 0, 6, 4]),# a
  Vertex([0, 6, 1]),      # b
  Vertex([5, 3, 2]),      # c
  Vertex([5, 3, 4]),      # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^0c^1 \rightarrow {}^4a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

The Fatgraph $G_{7,7}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 6, 1]),# a
  Vertex([5, 3, 2]),      # b
  Vertex([5, 3, 4]),      # c
  Vertex([0, 6, 4]),      # d
])
```

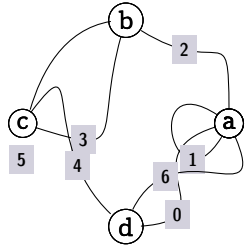
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^3a^4 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^0c^1 \rightarrow ^2c^0 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^1b^2 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Differentials

$$D(G_{7,7}^{(0)}) = -G_{6,1}^{(2)}$$

The Fatgraph $G_{7,8}$ (1 orientable marking)

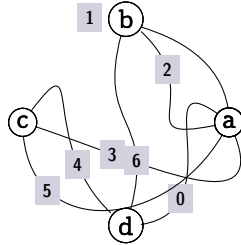


```
Fatgraph([
  Vertex([2, 1, 0, 1, 6]),# a
  Vertex([5, 3, 2]),      # b
  Vertex([5, 3, 4]),      # c
  Vertex([0, 6, 4]),      # d
])
```

Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^0b^1 \rightarrow ^1a^2 \rightarrow ^1c^2 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^1d^2 \rightarrow ^1b^2)$$

The Fatgraph $G_{7,9}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 5, 3]),# a
  Vertex([1, 6, 2]),      # b
  Vertex([5, 3, 4]),      # c
  Vertex([0, 6, 4]),      # d
])
```

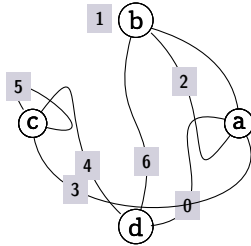
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^4a^0 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,9}^{(0)}) = +G_{6,0}^{(0)} + 2G_{6,0}^{(1)}$$

The Fatgraph $G_{7,10}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 6, 2]),  # b
  Vertex([3, 5, 4, 5]),# c
  Vertex([0, 6, 4]),  # d
])
```

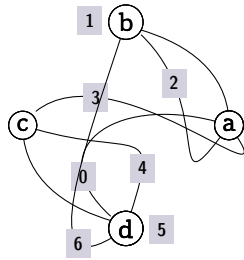
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3a^0 \rightarrow {}^2b^0 \rightarrow {}^3c^0 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,10}^{(0)}) = +G_{6,0}^{(0)} + G_{6,1}^{(2)}$$

The Fatgraph $G_{7,11}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 6, 2]),    # b
  Vertex([5, 4, 3]),    # c
  Vertex([5, 4, 0, 6]),# d
])
```

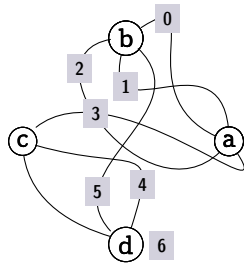
Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,11}^{(0)}) = -G_{6,0}^{(0)} + 2G_{6,0}^{(1)}$$

The Fatgraph $G_{7,12}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([2, 1, 5, 0]),# b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
])
```

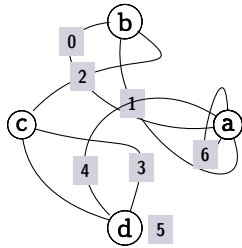
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,12}^{(0)}) = +G_{6,0}^{(1)}$$

The Fatgraph $G_{7,13}$ (1 orientable marking)



```
Fatgraph([
  Vertex([6, 4, 0, 6, 1]),# a
  Vertex([0, 1, 2]),      # b
  Vertex([5, 3, 2]),      # c
  Vertex([5, 3, 4]),      # d
])
```

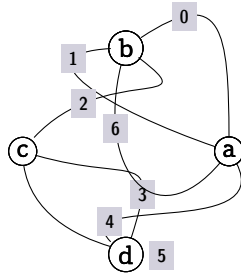
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^4a^0 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

Differentials

$$D(G_{7,13}^{(0)}) = +G_{6,1}^{(2)}$$

The Fatgraph $G_{7,14}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 1, 6, 4]),# a
  Vertex([1, 6, 2, 0]),# b
  Vertex([5, 3, 2]),    # c
  Vertex([5, 3, 4]),    # d
])
```

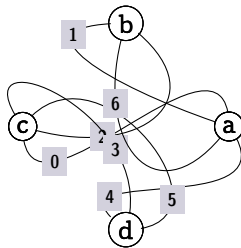
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^3a^0 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	b	a	d	c	0	1	4	3	2	5	6	α

The Fatgraph $G_{7,15}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 6, 4]),# a
  Vertex([1, 6, 2]),    # b
  Vertex([0, 2, 5, 3]),# c
  Vertex([5, 3, 4]),    # d
])
```

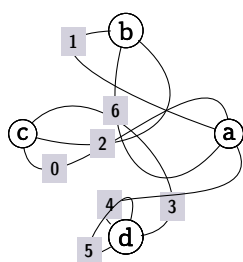
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^0b^1 \rightarrow {}^3a^0 \rightarrow {}^2b^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,15}^{(0)}) = +G_{6,0}^{(1)}$$

The Fatgraph $G_{7,16}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 6, 4]),# a
  Vertex([1, 6, 2]),   # b
  Vertex([0, 2, 3]),   # c
  Vertex([3, 5, 4, 5]),# d
])
```

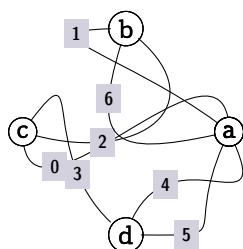
Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^3a^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,16}^{(0)}) = -G_{6,0}^{(1)}$$

The Fatgraph $G_{7,17}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 6, 5, 4]),# a
  Vertex([1, 6, 2]),      # b
  Vertex([0, 2, 3]),      # c
  Vertex([5, 4, 3]),      # d
])
```

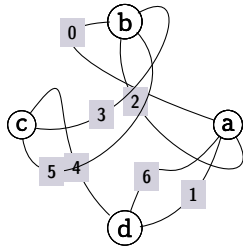
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^4a^0 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

Differentials

$$D(G_{7,17}^{(0)}) = -G_{6,0}^{(0)} + G_{6,1}^{(2)}$$

The Fatgraph $G_{7,18}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 6, 2]),# a
  Vertex([0, 2, 5, 3]),# b
  Vertex([5, 3, 4]),    # c
  Vertex([1, 6, 4]),    # d
])
```

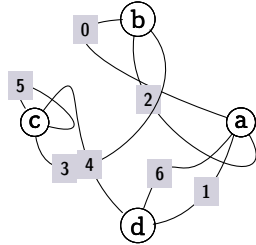
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,18}^{(0)}) = -G_{6,0}^{(0)}$$

The Fatgraph $G_{7,19}$ (1 orientable marking)

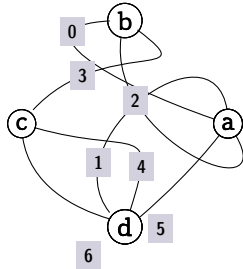


```
Fatgraph([
  Vertex([1, 0, 6, 2]),# a
  Vertex([0, 2, 3]),  # b
  Vertex([3, 5, 4, 5]),# c
  Vertex([1, 6, 4]),  # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^3c^0 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,20}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 6, 2]),# a
  Vertex([0, 2, 3]),  # b
  Vertex([5, 4, 3]),  # c
  Vertex([5, 4, 1, 6]),# d
])
```

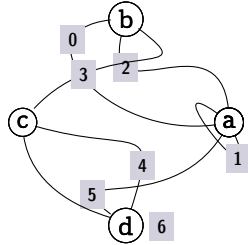
Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^0d^1 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{7,20}^{(0)}) = +2G_{6,0}^{(1)}$$

The Fatgraph $G_{7,21}$ (1 orientable marking)



```
Fatgraph([
  Vertex([2, 1, 0, 5, 1]),# a
  Vertex([0, 2, 3]),      # b
  Vertex([6, 4, 3]),      # c
  Vertex([6, 4, 5]),      # d
])
```

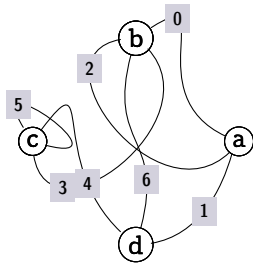
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^3a^4 \rightarrow ^0c^1 \rightarrow ^4a^0 \rightarrow ^2a^3 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^1a^2 \rightarrow ^1c^2 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

Differentials

$$D(G_{7,21}^{(0)}) = +G_{6,0}^{(0)} - 2G_{6,0}^{(1)}$$

The Fatgraph $G_{7,22}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 6, 3, 0]),   # b
  Vertex([3, 5, 4, 5]),   # c
  Vertex([1, 6, 4]),      # d
])
```

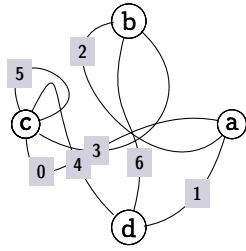
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^3c^0 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^1b^2 \rightarrow ^3b^0 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2c^3 \rightarrow ^2b^3 \rightarrow ^0b^1 \rightarrow ^1d^2 \rightarrow ^1c^2)$$

Differentials

$$D(G_{7,22}^{(0)}) = -5G_{6,0}^{(1)}$$

The Fatgraph $G_{7,23}$ (1 orientable marking)

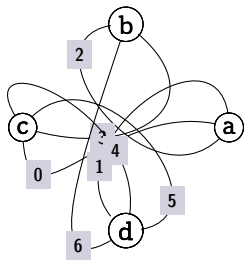


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 6, 3]),      # b
  Vertex([0, 3, 5, 4, 5]),# c
  Vertex([1, 6, 4]),      # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^4c^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3c^4 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,24}$ (1 orientable marking)

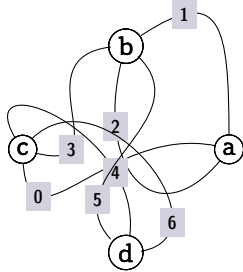


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 6, 3]),      # b
  Vertex([0, 3, 5, 4]),# c
  Vertex([5, 4, 1, 6]),# d
])
```

Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1b^2 \rightarrow {}^1d^2 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^2c^3)$$

The Fatgraph $G_{7,25}$ (1 orientable marking)

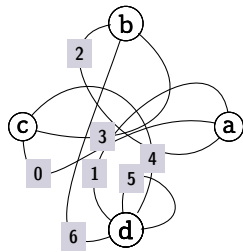


```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([3, 2, 5, 1]),# b
  Vertex([0, 3, 6, 4]),# c
  Vertex([6, 4, 5]),    # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^0b^1 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,26}$ (1 orientable marking)

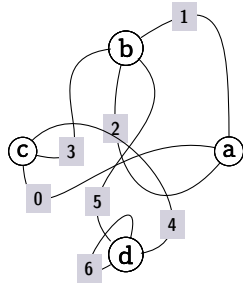


```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 6, 3]),    # b
  Vertex([0, 3, 4]),    # c
  Vertex([5, 4, 5, 1, 6]),# d
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^4d^0 \rightarrow {}^1b^2 \rightarrow {}^3d^4 \rightarrow {}^0c^1 \rightarrow {}^2d^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,27}$ (1 orientable marking)

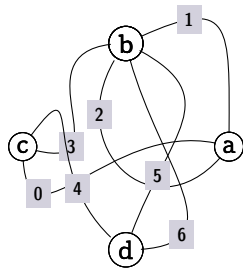


```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([3, 2, 5, 1]),# b
  Vertex([0, 3, 4]),    # c
  Vertex([4, 6, 5, 6]),# d
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1d^2 \rightarrow {}^0b^1 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^3d^0 \rightarrow {}^0d^1 \rightarrow {}^2c^0 \rightarrow {}^2d^3 \rightarrow {}^1b^2)$$

The Fatgraph $G_{7,28}$ (1 orientable marking)

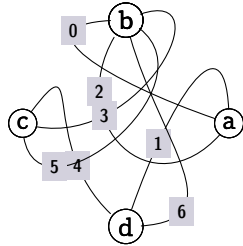


```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([3, 2, 6, 5, 1]),# b
  Vertex([0, 3, 4]),    # c
  Vertex([6, 5, 4]),    # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^4b^0 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^3b^4 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^3 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

The Fatgraph $G_{7,29}$ (1 orientable marking)

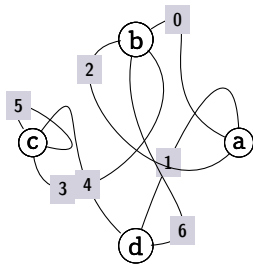


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([0, 2, 6, 5, 3]),# b
  Vertex([5, 3, 4]),      # c
  Vertex([6, 1, 4]),      # d
])
```

Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^2b^3 \rightarrow ^4b^0 \rightarrow ^0c^1 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2c^0 \rightarrow ^1c^2 \rightarrow ^3b^4 \rightarrow ^0d^1 \rightarrow ^0b^1 \rightarrow ^1d^2 \rightarrow ^1b^2)$$

The Fatgraph $G_{7,30}$ (1 orientable marking)

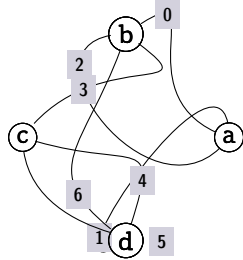


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 6, 3, 0]),  # b
  Vertex([3, 5, 4, 5]),  # c
  Vertex([6, 1, 4]),      # d
])
```

Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^0c^1 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^3c^0 \rightarrow ^1b^2 \rightarrow ^3b^0 \rightarrow ^1c^2 \rightarrow ^0d^1 \rightarrow ^2c^3 \rightarrow ^2b^3 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

The Fatgraph $G_{7,31}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 6, 3, 0]), # b
  Vertex([5, 4, 3]),    # c
  Vertex([5, 4, 6, 1]), # d
])
```

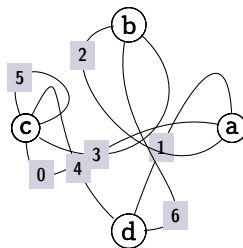
Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^3d^0 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2 \rightarrow {}^2d^3)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	c	d	a	b	5	3	4	1	2	0	6	α

The Fatgraph $G_{7,32}$ (1 orientable marking)

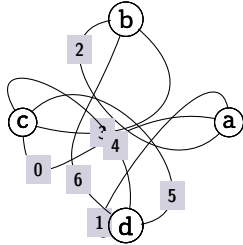


```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 6, 3]),    # b
  Vertex([0, 3, 5, 4, 5]), # c
  Vertex([6, 1, 4]),    # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^4c^0 \rightarrow {}^2c^3 \rightarrow {}^0a^1 \rightarrow {}^1d^2 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^3c^4 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^2b^0)$$

The Fatgraph $G_{7,33}$ (1 orientable marking)

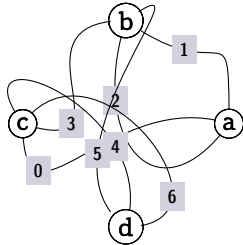


```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 6, 3]), # b
  Vertex([0, 3, 5, 4]), # c
  Vertex([5, 4, 6, 1]), # d
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1d^2 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^3d^0 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^2c^3 \rightarrow {}^2d^3)$$

The Fatgraph $G_{7,34}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([3, 2, 1, 5]), # b
  Vertex([0, 3, 6, 4]), # c
  Vertex([6, 4, 5]), # d
])
```

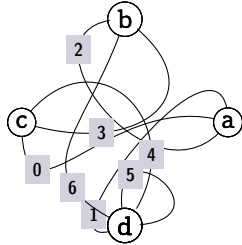
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^2b^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	d	c	b	a	5	4	6	3	1	0	2	α

The Fatgraph $G_{7,35}$ (1 orientable marking)

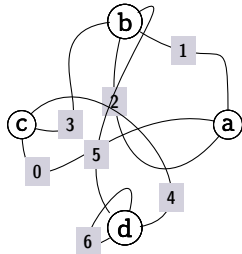


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 6, 3]),      # b
  Vertex([0, 3, 4]),      # c
  Vertex([5, 4, 5, 6, 1]),# d
])
```

Boundary cycles

$$\alpha = (^2a^0 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^4d^0 \rightarrow ^1b^2 \rightarrow ^3d^4 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2d^3 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

The Fatgraph $G_{7,36}$ (1 orientable marking)

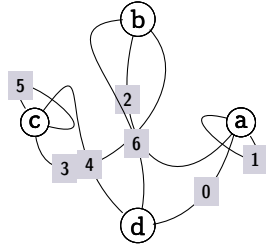


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([3, 2, 1, 5]),  # b
  Vertex([0, 3, 4]),      # c
  Vertex([4, 6, 5, 6]),  # d
])
```

Boundary cycles

$$\alpha = (^3d^0 \rightarrow ^2a^0 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2c^0 \rightarrow ^2b^3 \rightarrow ^3b^0 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2d^3 \rightarrow ^0b^1 \rightarrow ^1d^2 \rightarrow ^1b^2)$$

The Fatgraph $G_{7,37}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([6, 2, 3]),   # b
  Vertex([3, 5, 4, 5]),# c
  Vertex([0, 6, 4]),   # d
])
```

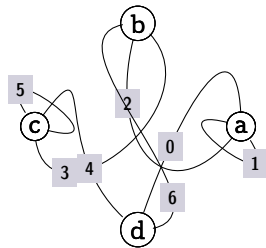
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^3c^0 \rightarrow {}^2a^3 \rightarrow {}^3a^0 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	c	d	a	b	3	5	4	0	2	1	6	α

The Fatgraph $G_{7,38}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([6, 2, 3]),   # b
  Vertex([3, 5, 4, 5]),# c
  Vertex([6, 0, 4]),   # d
])
```

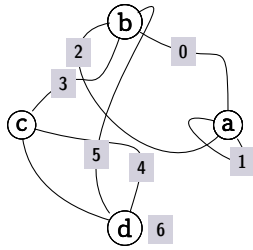
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^2b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1	a	d	c	b	2	1	0	4	3	5	6	α

The Fatgraph $G_{7,39}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([2, 3, 0, 5]),# b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
])
```

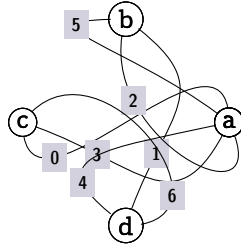
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2b^3 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^0b^1 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1	a	b	d	c	2	1	0	5	4	3	6	α

The Fatgraph $G_{7,40}$ (1 orientable marking)

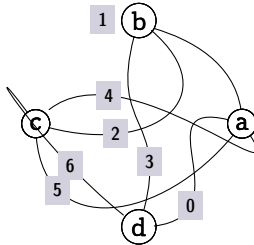


```
Fatgraph([
  Vertex([0, 5, 4, 3, 2]),# a
  Vertex([5, 2, 1]),      # b
  Vertex([0, 3, 6]),      # c
  Vertex([6, 1, 4]),      # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^4a^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,41}$ (1 orientable marking)

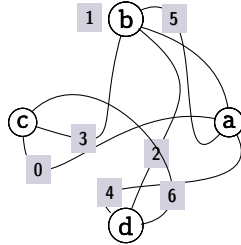


```
Fatgraph([
  Vertex([1, 0, 5, 4]),# a
  Vertex([1, 3, 2]),  # b
  Vertex([5, 2, 4, 6]),# c
  Vertex([0, 3, 6]),  # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3c^0 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^2b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,42}$ (1 orientable marking)

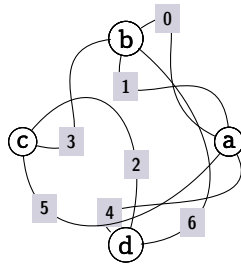


```
Fatgraph([
  Vertex([1, 0, 5, 4]),# a
  Vertex([1, 3, 2, 5]),# b
  Vertex([0, 3, 6]),    # c
  Vertex([6, 2, 4]),    # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,43}$ (1 orientable marking)

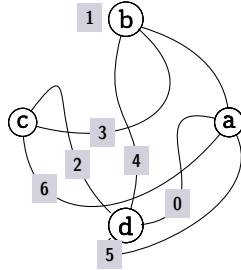


```
Fatgraph([
  Vertex([1, 0, 5, 4]),# a
  Vertex([3, 1, 6, 0]),# b
  Vertex([5, 3, 2]),   # c
  Vertex([6, 2, 4]),   # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,44}$ (1 orientable marking)

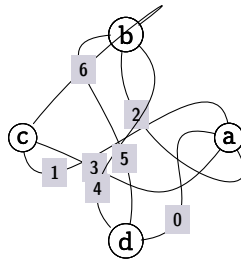


```
Fatgraph([
  Vertex([1, 0, 6, 5]),# a
  Vertex([1, 4, 3]),    # b
  Vertex([6, 3, 2]),    # c
  Vertex([0, 4, 2, 5]),# d
])
```

Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^1c^2 \rightarrow {}^2d^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,45}$ (1 orientable marking)

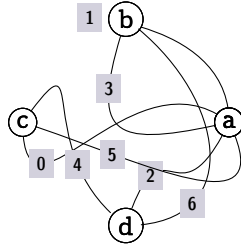


```
Fatgraph([
  Vertex([1, 0, 3, 2]),# a
  Vertex([5, 2, 4, 6]),# b
  Vertex([1, 3, 6]),   # c
  Vertex([0, 5, 4]),   # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^3b^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^1b^2 \rightarrow {}^1d^2)$$

The Fatgraph $G_{7,46}$ (1 orientable marking)

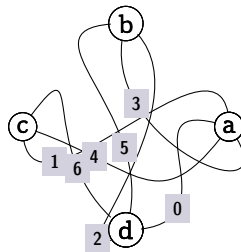


```
Fatgraph([
  Vertex([1, 0, 3, 2, 5]),# a
  Vertex([1, 3, 6]),      # b
  Vertex([0, 5, 4]),      # c
  Vertex([6, 2, 4]),      # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3a^4 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^4a^0 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

The Fatgraph $G_{7,47}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 4, 3]),# a
  Vertex([5, 3, 2]),  # b
  Vertex([1, 4, 6]),  # c
  Vertex([0, 5, 6, 2]),# d
])
```

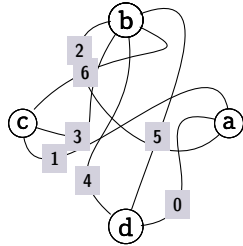
Boundary cycles

$$\alpha = ({}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^1d^2 \rightarrow {}^0c^1 \rightarrow {}^3d^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^2d^3)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	d	c	b	a	0	2	1	6	5	4	3	α

The Fatgraph $G_{7,48}$ (1 orientable marking)

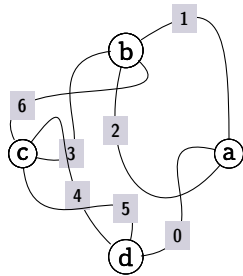


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 3, 4, 6, 5]),# b
  Vertex([1, 3, 6]),      # c
  Vertex([0, 5, 4]),      # d
])
```

Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^2b^3 \rightarrow ^4b^0 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2c^0 \rightarrow ^1b^2 \rightarrow ^0c^1 \rightarrow ^3b^4 \rightarrow ^0d^1 \rightarrow ^0b^1 \rightarrow ^1d^2)$$

The Fatgraph $G_{7,49}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([3, 2, 6, 1]),# b
  Vertex([5, 3, 4, 6]),# c
  Vertex([0, 5, 4]),      # d
])
```

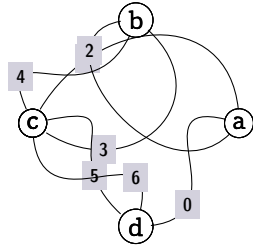
Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^2b^3 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^3c^0 \rightarrow ^0b^1 \rightarrow ^3b^0 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2c^3 \rightarrow ^1b^2 \rightarrow ^1d^2)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	d	c	b	a	0	4	5	6	1	2	3	α

The Fatgraph $G_{7,50}$ (1 orientable marking)

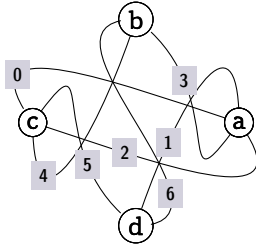


```
Fatgraph([
  Vertex([1, 0, 2]),      # a
  Vertex([2, 4, 3]),      # b
  Vertex([6, 3, 5, 1, 4]),# c
  Vertex([0, 6, 5]),      # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^4c^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1d^2 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^3c^4)$$

The Fatgraph $G_{7,51}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 3, 2]),# a
  Vertex([6, 4, 3]),  # b
  Vertex([4, 2, 5, 0]),# c
  Vertex([6, 1, 5]),  # d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^3c^0 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0c^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3a^0 \rightarrow {}^2b^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

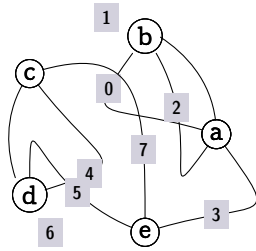
Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	6	α
A_1^\dagger	c	d	a	b	2	4	0	5	1	3	6	α

Fatgraphs with 8 edges / 5 vertices

There are 29 unmarked fatgraphs in this section, originating 29 marked fatgraphs (28 orientable, and 1 nonorientable).

The Fatgraph $G_{8,0}$ (1 orientable marking)

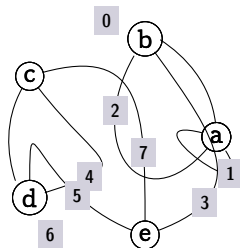


```
Fatgraph([
  Vertex([1, 0, 2, 3]), # a
  Vertex([1, 0, 2]),    # b
  Vertex([6, 4, 7]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([3, 7, 5]),    # e
])
```

Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1c^3)$$

The Fatgraph $G_{8,1}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]), # a
  Vertex([0, 2, 3]),    # b
  Vertex([6, 4, 7]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([3, 7, 5]),    # e
])
```

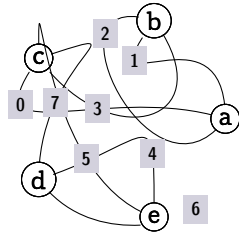
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1c^0 \rightarrow {}^0e^1)$$

Differentials

$$D(G_{8,1}^{(0)}) = +G_{7,0}^{(0)} - 2G_{7,1}^{(2)}$$

The Fatgraph $G_{8,2}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 1, 3]), # b
  Vertex([0, 3, 7, 5]), # c
  Vertex([6, 4, 7]), # d
  Vertex([6, 4, 5]), # e
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^0 \rightarrow {}^2d^0)$$

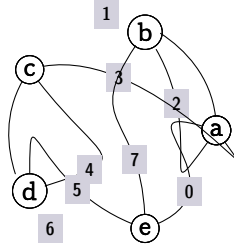
Automorphisms

A_0	a	b	c	d	e	0	1	2	3	4	5	6	7	α
A_1	d	e	c	a	b	7	4	6	5	1	3	2	0	α

Differentials

$$D(G_{8,2}^{(0)}) = +G_{7,0}^{(1)} - G_{7,1}^{(2)}$$

The Fatgraph $G_{8,3}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 7, 2]),   # b
  Vertex([6, 4, 3]),   # c
  Vertex([6, 4, 5]),   # d
  Vertex([0, 7, 5]),   # e
])
```

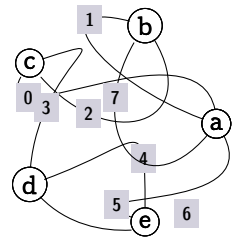
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1e^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1c^0)$$

Differentials

$$D(G_{8,3}^{(0)}) = -G_{7,0}^{(0)} + 2G_{7,0}^{(1)}$$

The Fatgraph $G_{8,4}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 7, 5]),# a
  Vertex([1, 7, 2]),   # b
  Vertex([0, 2, 3]),   # c
  Vertex([6, 4, 3]),   # d
  Vertex([6, 4, 5]),   # e
])
```

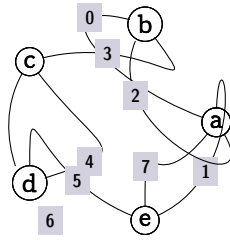
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^0e^1 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^3a^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1c^0)$$

Differentials

$$D(G_{8,4}^{(0)}) = -G_{7,2}^{(3)} - G_{7,4}^{(5)}$$

The Fatgraph $G_{8,5}$ (1 orientable marking)

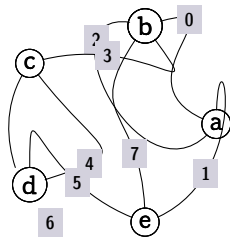


```
Fatgraph([
  Vertex([1, 0, 7, 2]), # a
  Vertex([0, 2, 3]),    # b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([1, 7, 5]),    # e
])
```

Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1c^3)$$

The Fatgraph $G_{8,6}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3, 0]), # b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([1, 7, 5]),    # e
])
```

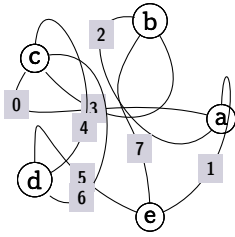
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^1e^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^2d^0 \rightarrow {}^0d^1 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1d^2)$$

Differentials

$$D(G_{8,6}^{(0)}) = -G_{7,2}^{(3)} - G_{7,4}^{(5)}$$

The Fatgraph $G_{8,7}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3]),    # b
  Vertex([0, 3, 6, 4]), # c
  Vertex([6, 4, 5]),    # d
  Vertex([1, 7, 5]),    # e
])
```

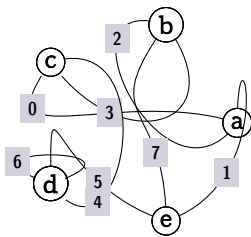
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1b^2 \rightarrow {}^1d^2 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2d^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^2c^0)$$

Differentials

$$D(G_{8,7}^{(0)}) = +G_{7,0}^{(1)} + G_{7,2}^{(3)} - G_{7,7}^{(8)}$$

The Fatgraph $G_{8,8}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3]),    # b
  Vertex([0, 3, 4]),    # c
  Vertex([4, 6, 5, 6]), # d
  Vertex([1, 7, 5]),    # e
])
```

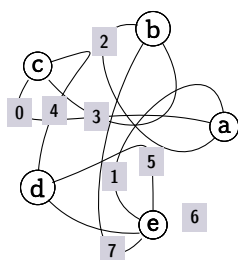
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^1e^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1d^2 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^3d^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^0b^1 \rightarrow {}^2a^0)$$

Differentials

$$D(G_{8,8}^{(0)}) = -G_{7,0}^{(0)} + G_{7,3}^{(4)} + G_{7,6}^{(7)} - G_{7,8}^{(9)}$$

The Fatgraph $G_{8,9}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 7, 3]), # b
  Vertex([0, 3, 4]), # c
  Vertex([6, 5, 4]), # d
  Vertex([6, 5, 1, 7]), # e
])
```

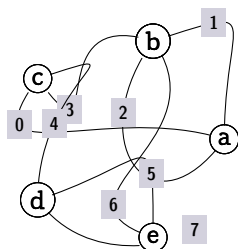
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3e^0 \rightarrow {}^0e^1 \rightarrow {}^0c^1 \rightarrow {}^1e^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^2e^3 \rightarrow {}^1d^1)$$

Differentials

$$D(G_{8,9}^{(0)}) = -G_{7,0}^{(1)} + G_{7,4}^{(5)} + G_{7,7}^{(8)}$$

The Fatgraph $G_{8,10}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([3, 2, 6, 1]), # b
  Vertex([0, 3, 4]), # c
  Vertex([7, 5, 4]), # d
  Vertex([7, 5, 6]), # e
])
```

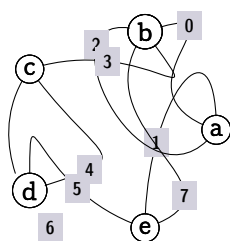
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^0e^1 \rightarrow {}^2c^0 \rightarrow {}^1e^2 \rightarrow {}^1d^2 \rightarrow {}^1b^1)$$

Differentials

$$D(G_{8,10}^{(0)}) = +G_{7,2}^{(3)} + G_{7,5}^{(6)} - G_{7,10}^{(11)}$$

The Fatgraph $G_{8,11}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 7, 3, 0]), # b
  Vertex([6, 4, 3]), # c
  Vertex([6, 4, 5]), # d
  Vertex([7, 1, 5]), # e
])
```

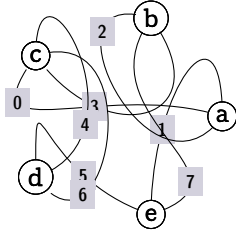
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^0e^1 \rightarrow {}^2c^0 \rightarrow {}^1e^2 \rightarrow {}^1d^1)$$

Differentials

$$D(G_{8,11}^{(0)}) = +G_{7,3}^{(4)} + G_{7,9}^{(10)} - G_{7,11}^{(12)}$$

The Fatgraph $G_{8,12}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3]),    # b
  Vertex([0, 3, 6, 4]), # c
  Vertex([6, 4, 5]),    # d
  Vertex([7, 1, 5]),    # e
])
```

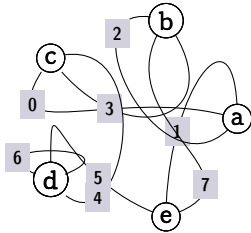
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1d^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^2c^0)$$

Differentials

$$D(G_{8,12}^{(0)}) = +G_{7,4}^{(5)} - G_{7,5}^{(6)} + G_{7,10}^{(11)}$$

The Fatgraph $G_{8,13}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3]),    # b
  Vertex([0, 3, 4]),    # c
  Vertex([4, 6, 5, 6]), # d
  Vertex([7, 1, 5]),    # e
])
```

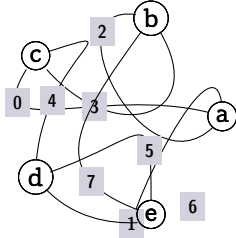
Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^1e^2 \rightarrow {}^1a^0)$$

Differentials

$$D(G_{8,13}^{(0)}) = -G_{7,4}^{(5)} - G_{7,6}^{(7)} + G_{7,9}^{(10)} - G_{7,12}^{(13)} - G_{7,13}^{(14)}$$

The Fatgraph $G_{8,14}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 7, 3]),    # b
  Vertex([0, 3, 4]),    # c
  Vertex([6, 5, 4]),    # d
  Vertex([6, 5, 7, 1]),# e
])
```

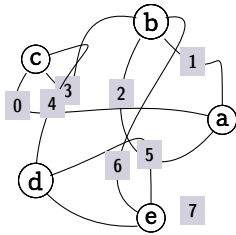
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^1e^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^3 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^3e^0 \rightarrow {}^1a^0)$$

Differentials

$$D(G_{8,14}^{(0)}) = -G_{7,5}^{(6)} - G_{7,7}^{(8)} + G_{7,10}^{(11)} - G_{7,14}^{(15)}$$

The Fatgraph $G_{8,15}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([3, 2, 1, 6]),# b
  Vertex([0, 3, 4]),    # c
  Vertex([7, 5, 4]),    # d
  Vertex([7, 5, 6]),    # e
])
```

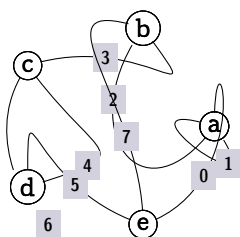
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2b^3 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^0e^1 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^2 \rightarrow {}^1b^2)$$

Differentials

$$D(G_{8,15}^{(0)}) = +G_{7,0}^{(1)} + G_{7,14}^{(15)}$$

The Fatgraph $G_{8,16}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([7, 2, 3]),    # b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([0, 7, 5]),    # e
])
```

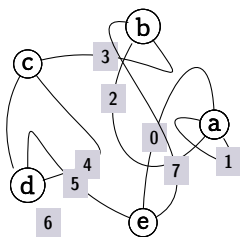
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^3a^0 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^1a^2 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1c$$

Differentials

$$D(G_{8,16}^{(0)}) = +G_{7,2}^{(3)} - G_{7,7}^{(8)} - G_{7,8}^{(9)} + G_{7,11}^{(12)} - G_{7,12}^{(13)}$$

The Fatgraph $G_{8,17}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 2, 1]), # a
  Vertex([7, 2, 3]),    # b
  Vertex([6, 4, 3]),    # c
  Vertex([6, 4, 5]),    # d
  Vertex([7, 0, 5]),    # e
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^0e^1 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1c$$

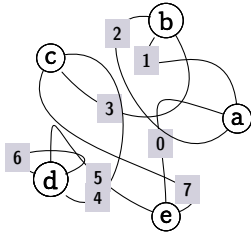
Automorphisms

A_0	a	b	c	d	e	0	1	2	3	4	5	6	7	α
A_1	a	e	d	c	b	2	1	0	5	4	3	6	7	α

Differentials

$$D(G_{8,17}^{(0)}) = -G_{7,0}^{(0)} + G_{7,7}^{(8)} - G_{7,13}^{(14)}$$

The Fatgraph $G_{8,18}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 1, 3]), # b
  Vertex([7, 3, 4]), # c
  Vertex([4, 6, 5, 6]), # d
  Vertex([7, 0, 5]), # e
])
```

Boundary cycles

$$\alpha = ({}^3d^0 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2d^3 \rightarrow {}^2b^0 \rightarrow {}^2c^0 \rightarrow {}^1e^2 \rightarrow {}^1a^2 \rightarrow \dots)$$

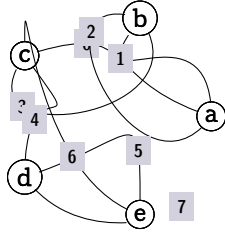
Automorphisms

A_0	a	b	c	d	e	0	1	2	3	4	5	6	7	α
A_1	b	a	e	d	c	3	1	2	0	5	4	6	7	α

Differentials

$$D(G_{8,18}^{(0)}) = -G_{7,0}^{(1)} - G_{7,14}^{(15)}$$

The Fatgraph $G_{8,19}$ (non-orientable, no orientable markings)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([2, 1, 3]), # b
  Vertex([3, 4, 0, 6]), # c
  Vertex([7, 5, 4]), # d
  Vertex([7, 5, 6]), # e
])
```

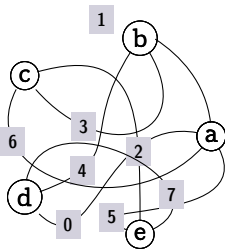
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^0c^1 \rightarrow {}^1c^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^1a^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^3c^0 \rightarrow {}^1d^2 \rightarrow {}^0d^1 \rightarrow {}^2c^3 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^2b^0)$$

Automorphisms

A_0	a	b	c	d	e	0	1	2	3	4	5	6	7	α
A_1^\dagger	e	d	c	a	b	6	5	7	4	0	1	3	2	α
A_2	b	a	c	e	d	3	1	2	0	6	5	4	7	α
A_3^\dagger	d	e	c	b	a	4	5	7	6	3	1	0	2	α

The Fatgraph $G_{8,20}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 6, 5]), # a
  Vertex([1, 4, 3]), # b
  Vertex([6, 3, 2]), # c
  Vertex([0, 4, 7]), # d
  Vertex([7, 2, 5]), # e
])
```

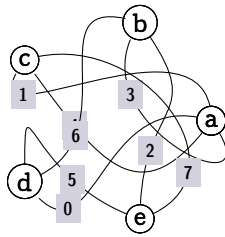
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2a^3 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1c^0)$$

Differentials

$$D(G_{8,20}^{(0)}) = +G_{7,2}^{(3)} - G_{7,5}^{(6)} - G_{7,10}^{(11)}$$

The Fatgraph $G_{8,21}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 4, 3]),# a
  Vertex([6, 3, 2]),   # b
  Vertex([1, 4, 7]),   # c
  Vertex([0, 6, 5]),   # d
  Vertex([7, 2, 5]),   # e
])
```

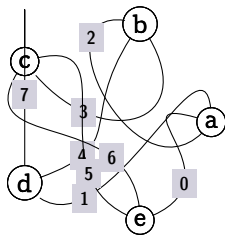
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^0e^1 \rightarrow {}^1c^2 \rightarrow {}^3a^0 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^2a^3 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1c^3)$$

Differentials

$$D(G_{8,21}^{(0)}) = +G_{7,3}^{(4)} - G_{7,9}^{(10)} + G_{7,11}^{(12)}$$

The Fatgraph $G_{8,22}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),   # a
  Vertex([2, 4, 3]),   # b
  Vertex([6, 3, 5, 7]),# c
  Vertex([1, 4, 7]),   # d
  Vertex([0, 6, 5]),   # e
])
```

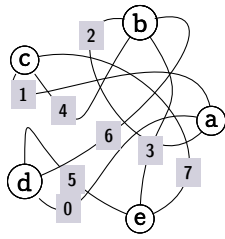
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^3c^0 \rightarrow {}^1b^2 \rightarrow {}^2d^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^3 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^3)$$

Differentials

$$D(G_{8,22}^{(0)}) = +G_{7,4}^{(5)} + G_{7,5}^{(6)} + G_{7,10}^{(11)}$$

The Fatgraph $G_{8,23}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 4, 3, 6]), # b
  Vertex([1, 4, 7]),    # c
  Vertex([0, 6, 5]),    # d
  Vertex([7, 3, 5]),    # e
])
```

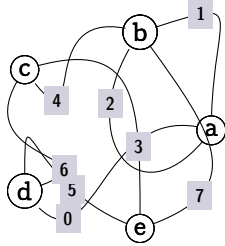
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^0e^1 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^3)$$

Differentials

$$D(G_{8,23}^{(0)}) = +G_{7,5}^{(6)} - G_{7,15}^{(16)}$$

The Fatgraph $G_{8,24}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([4, 2, 7, 1]), # b
  Vertex([6, 4, 3]),    # c
  Vertex([0, 6, 5]),    # d
  Vertex([7, 3, 5]),    # e
])
```

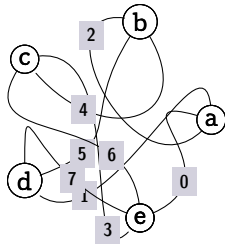
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^0b^1 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^0d^1 \rightarrow {}^0e^1 \rightarrow {}^1b^2 \rightarrow {}^1e^2 \rightarrow {}^1d^3 \rightarrow {}^2d^0)$$

Differentials

$$D(G_{8,24}^{(0)}) = +G_{7,6}^{(7)} - G_{7,9}^{(10)} - G_{7,11}^{(12)} - G_{7,16}^{(17)} - G_{7,17}^{(18)}$$

The Fatgraph $G_{8,25}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),    # a
  Vertex([2, 5, 4]),    # b
  Vertex([6, 4, 3]),    # c
  Vertex([1, 5, 7]),    # d
  Vertex([0, 6, 7, 3]), # e
])
```

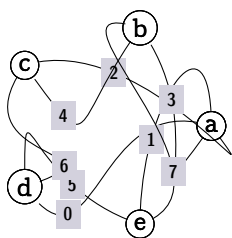
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^2e^3 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^1e^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^3e^0 \rightarrow {}^1d^3 \rightarrow {}^2d^0)$$

Differentials

$$D(G_{8,25}^{(0)}) = +G_{7,7}^{(8)} - 2G_{7,10}^{(11)} + G_{7,13}^{(14)}$$

The Fatgraph $G_{8,26}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 3, 2]), # a
  Vertex([7, 4, 3]),    # b
  Vertex([6, 4, 2]),    # c
  Vertex([0, 6, 5]),    # d
  Vertex([7, 1, 5]),    # e
])
```

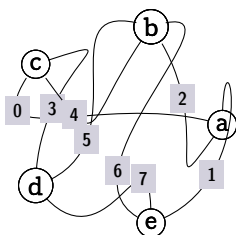
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^0e^1 \rightarrow {}^1c^2 \rightarrow {}^2a^3 \rightarrow {}^1e^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^1b^2 \rightarrow {}^0b^1 \rightarrow {}^1c$$

Differentials

$$D(G_{8,26}^{(0)}) = -G_{7,0}^{(1)} + G_{7,2}^{(3)} - G_{7,13}^{(14)}$$

The Fatgraph $G_{8,27}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]), # a
  Vertex([5, 4, 2, 6]), # b
  Vertex([0, 4, 3]), # c
  Vertex([7, 5, 3]), # d
  Vertex([1, 7, 6]), # e
])
```

Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^2b^3 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^1b^2 \rightarrow {}^3b^0 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^2d^0 \rightarrow {}^0d^1 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^1)$$

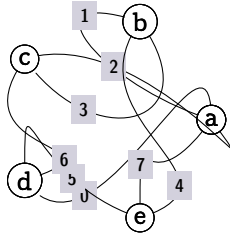
Automorphisms

A_0	a	b	c	d	e	0	1	2	3	4	5	6	7	α
A_1	d	b	e	a	c	7	3	5	1	6	2	4	0	α

Differentials

$$D(G_{8,27}^{(0)}) = +G_{7,6}^{(7)} + G_{7,8}^{(9)} - G_{7,9}^{(10)} - G_{7,11}^{(12)} - G_{7,16}^{(17)}$$

The Fatgraph $G_{8,28}$ (1 orientable marking)



```
Fatgraph([
  Vertex([0, 1, 7, 2]), # a
  Vertex([1, 4, 3]),    # b
  Vertex([6, 3, 2]),    # c
  Vertex([0, 6, 5]),    # d
  Vertex([4, 7, 5]),    # e
])
```

Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2d^0 \rightarrow {}^0c^1 \rightarrow {}^2a^3 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2c^0 \rightarrow {}^3a^0 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0b^1 \rightarrow {}^1e^2 \rightarrow {}^1d^2 \rightarrow {}^1b^2 \rightarrow {}^0e^3 \rightarrow {}^0c^2 \rightarrow {}^1a^3 \rightarrow {}^2d^1 \rightarrow {}^0e^1)$$

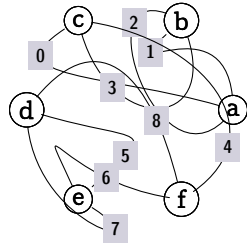
Differentials

$$D(G_{8,28}^{(0)}) = +G_{7,0}^{(0)} + G_{7,3}^{(4)} - G_{7,6}^{(7)} - G_{7,17}^{(18)}$$

Fatgraphs with 9 edges / 6 vertices

There are 9 unmarked fatgraphs in this section, originating 9 marked fatgraphs (all of them orientable).

The Fatgraph $G_{9,0}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 3]),# b
  Vertex([0, 3, 4]),# c
  Vertex([7, 5, 8]),# d
  Vertex([7, 5, 6]),# e
  Vertex([4, 8, 6]),# f
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2f^0 \rightarrow {}^0f^1 \rightarrow {}^0c^1 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1f^2 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^1c^2 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

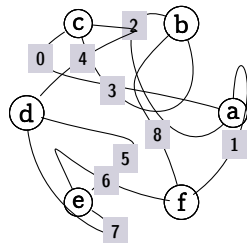
Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	d	e	f	a	b	c	8	5	7	6	4	1	3	2	0	α

Differentials

$$D(G_{9,0}^{(0)}) = +G_{8,0}^{(0)}$$

The Fatgraph $G_{9,1}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 8, 3]),# b
  Vertex([0, 3, 4]),# c
  Vertex([7, 5, 4]),# d
  Vertex([7, 5, 6]),# e
  Vertex([1, 8, 6]),# f
])
```

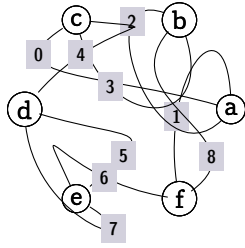
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2f^0 \rightarrow {}^0f^1 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^1e^2 \rightarrow {}^0a^1 \rightarrow {}^1f^2 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

Differentials

$$D(G_{9,1}^{(0)}) = -G_{8,0}^{(0)}$$

The Fatgraph $G_{9,2}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 8, 3]),# b
  Vertex([0, 3, 4]),# c
  Vertex([7, 5, 4]),# d
  Vertex([7, 5, 6]),# e
  Vertex([8, 1, 6]),# f
])
```

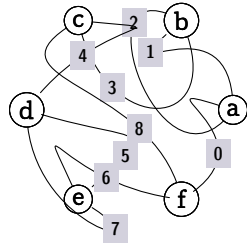
Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1f^2 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0f^1 \rightarrow {}^0a^1 \rightarrow {}^2f^0 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

Differentials

$$D(G_{9,2}^{(0)}) = -G_{8,0}^{(1)} + 2G_{8,1}^{(2)} - G_{8,2}^{(3)}$$

The Fatgraph $G_{9,3}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 3]),# b
  Vertex([8, 3, 4]),# c
  Vertex([7, 5, 4]),# d
  Vertex([7, 5, 6]),# e
  Vertex([0, 8, 6]),# f
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^2f^0 \rightarrow {}^0f^1 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0a^1 \rightarrow {}^1f^2 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

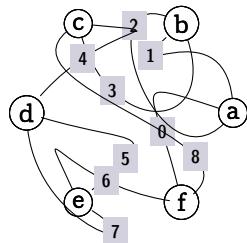
Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	d	e	f	a	b	c	4	5	7	6	0	1	3	2	8	α

Differentials

$$D(G_{9,3}^{(0)}) = +2G_{8,0}^{(0)}$$

The Fatgraph $G_{9,4}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 3]),# b
  Vertex([8, 3, 4]),# c
  Vertex([7, 5, 4]),# d
  Vertex([7, 5, 6]),# e
  Vertex([8, 0, 6]),# f
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \rightarrow {}^2a^0 \rightarrow {}^1f^2 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0b^1 \rightarrow {}^0f^1 \rightarrow {}^0a^1 \rightarrow {}^2f^0 \rightarrow {}^1b^2 \rightarrow {}^0e^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

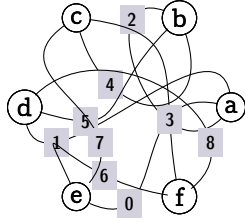
Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	b	a	f	e	d	c	3	1	2	0	6	5	4	7	8	α

Differentials

$$D(G_{9,4}^{(0)}) = +2G_{8,4}^{(5)}$$

The Fatgraph $G_{9,5}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 5, 4]),# b
  Vertex([7, 4, 3]),# c
  Vertex([1, 5, 8]),# d
  Vertex([0, 7, 6]),# e
  Vertex([8, 3, 6]),# f
])
```

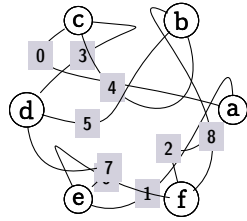
Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^1f^2 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2f^0 \rightarrow {}^1b^2 \rightarrow {}^0f^1 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^2d^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^2$$

Differentials

$$D(G_{9,5}^{(0)}) = -G_{8,0}^{(0)} + G_{8,3}^{(4)} + G_{8,6}^{(7)} + G_{8,8}^{(9)}$$

The Fatgraph $G_{9,6}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([8, 5, 4]),# b
  Vertex([0, 4, 3]),# c
  Vertex([7, 5, 3]),# d
  Vertex([1, 7, 6]),# e
  Vertex([8, 2, 6]),# f
])
```

Boundary cycles

$$\alpha = ({}^0e^1 \rightarrow {}^2a^0 \rightarrow {}^0f^1 \rightarrow {}^1c^2 \rightarrow {}^1a^2 \rightarrow {}^0a^1 \rightarrow {}^2f^0 \rightarrow {}^1b^2 \rightarrow {}^1f^2 \rightarrow {}^2e^0 \rightarrow {}^0c^1 \rightarrow {}^2c^0 \rightarrow {}^2d^0 \rightarrow {}^0d^1 \rightarrow {}^2b^0 \rightarrow {}^0$$

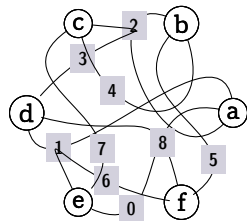
Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	d	f	e	a	c	b	7	3	5	1	6	2	4	0	8	α

Differentials

$$D(G_{9,6}^{(0)}) = +G_{8,3}^{(4)} + G_{8,9}^{(10)} + G_{8,11}^{(12)}$$

The Fatgraph $G_{9,7}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 5, 4]),# b
  Vertex([7, 4, 3]),# c
  Vertex([1, 8, 3]),# d
  Vertex([0, 7, 6]),# e
  Vertex([5, 8, 6]),# f
])
```

Boundary cycles

$$\alpha = (^0e^1 \rightarrow ^2a^0 \rightarrow ^1f^2 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2f^0 \rightarrow ^0b^1 \rightarrow ^0f^1 \rightarrow ^2e^0 \rightarrow ^0c^1 \rightarrow ^2d^0 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^1$$

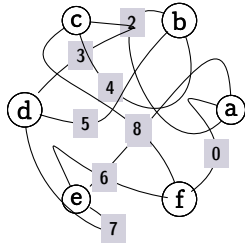
Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	c	b	f	d	e	a	7	3	4	8	5	2	0	6	1	α
A_2	f	b	a	d	e	c	6	8	5	1	2	4	7	0	3	α

Differentials

$$D(G_{9,7}^{(0)}) = +G_{8,4}^{(5)} - G_{8,6}^{(7)} + G_{8,9}^{(10)} + G_{8,13}^{(14)} + G_{8,14}^{(15)}$$

The Fatgraph $G_{9,8}$ (1 orientable marking)



```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 5, 4]),# b
  Vertex([8, 4, 3]),# c
  Vertex([7, 5, 3]),# d
  Vertex([7, 1, 6]),# e
  Vertex([0, 8, 6]),# f
])
```

Boundary cycles

$$\alpha = (^2d^0 \rightarrow ^2a^0 \rightarrow ^0f^1 \rightarrow ^1c^2 \rightarrow ^1a^2 \rightarrow ^0a^1 \rightarrow ^2f^0 \rightarrow ^0b^1 \rightarrow ^0e^1 \rightarrow ^1f^2 \rightarrow ^2e^0 \rightarrow ^0c^1 \rightarrow ^0d^1 \rightarrow ^2b^0 \rightarrow ^2c^0 \rightarrow ^1$$

Automorphisms

A_0	a	b	c	d	e	f	0	1	2	3	4	5	6	7	8	α
A_1	d	e	f	a	b	c	3	5	7	0	6	1	4	2	8	α

Differentials

$$D(G_{9,8}^{(0)}) = +G_{8,5}^{(6)} - G_{8,7}^{(8)} + G_{8,10}^{(11)} - G_{8,12}^{(13)} + G_{8,15}^{(16)} + G_{8,16}^{(17)}$$