Fatgraphs of $M_{1,2}$

Automatically generated by FatGHoL 5.4
(See: `http://fatghol.googlecode.com/`)

2012-02-09

There are a total of 24 undecorated fatgraphs in the Kontsevich graph complex of $M_{1,2}$, originating 43 marked ones.

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Notation

We denote $G_{m,j}$ the j-th graph in the set of undecorated fatgraphs with m edges; the symbol $G^{(k)}_{m,j}$ denotes the k-th inequivalent marking of $G_{m,j}$.

Fatgraph vertices are marked with lowercase latin letters "a", "b", "c", etc.; edges are marked with an arabic numeral starting from "1"; boundary cycles are denoted by lowercase greek letters "α", "β", etc.

Automorphisms are specified by their action on the set of vertices, edges, and boundary cycles: for each automorphism A_k, a table line lists how it permutes vertices, edges and boundary cycles relative to the identity morphism A_0. The automorphism table is printed only if the automorphism group is non-trivial.

Automorphisms that reverse the orientation of the unmarked fatgraph are indicated with a "†" symbol in the automorphism table; those that reverse the orientation of the marked fatgraphs are distinguished with a "‡" sign.

If a fatgraph is orientable, a "Markings" section lists all the inequivalent ways of assigning distinct numbers $\{0, \dots, n-1\}$ to the boundary cycles; this is of course a set of representatives for the orbits of $\mathfrak{S}_n$ under the action of $\mathrm{Aut}(G)$.

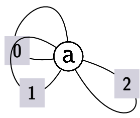
A separate section lists the differential of marked fatgraphs; graphs with null differential are omitted. If no marked fatgraph has a non-zero differential, the entire section is dropped.

Boundary cycles are specified using a "sequence of corners" notation: each corner is represented as ${}^{p}L^{q}$ where L is a latin letter indicating a vertex, and p, q are the attachment indices of the incoming and outgoing edges, respectively. Attachment indices match the Python representation of the vertex: e.g., if a=`Vertex([0,0,1])`, the two legs of edge 0 have attachment indices 0 and 1, and the boundary cycle enclosed by them is represented by the (single) corner ${}^{0}a^{1}$.

Fatgraphs with 3 edges / 1 vertex

There are 3 unmarked fatgraphs in this section, originating 8 marked fatgraphs (3 orientable, and 5 nonorientable).

The Fatgraph $G_{3,0}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 0, 1, 2, 2]),# a
])
```

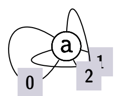
Boundary cycles

$$\alpha = ({}^2a^3 \to {}^3a^4 \to {}^1a^2 \to {}^0a^1 \to {}^5a^0)$$
$$\beta = ({}^4a^5)$$

Markings

Fatgraph $G_{3,0}$ only has the identity automorphism, so the marked fatgraphs $G_{3,0}^{(0)}$ to $G_{3,0}^{(2)}$ are formed by decorating boundary cycles of $G_{3,0}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{3,1}$ (non-orientable, no orientable markings)




```
Fatgraph([
  Vertex([2, 0, 1, 0, 2, 1]),# a
])
```

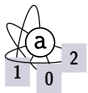
Boundary cycles

$$\alpha = ({}^3a^4 \to {}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^2a^3 \to {}^4a^5 \to {}^5a^0)$$

Automorphisms

A_0	a	0	1	2	α	β
$A_1{}^{\dagger}$	a	2	1	0	α	β

The Fatgraph $G_{3,2}$ (1 orientable marking)




```
Fatgraph([
  Vertex([1, 0, 2, 1, 0, 2]),# a
])
```

Boundary cycles

$$\alpha = ({}^2a^3 \to {}^4a^5 \to {}^0a^1)$$
$$\beta = ({}^1a^2 \to {}^3a^4 \to {}^5a^0)$$

Automorphisms

A_0	a	0	1	2	α	β
$A_1{}^{\ddagger}$	a	2	0	1	β	α
A_2	a	1	2	0	α	β
$A_3{}^{\ddagger}$	a	0	1	2	β	α
A_4	a	2	0	1	α	β
$A_5{}^{\ddagger}$	a	1	2	0	β	α

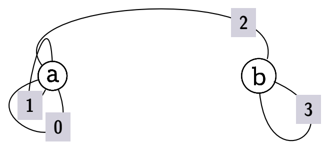
Markings

	$G^{(0)}_{3,2}$
α	0
β	1

Fatgraphs with 4 edges / 2 vertices

There are 8 unmarked fatgraphs in this section, originating 24 marked fatgraphs (10 orientable, and 14 nonorientable).

The Fatgraph $G_{4,0}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 2, 0, 1, 0]),# a
  Vertex([3, 3, 2]),      # b
])
```

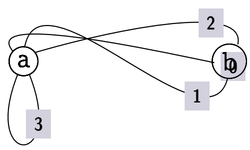
Boundary cycles

$$\alpha = ({}^{3}a^{4} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{4}a^{0} \to {}^{2}a^{3} \to {}^{2}b^{0} \to {}^{1}b^{2})$$
$$\beta = ({}^{0}b^{1})$$

Markings

Fatgraph $G_{4,0}$ only has the identity automorphism, so the marked fatgraphs $G_{4,0}^{(0)}$ to $G_{4,0}^{(2)}$ are formed by decorating boundary cycles of $G_{4,0}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,1}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2, 3, 3]),# a
  Vertex([1, 0, 2]),      # b
])
```

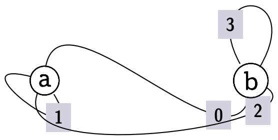
Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{0}a^{1} \to {}^{0}b^{1} \to {}^{1}a^{2} \to {}^{2}b^{0} \to {}^{4}a^{0} \to {}^{1}b^{2})$$
$$\beta = ({}^{3}a^{4})$$

Markings

Fatgraph $G_{4,1}$ only has the identity automorphism, so the marked fatgraphs $G_{4,1}^{(0)}$ to $G_{4,1}^{(2)}$ are formed by decorating boundary cycles of $G_{4,1}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,2}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([0, 2, 3, 3]),# b
])
```

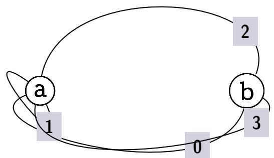
Boundary cycles

$$\alpha = ({}^{3}a^{0} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{1}b^{2} \to {}^{2}a^{3} \to {}^{3}b^{0} \to {}^{0}b^{1})$$
$$\beta = ({}^{2}b^{3})$$

Markings

Fatgraph $G_{4,2}$ only has the identity automorphism, so the marked fatgraphs $G_{4,2}^{(0)}$ to $G_{4,2}^{(2)}$ are formed by decorating boundary cycles of $G_{4,2}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,3}$ (2 orientable markings)




```
Fatgraph([
  Vertex([2, 1, 0, 3, 1]),# a
  Vertex([0, 3, 2]),      # b
])
```

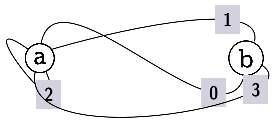
Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{2}b^{0} \to {}^{1}b^{2} \to {}^{0}a^{1} \to {}^{4}a^{0})$$
$$\beta = ({}^{1}a^{2} \to {}^{3}a^{4} \to {}^{0}b^{1})$$

Markings

Fatgraph $G_{4,3}$ only has the identity automorphism, so the marked fatgraphs $G_{4,3}^{(0)}$ to $G_{4,3}^{(2)}$ are formed by decorating boundary cycles of $G_{4,3}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,4}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 2, 1, 3, 2]),# a
  Vertex([0, 3, 1]),      # b
])
```

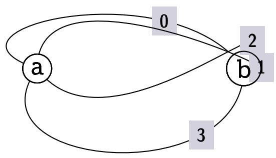
Boundary cycles

$$\alpha = ({}^{3}a^{4} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{0}b^{1} \to {}^{2}b^{0} \to {}^{4}a^{0})$$
$$\beta = ({}^{2}a^{3} \to {}^{1}b^{2})$$

Markings

Fatgraph $G_{4,4}$ only has the identity automorphism, so the marked fatgraphs $G_{4,4}^{(0)}$ to $G_{4,4}^{(2)}$ are formed by decorating boundary cycles of $G_{4,4}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{4,5}$ (non-orientable, no orientable markings)




```
Fatgraph([
  Vertex([1, 0, 3, 2]),# a
  Vertex([3, 1, 2, 0]),# b
])
```

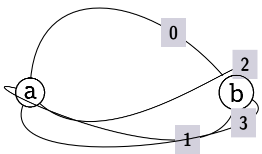
Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{0}a^{1} \to {}^{0}b^{1} \to {}^{1}a^{2} \to {}^{3}b^{0} \to {}^{2}b^{3})$$
$$\beta = ({}^{3}a^{0} \to {}^{1}b^{2})$$

Automorphisms

A_0	a	b	0	1	2	3	α	β
$A_1{}^{\dagger}$	b	a	0	2	1	3	α	β

The Fatgraph $G_{4,6}$ (non-orientable, no orientable markings)




```
Fatgraph([
  Vertex([0, 1, 3, 2]),# a
  Vertex([1, 3, 2, 0]),# b
])
```

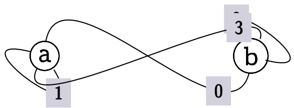
Boundary cycles

$$\alpha = ({}^{2}a^{3} \rightarrow {}^{2}b^{3} \rightarrow {}^{0}a^{1} \rightarrow {}^{0}b^{1})$$
$$\beta = ({}^{1}a^{2} \rightarrow {}^{3}a^{0} \rightarrow {}^{3}b^{0} \rightarrow {}^{1}b^{2})$$

Automorphisms

A_0	a	b	0	1	2	3	α	β
$A_1{}^{\dagger\ddagger}$	a	b	1	3	0	2	β	α
A_2	a	b	3	2	1	0	α	β
$A_3{}^{\dagger\ddagger}$	a	b	2	0	3	1	β	α
$A_4{}^{\dagger}$	b	a	1	3	0	2	α	β
$A_5{}^{\ddagger}$	b	a	3	2	1	0	β	α
$A_6{}^{\dagger}$	b	a	2	0	3	1	α	β
$A_7{}^{\ddagger}$	b	a	0	1	2	3	β	α

The Fatgraph $G_{4,7}$ (non-orientable, no orientable markings)




```
Fatgraph([
  Vertex([0, 1, 3, 1]),# a
  Vertex([0, 2, 3, 2]),# b
])
```

Boundary cycles

$$\alpha = ({}^3a^0 \to {}^3b^0 \to {}^0a^1 \to {}^0b^1)$$
$$\beta = ({}^1a^2 \to {}^2a^3 \to {}^1b^2 \to {}^2b^3)$$

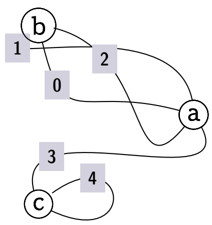
Automorphisms

A_0	a	b	0	1	2	3	α	β
$A_1{}^{\dagger\ddagger}$	a	b	3	1	2	0	β	α
$A_2{}^{\dagger}$	b	a	0	2	1	3	α	β
$A_3{}^{\ddagger}$	b	a	3	2	1	0	β	α

Fatgraphs with 5 edges / 3 vertices

There are 8 unmarked fatgraphs in this section, originating 30 marked fatgraphs (15 orientable, and 15 nonorientable).

The Fatgraph $G_{5,0}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2, 3]),# a
  Vertex([1, 0, 2]),   # b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{2}c^{0} \to {}^{3}a^{0} \to {}^{1}c^{2} \to {}^{2}b^{0} \to {}^{1}b^{2} \to {}^{0}b^{1})$$
$$\beta = ({}^{0}c^{1})$$

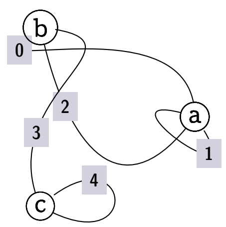
Markings

Fatgraph $G_{5,0}$ only has the identity automorphism, so the marked fatgraphs $G_{5,0}^{(0)}$ to $G_{5,0}^{(2)}$ are formed by decorating boundary cycles of $G_{5,0}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,0}^{(0)}) = +G_{4,0}^{(0)} \qquad D(G_{5,0}^{(1)}) = +G_{4,0}^{(1)}$$

The Fatgraph $G_{5,1}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 2, 1]),# a
  Vertex([0, 2, 3]),   # b
  Vertex([4, 4, 3]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{3}a^{0} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{1}b^{2} \to {}^{2}a^{3} \to {}^{1}c^{2} \to {}^{2}c^{0} \to {}^{2}b^{0} \to {}^{0}b^{1})$$
$$\beta = ({}^{0}c^{1})$$

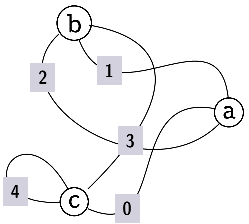
Markings

Fatgraph $G_{5,1}$ only has the identity automorphism, so the marked fatgraphs $G_{5,1}^{(0)}$ to $G_{5,1}^{(2)}$ are formed by decorating boundary cycles of $G_{5,1}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,1}^{(0)}) = -G_{4,0}^{(0)} \qquad D(G_{5,1}^{(1)}) = -G_{4,0}^{(1)}$$

The Fatgraph $G_{5,2}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),   # a
  Vertex([2, 1, 3]),   # b
  Vertex([0, 3, 4, 4]),# c
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \to {}^0c^1 \to {}^1a^2 \to {}^0a^1 \to {}^3c^0 \to {}^1b^2 \to {}^1c^2 \to {}^2b^0 \to {}^0b^1)$$
$$\beta = ({}^2c^3)$$

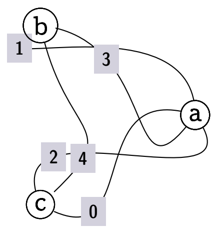
Markings

Fatgraph $G_{5,2}$ only has the identity automorphism, so the marked fatgraphs $G_{5,2}^{(0)}$ to $G_{5,2}^{(2)}$ are formed by decorating boundary cycles of $G_{5,2}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,2}^{(0)}) = +2G_{4,0}^{(0)} \qquad D(G_{5,2}^{(1)}) = +2G_{4,0}^{(1)}$$

The Fatgraph $G_{5,3}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 3, 2]),# a
  Vertex([1, 4, 3]),   # b
  Vertex([0, 4, 2]),   # c
])
```

Boundary cycles

$$\alpha = ({}^2a^3 \to {}^0a^1 \to {}^2c^0 \to {}^1a^2 \to {}^0c^1 \to {}^2b^0 \to {}^1b^2)$$
$$\beta = ({}^3a^0 \to {}^1c^2 \to {}^0b^1)$$

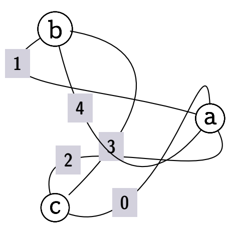
Markings

Fatgraph $G_{5,3}$ only has the identity automorphism, so the marked fatgraphs $G_{5,3}^{(0)}$ to $G_{5,3}^{(2)}$ are formed by decorating boundary cycles of $G_{5,3}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$D(G_{5,3}^{(0)}) = +G_{4,0}^{(2)}$ $\qquad$ $D(G_{5,3}^{(1)}) = +G_{4,0}^{(2)}$

The Fatgraph $G_{5,4}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 4, 2]),# a
  Vertex([1, 4, 3]),   # b
  Vertex([0, 3, 2]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{2}a^{3} \to {}^{2}c^{0} \to {}^{0}a^{1} \to {}^{0}b^{1})$$
$$\beta = ({}^{1}c^{2} \to {}^{1}a^{2} \to {}^{1}b^{2} \to {}^{3}a^{0} \to {}^{0}c^{1} \to {}^{2}b^{0})$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	α	β
A_1	a	c	b	4	2	1	3	0	α	β

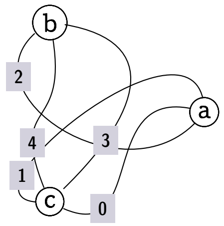
Markings

Fatgraph $G_{5,4}$ only has the identity automorphism, so the marked fatgraphs $G_{5,4}^{(0)}$ to $G_{5,4}^{(2)}$ are formed by decorating boundary cycles of $G_{5,4}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{5,4}^{(0)}) = +2G_{4,0}^{(0)} \qquad D(G_{5,4}^{(1)}) = +2G_{4,0}^{(1)}$$

The Fatgraph $G_{5,5}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),   # a
  Vertex([2, 4, 3]),   # b
  Vertex([0, 3, 4, 1]),# c
])
```

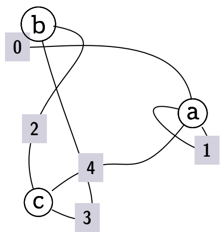
Boundary cycles

$$\alpha = ({}^{2}a^{0} \to {}^{0}c^{1} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{0}b^{1} \to {}^{3}c^{0} \to {}^{2}b^{0} \to {}^{2}c^{3})$$
$$\beta = ({}^{1}c^{2} \to {}^{1}b^{2})$$

Markings

Fatgraph $G_{5,5}$ only has the identity automorphism, so the marked fatgraphs $G_{5,5}^{(0)}$ to $G_{5,5}^{(2)}$ are formed by decorating boundary cycles of $G_{5,5}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{5,6}$ (2 orientable markings)




```
Fatgraph([
  Vertex([0, 1, 4, 1]),# a
  Vertex([0, 3, 2]),   # b
  Vertex([3, 4, 2]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{1}c^{2} \to {}^{3}a^{0} \to {}^{1}a^{2} \to {}^{0}a^{1} \to {}^{0}b^{1} \to {}^{2}a^{3} \to {}^{0}c^{1} \to {}^{2}b^{0})$$
$$\beta = ({}^{2}c^{0} \to {}^{1}b^{2})$$

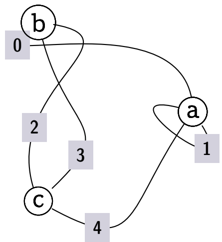
Automorphisms

A_0	a	b	c	0	1	2	3	4	α	β
A_1	a	c	b	4	1	3	2	0	α	β

Markings

Fatgraph $G_{5,6}$ only has the identity automorphism, so the marked fatgraphs $G_{5,6}^{(0)}$ to $G_{5,6}^{(2)}$ are formed by decorating boundary cycles of $G_{5,6}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{5,7}$ (non-orientable, 1 orientable marking)




```
Fatgraph([
  Vertex([0, 1, 4, 1]),# a
  Vertex([0, 3, 2]),   # b
  Vertex([4, 3, 2]),   # c
])
```

Boundary cycles

$$\alpha = ({}^{3}a^{0} \to {}^{2}b^{0} \to {}^{0}a^{1} \to {}^{1}c^{2} \to {}^{0}b^{1})$$
$$\beta = ({}^{1}a^{2} \to {}^{2}a^{3} \to {}^{2}c^{0} \to {}^{0}c^{1} \to {}^{1}b^{2})$$

Automorphisms

A_0	a	b	c	0	1	2	3	4	α	β
$A_1{}^{\dagger\ddagger}$	a	c	b	4	1	2	3	0	β	α

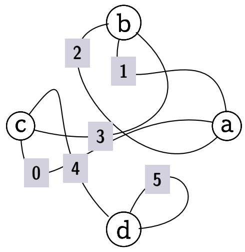
Markings

	$G_{5,7}^{(0)}$
α	0
β	1

Fatgraphs with 6 edges / 4 vertices

There are 5 unmarked fatgraphs in this section, originating 18 marked fatgraphs (9 orientable, and 9 nonorientable).

The Fatgraph $G_{6,0}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 3]),# b
  Vertex([0, 3, 4]),# c
  Vertex([5, 5, 4]),# d
])
```

Boundary cycles

$\alpha = ({}^2d^0 \to {}^2a^0 \to {}^0c^1 \to {}^1a^2 \to {}^0a^1 \to {}^1b^2 \to {}^1c^2 \to {}^2b^0 \to {}^2c^0 \to {}^0b^1 \to {}^1d^2)$
$\beta = ({}^0d^1)$

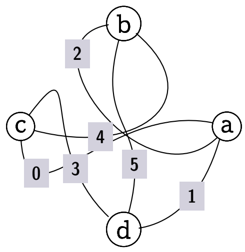
Markings

Fatgraph $G_{6,0}$ only has the identity automorphism, so the marked fatgraphs $G_{6,0}^{(0)}$ to $G_{6,0}^{(2)}$ are formed by decorating boundary cycles of $G_{6,0}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$D(G_{6,0}^{(0)}) = -G_{5,0}^{(0)} - G_{5,0}^{(2)}$ $\qquad$ $D(G_{6,0}^{(1)}) = -G_{5,0}^{(1)} - G_{5,1}^{(3)}$

The Fatgraph $G_{6,1}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 5, 4]),# b
  Vertex([0, 4, 3]),# c
  Vertex([1, 5, 3]),# d
])
```

Boundary cycles

$$\alpha = ({}^2d^0 \to {}^2a^0 \to {}^1c^2 \to {}^0a^1 \to {}^1b^2 \to {}^0c^1 \to {}^0d^1 \to {}^2b^0)$$
$$\beta = ({}^1a^2 \to {}^2c^0 \to {}^1d^2 \to {}^0b^1)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β
A_1	b	d	a	c	2	4	5	0	1	3	α	β
A_2	c	a	d	b	3	4	0	5	1	2	α	β
A_3	d	c	b	a	5	1	3	2	4	0	α	β

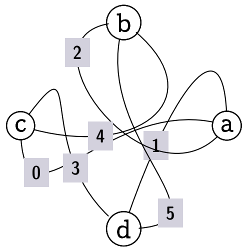
Markings

Fatgraph $G_{6,1}$ only has the identity automorphism, so the marked fatgraphs $G_{6,1}^{(0)}$ to $G_{6,1}^{(2)}$ are formed by decorating boundary cycles of $G_{6,1}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

Differentials

$$D(G_{6,1}^{(0)}) = +2G_{5,0}^{(0)} - G_{5,1}^{(4)} \qquad D(G_{6,1}^{(1)}) = +2G_{5,0}^{(1)} - G_{5,2}^{(5)}$$

The Fatgraph $G_{6,2}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 5, 4]),# b
  Vertex([0, 4, 3]),# c
  Vertex([5, 1, 3]),# d
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \to {}^1a^2 \to {}^0a^1 \to {}^2c^0 \to {}^0c^1 \to {}^0d^1 \to {}^2b^0 \to {}^0b^1 \to {}^1d^2)$$
$$\beta = ({}^2d^0 \to {}^1c^2 \to {}^1b^2)$$

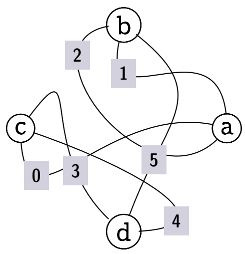
Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β
A_1	a	d	b	c	2	0	1	4	5	3	α	β
A_2	a	c	d	b	1	2	0	5	3	4	α	β

Markings

Fatgraph $G_{6,2}$ only has the identity automorphism, so the marked fatgraphs $G_{6,2}^{(0)}$ to $G_{6,2}^{(2)}$ are formed by decorating boundary cycles of $G_{6,2}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{6,3}$ (2 orientable markings)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 5]),# b
  Vertex([0, 4, 3]),# c
  Vertex([4, 5, 3]),# d
])
```

Boundary cycles

$$\alpha = ({}^2a^0 \to {}^1a^2 \to {}^0a^1 \to {}^1b^2 \to {}^0c^1 \to {}^0d^1 \to {}^2b^0 \to {}^2c^0 \to {}^0b^1 \to {}^1d^2)$$
$$\beta = ({}^2d^0 \to {}^1c^2)$$

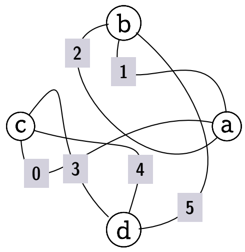
Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β
A_1	b	a	d	c	5	1	2	4	3	0	α	β

Markings

Fatgraph $G_{6,3}$ only has the identity automorphism, so the marked fatgraphs $G_{6,3}^{(0)}$ to $G_{6,3}^{(2)}$ are formed by decorating boundary cycles of $G_{6,3}$ with all permutations of $(0, 1)$ in lexicographic order. See Section "Markings of fatgraphs with trivial automorphisms" for a complete table.

The Fatgraph $G_{6,4}$ (non-orientable, 1 orientable marking)




```
Fatgraph([
  Vertex([1, 0, 2]),# a
  Vertex([2, 1, 5]),# b
  Vertex([0, 4, 3]),# c
  Vertex([5, 4, 3]),# d
])
```

Boundary cycles

$$\alpha = ({}^1a^2 \to {}^0a^1 \to {}^0b^1 \to {}^0c^1 \to {}^2c^0 \to {}^1d^2)$$
$$\beta = ({}^2d^0 \to {}^2a^0 \to {}^1b^2 \to {}^1c^2 \to {}^0d^1 \to {}^2b^0)$$

Automorphisms

A_0	a	b	c	d	0	1	2	3	4	5	α	β
$A_1{}^{\dagger\ddagger}$	b	a	d	c	5	1	2	3	4	0	β	α
A_2	c	d	a	b	0	3	4	1	2	5	α	β
$A_3{}^{\dagger\ddagger}$	d	c	b	a	5	3	4	1	2	0	β	α

Markings

	$G_{6,4}^{(0)}$
α	0
β	1

Markings of fatgraphs with trivial automorphisms

This appendix shows the numbering of marked fatgraphs when the base unmarked fatgraph G has only the trivial automorphism.

	$G^{(0)}$	$G^{(1)}$
α	0	1
β	1	0