610_HW1

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Question 1: OLS Estimation

a)

Consider the case of a single μ_j :

$$\hat{\mu}_j = argmin_{\mu_j} \sum_{j=1}^{J} \sum_{i=1}^{n_j} (y_{ij} - \mu_j)^2$$

$$\frac{\partial}{\partial \mu_j} = 0 \;\Rightarrow\; -2 \sum_{i=1}^{n_j} (y_{ij} - \mu_j) = 0$$

$$\sum_{i=1}^{n_j} y_{ij} = \sum_{i=1}^{n_j} \mu_j = n_j \mu_j$$

$$\Rightarrow\; \hat{\mu_j} = \frac{1}{n_j} \sum_{i=1}^{n_j} y_{ij} = \bar{y_j}$$

By symmetry this result holds for all $\mu_{j's}$

b) i.

$$\hat{\mu} = argmin_{\mu} \sum_{j=1}^{J} \sum_{i=1}^{n} (y_{ij} - \mu - \alpha_j)^2$$

$$\frac{\partial}{\partial \mu} = 0 \;\Rightarrow\; -2 \sum_{j=1}^{J} \sum_{i=1}^{n} (y_{ij} - \mu - \alpha_j) = 0$$

$$\sum_{j=1}^{J} \sum_{i=1}^{n} y_{ij} \;-\; (nJ\mu) \;-\; n \sum_{j=1}^{J} \alpha_j = 0$$

$$\sum_{j=1}^{J} \sum_{i=1}^{n} y_{ij} = nJ\mu$$

$$\Rightarrow \hat{\mu} = \frac{\sum_{j=1}^{J} \sum_{i=1}^{n} y_{ij}}{nJ} = \bar{y}..$$

ii.

$$\frac{1}{J} \sum_{j=1}^{J} \hat{\mu}_j = \frac{1}{J} \sum_{j=1}^{J} \bar{y}_j = \frac{1}{nJ} \sum_{j=1}^{J} \sum_{i=1}^{n} y_{ij} = \bar{y}..$$

iii.

Consider the case of a single α_j :

$$\hat{\alpha}_j = argmin_{\alpha_j} \sum_{j=1}^{J} \sum_{i=1}^{n} (y_{ij} - \mu - \alpha_j)^2$$

$$\frac{\partial}{\partial \alpha_j} = 0 \;\Rightarrow\; -2 \sum_{i=1}^{n} (y_{ij} - \mu - \alpha_j) = 0$$

$$\sum_{i=1}^{n} y_{ij} - n\mu - n\alpha_j = 0$$

$$\sum_{i=1}^{n} y_{ij} - n\mu = n\alpha_j$$

$$\Rightarrow\; \hat{\alpha}_j = \frac{\sum_{i=1}^{n} y_{ij}}{n} - \hat{\mu} \;=\; \hat{\mu_j} - \hat{\mu} \;=\; \bar{y}_j - \bar{y}..$$

By symmetry this result holds for all $\alpha_{j's}$

c)

$$y = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{21} \\ y_{22} \\ y_{23} \\ y_{31} \\ y_{32} \\ y_{33} \end{bmatrix} \quad x = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$$

$$solve\ (X^T X)^{-1} X^T y$$

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix},\ (X^T X)^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$X^T y = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{21} \\ y_{22} \\ y_{23} \\ y_{31} \\ y_{32} \\ y_{33} \end{bmatrix} = \begin{bmatrix} y_{11} + y_{12} + y_{13} \\ y_{21} + y_{22} + y_{23} \\ y_{31} + y_{32} + y_{33} \end{bmatrix}$$

$$(X^T X)^{-1} X^T y = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} * \begin{bmatrix} y_{11} + y_{12} + y_{13} \\ y_{21} + y_{22} + y_{23} \\ y_{31} + y_{32} + y_{33} \end{bmatrix} = \begin{bmatrix} \frac{y_{11}+y_{12}+y_{13}}{3} \\ \frac{y_{21}+y_{22}+y_{23}}{3} \\ \frac{y_{31}+y_{32}+y_{33}}{3} \end{bmatrix} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \end{bmatrix}$$

Question 2: Game of Thrones

2.1: Exploratory data analysis of screentime, including average screentime per gender as well as total screentime for all actors of a given gender.

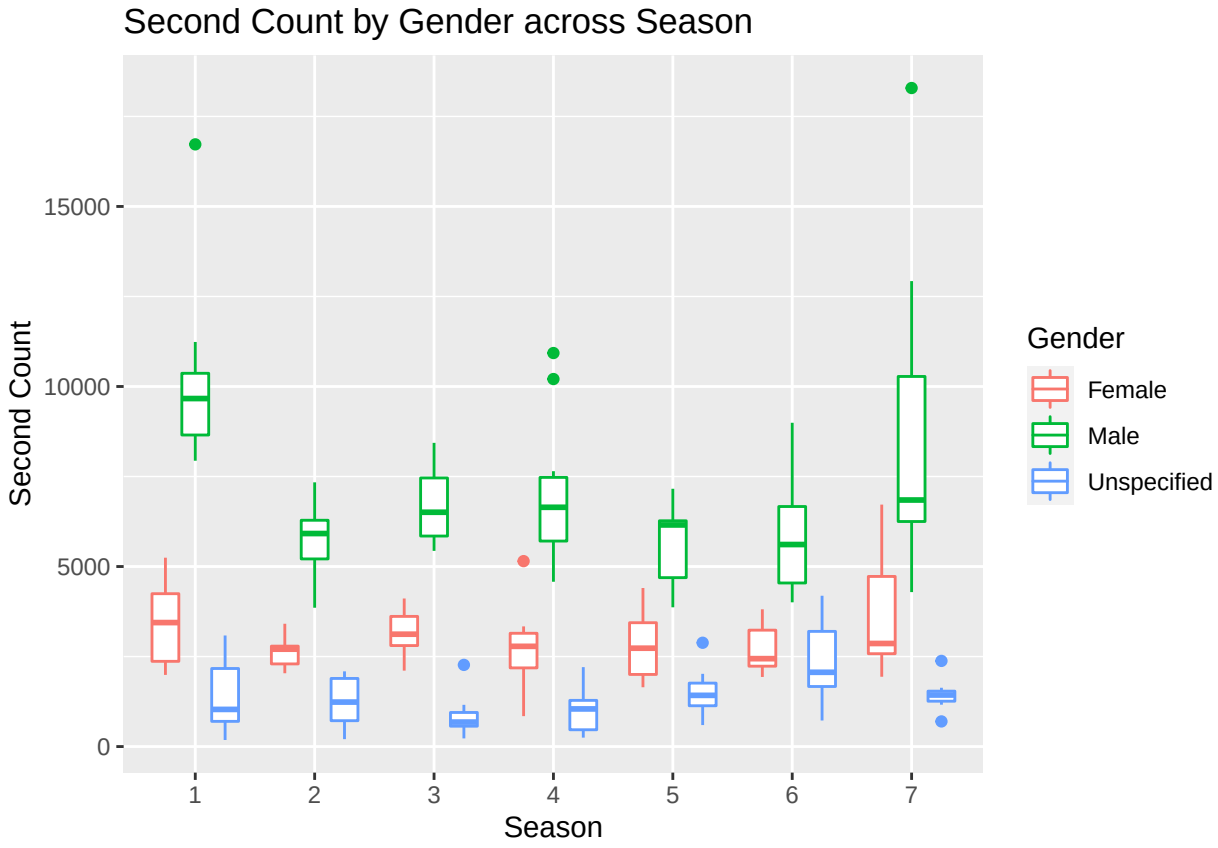
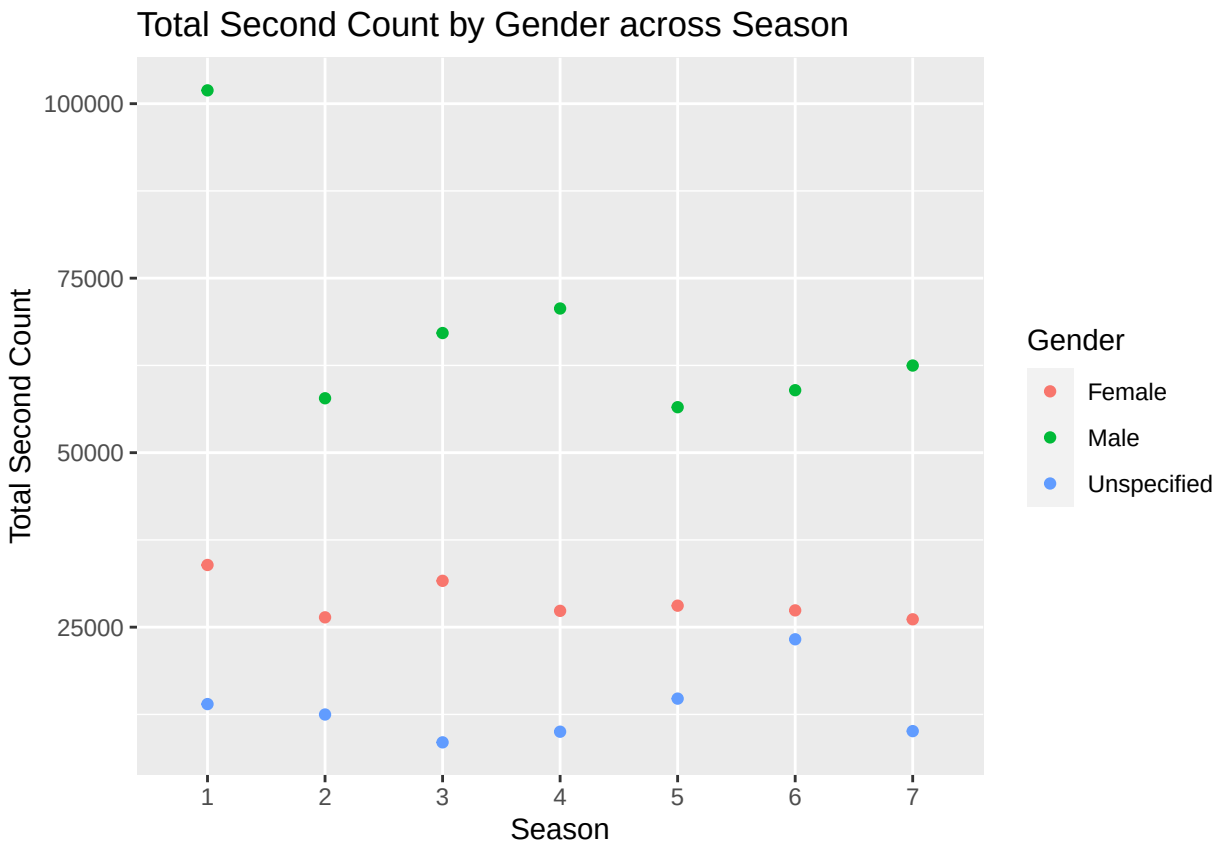


Table 1: Total Screentime

Season	Female	Male	Unspecified
1	33908	101907	13982
2	26408	57796	12487
3	31640	67142	8499
4	27339	70655	10024
5	28078	56517	14764
6	27407	58949	23264
7	26129	62484	10108



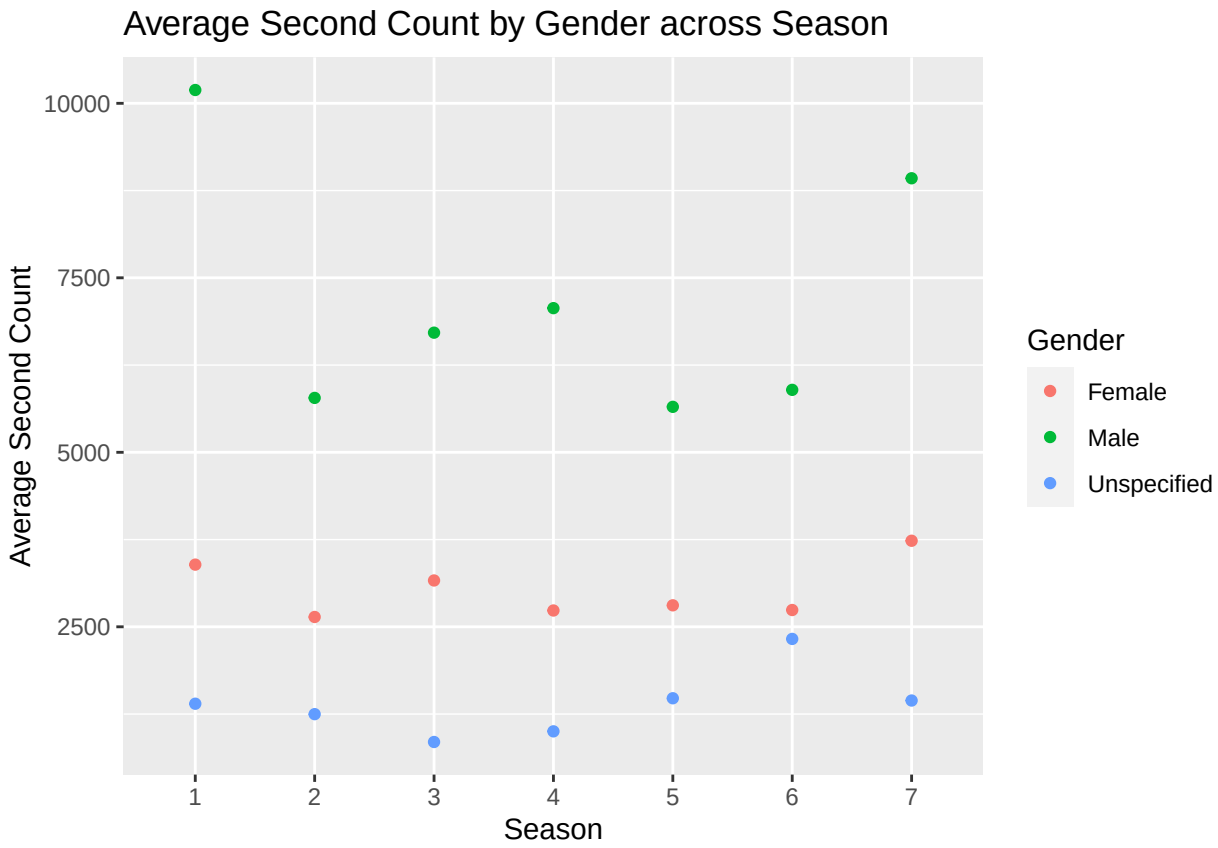


Table 2: Average Screentime

Season	Female	Male	Unspecified
1	3390.800	10190.700	1398.2
2	2640.800	5779.600	1248.7
3	3164.000	6714.200	849.9
4	2733.900	7065.500	1002.4
5	2807.800	5651.700	1476.4
6	2740.700	5894.900	2326.4
7	3732.714	8926.286	1444.0

2.2: A clear specification of the model using an equation, including clear specification of any modeling assumptions.

$$y_{ijk} = \mu + \alpha_j I(Season = j) + \beta_k I(Gender = k) + \gamma_{jk} I(Season = j \text{ and } Gender = k) + \epsilon_{ijk}$$

$$Assumptions: \ \alpha_1 = 0, \ \beta_1 = 0, \ \gamma_{j1} = 0 \ \forall \ j = 1, .., J, \ \gamma_{1k} = 0 \ \forall \ k = 1, .., K, \quad \epsilon_{ijk} \sim N(0, \sigma^2)$$

2.3: A clearly-labeled table providing point and interval estimates for each parameter in the linear predictor of your model.

Table 3: Parameter Estimates

Parameters	Coefficients	Lowerbound_2.5	Upperbound_97.5
Intercept	3390.80	2468.83	4312.77
S2	-750.00	-2053.87	553.87
S3	-226.80	-1530.67	1077.07
S4	-656.90	-1960.77	646.97
S5	-583.00	-1886.87	720.87
S6	-650.10	-1953.97	653.77
S7	341.91	-1094.88	1778.71
Male	6799.90	5496.03	8103.77
Unsp	-1992.60	-3296.47	-688.73
S2:Male	-3661.10	-5505.05	-1817.15
S3:Male	-3249.70	-5093.65	-1405.75
S4:Male	-2468.30	-4312.25	-624.35
S5:Male	-3956.00	-5799.95	-2112.05
S6:Male	-3645.70	-5489.65	-1801.75
S7:Male	-1606.33	-3638.26	425.60
S2:Unsp	600.50	-1243.45	2444.45
S3:Unsp	-321.50	-2165.45	1522.45
S4:Unsp	261.10	-1582.85	2105.05
S5:Unsp	661.20	-1182.75	2505.15
S6:Unsp	1578.30	-265.65	3422.25
S7:Unsp	-296.11	-2328.05	1735.82

2.4: Clear specification of any hypothesis tests or other inferential techniques used to evaluate the questions posed above.

The summary F statistics as well as p-values from the output of the ANOVA summary are statistically significant for all three parameters, with gender being overwhelmingly significant. This leads to a rejection of the null hypothesis that there is no difference across groups. Point and interval estimates of parameters and Tukey means were used to highlight specific points of interest in evaluating the question at hand. Interval estimates that did not include 0, helped identify which parameters and comparisons were of specific importance. Season to season differences and differences in interaction terms that did not differ in gender did not appear to be significant. The differences between groups that standout as statistically significant are those that contain male vs female comparisons and male vs unspecified comparisons. Specifically, males are spending a lot more time on screen than both female and unspecified characters, and this trend holds regardless of season.

```
##                Df    Sum Sq   Mean Sq F value   Pr(>F)
## Season          6 8.007e+07  13345037   6.113 7.54e-06 ***
## Gender          2 1.160e+09 579999046 265.672  < 2e-16 ***
## Season:Gender  12 1.153e+08   9609263   4.402 3.72e-06 ***
## Residuals     180 3.930e+08   2183140
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = seccount ~ Season * Gender, data = gotscreen)
##
## $Season
##            diff        lwr       upr     p adj
## 2-1 -1770.20000 -2907.8279 -632.5721 0.0001345
## 3-1 -1417.20000 -2554.8279 -279.5721 0.0049706
## 4-1 -1392.63333 -2530.2612 -255.0054 0.0062188
## 5-1 -1681.26667 -2818.8946 -543.6388 0.0003562
## 6-1 -1339.23333 -2476.8612 -201.6054 0.0099859
## 7-1  -292.23333 -1545.8382  961.3716 0.9927379
## 3-2   353.00000  -784.6279 1490.6279 0.9680712
## 4-2   377.56667  -760.0612 1515.1946 0.9557015
## 5-2    88.93333 -1048.6946 1226.5612 0.9999863
## 6-2   430.96667  -706.6612 1568.5946 0.9181976
## 7-2  1477.96667   224.3618 2731.5716 0.0098137
## 4-3    24.56667 -1113.0612 1162.1946 1.0000000
## 5-3  -264.06667 -1401.6946  873.5612 0.9929030
## 6-3    77.96667 -1059.6612 1215.5946 0.9999937
## 7-3  1124.96667  -128.6382 2378.5716 0.1104377
## 5-4  -288.63333 -1426.2612  848.9946 0.9885966
## 6-4    53.40000 -1084.2279 1191.0279 0.9999993
## 7-4  1100.40000  -153.2049 2354.0049 0.1268580
## 6-5   342.03333  -795.5946 1479.6612 0.9727037
## 7-5  1389.03333   135.4284 2642.6382 0.0193840
## 7-6  1047.00000  -206.6049 2300.6049 0.1689599
##
## $Gender
##                           diff       lwr       upr p adj
## Male-Female          4097.627  3494.331  4700.923     0
## Unspecified-Female -1608.672 -2211.968 -1005.375     0
## Unspecified-Male   -5706.299 -6309.595 -5103.002     0
##
## $`Season:Gender`
##                                     diff           lwr          upr     p adj
## 2:Female-1:Female            -750.0000  -3144.548408  1644.548408 0.9999152
## 3:Female-1:Female            -226.8000  -2621.348408  2167.748408 1.0000000
## 4:Female-1:Female            -656.9000  -3051.448408  1737.648408 0.9999896
## 5:Female-1:Female            -583.0000  -2977.548408  1811.548408 0.9999986
## 6:Female-1:Female            -650.1000  -3044.648408  1744.448408 0.9999913
## 7:Female-1:Female             341.9143  -2296.749544  2980.578116 1.0000000
## 1:Male-1:Female              6799.9000   4405.351592  9194.448408 0.0000000
## 2:Male-1:Female              2388.8000     -5.748408  4783.348408 0.0513560
## 3:Male-1:Female              3323.4000    928.851592  5717.948408 0.0002283
## 4:Male-1:Female              3674.7000   1280.151592  6069.248408 0.0000192
## 5:Male-1:Female              2260.9000   -133.648408  4655.448408 0.0907480
```

```
## 6:Male-1:Female              2504.1000    109.551592  4898.648408 0.0294834
## 7:Male-1:Female              5535.4857   2896.821884  8174.149544 0.0000000
## 1:Unspecified-1:Female      -1992.6000  -4387.148408   401.948408 0.2507816
## 2:Unspecified-1:Female      -2142.1000  -4536.648408   252.448408 0.1468678
## 3:Unspecified-1:Female      -2540.9000  -4935.448408  -146.351592 0.0245051
## 4:Unspecified-1:Female      -2388.4000  -4782.948408     6.148408 0.0514515
## 5:Unspecified-1:Female      -1914.4000  -4308.948408   480.148408 0.3207486
## 6:Unspecified-1:Female      -1064.4000  -3458.948408  1330.148408 0.9900304
## 7:Unspecified-1:Female      -1946.8000  -4585.463830   691.863830 0.4771221
## 3:Female-2:Female             523.2000  -1871.348408  2917.748408 0.9999998
## 4:Female-2:Female              93.1000  -2301.448408  2487.648408 1.0000000
## 5:Female-2:Female             167.0000  -2227.548408  2561.548408 1.0000000
## 6:Female-2:Female              99.9000  -2294.648408  2494.448408 1.0000000
## 7:Female-2:Female            1091.9143  -1546.749544  3730.578116 0.9957947
## 1:Male-2:Female              7549.9000   5155.351592  9944.448408 0.0000000
## 2:Male-2:Female              3138.8000    744.251592  5533.348408 0.0007704
## 3:Male-2:Female              4073.4000   1678.851592  6467.948408 0.0000009
## 4:Male-2:Female              4424.7000   2030.151592  6819.248408 0.0000001
## 5:Male-2:Female              3010.9000    616.351592  5405.448408 0.0017235
## 6:Male-2:Female              3254.1000    859.551592  5648.648408 0.0003630
## 7:Male-2:Female              6285.4857   3646.821884  8924.149544 0.0000000
## 1:Unspecified-2:Female      -1242.6000  -3637.148408  1151.948408 0.9491128
## 2:Unspecified-2:Female      -1392.1000  -3786.648408  1002.448408 0.8659325
## 3:Unspecified-2:Female      -1790.9000  -4185.448408   603.648408 0.4498558
## 4:Unspecified-2:Female      -1638.4000  -4032.948408   756.148408 0.6250726
## 5:Unspecified-2:Female      -1164.4000  -3558.948408  1230.148408 0.9732447
## 6:Unspecified-2:Female       -314.4000  -2708.948408  2080.148408 1.0000000
## 7:Unspecified-2:Female      -1196.8000  -3835.463830  1441.863830 0.9874364
## 4:Female-3:Female            -430.1000  -2824.648408  1964.448408 1.0000000
## 5:Female-3:Female            -356.2000  -2750.748408  2038.348408 1.0000000
## 6:Female-3:Female            -423.3000  -2817.848408  1971.248408 1.0000000
## 7:Female-3:Female             568.7143  -2069.949544  3207.378116 0.9999998
## 1:Male-3:Female              7026.7000   4632.151592  9421.248408 0.0000000
## 2:Male-3:Female              2615.6000    221.051592  5010.148408 0.0166497
## 3:Male-3:Female              3550.2000   1155.651592  5944.748408 0.0000473
## 4:Male-3:Female              3901.5000   1506.951592  6296.048408 0.0000035
## 5:Male-3:Female              2487.7000     93.151592  4882.248408 0.0319783
## 6:Male-3:Female              2730.9000    336.351592  5125.448408 0.0089181
## 7:Male-3:Female              5762.2857   3123.621884  8400.949544 0.0000000
## 1:Unspecified-3:Female      -1765.8000  -4160.348408   628.748408 0.4781353
## 2:Unspecified-3:Female      -1915.3000  -4309.848408   479.248408 0.3198856
## 3:Unspecified-3:Female      -2314.1000  -4708.648408    80.448408 0.0720569
## 4:Unspecified-3:Female      -2161.6000  -4556.148408   232.948408 0.1361558
## 5:Unspecified-3:Female      -1687.6000  -4082.148408   706.948408 0.5682852
## 6:Unspecified-3:Female       -837.6000  -3232.148408  1556.948408 0.9995624
## 7:Unspecified-3:Female      -1720.0000  -4358.663830   918.663830 0.7112441
## 5:Female-4:Female              73.9000  -2320.648408  2468.448408 1.0000000
## 6:Female-4:Female               6.8000  -2387.748408  2401.348408 1.0000000
```

```
## 7:Female-4:Female            998.8143  -1639.849544  3637.478116 0.9986836
## 1:Male-4:Female             7456.8000   5062.251592  9851.348408 0.0000000
## 2:Male-4:Female             3045.7000    651.151592  5440.248408 0.0013888
## 3:Male-4:Female             3980.3000   1585.751592  6374.848408 0.0000019
## 4:Male-4:Female             4331.6000   1937.051592  6726.148408 0.0000001
## 5:Male-4:Female             2917.8000    523.251592  5312.348408 0.0030338
## 6:Male-4:Female             3161.0000    766.451592  5555.548408 0.0006677
## 7:Male-4:Female             6192.3857   3553.721884  8831.049544 0.0000000
## 1:Unspecified-4:Female     -1335.7000  -3730.248408  1058.848408 0.9035398
## 2:Unspecified-4:Female     -1485.2000  -3879.748408   909.348408 0.7876026
## 3:Unspecified-4:Female     -1884.0000  -4278.548408   510.548408 0.3506337
## 4:Unspecified-4:Female     -1731.5000  -4126.048408   663.048408 0.5174309
## 5:Unspecified-4:Female     -1257.5000  -3652.048408  1137.048408 0.9431363
## 6:Unspecified-4:Female      -407.5000  -2802.048408  1987.048408 1.0000000
## 7:Unspecified-4:Female     -1289.9000  -3928.563830  1348.763830 0.9717449
## 6:Female-5:Female            -67.1000  -2461.648408  2327.448408 1.0000000
## 7:Female-5:Female            924.9143  -1713.749544  3563.578116 0.9995491
## 1:Male-5:Female             7382.9000   4988.351592  9777.448408 0.0000000
## 2:Male-5:Female             2971.8000    577.251592  5366.348408 0.0021903
## 3:Male-5:Female             3906.4000   1511.851592  6300.948408 0.0000034
## 4:Male-5:Female             4257.7000   1863.151592  6652.248408 0.0000002
## 5:Male-5:Female             2843.9000    449.351592  5238.448408 0.0046904
## 6:Male-5:Female             3087.1000    692.551592  5481.648408 0.0010709
## 7:Male-5:Female             6118.4857   3479.821884  8757.149544 0.0000000
## 1:Unspecified-5:Female     -1409.6000  -3804.148408   984.948408 0.8527096
## 2:Unspecified-5:Female     -1559.1000  -3953.648408   835.448408 0.7131386
## 3:Unspecified-5:Female     -1957.9000  -4352.448408   436.648408 0.2805474
## 4:Unspecified-5:Female     -1805.4000  -4199.948408   589.148408 0.4337613
## 5:Unspecified-5:Female     -1331.4000  -3725.948408  1063.148408 0.9060913
## 6:Unspecified-5:Female      -481.4000  -2875.948408  1913.148408 1.0000000
## 7:Unspecified-5:Female     -1363.8000  -4002.463830  1274.863830 0.9510021
## 7:Female-6:Female            992.0143  -1646.649544  3630.678116 0.9988001
## 1:Male-6:Female             7450.0000   5055.451592  9844.548408 0.0000000
## 2:Male-6:Female             3038.9000    644.351592  5433.448408 0.0014489
## 3:Male-6:Female             3973.5000   1578.951592  6368.048408 0.0000020
## 4:Male-6:Female             4324.8000   1930.251592  6719.348408 0.0000001
## 5:Male-6:Female             2911.0000    516.451592  5305.548408 0.0031594
## 6:Male-6:Female             3154.2000    759.651592  5548.748408 0.0006977
## 7:Male-6:Female             6185.5857   3546.921884  8824.249544 0.0000000
## 1:Unspecified-6:Female     -1342.5000  -3737.048408  1052.048408 0.8994136
## 2:Unspecified-6:Female     -1492.0000  -3886.548408   902.548408 0.7811554
## 3:Unspecified-6:Female     -1890.8000  -4285.348408   503.748408 0.3438276
## 4:Unspecified-6:Female     -1738.3000  -4132.848408   656.248408 0.5095952
## 5:Unspecified-6:Female     -1264.3000  -3658.848408  1130.248408 0.9402488
## 6:Unspecified-6:Female      -414.3000  -2808.848408  1980.248408 1.0000000
## 7:Unspecified-6:Female     -1296.7000  -3935.363830  1341.963830 0.9701801
## 1:Male-7:Female             6457.9857   3819.321884  9096.649544 0.0000000
## 2:Male-7:Female             2046.8857   -591.778116  4685.549544 0.3777664
```

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## 3:Male-7:Female              2981.4857    342.821884  5620.149544 0.0102529
## 4:Male-7:Female              3332.7857    694.121884  5971.449544 0.0015850
## 5:Male-7:Female              1918.9857   -719.678116  4557.649544 0.5059705
## 6:Male-7:Female              2162.1857   -476.478116  4800.849544 0.2767882
## 7:Male-7:Female              5193.5714   2331.538665  8055.604192 0.0000001
## 1:Unspecified-7:Female      -2334.5143  -4973.178116   304.149544 0.1606558
## 2:Unspecified-7:Female      -2484.0143  -5122.678116   154.649544 0.0933600
## 3:Unspecified-7:Female      -2882.8143  -5521.478116  -244.150456 0.0166049
## 4:Unspecified-7:Female      -2730.3143  -5368.978116   -91.650456 0.0335864
## 5:Unspecified-7:Female      -2256.3143  -4894.978116   382.349544 0.2079804
## 6:Unspecified-7:Female      -1406.3143  -4044.978116  1232.349544 0.9349447
## 7:Unspecified-7:Female      -2288.7143  -5150.747049   573.318477 0.3202922
## 2:Male-1:Male               -4411.1000  -6805.648408 -2016.551592 0.0000001
## 3:Male-1:Male               -3476.5000  -5871.048408 -1081.951592 0.0000796
## 4:Male-1:Male               -3125.2000  -5519.748408  -730.651592 0.0008405
## 5:Male-1:Male               -4539.0000  -6933.548408 -2144.451592 0.0000000
## 6:Male-1:Male               -4295.8000  -6690.348408 -1901.251592 0.0000002
## 7:Male-1:Male               -1264.4143  -3903.078116  1374.249544 0.9770557
## 1:Unspecified-1:Male        -8792.5000 -11187.048408 -6397.951592 0.0000000
## 2:Unspecified-1:Male        -8942.0000 -11336.548408 -6547.451592 0.0000000
## 3:Unspecified-1:Male        -9340.8000 -11735.348408 -6946.251592 0.0000000
## 4:Unspecified-1:Male        -9188.3000 -11582.848408 -6793.751592 0.0000000
## 5:Unspecified-1:Male        -8714.3000 -11108.848408 -6319.751592 0.0000000
## 6:Unspecified-1:Male        -7864.3000 -10258.848408 -5469.751592 0.0000000
## 7:Unspecified-1:Male        -8746.7000 -11385.363830 -6108.036170 0.0000000
## 3:Male-2:Male                 934.6000  -1459.948408  3329.148408 0.9980176
## 4:Male-2:Male                1285.9000  -1108.648408  3680.448408 0.9303925
## 5:Male-2:Male                -127.9000  -2522.448408  2266.648408 1.0000000
## 6:Male-2:Male                 115.3000  -2279.248408  2509.848408 1.0000000
## 7:Male-2:Male                3146.6857    508.021884  5785.349544 0.0043820
## 1:Unspecified-2:Male        -4381.4000  -6775.948408 -1986.851592 0.0000001
## 2:Unspecified-2:Male        -4530.9000  -6925.448408 -2136.351592 0.0000000
## 3:Unspecified-2:Male        -4929.7000  -7324.248408 -2535.151592 0.0000000
## 4:Unspecified-2:Male        -4777.2000  -7171.748408 -2382.651592 0.0000000
## 5:Unspecified-2:Male        -4303.2000  -6697.748408 -1908.651592 0.0000001
## 6:Unspecified-2:Male        -3453.2000  -5847.748408 -1058.651592 0.0000937
## 7:Unspecified-2:Male        -4335.6000  -6974.263830 -1696.936170 0.0000027
## 4:Male-3:Male                 351.3000  -2043.248408  2745.848408 1.0000000
## 5:Male-3:Male               -1062.5000  -3457.048408  1332.048408 0.9902352
## 6:Male-3:Male                -819.3000  -3213.848408  1575.248408 0.9996820
## 7:Male-3:Male                2212.0857   -426.578116  4850.749544 0.2387008
## 1:Unspecified-3:Male        -5316.0000  -7710.548408 -2921.451592 0.0000000
## 2:Unspecified-3:Male        -5465.5000  -7860.048408 -3070.951592 0.0000000
## 3:Unspecified-3:Male        -5864.3000  -8258.848408 -3469.751592 0.0000000
## 4:Unspecified-3:Male        -5711.8000  -8106.348408 -3317.251592 0.0000000
## 5:Unspecified-3:Male        -5237.8000  -7632.348408 -2843.251592 0.0000000
## 6:Unspecified-3:Male        -4387.8000  -6782.348408 -1993.251592 0.0000001
## 7:Unspecified-3:Male        -5270.2000  -7908.863830 -2631.536170 0.0000000
```

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## 5:Male-4:Male                -1413.8000  -3808.348408   980.748408 0.8494287
## 6:Male-4:Male                -1170.6000  -3565.148408  1223.948408 0.9717363
## 7:Male-4:Male                 1860.7857   -777.878116  4499.449544 0.5670839
## 1:Unspecified-4:Male         -5667.3000  -8061.848408 -3272.751592 0.0000000
## 2:Unspecified-4:Male         -5816.8000  -8211.348408 -3422.251592 0.0000000
## 3:Unspecified-4:Male         -6215.6000  -8610.148408 -3821.051592 0.0000000
## 4:Unspecified-4:Male         -6063.1000  -8457.648408 -3668.551592 0.0000000
## 5:Unspecified-4:Male         -5589.1000  -7983.648408 -3194.551592 0.0000000
## 6:Unspecified-4:Male         -4739.1000  -7133.648408 -2344.551592 0.0000000
## 7:Unspecified-4:Male         -5621.5000  -8260.163830 -2982.836170 0.0000000
## 6:Male-5:Male                  243.2000  -2151.348408  2637.748408 1.0000000
## 7:Male-5:Male                 3274.5857    635.921884  5913.249544 0.0021924
## 1:Unspecified-5:Male         -4253.5000  -6648.048408 -1858.951592 0.0000002
## 2:Unspecified-5:Male         -4403.0000  -6797.548408 -2008.451592 0.0000001
## 3:Unspecified-5:Male         -4801.8000  -7196.348408 -2407.251592 0.0000000
## 4:Unspecified-5:Male         -4649.3000  -7043.848408 -2254.751592 0.0000000
## 5:Unspecified-5:Male         -4175.3000  -6569.848408 -1780.751592 0.0000004
## 6:Unspecified-5:Male         -3325.3000  -5719.848408  -930.751592 0.0002253
## 7:Unspecified-5:Male         -4207.7000  -6846.363830 -1569.036170 0.0000066
## 7:Male-6:Male                 3031.3857    392.721884  5670.049544 0.0079749
## 1:Unspecified-6:Male         -4496.7000  -6891.248408 -2102.151592 0.0000000
## 2:Unspecified-6:Male         -4646.2000  -7040.748408 -2251.651592 0.0000000
## 3:Unspecified-6:Male         -5045.0000  -7439.548408 -2650.451592 0.0000000
## 4:Unspecified-6:Male         -4892.5000  -7287.048408 -2497.951592 0.0000000
## 5:Unspecified-6:Male         -4418.5000  -6813.048408 -2023.951592 0.0000001
## 6:Unspecified-6:Male         -3568.5000  -5963.048408 -1173.951592 0.0000415
## 7:Unspecified-6:Male         -4450.9000  -7089.563830 -1812.236170 0.0000012
## 1:Unspecified-7:Male         -7528.0857 -10166.749544 -4889.421884 0.0000000
## 2:Unspecified-7:Male         -7677.5857 -10316.249544 -5038.921884 0.0000000
## 3:Unspecified-7:Male         -8076.3857 -10715.049544 -5437.721884 0.0000000
## 4:Unspecified-7:Male         -7923.8857 -10562.549544 -5285.221884 0.0000000
## 5:Unspecified-7:Male         -7449.8857 -10088.549544 -4811.221884 0.0000000
## 6:Unspecified-7:Male         -6599.8857  -9238.549544 -3961.221884 0.0000000
## 7:Unspecified-7:Male         -7482.2857 -10344.318477 -4620.252951 0.0000000
## 2:Unspecified-1:Unspecified   -149.5000  -2544.048408  2245.048408 1.0000000
## 3:Unspecified-1:Unspecified   -548.3000  -2942.848408  1846.248408 0.9999995
## 4:Unspecified-1:Unspecified   -395.8000  -2790.348408  1998.748408 1.0000000
## 5:Unspecified-1:Unspecified     78.2000  -2316.348408  2472.748408 1.0000000
## 6:Unspecified-1:Unspecified    928.2000  -1466.348408  3322.748408 0.9981899
## 7:Unspecified-1:Unspecified     45.8000  -2592.863830  2684.463830 1.0000000
## 3:Unspecified-2:Unspecified   -398.8000  -2793.348408  1995.748408 1.0000000
## 4:Unspecified-2:Unspecified   -246.3000  -2640.848408  2148.248408 1.0000000
## 5:Unspecified-2:Unspecified    227.7000  -2166.848408  2622.248408 1.0000000
## 6:Unspecified-2:Unspecified   1077.7000  -1316.848408  3472.248408 0.9884969
## 7:Unspecified-2:Unspecified    195.3000  -2443.363830  2833.963830 1.0000000
## 4:Unspecified-3:Unspecified    152.5000  -2242.048408  2547.048408 1.0000000
## 5:Unspecified-3:Unspecified    626.5000  -1768.048408  3021.048408 0.9999953
## 6:Unspecified-3:Unspecified   1476.5000   -918.048408  3871.048408 0.7957178
```
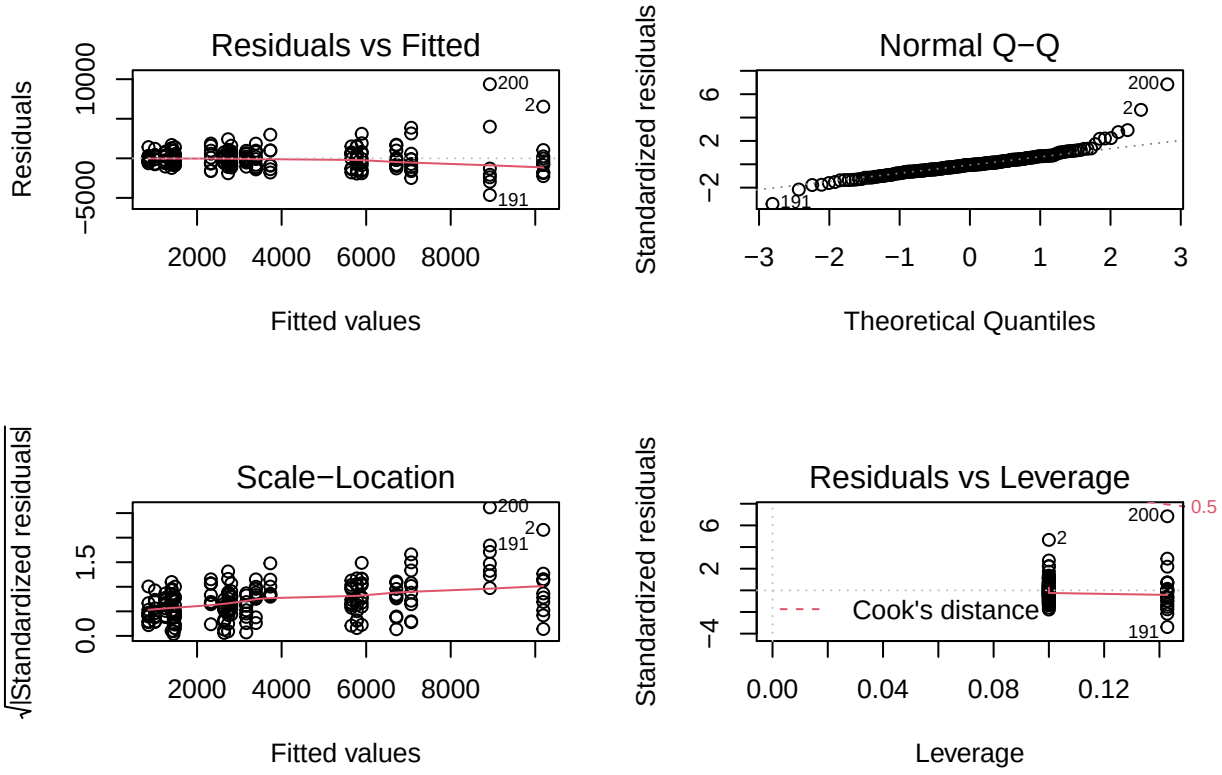

```
## 7:Unspecified-3:Unspecified   594.1000  -2044.563830  3232.763830 0.9999996
## 5:Unspecified-4:Unspecified   474.0000  -1920.548408  2868.548408 1.0000000
## 6:Unspecified-4:Unspecified  1324.0000  -1070.548408  3718.548408 0.9103782
## 7:Unspecified-4:Unspecified   441.6000  -2197.063830  3080.263830 1.0000000
## 6:Unspecified-5:Unspecified   850.0000  -1544.548408  3244.548408 0.9994602
## 7:Unspecified-5:Unspecified   -32.4000  -2671.063830  2606.263830 1.0000000
## 7:Unspecified-6:Unspecified  -882.4000  -3521.063830  1756.263830 0.9997727
```

2.5: Evidence of adequacy of model fit and evaluation of suitability of any assumptions.

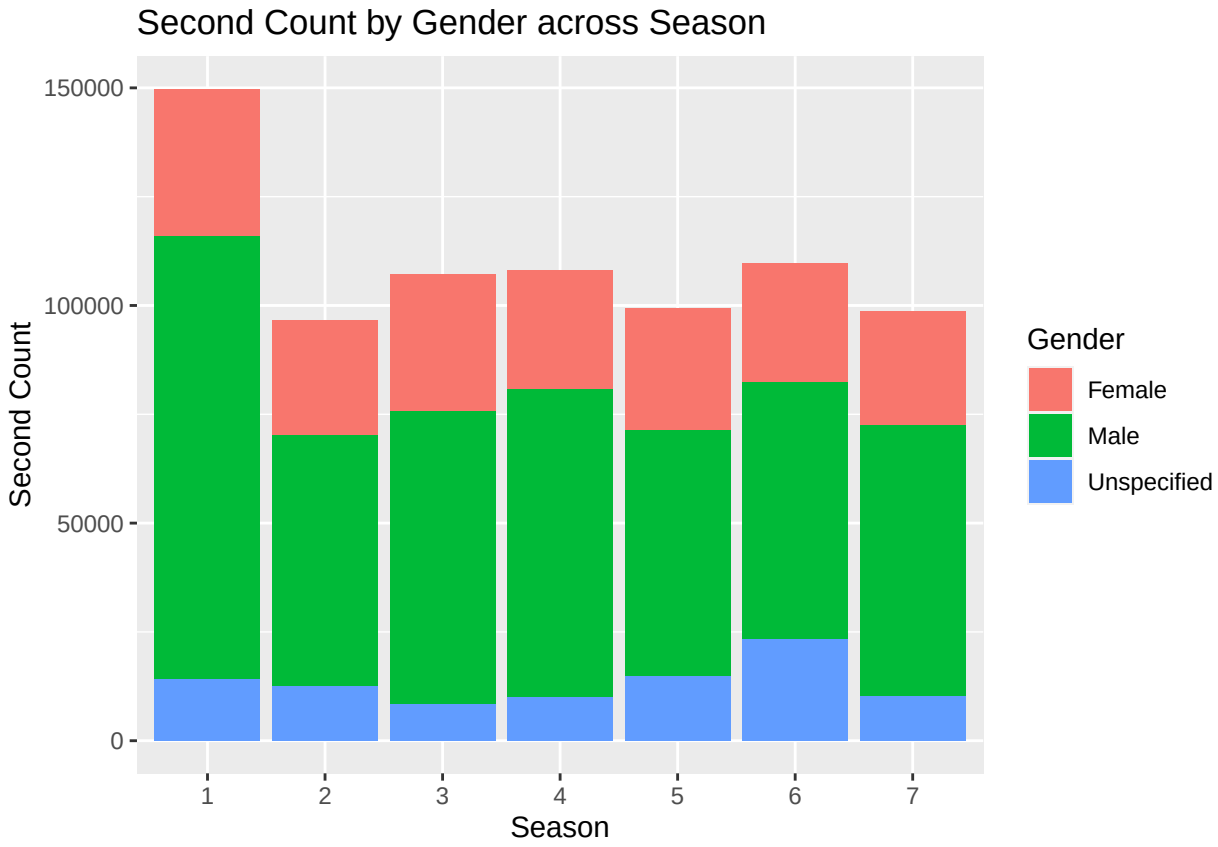
Our model fit is adequate, and our assumptions are suitable. The Residuals vs Fitted plot suggests that the residuals roughly fit a linear pattern. The QQ-Plot has slight deviation in the tails suggesting some skewness, but overall it seems that the residuals fall in line with a normal distribution. The Scale-Location plot shows a moderately flat line, which indicates equal variance. Finally, the Residuals vs Leverage plot shows no points of heavy influence, as no points fall outside of the dashed red lines (Cook's Distance).

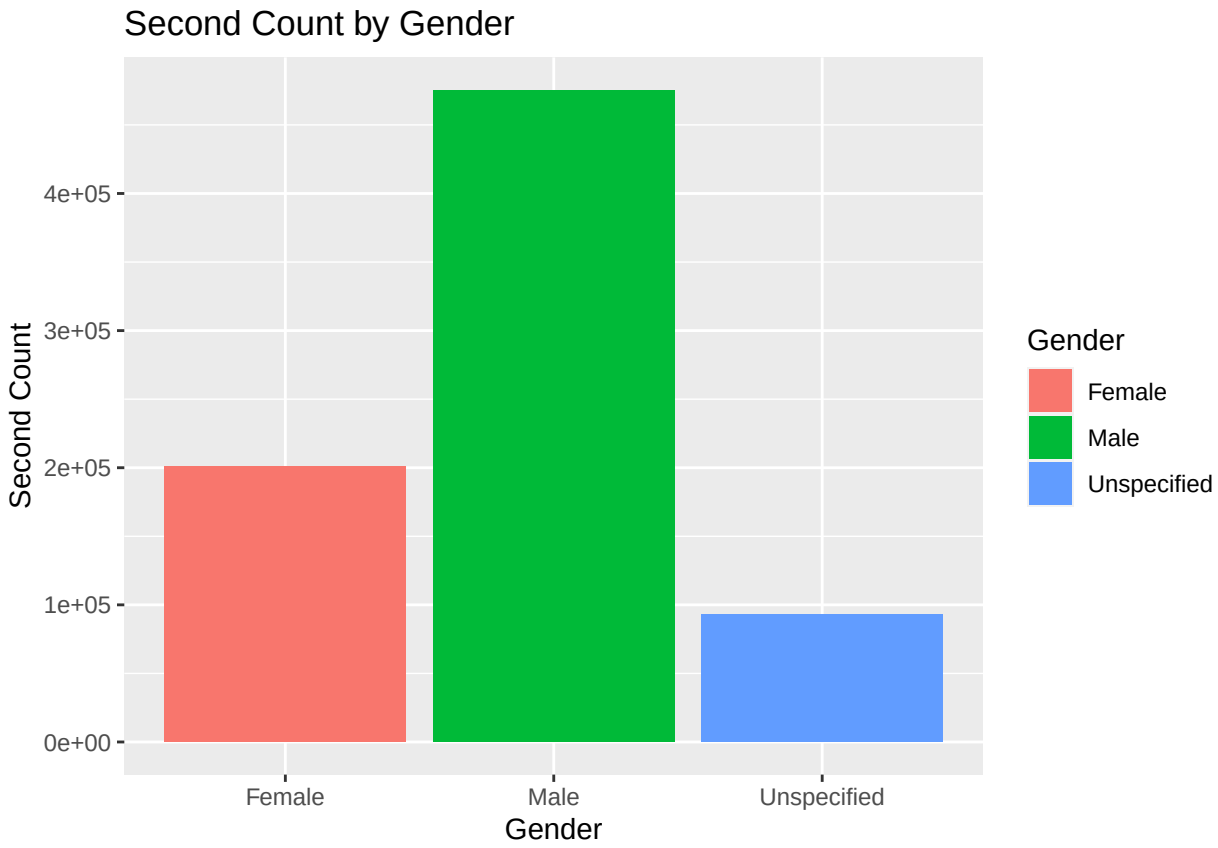


2.6: Clear description of results in language accessible to the average fan of the show, including graphical displays as appropriate. Comment on any insights that may differ between exploratory data analysis and analysis using the ANOVA model, along with reasons why these insights may differ.

The results of my analysis suggest that male actors spend significantly more time on the screen than their female and unspecified counterparts. Season to season the exploratory data analysis demonstrates that this seems to be a common theme across all seasons. Season 1 shows slightly more male dominance, but the other 6 seasons are fairly consistent. The ANOVA output indicates season as statistically significant, but as mentioned above this is not obvious in the EDA/Tukey

Output. This is likely because Tukey adjusts for multiple comparisons whereas the ANOVA output does not. The plots below reiterates my conclusion above that male actors dominate the screentime both seasonally and over the series as a whole.





Question 3: Contrasts

a)

$$Model\ 1 : y_{ij} = \mu_j + \epsilon_{ij}$$
$$Model\ 2 : y_{ij} = \mu + \alpha_1 I(j = 1) + \alpha_2 I(j = 2) + \epsilon_{ij}$$

$$\mu_1 = \mu + \alpha_1$$
$$\mu_2 = \mu + \alpha_2$$
$$\mu_3 = \mu$$

$$\mu_1 - \mu_2 = \alpha_1 - \alpha_2,\ \mu_1 - \mu_3 = \alpha_1,\ \mu_2 - \mu_3 = \alpha_2$$

b)

$$Model\ 1 : \mu_1 = 7096.269,\quad \mu_2 = 1389.970,\quad \mu_3 = 2998.642$$
$$Model\ 2 : \mu = 2998.642,\quad \alpha_1 = 4097.627,\quad \alpha_2 = -1608.672$$

$\mu_1 - \mu_2 = \alpha_1 - \alpha_2 \Rightarrow 7096.269 - 1389.970 = 5706.299,\ 4097.627 - (-1608.672) = 5706.299$
$\mu_1 - \mu_3 = \alpha_1 \Rightarrow 7096.269 - 2998.642 = 4097.627,\ 4097.627$
$\mu_2 - \mu_3 = \alpha_2 \Rightarrow 1389.970 - 2998.642 = -1608.672, -1608.672$

Question 4

$$Proof\ by\ Contradiction:$$

$$Suppose:\ X \sim N(0,1), \qquad Z = \begin{cases} 1 & w.p.\ 0.5 \\ -1 & w.p.\ 0.5 \end{cases} \qquad Y = ZX$$

$$\begin{aligned} Pr(Y \leq x) &= E(Pr(Y \leq x\ |Z)) \\ &= Pr(X \leq x)Pr(Z = 1) + Pr(-X \leq x)Pr(Z = -1) \\ &= \Phi(x)(0.5) + \Phi(x)(0.5) \\ &= \Phi(x) \end{aligned}$$

$$Then,\ Y \sim N(0,1)$$

$$\begin{aligned} Cov(X,Y) &= E(XY) - E(X)E(Y) \\ &= E(X^2Z) - (0)(0) \\ &= E(X^2)E(Z)\ (because\ X\ and\ Z\ are\ Independent) \\ &= (1)(0) \\ &= 0 \end{aligned}$$

X and Y are univariate normal distributions with $Cov(X,Y) = 0$ *but X and Y are not independent*