

Senior Syllabus

Mathematics B

2008 (amended 2014)



Statement on choice of Mathematics subjects

There are three Authority subjects available to schools for students of senior Mathematics. Choice of subject may be helped by considering the following statements on prior study and future pathways.

The *Senior Syllabus in Mathematics A* is a recommended precursor to further study and training in the technical trades such as toolmaking, sheet-metal working, fitting and turning, carpentry and plumbing, auto mechanics, tourism and hospitality, and administrative and managerial employment in a wide range of industries. It is also suitable as a precursor to tertiary studies in subjects with moderate demand in mathematics.

The *Senior Syllabus in Mathematics B* is a recommended precursor to tertiary studies in subjects with high demand in mathematics, especially in the areas of science, medicine, mining and engineering, information technology, mathematics, finance, and business and economics.

The *Senior Syllabus in Mathematics C* is a recommended companion subject to Mathematics B. It provides additional preparation for tertiary studies in subjects with high demand in mathematics, especially in the areas of science, medicine, mining and engineering, information technology, mathematics, finance, and business and economics.

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Mathematics B Senior Syllabus 2008 (amended 2014)

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Summary of syllabus amendments January 2014

The following table outlines the amendments made to Mathematics B Senior Syllabus 2008. These amendments are a consequence of the directions of the Minister as outlined in the *Queensland Government Response to the Education and Innovation Committee Report No. 25: The assessment methods used in senior mathematics, chemistry and physics in Queensland schools*.

<i>Syllabus section</i>	<i>2014 update</i>
Section 1: Rationale	<p>Mathematics B aims to provide the opportunity for students to participate more fully in lifelong learning. This subject provides a foundation for further studies in disciplines within which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, Mathematics B is designed for students whose future pathways may involve mathematics and statistics, and their application, in a range of disciplines at the tertiary level, including:</p> <ul style="list-style-type: none"> • mathematics and science education • natural and physical sciences, especially physics and chemistry • medical and health sciences, including human biology, biomedical, nanoscience and forensics • engineering sciences, including avionics, chemical, civil, communications, electrical, mechanical and mining • information technology and computer science, including electronic and software • mathematical applications in: <ul style="list-style-type: none"> – energy and resources — management and conservation – climatology – design and built environment – industry, manufacturing and trades – business and tourism – primary industries and environment – economics and commerce – statistics and data analysis • pure mathematics.
Section 6.3.4 Authentication of student work	<p>It is essential that judgments of student achievement be made on genuine student assessment responses. Teachers must take reasonable steps to ensure that each student's work is their own, particularly where students have access to electronic resources or when they are preparing responses to collaborative tasks.</p> <p>The QSA's A–Z of Senior Moderation contains a strategy for authenticating student work <www.qsa.qld.edu.au/10773.html>. This provides information about various methods teachers can use to monitor that students' work is their own. Particular methods outlined include:</p> <ul style="list-style-type: none"> • teachers seeing plans and drafts of student work • student production and maintenance of evidence for the development of responses • student acknowledgment of resources used. <p>Teachers must ensure students use consistent, accepted conventions of in-text citation and referencing, where appropriate.</p>
Section 6.8: Requirements for a verification folio	<ul style="list-style-type: none"> • a student profile completed to date.

1. Rationale

Mathematics is an integral part of a general education. It enhances an understanding of the world and the quality of participation in a rapidly changing society. It is a truly international system for the communication of ideas and concepts, and has developed over many thousands of years through contributions by scholars of both ancient and present-day cultures around the world.

Mathematics is:

- a unique and powerful way of viewing the world to investigate patterns, order, generality and uncertainty
- a way of thinking in which problems are explored through observation, reflection, and logical, inductive or deductive reasoning
- a powerful, concise and unambiguous symbolic system with written, spoken and visual components
- a creative activity with its own intrinsic value, involving invention, intuition and exploration.

Mathematics B involves the study of mathematical functions and their applications, differential and integral calculus and applied statistical analysis. These are used to develop:

- knowledge and skills in advanced computation and algebraic methods and procedures
- mathematical modelling and problem-solving strategies and skills
- the capacity to justify mathematical arguments and make decisions
- the capacity to communicate about mathematics in a variety of forms.

Why study this subject?

Mathematics B aims to provide the opportunity for students to participate more fully in lifelong learning. This subject provides a foundation for further studies in disciplines within which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, Mathematics B is designed for students whose future pathways may involve mathematics and statistics, and their application, in a range of disciplines at the tertiary level, including:

- mathematics and statistics
- mathematics and science education
- natural and physical sciences
- medical and health sciences, including human biology, biomedical, nanoscience and forensics
- engineering sciences, including avionics, chemical, civil, communications, electrical, mechanical and mining
- information technology and computer science, including electronic and software

- mathematical applications in:
 - energy and resources — management and conservation
 - climatology
 - design and built environment
 - industry, manufacturing and trades
 - business and tourism
 - primary industries and environment
 - economics and commerce
 - statistics and data analysis.
- pure mathematics.

Key competencies

Mathematics B provides opportunities for the development of the key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The seven key competencies are:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems
- using technology.

2. Global aims

The global aims are statements of the long-term achievements, attitudes and values that are developed by students through studying Mathematics B but which are not directly assessed by the school.

By the end of this course, students should develop:

- broad mathematical knowledge and skills
- the ability to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis and solve problems with confidence
- an awareness of the uncertain nature of their world and be able to use mathematics to help make informed decisions in life-related situations
- an understanding of the diverse applications of mathematics
- an ability to comprehend mathematical information which is presented in a variety of forms
- an ability to communicate mathematical information in a variety of forms
- an ability to use mathematical procedures to justify conclusions
- an ability to benefit from the availability of a wide range of technologies
- an ability to choose and use mathematical instruments appropriately
- positive attitudes to the learning and practice of mathematics.

3. General objectives

3.1 Introduction

The general objectives of this course are organised into four categories:

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification
- Affective.

3.2 Objectives

The general objectives for each of the categories are detailed below. These general objectives incorporate several key competencies. The first three categories of objectives, *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, are linked to the exit criteria in Section 6.6.

Knowledge and procedures

The objectives of this category involve recalling and using results and procedures across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology.

Modelling and problem solving

The objectives of this category involve the uses of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.

Communication and justification

The objectives of this category involve presentation, communication (using both mathematical and everyday language), logical arguments, interpretation and justification of mathematics across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both, when appropriate.

Affective

Affective objectives refer to attitudes, values and feelings which this subject aims to develop in students. Affective objectives are not assessed for the award of exit levels of achievement.

By the end of the course, students should appreciate the:

- diverse applications of mathematics
- precise language and structure of mathematics
- nature of proof
- diverse evolutionary nature of mathematics and the wide range of mathematics-based vocations
- contribution of mathematics to human culture and progress
- power and value of mathematics.

3.3 Principles of a balanced course

The categories of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* incorporate principles of application, technology, initiative, and complexity. Each of the principles has a continuum for the particular aspects of mathematics it represents. A balanced course of study developed from this syllabus must give expression to these principles over the two years. It is expected that all students will be given the opportunity to experience mathematics along the continuum within each of the principles outlined below.

Application

Students must have the opportunity to recognise the usefulness of mathematics through its application, and the power of mathematics that comes from the capacity to abstract and generalise. Thus students' learning experiences and assessment programs must include mathematical tasks that demonstrate a balance across the range from life-related to pure abstraction.

Technology

A range of technological tools must be used in the learning and assessment experiences offered in this course. This ranges from pen and paper, measuring instruments and tables through to higher technologies such as computers and graphing calculators, including those that allow for algebraic manipulations. The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator.

Initiative

Learning experiences and the corresponding assessment must provide students with the opportunity to demonstrate their capability when dealing with tasks that range from well-rehearsed (routine) through to those that require demonstration of insight and creativity (non-routine).

Complexity

Students must be provided with the opportunity to work on simple, single-step tasks through to complex tasks. Complexity may derive from either the nature of the concepts involved or from the number of ideas or procedures that must be sequenced in order to produce an appropriate conclusion.

4. Course organisation

4.1 Introduction

The subject matter has been organised into seven topics, which are discussed in detail in Section 5. All topics must be studied. The order in which topics are presented *does not imply* a teaching sequence. The topics are:

- Introduction to functions
- Rates of change
- Periodic functions and applications
- Exponential and logarithmic functions and applications
- Optimisation
- Introduction to integration
- Applied statistical analysis.

Throughout the course, certain fundamental knowledge and procedures are required. Some of these have been identified and listed in Section 8, “Quantitative concepts and skills”. Time should be provided to revise the fundamental knowledge and procedures within topics as they are required. This maintenance takes time, and should be allowed for in designing the course sequence.

4.2 Time allocation

The minimum number of hours of timetabled school time, including assessment, for a course of study developed from this syllabus is 55 hours per semester. A course of study will usually be completed over two years (220 hours).

Notional times are given for each topic. These times are included as a guide, and minor variations of these times may occur.

4.3 Sequencing

After considering the subject matter and the appropriate range of learning experiences to enable the general objectives to be achieved, a *spiralling* and *integrated* sequence should be developed which allows students to see links between the different topics of mathematics included in the course rather than seeing them as discrete.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence, but some topics include subject matter that is developed and extended in the subject matter of other topics. The school’s sequence should be designed so that the subject matter is revisited and spiralled to allow students to internalise their knowledge before developing it further.

The following guidelines for the sequencing of subject matter should be referred to when developing a sequence for the course.

- No subject matter should be studied before the relevant prerequisite material has been covered.

- The sequencing of subject matter may depend on the importance placed by schools on students being able to make decisions about the mathematics suitable to their needs. For example, the Year 11 sections of the sequences for Mathematics A and Mathematics B may need to be developed together.
- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered appropriately.
- Subject matter across topics should be linked when possible.
- Sequencing may be constrained by a school's ability to provide physical resources.
- Time will be needed for maintaining quantitative concepts and skills.

4.4 Technology

The advantage of mathematics-enabled technology in the mathematics classroom is that it allows for the exploration of the concepts and processes of mathematics. Graphing calculators and spreadsheeting for example, let students explore and investigate; they help students understand concepts, and they complement traditional approaches to teaching.

More specifically, the mathematics-enabled technology allows students to tackle more diverse, life-related problems. Real-life optimisation problems are more easily solved with this technology. It may be used in statistics to investigate larger datasets and rapidly produce a variety of graphical displays and summary statistics, thus freeing students to look for patterns, to detect anomalies in the data and to make informed comments.

The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator. Although student ownership of graphing calculators is not a requirement, *regular and frequent student access* to appropriate technology *is necessary* to enable students to develop the full range of skills required for successful problem solving during their course of study. Use of graphing calculators or computers will significantly enhance the learning outcomes of this syllabus.

To meet the requirements of this syllabus schools should consider the use of:

- general purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheeting software
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- hand-held (calculator) technologies designed for mathematics teaching and learning, e.g. graphics calculators with and without algebraic manipulation or dynamic geometry facilities.

Complete dependence on calculator and computer technologies at the expense of students demonstrating algebraic facility may not satisfy syllabus requirements for *Knowledge and procedures*.

4.5 Composite classes

In some schools, it may be necessary to combine students into a composite Year 11 and 12 class. This syllabus provides teachers with an opportunity to develop a course of study that caters for a variety of circumstances such as combined Year 11 and 12 classes, combined campuses, or modes of delivery involving periods of student-directed study.

The multilevel nature of such classes can prove advantageous to the teaching and learning process because:

- it provides opportunities for peer teaching
- it allows teachers to maximise the flexibility of the syllabus
- it provides opportunities for a mix of multilevel group work, and for independent work on appropriate occasions
- learning experiences and assessment can be structured to allow students to consider the key concepts and ideas at the level appropriate to their needs, in both Year 11 and Year 12.

The following guidelines may prove helpful in designing a course of study for a composite class:

- The course of study could be written in a Year A/Year B format, if the school intends to teach the same topic to both cohorts.
- Place a topic at the beginning of each year that will allow new Year 11 students easy entry into the course.
- Learning experiences and assessment items need to cater for both year levels throughout the course. Even though tasks may be similar for both year levels, it is recommended that more extended and/or complex tasks be used with Year 12 students.

4.6 Work program requirements

A work program is the school's plan of how the course will be delivered and assessed based on the school's interpretation of the syllabus. It allows for the special characteristics of the individual school and its students.

The school's work program must meet all syllabus requirements and must demonstrate that there will be sufficient scope and depth of student learning to meet the general objectives and the exit criteria.

The requirements for work program approval can be accessed on our website, <www.qsa.qld.edu.au>. This information should be consulted before writing a work program. The requirements for work program approval may be updated periodically.

5. Topics

5.1 Introduction

Each topic has a focus statement, subject matter and suggested learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus


This section highlights the intent of the syllabus with respect to the topic and indicates how students should be encouraged to develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, but the order in which it is presented is not necessarily intended to imply a teaching sequence.

Learning experiences

This section provides some suggested learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. The numbers provided with the subject matter link to suggested learning experiences. Included are experiences which involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for *Modelling and problem solving*. The listed learning experiences may require students to work individually, in small groups or as a class.

The learning experiences are suggestions only and are not prescriptive. Schools are encouraged to develop further learning experiences, especially those that relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of learning experiences that students will encounter should be shown in the work program. Learning experiences which have a technology component beyond the use of a scientific calculator have been labelled by the use of an icon ().

N.B. The learning experiences must provide students with the opportunity to experience mathematics along the continuum within each of the principles of a balanced course (see Section 3.3).

Some of the key competencies, predominantly *Using mathematical ideas and techniques*, *Solving problems*, and *Using technology*, are to be found in the learning experiences within the topic areas. Opportunities are provided for the development of key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The key competencies of *Collecting, analysing and organising information*, *Planning and organising activities*, and *Working with others and in teams*, also feature in some of the learning experiences.

5.2 The topics

The order in which topics and items within topics are given do not imply a teaching sequence. Numbers listed after each item of subject matter refer to suggested learning experiences (SLEs).

Introduction to functions (notional time 35 hours)

Focus

Students should be encouraged to develop an understanding and appreciation of relationships between variables, be conversant with the three methods of representation (algebraic, graphical, and numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of functions, sketching, investigating shapes and relationships, and the general forms of functions. The use of technology should help students in these processes.

Subject matter

- concepts of function, domain and range (suggested learning experiences (SLEs) 1, 2, 3, 4)
- ordered pairs, tables, graphs and equations as representations of functions and relations (SLEs 1, 2, 3, 4)
- graphs as a representation of the points whose coordinates satisfy an equation (SLEs 1, 3, 4, 5, 6, 13)
- distinction between functions and relations (SLEs 1, 2)
- distinctions between continuous functions, discontinuous functions and discrete functions (SLEs 1, 3, 7)
- general shapes of functions, including:
 - polynomials up to degree 4
 - reciprocal functions
 - absolute value functions (SLEs 4, 5, 13)
- relationships between the graph of $f(x)$ and the graphs of $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$ for both positive and negative values of the constant a (SLE 5)
- practical applications:
 - polynomials up to degree 2
 - reciprocal functions
 - absolute value functions (SLEs 6, 7, 8, 9, 10, 13)
- solutions to simultaneous equations in two variables:
 - graphically, using technology
 - algebraically (linear and quadratic equations only) (SLEs 6, 7, 8)
- composition of two functions (SLE 12)
- concept of the inverse of a function (SLE 14).

Suggested learning experiences (SLEs)

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Find the domain and range of functions in mathematical and life-related contexts, given data in a variety of forms such as graphs, tables of values, mathematical expressions or descriptions of situations.

2. Use the vertical line test to determine whether a relation is a function.
3. Investigate the *shapes* of common relations which are not functions, for example circles and ellipses.
4. Examine the general shapes of polynomial functions of the type $y = x^n$, $n = 1, 2, 3, 4$.
5. ■ Using $f(x) = x^n$, for $n = 1$ to 4 , investigate the relationships between the graph of $f(x)$ and the graphs of $f(x) + a$, $f(x + a)$, $af(x)$ and $f(ax)$ by means of a graphing calculator and algebraic methods.
 - Use a graphing calculator to investigate the shapes of different functions.
 - Use a graphing calculator to investigate possible functions for data.
 - Investigate the difficulties encountered in using a graphing calculator or computer software to draw graphs of relations which are not functions.
6. Investigate the number of times a straight line intersects the graph of a polynomial of degree n , $n = 1$ to 4 .
7. ■ Practical applications of linear functions (e.g. taxi fares, mobile phone charges, break-even analysis), for example:
 - calculate the amount of simple interest generated over a given period using a graphing calculator or a suitable computer software package; plot these discrete values and generate a function which can be used to represent all values.
8. Practical applications of quadratic functions, for example:
 - locate the position algebraically, numerically and graphically of the highest point on a projectile path defined by a quadratic function
 - use quadratic functions in life-related situations such as: find the dose of a chemical needed to obtain a 50% kill of insects, if the percentage of insects killed, p , is related to the dose level, x , by the equation, $p = a + bx + cx^2$ where a , b and c are constants with values such as 2, -1 and 3
 - by approximating the surface area of a living being as a function of the square of its linear dimension, and the volume as a function of the cube of its linear dimension, investigate the limitations on the sizes of living beings in different environments
 - investigate how closely a quadratic function approximates (a) the shape of a hanging chain and (b) the curve of the cables of a suspension bridge; suggest an explanation for any difference between the two results.
9. Use reciprocal relationships in practical contexts such as conversion from miles per gallon to litres per 100 kilometres.
10. Practical applications of absolute functions; for example, light reflection, balls on a pool table, Sashiko quilt patterns, art deco.
11. ■ Collect a dataset and consider the most appropriate mathematical model for that set. Use technology and pen-and-paper methods to build and verify the model.
12. Devise a procedure for producing the graph, table or algebraic expression for the sum of two functions $f(x) + g(x)$, and the composite function $f[g(x)]$ given graphs, tables or algebraic expressions of two functions, $f(x)$ and $g(x)$.
13. ■ Use dynamic graphing software to support the study of technology.

14. Examine the relationship between functions and their inverse function, for example:

$$f(x) = x^2 \text{ and } f^{-1}(x) = \pm\sqrt{x}$$

$$f(x) = x^3 \text{ and } f^{-1}(x) = \sqrt[3]{x}$$

Rates of change (notional time 35 hours)

Focus

Students should be encouraged to develop an understanding of average and instantaneous rates of change and of the derivative as a function. This understanding should be developed using algebraic and graphical approaches. Students should be expected to apply the rules for differentiation and interpret the results. The use of technology should help students in these processes.

Subject matter

- concept of rate of change (SLEs 1, 2)
- calculation of average rates of change in both practical and purely mathematical situations (SLEs 1, 2)
- interpretation of the average rate of change as the gradient of the secant (SLEs 1, 2)
- understanding of a limit in simple situations (SLEs 3, 4)

N.B. Calculations using limit theorems are not required.

- definition of the derivative of a function at a point (SLEs 5, 6, 7)
- derivative of simple algebraic functions from first principles (SLEs 3, 4, 5)
- evaluation of the derivative of a function at a point (SLEs 5, 6, 7)
- interpretation of instantaneous rate of change at a point as the gradient of a tangent and as the derivative at that point (SLEs 1, 2, 5, 10)
- rules for differentiation including:

$$\frac{d}{dp} p^n \text{ for rational values of } n$$

$$\frac{d}{dr} [k f(r)]$$

$$\frac{d}{ds} [f(s) + g(s)]$$




$$\frac{d}{dt} [f(t) g(t)] \quad (\text{Product rule})$$

$$\frac{d}{dx} f[g(x)] \quad (\text{Chain rule})$$

- interpretation of the derivative as the gradient function (SLEs 1, 2, 5, 6, 7)
- practical applications of instantaneous rates of change (SLEs 1, 2, 5–12).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Determine average and instantaneous speeds from a distance–time graph.
2.  Use data-loggers and motion detectors with hand-held graphics calculators to explore average and instantaneous rates of change in distance–time graphs.
3. Use a numerical technique to estimate a limit or an average rate of change.
4. Use simple algebraic techniques to explore limits.
5. Graph a function and its gradient function; relate the features of each to the other.
6.  Use a calculator or computer to investigate the behaviour of a tangent to a curve and its relationship to the curvature at that point.
7.  Use an electronic spreadsheet to investigate the gradient of a secant as one of the points approaches the other (i.e. as the secant approaches the tangent).
8. Determine the instantaneous rate of change of a variable with respect to another variable in life-related situations given the mathematical model, such as:
 - the rate of population change with respect to time
 - the rate of change of resistance in a wire with respect to temperature
 - the rate of change of the surface area of an object with respect to volume
 - the rate of change of a cost function with respect to the number of items produced.
9. Find the equation of the tangent to a curve under various given conditions.
10. Compare the evaporation rate of water in open containers of varying cross sections.
11. Investigate the concept of marginal costs/profits related to the derivative of a cost/profit function.
12. Applications of the chain rule to life-related situations involving three variables.

Periodic functions and applications (notional time 30 hours)***Focus***

Students should be encouraged to develop an understanding and appreciation of periodic functions, be conversant with the three methods of representation (algebraic, graphical, numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of periodic functions, sketching, investigating shapes and relationships, and the general forms of periodic functions. The use of technology should help students in these processes. Trigonometric identities need not be developed beyond the Pythagorean identity.

Subject matter

- definition of a radian and its relationship with degrees (SLE 6)
- trigonometry, including the definition and practical applications of the sine, cosine and tangent ratios (SLEs 1, 2)
- simple practical applications of the sine and cosine rules (the ambiguous case is not essential) (SLEs 1, 2)
- definition of a periodic function, the period and amplitude (SLEs 3, 4, 8)

- definition of the trigonometric functions \sin , \cos and \tan of any angle in degrees and in radians (SLEs 3, 7)
- graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for any angle in degrees ($-360^\circ \leq x \leq 360^\circ$) and in radians ($-2\pi \leq x \leq 2\pi$) (SLEs 3, 7, 11, 16)
- significance of the constants A , B , C and D on the graphs of $y = A \sin B(x + C) + D$, $y = A \cos B(x + C) + D$ (SLEs 5, 10, 12)
- applications of periodic functions (SLEs 4, 5, 8, 13, 16, 17)
- Pythagorean identity $\sin^2 x + \cos^2 x = 1$ (SLE 9)
- solution of trigonometric equations within a specified domain
 - algebraically in simple situations (multiple angles are not essential) (SLE 9)
 - using technology to any complexity
- derivatives of functions involving $\sin x$ and $\cos x$ (SLEs 8, 11, 14)
- applications of the derivatives of $\sin x$ and $\cos x$ in life-related situations (SLE 8).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Use sine, cosine and tangent ratios to determine lengths/distances and magnitudes of angles in life-related situations such as:
 - guy ropes for tents or flagpoles
 - distances across rivers and valleys.
2. Use sine and cosine rules to solve triangles in two- and three-dimensional contexts and determine lengths/distances and the magnitude of angles in life-related situations such as:
 - the distance between two ships given the distances and bearings to a fixed point
 - the height of a tower given the direction and angles of inclination from two fixed locations of known distance apart.
3. Investigate the repetitive nature of daily temperature and human pulse.
4. Find the period, amplitude and frequency of periodic functions involving sine and cosine given their graphs and/or equations.
5. Find the period, amplitude and frequency of trigonometric functions which are used to model phenomena such as biorhythms, tide heights.
6. Explore the exact values of trigonometric ratios, including the angles 0 , $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, etc.
7. Using the concept of the unit circle:
 - evaluate the trigonometric functions \sin , \cos and \tan of any angle in degrees and radians
 - explore the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$
8. Use practical applications of periodic functions, for example:
 - investigate the periodic motion of a mass on the end of a spring; given the mathematical model of the displacement from a fixed position, find mathematical representations of its velocity and acceleration; solve these simple trigonometric equations to find displacement, velocity and acceleration at a given time during the periodic motion

- calculate the rate at which the water level is changing on a vertical marker given a sine function as a model of tide height
 - investigate the periodic motion of a pendulum; mathematically model the motion of a pendulum using trigonometric equations
 - explore the period, amplitude and frequency of periodic (oscillatory) phenomena including planetary motion, hormone cycles, ECGs, Halley's comet, reciprocating motion
 - investigate the path of a point on a moving bicycle wheel.
9. Find solutions of trigonometric equations for $-2\pi \leq \theta \leq 2\pi$ such as $2\sin \theta = -0.7$, $2\sin^2 \theta = \cos \theta$, $\cos \theta = \sqrt{3}/2$.
 10. ■ Investigate the effect of the constants A , B , C and D on the graphs of $y = A\sin B(x + C) + D$, $y = A\cos B(x + C) + D$ using graphing calculators.
 11. Sketch the graphs $y = \sin x$, $y = \cos x$ and $y = \tan x$ for any angle in degrees ($-360^\circ \leq x \leq 360^\circ$) and in radians in the range $-2\pi \leq x \leq 2\pi$.
 12. Use the graph of a sinusoidal function to develop the corresponding algebraic form.
 13. Plot the tide heights at a specified point over a 24-hour period.
 14. Develop the derivative of $\sin x$ from the graph of $\cos x$ and vice versa by examination of the gradient of the tangent.
 15. Plot, as a function of the date, the elapsed time between sunrise and sunset for capital cities in Australia.
 16. Investigate daily electrical energy consumption over a period of time.
 17. ■ Use computer software or a graphing calculator to investigate more complicated periodic functions, for example

$$\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \dots$$

18. ■ Use a graphing calculator or computer package to investigate the superposition of two almost equal frequencies of sinusoidal waves to produce "beats", and identify the properties of the resulting function.

Exponential and logarithmic functions and applications (notional time 35 hours)

Focus

Students should be encouraged to develop an understanding and appreciation of exponential and logarithmic functions and the relationships between them. They should be conversant with the three methods of representation (algebraic, graphical, numerical). Emphasis should be placed on the application of these functions to solve problems in a range of life-related situations (e.g. finance and investment, growth and decay). The use of technology should help students in these processes.

It is not intended that great emphasis be placed on simplification of expressions involving indices or logarithms.


Subject matter

- index laws and definitions (SLEs 1, 3, 5, 9)
- definitions of a^x and $\log_a x$, for $a > 1$ (SLE 1)
- logarithmic laws and definitions (SLEs 1, 2, 9)
- definition of the exponential function e^x (SLEs 4, 6)
- graphs of, and the relationships between, $y = a^x$, $y = \log_a x$ for $a = e$ and other values of a (SLEs 3, 6, 12, 14, 16, 17, 18)
- graphs of $y = e^{kx}$ for $k \neq 0$ (SLEs 3, 6, 8, 14, 17, 18)
- solution of equations involving indices (SLEs 5, 8, 9, 11)
- use of logarithms to solve equations involving indices (SLEs 8, 9, 11, 13)
- development of algebraic models from appropriate datasets using logarithms and/or exponents (SLEs 2, 5, 7, 10, 17, 18)
- derivatives of exponential and logarithmic functions for base e (SLEs 3, 6, 7)
- applications of exponential and logarithmic functions, and the derivative of exponential functions (SLEs 2, 5, 7, 8, 10, 11, 13, 14)
- applications of geometric progressions to compound interest including past, present and future values (SLEs 19, 20, 21)
- applications of geometric progressions to annuities and amortising a loan (SLEs 22–32).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Use the identities $x = a^{\log_a x}$ and $y = \log_a(a^y)$ to investigate the logarithmic and index laws and definitions.
2. Investigate life-related situations that can be modelled by simple exponential functions, e.g. applications of Newton's Law of Cooling, concentration against time in chemistry, carbon dating in archaeology, and decrease of atmospheric pressure with altitude.
3. ■ Use a graphing calculator or computer software to investigate the shapes of exponential and logarithmic functions, their transformations and their derivatives.
4. Investigate the derivative of the function a^x and identify the significance of the exponential constant e .
5. Investigate the role of indices (or powers) in the establishment of formulas in financial matters such as compound interest, time required to repay a loan for given repayments, and rate of interest.
6. Graph the derivative of a growth function or a decay function and interpret the result.
7. Investigate change such as radioactive decay, growth of bacteria, or growth of an epidemic, where the rate of change is proportional to the amount of material left or the current population size.
8. Consider that the proportion of a radioactive material remaining after time t has elapsed is e^{-kt} , where k is a positive constant; investigate the relationship between k and the half-life of the material.

9. Use logarithms to solve equations involving indices such as:
 - consider the probability of obtaining at least one head in n tosses of a fair coin, and then find how many tosses are required for this probability to be at least 0.9.
 - consider the time taken for an investment to double for a given compound interest rate.
10. Consider that the difference between assuming that running time is proportional to distance and assuming that \log (running time) is proportional to \log (distance); interpret the value of the constant of proportionality in the second model (athletes' world record times may be of interest in this context).
11. Investigate logarithmic scales, for example, decibels, Mohs scale of hardness, Richter scale, and pH.
12. Plot the logarithms of some apparent growth functions, for example car registrations over time, to produce a near-linear graph.
13. Investigate the time at which the quantity of the intermediate substance reaches a maximum in a simple two-step radioactive decay, i.e. the original substance decays to an intermediate substance which in turn decays to an inert substance.
14. Plot the logarithm of the population of Australia at censuses (a) from 1891 to 1933 (b) from 1947 to 1971 and (c) from 1971 to 1991; recognise that the linear tendencies of the plot indicate power/exponential relationships in the original.
15. By graphing the logarithm of the distance of planets from the sun against the logarithm of the time of revolution about the sun, investigate the relationship between the variables.
16. Plot log-log graphs to test data that can be modelled by power functions, then form the equation $\log_a y = k \log_a x + c$ to derive the power function $y = bx^k$.
17. Plot semi-log graphs to test a dataset that can be modelled by an exponential equation, then form the equation $\log_b y = kx + c$ to derive the exponential function $y = ar^x$.
18. Use a graph and/or a table of values created by a spreadsheet or a calculator to compare interest accrued and yearly balances for an investment over a number of years where (a) a flat rate applies, and (b) compound interest applies.
19. Apply the compound interest formula to the \sum of a geometric progression to develop a formula for the future value and present value of an annuity.
20. Develop the formula for compound interest; solve a range of financial problems involving this formula.
21. Construct a loan repayment schedule showing principal, interest charged and balance owing.
22. Solve a range of financial problems involving annuities formulas.
23. Calculate mortgage repayments of a home loan given the interest rate and the term of the loan.
24. Calculate the term of the loan given the interest rate and the mortgage payment.
25. Calculate the effective interest rate given an annual nominal interest rate and the compounding period.
26.  Use a spreadsheet or a calculator to examine the effect of changing the interest rate, term or repayment on a housing loan.

27. Prepare a list of charges, interest rates and conditions for investment in different commercial institutions; justify the selection of one of these to match a given financial scenario.
28. Investigate the use of tables by financial institutions in annuity calculations.
29. Discuss responses of newspaper financial columnists to financial questions.
30. Solve problems involving sinking funds designed for future purchases.
31. Investigate the effect of taxation on a variety of investment situations such as superannuation and other cash investments.

Introduction to integration (notional time 25 hours)

Focus

Students should be encouraged to develop an understanding of the concept of integration as a process by which a “whole” can be obtained from the summation of a large number of parts. This understanding should be developed using numerical and analytical techniques, in life-related situations as well as in purely mathematical situations. The emphasis in the topic should be on the applications of integration rather than on developing a large repertoire of techniques. The use of technology should help students in these processes.

Subject matter

- definition of the definite integral and its relation to the area under a curve (SLEs 1–7)
- the value of the limit of a sum as a definite integral (SLEs 1–7)
- definition of the indefinite integral (SLEs 1–8)
- rules for integration including (SLEs 1–8)

$$\int a f(x) dx$$

$$\int [f(x) \pm g(x)] dx$$

$$\int f(ax + b) dx$$

- indefinite integrals of simple polynomial functions, simple exponential functions, $\sin(ax + b)$, $\cos(ax + b)$ and $\frac{1}{ax + b}$ (SLEs 1–5, 7, 8)
- use of integration to find area (SLEs 1, 2, 4, 5, 7, 8, 10)
- practical applications of the integral (SLEs 1, 3, 4, 5, 7, 8, 10, 11, 15)
- trapezoidal rule for the approximation of a value of a definite integral numerically (SLEs 4, 5, 6, 12, 13, 14).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Use integration to calculate the areas of regions by finding the area under the curve for suitably chosen functions including functions which intersect the x axis within the given interval.
2. Use integration to calculate the area enclosed by two intersecting curves.

3. Investigate the motion of a falling body in terms of its displacement and velocity as functions of time neglecting air resistance.
4. From a velocity time function (or graph) determine a distance or displacement function (or graph); interpret the result.
5. From an acceleration time function (or graph) determine a velocity function (or graph); interpret the result.
6. Apply the trapezoidal rule to integrals of known values and compare the approximate solutions with the exact solutions; investigate the variation in accuracy with the number of strips chosen.
7. Calculate the volume of simply shaped objects by summing the volumes of a set of thin slabs of simple geometry (the use of the formula for volumes of solids of revolution is not intended).
8. Determine the function Q given a rate of growth or decay of some quantity Q such as quantity of bacteria, size of epidemic, drug concentration, population size as a simple function of time or Q alone; interpret.
9. Determine the volume of water that could be contained in a trough of given length and parabolic cross-section.
10. Investigate the Monte Carlo technique for definite integrals and area.
11. Investigate the method for finding the volume of timber in a tree trunk by using several functions to describe the shape of the trunk. A typical taper equation to describe a tree could consist of a quadratic equation for the base, a linear equation for the main part of the trunk, and a second quadratic equation for the tip of the tree.
12. Calculate the distance travelled in a car by taking speedometer readings at regular intervals and then using these readings in a numerical formula; check the accuracy of the result by using the tripmeter.
13. Calculate the approximate volume of fill to be removed in the construction of a road cutting by approximating the cross-sectional area using a numerical method.
14. Investigate the accuracy of the trapezoidal rule applied as a composite rule with strip widths chosen according to the different slopes of the integrand.
15. Investigate consumer surplus and producer surplus in relation to supply and demand functions.
16. Use integration to determine appropriate values in contexts, such as learning curves, consumer curves, Lorenz curves.

Applied statistical analysis (notional time 25 hours)

Focus

Students should be encouraged to develop a working knowledge of the concepts involved in describing, summarising, comparing and modelling data, and of some elementary concepts in using data to estimate probabilities and parameters, and to answer simple questions. Students should be encouraged to develop skills in interpreting and commenting on data in context. **It is expected that calculators (or computers) will be used routinely for calculations and graphical displays.**

Subject matter

- identification of variables and types of variables and data (continuous and discrete); practical aspects of collection and entry of data (SLEs 1, 2, 4, 5, 6, 13, 14, 15, 20)
- select and use in context appropriate graphical and tabular displays for different types of data including pie charts, barcharts, tables, histograms, stem-and-leaf and box plots (SLEs 1, 2, 3, 10, 13)
- use of summary statistics including mean, median, standard deviation and interquartile distance as appropriate descriptors of features of data in context (SLEs 1, 2, 3, 9, 10, 11, 12, 14, 15)
- use of graphical displays and summary statistics in describing key features of data, particularly in comparing datasets and exploring possible relationships (SLEs 1, 2, 3, 9–14)
- use of relative frequencies to estimate probabilities; the notion of probabilities of individual values for discrete variables and intervals for continuous variables (SLEs 5, 6, 15, 16, 17)
- probability distribution and expected value for a discrete variable (SLEs 6, 16, 17)
- identification of the binomial situation and use of tables or technology for binomial probabilities (SLEs 5, 6, 7)
- concept of a probability distribution for a continuous random variable; notion of expected value and median for a continuous variable (SLEs 3, 12, 17, 18)
- the normal model and use of standard normal tables or technology (SLEs 8, 9, 10)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Organise a set of real data (both continuous and discrete) into an understandable form using a variety of approaches such as summary statistics and graphical displays.
2. Given a set of data, produce a concise summary of the main information in the data, referring to graphical displays and summary statistics.
3. Use graphical displays on the same scale to give an effective visual comparison between two or more datasets, and comment on general comparative features, making allowance for variation.
4. Identify the effect of different sampling situations in pursuit of a random sample, e.g. Gallup poll compared with a phone-in poll.
5. Identify situations with events that could be assumed to be equally likely such as birthdays, month of birth, male/female births.
6. Identify discrete variables and estimate probabilities of their values from data and/or model probabilities from assumptions; for example, the number of girls in families of two or three children by listing probabilities.
7. Examine a number of situations for which the binomial is appropriate and use binomial probabilities; for example, with $n = 3$ or 4, write down the probability of each value of the variable, calculate the expected value and relate it to the example.
8. Consider a number of life-related situations where a normal distribution may be assumed, and standardise variables.

9. Examine the use of summary statistics in, for example, newspapers, articles, TV programs such as weather reports and advertisements, government reports.
10. Compare two sets of data (e.g. male and female samples) from the same population and from different populations using graphical and empirical methods such as box plots and z-scores.
11. Examine reports by the Real Estate Institute (e.g. house prices in different areas) and explain their choice of measure of central tendency.
12. Compare the effects of an outlier on a variety of summary statistics.
13. Use a computer database to store, sort and graphically display data.
14. Discuss different sampling situations and possible difficulties and sources of bias, e.g. due to such things as poor questionnaire design, a lack of random sampling or to practical difficulties such as survey interviewer influence.
15. Discuss why it is easier to estimate parameters such as the proportion of women who work full-time rather than the proportion of full-time workers who are women.
16. Use the tabled information given in the newspaper about previous Gold Lotto draws to determine whether the numbers are drawn at random, i.e. whether the numbers follow a uniform probability distribution model.
17. Ask a group of people to try to generate random numbers between, say, 0 and 50, and use graphical displays to investigate how successful they were, e.g. use a histogram to check rectangular shape, and plot numbers in order of generation to check on trends or patterns.
18. Examine the variability of data by the use of a calculator or a computer package to simulate a number of samples of data from the same distribution and look at the variation in the summary statistics that are estimating population parameters.
19. Explain the relationship between the area under a curve or histogram and probabilities.
20. Examine data collected by a survey or by observation or an experiment to check for recording or measurement error to decide how to handle non-compliant responses or observations and to prepare data for entry.

Optimisation (notional time 30 hours)

Focus

Students should be encouraged to develop an understanding of the use of differentiation as a tool in a range of situations which involve the optimisation of continuous functions. This understanding should be developed using algebraic and graphical approaches. The use of technology should help students in these processes.

Parts of this topic are seen as the culmination of the entire course and provide opportunities to link many of the preceding topics.

Subject matter

- positive and negative values of the derivative as an indication of the points at which the function is increasing or decreasing (SLEs 2, 3)
- zero values of the derivative as an indication of stationary points (SLE 4)
- concept of relative maxima and minima and greatest and least value of functions (SLE 2)
- methods of determining the nature of stationary points (SLEs 1–6, 9)
- greatest and least values of a function in a given interval (SLEs 1–6, 9)

- recognition of the problem to be optimised (maximised or minimised) (SLEs 1–9)
- identification of variables and construction of the function to be optimised (SLEs 1–9)
- applications of the derivative to optimisation in life-related situations using a variety of function types (SLEs 1–8)
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem (SLEs 1–9).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. ■ Use life-related situations (with and without technology) such as enclosing a rectangular area with a fixed length of fencing, to demonstrate the need for calculus to determine optimal values.
2. ■ Interpret a table of data values as to the rate of increase, greatest and least values taken by a smoothly varying function.
3. Interpret a graph as to rates of increase or decrease of a function; relate these observations to the behaviour of the derivative.
4. Use zero values of the derivative, or the value of the second derivative, to find local optima and points of horizontal inflection in curve sketching of several function types (the solutions should not be dependent on the factor theorem).
5. Maximise areas subject to restrictions on their perimeters; elimination of constraint variables will involve only simple algebra.
6. Minimise perimeters subject to constraints on area; elimination of constraint variables will involve only simple algebra.
7. Investigate situations finding optimal quantities and/or optimal costs such as the optimal use of materials used for the manufacture of various containers of simple shapes.
8. Investigate examples of shortest travel time, and examples in which the velocity varies according to the mode of travel.
9. ■ Use electronic spreadsheets and/or graphing calculators to investigate optimal points and optimal values in life-related situations, especially those involving more complex algebra.

6. Assessment

The purposes of assessment are to provide feedback to students and parents about learning that has occurred, to provide feedback to teachers about the teaching and learning processes, and to provide information on which to base judgments about how well students meet the general objectives of the course. In designing an assessment program, it is important that the assessment tasks, conditions and criteria are compatible with the general objectives and the learning experiences. Assessment then is an integral aspect of a course of study. It can be formative or summative. The distinction between formative and summative assessment lies in the purpose for which that assessment is used.

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify the students' strengths and weaknesses so students may improve their achievement and better manage their own learning. The formative techniques used should be similar to summative assessment techniques, which students will meet later in the course. This provides students with experience in responding to particular types of tasks, under appropriate conditions. Feedback on any early assessment tasks may be used in a formative sense to assist students' preparation for later assessment tasks.

Summative assessment, while also providing feedback to students, parents and teachers, provides cumulative information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment techniques and instruments/tasks to be used, when they will be administered, and how they contribute to the determination of exit levels of achievement. Students' achievements are matched to the standards of exit criteria, which are derived from the general objectives of the course. Thus, summative assessment provides the information for certification at the end of the course.

6.1 Underlying principles of exit assessment

The policy on exit assessment requires consideration to be given to the following principles when devising an assessment program for the two-year course of study.

- Information is gathered through a process of continuous assessment.
- Balance of assessments is a balance over the course of study and not necessarily a balance over a semester or between semesters.
- Exit achievement levels are devised from student achievement in all areas identified in the syllabus as being mandatory.
- Assessment of a student's achievement is in the significant aspects of the course of study identified in the syllabus and the school's work program.
- Selective updating of a student's profile of achievement is undertaken over the course of study.
- Exit assessment is devised to provide the fullest and latest information on a student's achievement in the course of study.

These principles are to be considered together and not individually in the development of an assessment program. Exit assessment must satisfy concurrently the six principles associated with it.

Continuous assessment

The major operating principle is “continuous assessment”. The process of continuous assessment provides the framework in which all the other five principles of balance, mandatory aspects of the syllabus, significant aspects of the course, selective updating, and fullest and latest information exist and operate.

This is the means by which assessment instruments are administered at suitable intervals and by which information on student achievement is collected. It involves a continuous gathering of information and the making of judgments in terms of the stated criteria and standards throughout a two-year course of study.

Decisions about levels of achievement are based on information gathered, through the process of continuous assessment, at points in the course of study appropriate to the organisation of the learning experiences. Levels of achievement must not be based on students’ responses to a single assessment task at the end of a course or instruments set at arbitrary intervals that are unrelated to the developmental course of study.

Balance

Balance of assessments is a balance over the course of study and not necessarily a balance within a semester or between semesters.

Within the two-year course for Mathematics B it is necessary to establish a suitable balance in the general objectives, assessment techniques and instruments/tasks, conditions and across the criteria. The exit criteria are to have equal emphasis across the range of summative assessment. The exit assessment program must ensure an appropriate balance over the course of study as a whole.

Mandatory aspects of the syllabus

Judgment of student achievement at exit from a two-year course of study must be derived from information gathered about student achievement in those aspects stated in the syllabus as being mandatory, namely:

- the general objectives of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*
- the seven topics, *Introduction to functions*, *Rates of change*, *Periodic functions and applications*, *Exponential and logarithmic functions and applications*, *Optimisation*, *Introduction to integration*, and *Applied statistical analysis*.

The exit criteria and standards stated in Sections 6.6 and 6.9 respectively must be used to make the judgment of student achievement at exit from a two-year course of study.

Significant aspects of the course of study

Significant aspects refer to those areas in the school’s course of study permitted by the syllabus. Significant aspects can complement mandatory aspects or be in addition to them. They will be determined by the context of the school and the needs of students at that school to provide choice of learning experiences appropriate to the location of the school, the local environment and the resources available.

The significant aspects must be consistent with the general objectives of the syllabus and complement the developmental nature of learning in the course over two years.

Selective updating

In conjunction with the principle of fullest and latest information, information on student achievement should be selectively updated throughout the course.

Selective updating is related to the developmental nature of the course of study and operates within the context of continuous assessment. As subject matter is treated at increasing levels of complexity, assessment information gathered at earlier stages of the course may no longer be representative of student achievement. The information therefore should be selectively and continually updated (not averaged) to accurately reflect student achievement.

The following conceptions of the principle of selective updating apply:

- A systemic whole subject-group approach in which considerations about the whole group of students are made according to the developmental nature of the course and, in turn, the assessment program. In this conception, developmental aspects of the course are revisited so that later summative assessment replaces earlier formative information.
- An act of decision-making about individual students — deciding from a set of assessment results the subset which meets study area specification requirements and typically represents a student's achievements, thus forming the basis for a decision about a level of achievement. In the application of decisions about individual students, the set of assessment results does not have to be the same for all students. However, the subset which represents the typical achievement of a student must conform to the parameters outlined in the school's study plan for the strand.

Selective updating must not involve students reworking and resubmitting previously graded assessment tasks. Opportunities may be provided for students to complete and submit additional tasks. Such tasks may provide information for making judgments where achievement on an earlier task was unrepresentative or atypical, or there was insufficient information upon which to base a judgment.

Fullest and latest information

Judgments about student achievement made at exit from a school course of study must be based on the fullest and latest information available. This information is recorded on a student profile.

"Fullest" refers to information about student achievement gathered across the range of general objectives. "Latest" refers to information about student achievement gathered from the most recent period in which the general objectives are assessed. As the assessment program in Mathematics B is developmental, fullest and latest information will most likely come from Year 12.

Information recorded on a student profile will consist of the latest assessment data on mandatory and significant aspects of the course, which includes the data gathered in the summative assessment program that is not superseded.

6.2 Planning an assessment program

At the end of Year 12, judgments are made about how students have achieved in relation to the standards stated in the syllabus for each of the criteria. These summative judgments are based on achievement in each of the general objectives.

When planning an assessment program, schools must consider:

- general objectives (Section 3)
- principles of a balanced course (Section 3.3)

- the learning experiences (Section 5)
- the underlying principles of assessment (Section 6.1)
- a variety of assessment techniques and instruments over the two-year course (Section 6.4)
- conditions under which the assessment is implemented
- the exit criteria and standards (Sections 6.6 and 6.9)
- verification folio requirements, especially the number and the nature of student responses to assessment tasks to be included (Section 6.8)
- minimum assessment necessary to reach a valid judgment of the student's standard of achievement.

Students should be conversant with the assessment techniques and have knowledge of the criteria to be used in assessment instruments.

6.3 Implementing assessment

Assessment instruments are developed by the school to provide:

- information on which teachers may make judgments about student achievement of the general objectives
- a level of challenge suitable for the whole range of students.

An assessment instrument is accompanied by:

- a statement of the conditions of assessment that apply (Section 6.3.1)
- a detailed description of the instrument (Section 6.3.2)
- a detailed criteria sheet (Section 6.3.3)
- details of procedures for authentication of student responses (Section 6.3.4).

6.3.1 Conditions of assessment

Across the whole assessment program, teachers should establish a range of conditions. This can be done by systematically varying the factors that are most significant in establishing the conditions for an instrument, namely:

- the time allowed to prepare and complete the response
- access to resources, material and human, during the preparation for and completion of the instrument.

Every instrument description must include clear statements of the assessment conditions that apply. These may include:

- time available for the preparation and completion of the response
- resources accessible and available (both material and human) during the preparation for and completion of the response
- location for the preparation and completion of the response, e.g. in class, at home
- whether the response is to be an individual or group production
- the strategy used to ensure student authorship and ownership, e.g. the degree of teacher supervision and teacher monitoring that will apply.

6.3.2 Instrument descriptions

Instrument descriptions are to:

- state all instrument requirements, including the length and conditions
- be congruent with the general objectives of the syllabus, the standards associated with exit criteria and the school work program; this congruence ensures the essential relationship between learning, teaching and assessment practices.

6.3.3 Criteria sheets

Where criteria sheets specific to each instrument are developed, they should be provided to students before undertaking assessment.

An instrument-specific criteria sheet:

- should be derived from the exit criteria
- must describe/state standards consistent with those associated with exit criteria (see section 6.9)
- should provide a clear expectation of how standards will be demonstrated
- should inform teaching and learning practice.

Once the student has completed an assessment instrument, the criteria sheet:

- must indicate student achievement
- is used to inform teacher judgment about student achievement
- may provide students with the opportunity to develop self-evaluative expertise.

The extent to which the exit standards are reflected in the criteria sheet will vary according to the general objectives associated with the instrument and according to the stage in the course at which the instrument is undertaken.

6.3.4 Authentication of student work

It is essential that judgments of student achievement be made on genuine student assessment responses. Teachers must take reasonable steps to ensure that each student's work is their own, particularly where students have access to electronic resources or when they are preparing responses to collaborative tasks.

The QSA's A–Z of Senior Moderation contains a strategy on authenticating student work <www.qsa.qld.edu.au/10773.html>. This provides information about various methods teachers can use to monitor that students' work is their own. Particular methods outlined include:

- teachers seeing plans and drafts of student work
- student production and maintenance of evidence for the development of responses
- student acknowledgment of resources used.

Teachers must ensure students use consistent, accepted conventions of in-text citation and referencing, where appropriate.

6.4 Assessment techniques

Assessment techniques in this syllabus are grouped under categories. The following categories of assessment techniques may be considered:

- extended modelling and problem-solving tasks
- reports
- supervised tests.

Assessment of student achievement should not be seen as a separate activity, but as an integral part of the developmental learning process which reflects the learning experiences of students. There should be variety and balance in the types of assessment instruments used, thereby enabling students with different learning styles to demonstrate their understanding.

An extended modelling and problem-solving task or a report or similar must be included at least twice each year. These should contribute significantly to the decision-making process in each of the three exit criteria.

6.4.1 Category: Extended modelling and problem-solving tasks

What is an extended modelling and problem-solving task?
<p>An extended modelling and problem-solving task is an assessment instrument developed in response to a mathematical task. It may require a response that involves mathematical language, appropriate calculations, tables of data, graphs and diagrams, and could involve standard Australian English.</p> <p>Students may provide a response to a specific task or issue that could be set in a context that highlights a real-life application of Mathematics.</p> <p>Aspects of each of the three criteria should be evident in the task.</p>
What might a student do to complete an extended modelling and problem-solving task?
<ul style="list-style-type: none"> • Analyse information and data from a variety of sources. • Process information to identify assumptions and parameters. • Interpret and synthesise data. • Explain relationships to develop and support mathematical arguments. • Reflect on and evaluate data collected, propositions, results and conclusions. • Communicate ideas.
What do teachers do when planning and implementing an extended modelling and problem-solving task?
<ul style="list-style-type: none"> • The teacher should provide the mathematical task. • Teachers must implement strategies to ensure authentication of student work. Some strategies are annotated notes in response to issues that emerged during the extended modelling and problem-solving task; teacher observation sheets; research checklists and referencing and reference lists. • Teachers may consult, negotiate and provide feedback prior to and during students' preparation of the report to provide ethical guidance and to monitor student work. Feedback and assistance should be provided judiciously. • When students undertake extended modelling and problem-solving tasks for the first time, scaffolding may be provided to help students complete the assessment. However, if the task is intended to demonstrate high initiative then the scaffolding provided should not specify the procedures, nor lead the student through a series of steps to reach a solution. • Scaffolding should be reduced from Year 11 to Year 12 to allow the student to better demonstrate the principle of initiative in the problem-solving process.

6.4.2 Category: Reports

What is a report?

A report is typically an extended response to a practical or investigative task such as:

- an experiment in which data are collected, analysed and modelled
- a mathematical investigation
- a field activity
- a project.

A mathematical report could comprise such forms as:

- a scientific report
- a proposal to a company or organisation
- a feasibility study.

The report and the activities leading to a report could be done individually and/or in groups; in class time and/or in the students' own time. A report will typically be in written form, or a combination of written and oral multimedia forms.

The report will generally include an introduction, analysis of results and data, conclusions drawn, justification, and, when necessary, appendixes and a bibliography and/or reference list.

Aspects of each of the three criteria should be evident in the task.

What might a student do to complete a report?

- Gather and sort information and data from a variety of sources.
- Process information to identify assumptions and parameters.
- Interpret, analyse and synthesise data.
- Explain relationships to develop and support mathematical arguments
- Reflect on and evaluate data collected, propositions, results and conclusions
- Communicate ideas.

What do teachers do when planning and implementing a report?

- The teacher suggests topics and provides some stimulus to trigger student interest.
- Teachers can provide the research question or it may be instigated by the student. In those instances teachers should negotiate with students to ensure the possibility of success. It is more likely that students will be able to generate their own research questions the further they progress in the course of study.
- Teachers may allow some class time for students to be able to effectively undertake each component of the report. Teachers may allow elements of the report to be conducted in small groups or pairs.
- Teachers must implement strategies to ensure authentication of student work. Some strategies are annotated notes in response to issues that emerged during the report; teacher observation sheets; checklists and referencing and reference lists.
- Teachers may consult, negotiate and give feedback prior to and during the report to provide ethical guidance and to monitor student work. Feedback and assistance should be provided judiciously.
- When students undertake reports for the first time scaffolding may be provided to help students complete the assessment. However, if the task is intended to demonstrate high initiative then the scaffolding provided should not specify the procedures, nor lead the student through a series of steps to reach a solution.
- Scaffolding should be reduced from Year 11 to Year 12 to allow the student to better demonstrate the principle of initiative in the problem-solving process.

6.4.3 Category: Supervised tests

What is a supervised test?

A supervised test is an assessment instrument that is conducted under supervised conditions. The supervised test commonly includes tasks requiring quantitative and/or qualitative responses. The tasks could be done individually and/or in groups.

The supervised test must provide adequate opportunities for students to demonstrate their level of expertise in Mathematics across the full range of standards in the syllabus.

The supervised test may be constructed from the following four types of techniques:

1. **Short items**

- requiring multiple-choice, single-word, sentence or short paragraph (up to 50 words) responses, written in mathematical language, symbols and/or standard Australian English

2. **Practical exercises**

- using graphs, tables, diagrams, data or the application of algorithms

3. **Paragraph responses**

- These are used when explanation of a greater complexity is required and are written in mathematical language, symbols and/or standard Australian English. Responses should be 50–150 words.

4. **Responses to seen or unseen stimulus materials**

- This may take the form of a series of short items, practical exercises and paragraph responses (see above).
- The question or statement is not provided before the assessment (unseen) and should focus on asking the students to evaluate and justify.
- Stimulus materials should be succinct enough to allow students to engage with them in the time provided for the supervised test. Perusal times may be required or if the stimulus materials are lengthy they may need to be shared with students prior to the administration of the supervised test.

What do teachers do when planning a supervised test?

The teacher should:

- construct questions that are unambiguous
- format the paper to allow for ease of reading and responding
- consider the individual needs of the students
- ensure the questions allow students to demonstrate the full range of standards
- ensure that formula sheets, if used, are supplied by the school and are common and constant across the cohort.
- consider whether students will have access to information previously stored in their calculator.

6.5 Special consideration

Guidance about the nature and appropriateness of special consideration and special arrangements for particular students may be found in the Authority's *Policy on Special Consideration in School-based Assessments in Senior Certification* (2006), available from <www.qsa.qld.edu.au> under Assessment > Senior assessment > Special consideration. This statement also provides guidance on responsibilities, principles and strategies that schools may need to consider in their school settings.

To enable special consideration to be effective for students so identified, it is important that schools plan and implement strategies in the early stages of an assessment program and not at the point of deciding levels of achievement. The special consideration might involve alternative teaching approaches, assessment plans and learning experiences.

6.6 Exit criteria

The following exit criteria must be used in making judgments about a student's level of achievement at exit from this course. They reflect the three assessable categories of general objectives of the syllabus as defined in Section 3.

Knowledge and procedures

This criterion refers to the student's ability to recall, access, select and apply mathematical definitions, rules and procedures, to demonstrate numerical and spatial sense and algebraic facility, with and without the use of mathematical technology.

Modelling and problem solving

This criterion refers to the student's ability to apply problem-solving strategies to investigate and model situations, generate and use data, analyse and interpret results in the context of problems to investigate the validity of mathematical arguments and models.

Communication and justification

This criterion refers to the student's ability to interpret, translate, communicate, present and justify mathematical arguments and propositions, using mathematical and everyday language and symbols to develop logical supported arguments.

6.7 Determining exit levels of achievement

On completion of the course of study, the school is required to award each student an exit level of achievement from one of the five categories:

Very High Achievement
High Achievement
Sound Achievement
Limited Achievement
Very Limited Achievement.

The school must award an exit standard for each of the criteria *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* based on the principles of assessment described in Section 6.1. The criteria are derived from the general objectives and are described in Section 3. The *typical* standards associated with the three exit criteria are described in the matrix in Section 6.9. When teachers are determining a standard for each criterion, *it is not always necessary* for the student to have met each descriptor for a particular standard; the standard awarded should be informed by how the qualities of the work match the descriptors overall.

For Year 11, particular standards descriptors may be selected from the matrix and/or adapted to suit the task. These standards are used to inform the teaching and learning process. For Year 12 tasks, students should be provided with opportunities to understand and become familiar with the expectations for exit. The *typical* standards are applied to the summative body of work selected for exit.

The seven key competencies* referred to in the Rationale are embedded in the descriptors in the standards matrix. The descriptors refer mainly to aspects of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*.

When standards have been determined in each of the criteria of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, The following table is used to award exit levels of achievement, where *A* represents the highest standard and *E* the lowest. The table indicates the *minimum combination of standards* across the criteria for each level.

Awarding exit levels of achievement

VHA	Standard <i>A</i> in any two criteria and no less than a <i>B</i> in the remaining criterion
HA	Standard <i>B</i> in any two criteria and no less than a <i>C</i> in the remaining criterion
SA	Standard <i>C</i> in any two criteria, one of which must be the <i>Knowledge and procedures</i> criterion, and no less than a <i>D</i> in the remaining criterion
LA	At least Standard <i>D</i> in any two criteria, one of which must be the <i>Knowledge and procedures</i> criterion
VLA	Standard <i>E</i> in the three criteria

6.8 Requirements for verification folio

A verification folio is a collection of a student's responses to assessment instruments on which the level of achievement is based. Each folio should contain a variety of assessment techniques demonstrating achievement in the criteria *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, over a range of topics. The variety of assessment techniques is necessary to provide a range of opportunities from which students may demonstrate achievement.

For information about preparing monitoring and verification submissions schools should refer to *Moderation Processes for Senior Certification* available at <www.qsa.qld.edu.au> under Assessment > Senior Assessment > Forms and procedures.

Students' verification folios for Mathematics B must contain:

- a minimum of four assessment instruments from Year 12, with at least one of these being an extended *Modelling and problem-solving* task or a report or similar.
- student responses to a minimum of four and a maximum of 10 summative assessment instruments
- where used, a criteria sheet for each assessment instrument which provides evidence of how students meet standards associated with the assessment criterion involved in that instrument
- formula sheets or other allowable materials used where appropriate
- a student profile completed to date.

6.8.1 Post-verification assessment

In addition to the contents of the verification folio, there must be subsequent summative assessment in the exit folio. In Mathematics B at least one instrument must be completed after verification. It is desirable for the assessment instrument to include all criteria.

* KC1: collecting, analysing and organising information; KC2: communicating ideas and information; KC3: planning and organising activities; KC4: working with others and in teams; KC5: using mathematical ideas and techniques; KC6: solving problems; KC7: using technology

6.9 Standards associated with exit criteria

Criterion	Standard A	Standard B	Standard C	Standard D	Standard E
<i>Knowledge and procedures</i>	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> use of stated rules and procedures in simple situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> statements of relevant mathematical facts
	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations 	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations 	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations 		
	<ul style="list-style-type: none"> numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<ul style="list-style-type: none"> numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations 	<ul style="list-style-type: none"> numerical calculations, spatial sense and algebraic facility in routine, simple life-related or abstract situations 	<ul style="list-style-type: none"> numerical sense, spatial sense and/or algebraic facility in routine or simple tasks 	
	<ul style="list-style-type: none"> appropriate selection and accurate use of technology 	<ul style="list-style-type: none"> appropriate selection and accurate use of technology 	<ul style="list-style-type: none"> selection and use of technology 	<ul style="list-style-type: none"> use of technology 	<ul style="list-style-type: none"> use of technology

<i>Criterion</i>	<i>Standard A</i>	<i>Standard B</i>	<i>Standard C</i>	<i>Standard D</i>	<i>Standard E</i>
<i>Modelling and problem solving</i>	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple tasks through to routine complex tasks in life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • evidence of simple problem-solving strategies in the context of problems 	<p>The student work has the following characteristic:</p> <ul style="list-style-type: none"> • evidence of simple mathematical procedures
	<ul style="list-style-type: none"> • identification of assumptions and their associated effects, parameters and/or variables 	<ul style="list-style-type: none"> • identification of assumptions, parameters and/or variables 			
	<ul style="list-style-type: none"> • use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations 	<ul style="list-style-type: none"> • use of data to synthesise mathematical models in simple situations and generation of data from mathematical models in simple through to complex situations 	<ul style="list-style-type: none"> • use of mathematical models to represent routine, simple situations and generate data 	<ul style="list-style-type: none"> • use of given simple mathematical models to generate data 	
	<ul style="list-style-type: none"> • investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems; the strengths and limitations of models, both given and developed 	<ul style="list-style-type: none"> • interpretation of results in the context of simple through to complex problems and mathematical models 	<ul style="list-style-type: none"> • interpretation of results in the context of routine, simple problems 		

<i>Criterion</i>	<i>Standard A</i>	<i>Standard B</i>	<i>Standard C</i>	<i>Standard D</i>	<i>Standard E</i>
<i>Communication and justification</i>	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> use of mathematical terminology, symbols or conventions in simple or routine situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> use of mathematical terminology, symbols or conventions
	<ul style="list-style-type: none"> organisation and presentation of information in a variety of representations 	<ul style="list-style-type: none"> organisation and presentation of information in a variety of representations 	<ul style="list-style-type: none"> organisation and presentation of information 	<ul style="list-style-type: none"> presentation of information 	<ul style="list-style-type: none"> presentation of information
	<ul style="list-style-type: none"> analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine 	<ul style="list-style-type: none"> analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine 	<ul style="list-style-type: none"> translation of information from one representation to another in simple routine situations 		
	<ul style="list-style-type: none"> use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language 	<ul style="list-style-type: none"> use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language 	<ul style="list-style-type: none"> use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language 		
	<ul style="list-style-type: none"> coherent, concise and logical justification of procedures, decisions and results 	<ul style="list-style-type: none"> coherent and logical justification of procedures, decisions and results 	<ul style="list-style-type: none"> justification of procedures, decisions or results 		
	<ul style="list-style-type: none"> justification of the reasonableness of results 				

7. Language education

Teachers of Senior English have a special responsibility for language education. However, it is the responsibility of all teachers to develop and monitor students' abilities to use the forms of language appropriate to their own subject areas. Their responsibility entails developing the following skills:

- ability in the selection and sequencing of information required in the various forms (such as reports, essays, interviews and seminar presentations)
- the use of technical terms and their definitions
- the use of correct grammar, spelling, punctuation and layout.

Mathematics B requires students to use language in a variety of ways — spoken, written, graphical, and symbolic. The responsibility for developing and monitoring students' abilities to use effectively the forms of language demanded by this course rests with the teachers of mathematics. This responsibility includes developing students' abilities to:

- select and sequence information
- manage the conventions related to the forms of communication used in Mathematics B (such as short responses, reports, multimedia presentations, seminars)
- use the specialised vocabulary and terminology related to Mathematics B
- use language conventions with reference to standard Australian English related to grammar, spelling, punctuation and presentation
- communicate effectively and efficiently.

Thus, when writing, reading, questioning, listening and talking about mathematics, teachers and students should use the specialised vocabulary related to the subject. Students should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions.

Assessment in Mathematics B needs to take into consideration appropriate use of language. Assessment instruments should use format and language that are familiar to students. Attention to language education within Mathematics B should assist students to meet the language components of the exit criteria, especially the criterion *Communication and justification*.

8. Quantitative concepts and skills

Success in dealing with issues and situations in life and work depends on the development and integration of a range of abilities, such as being able to:

- comprehend basic concepts and terms underpinning the areas of number, space, probability and statistics, measurement and algebra
- extract, convert or translate information given in numerical or algebraic forms, diagrams, maps, graphs or tables
- calculate, apply algebraic procedures, implement algorithms
- make use of calculators and computers
- use skills or apply concepts from one problem or one subject domain to another.

In all subjects students are to be encouraged to develop their understanding and to learn through the incorporation — to varying degrees — of mathematical strategies and approaches to tasks. Similarly, students should be presented with experiences that stimulate their mathematical interest and hone those quantitative skills that contribute to operating successfully within each of their subject domains.

Mathematics B focuses on the development and application of numerical and other mathematical concepts and skills. It provides a basis for the general development of such quantitative skills to prepare students to cope with the quantitative demands of their personal lives or to participate in a specific workplace environment.

The distinctive nature of Mathematics B will require that new mathematical concepts be introduced and new skills developed. Within appropriate learning contexts and experiences in the subject, opportunities are to be provided for revising, maintaining, and extending such skills and understandings.

The following quantitative knowledge and procedures will be required throughout the Mathematics B course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- simple interest
- basic algebraic manipulations
- identities, linear equations and inequalities
- gradient of a straight line
- equation of a straight line
- plotting points using Cartesian coordinates
- solutions of a quadratic equation
- graphs of quadratic functions
- tree diagrams as a tool for defining sample spaces and estimating probabilities
- the summation notation: $\sum_{i=1}^n x_i$

9. Educational equity

Equity means fair treatment of all. In developing work programs from this syllabus, schools should incorporate the following concepts of equity.

All young people in Queensland have a right to gain an education that meets their needs, and prepares them for active participation in creating a socially just, equitable and democratic global society. Schools need to provide opportunities for all students to demonstrate what they know and can do. All students, therefore, should have equitable access to educational programs and human and physical resources. Teachers should ensure that particular needs of the following groups of students are met: female students; male students; Aboriginal students; Torres Strait Islander students; students from non-English-speaking backgrounds; students with disabilities; students with gifts and talents; geographically isolated students; and students from low socioeconomic backgrounds.

Subject matter chosen should include, whenever possible, the contributions and experiences of all groups of people. Learning contexts and community needs and aspirations should also be considered. In choosing appropriate learning experiences teachers can introduce and reinforce non-racist, non-sexist, culturally sensitive and unprejudiced attitudes and behaviour. Learning experiences should encourage the participation of students with disabilities and accommodate different learning styles.

Resource materials used should recognise and value the contributions of both females and males to society and include social experiences of both genders. Resource materials should also reflect cultural diversity within the community and draw from the experiences of the range of cultural groups in the community.

To allow students to demonstrate achievement, barriers to equal opportunity need to be identified, investigated and removed. This may involve being proactive in finding the best ways to meet the diverse range of learning and assessment needs of students. The variety of assessment techniques in the work program should allow students of *all* backgrounds to demonstrate their knowledge and skills related to the criteria and standards stated in this syllabus. The syllabus criteria and standards should be applied in the same way to all students.

Teachers should consider equity policies of individual schools and schooling authorities, and may find the following resources useful for devising an inclusive work program:

ACACA 1996, *Guidelines for Assessment Quality and Equity*, available from <www.acaca.org.au>

ANTA 2004, *A Guide to Equity and the AQTF*, available from Australian Training Products Ltd <www.atpl.net.au>

QSA 2006, *Policy on Special Consideration in School-based Assessments in Senior Certification*, available from <www.qsa.qld.edu.au>

QSA 2006, *Policy Statement: Equity*, available from <www.qsa.qld.edu.au>

EQ 2005, *Inclusive education statement*, available from <<http://education.qld.gov.au/student-services/>>

EQ 2006, *Education Policy and Procedures Register: Inclusive education*, available from <<http://education.qld.gov.au/strategic/eppr/>>

QSCC 2001, *Equity Considerations for the development of curriculum and test material*, available from <www.qsa.qld.edu.au>

QCEC 2001, *Inclusive Practices in Queensland Catholic Schools*, available from <www.qcec.qld.catholic.edu.au>

10. Resources

Text and reference books

A wide variety of textbooks and resource materials that could be used as sources of information about Mathematics B are available. Book suppliers provide information regarding current publications.

World Wide Web

Many interactive and static websites can be used to enhance a course in Mathematics B and often include useful resources. Some relevant sites can be sourced through the QSA website. Interactive websites can be found by searching for *applets*. Some particularly useful sites include:

- Department of Education, Training and the Arts Maps and Compasses Resource Page
<www.education.qld.gov.au/curriculum/area/maths/compass/html/resources.html>
- Online Teachers' Resource Network
<www.otrnet.com.au/Weblinks/lesson_resources.html>
- The Australian Association of Mathematics Teachers
<www.aamt.edu.au>
- The Queensland Association of Mathematics Teachers
<www.qamt.org>

Newspaper reports

Newspapers can be a source of useful data. The compilation of news files on particular topics can broaden students' knowledge and provide a valuable source of material for developing assessment instruments.

Periodicals

Journals and periodicals provide current, relevant information. Journals and periodicals relevant to Mathematics B may include:

- *Australian Senior Mathematics Journal* — journal of the Australian Association of Mathematics Teachers, Inc.
- *Mathematics Education Research Journal* — journal of Mathematics Education Research Group of Australasia, Inc.
- *Teaching Mathematics* — journal of the Queensland Association of Mathematics Teachers
- *The ANZIAM Journal* (The Australian and New Zealand Industrial and Applied Mathematics Journal) — journal of the Australian Mathematics Society
- *The Australian Mathematics Teacher* — journal of the Australian Association of Mathematics Teachers, Inc.

School librarians should be able to help identify and locate other useful periodicals.

Electronic media and learning technology

A wide range of videos, DVDs and television recordings are available on a variety of topics related to Mathematics B. Various computer software programs and CD-ROMs may be useful for a course in Mathematics B, as learning tools, to gain access to information presented in a variety of forms, and to help students gain ICT skills. Educational program distributors can supply updated resource lists.

Organisations and community resources

A variety of government and community organisations provide personnel, advice, resources and information to assist in constructing and implementing a course in Mathematics B. Some of these include:

- Australian Bureau of Statistics
- Bureau of Meteorology
- Queensland Association of Mathematics Teachers.

Glossary

Abstraction

The process of extracting the underlying essence of a mathematical concept, removing dependence on the real-world contexts which might have inspired it.

Algebraic facility

The ability to model, manipulate, simplify, substitute, factorise, expand and solve some symbolic representation.

Algorithm

Process or set of rules to be used; a list of well-defined instructions for completing a task; step-by-step approach.

Amortising a loan

The repayment of a debt including principal and interest.

Analyse

To break up a whole into its parts, to examine in detail to determine the nature of, to look more deeply into and to detect the relationships between parts.

Annuity (pl. Annuities)

The accumulation of fixed payments at fixed intervals over a period of time.

Assessment instruments

Particular methods developed and used by a school to gather information about student achievement.

Assessment techniques

The methods (categories) identified in the syllabus (Section 6.4) to gather evidence about student achievement.

Conclusion

Final result or summing up; inference deduced from previous information; reasoned judgment.

Context

A context is a framework for linking concepts and learning experiences that enables students to identify and understand the application of Mathematics to their world. A context is a group of related situations, phenomena, technical applications and social issues likely to be encountered by students. A context provides a meaningful application of concepts in real-world situations.

Criterion (pl. Criteria)

A property, dimension or characteristic by which something is judged or appraised. In senior syllabuses, the criteria are the significant dimensions of the subject, described in the Rationale and used to categorise the general objectives and exit criteria. (Section 6.6).

Criteria sheets

Criteria sheets are developed from the standards associated with exit criteria to describe the attributes of student work anticipated at each level of achievement for the particular assessment instrument. (Section 6.3.3)

Data

In the context of the Mathematics B syllabus, data are thought of as to include documented information or evidence of any kind that lends itself to mathematical interpretation. Data may be quantitative or qualitative. (See Primary data, Secondary data, Qualitative data, Quantitative data.)

Deduce

Infer; reach a conclusion which is necessarily true provided a set of assumptions is true.

Depth

The development of knowledge and understandings from simple through to complex.

Describe

To give an account of in speech or writing; to convey an idea or impression of; characterise; to represent pictorially; depict; to trace the form or outline of.

Determine

To come to a resolution or decide.

Discuss

Consider a particular topic in speaking or writing; talk or write about a topic to reach a decision.

Estimate

Calculate an approximate amount or quantity.

Evaluate

Establishes the value, quality, importance, merit, relevance or appropriateness of information, data or arguments based in logic as opposed to subjective preference.

Exemplify

To show or illustrate using examples.

Exit level of achievement

The standard reached by students at exit judged by matching standards in student work with the exit criteria and standards stated in a syllabus. (Section 6.7)

Explain

Make clear or understandable, know in detail.

Formative assessment

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify the students' strengths and weaknesses so students may improve their achievement and better manage their own learning.

General objectives

General objectives are those which the school is intended to pursue directly and student achievement of these is assessed by the school. (Section 3)

Generalisation

Creation of a statement or formula by considering similar situations subject to the same constraints.

Example 1

By considering how a hot mass cools, students are able to generalise the statement *an exponential function* $y = ab^x + ta$ can be used to model the cooling of a hot mass to ambient temperature.

Example 2

Find the first and second derivatives of the function $y = \sin 2x$

Extend to find the third, fourth and fifth derivatives and generalise from your solutions to develop a rule that would help find the 100th derivative.

Solution

$$y = \sin 2x$$

$$y^i = 2 \cos 2x$$

$$y^{ii} = -4 \sin 2x$$

$$y^{iii} = -8 \cos 2x$$

$$y^{iv} = 16 \sin 2x$$

$$= 2^4 \sin 2x$$

$$y^v = 32 \cos 2x$$

$$2^5 \cos 2x$$

From the results, the original trigonometric function is regenerated every 4th derivative; the factor at the front of the function is the power of 2 raised to the same power as that of the derivative number; the sign of the derivative varies according to the sequence: +, −, −, +.

Thus the 100th derivative of $y = \sin 2x$ will be $y^{100} = 2^{100} \sin 2x$ since there are exactly 25 lots of 4.

Interpret

To give meaning to information presented in various forms — words, symbols, pictures, graphs etc.

Justify

Provide sound reasons based on logic or theory to support response; prove or show statements are just or reasonable; convince. Specifically the justification of procedures may include:

- providing evidence (words, diagrams, symbols etc.) to support processes used
- stating a generic formula before using it specifically
- providing reasoned, well-formed, logical sequence within a response.

Key competencies

The key competencies define skills essential for effective participation in adult life, including further education and employment.

Mathematical argument

A discussion in mathematical terms or language of some or all of the propositions, procedures, results or conclusions.

Mathematical model

Any depiction of a situation expressing a relationship between ideas in mathematical terms.

Mathematical modelling

The act of creating a mathematical model, which may involve the following steps:

- identify assumptions, parameters and/or variables
- interpret, clarify and analyse the problem
- develop strategies or identify procedures required to develop the model and solve the problem
- investigate the validity of the mathematical model.

Median boxplot (box-and-whisker plot)

A graphical presentation of some main features of a dataset. The simplest version of a box plot is formed by drawing a box extending from the lower to the upper quartiles, marking the median within that box, and drawing lines (called whiskers) from the box to the smallest and largest data points. There are slight variations in the possible ways of identifying the median and quartiles of data: these variations make very little difference except with small or sparse datasets.

Moderation

Moderation is the name given to the quality assurance process for senior secondary studies used by the QSA to ensure that:

- Authority subjects taught in schools are of the highest possible standards
- student results in the same subject match the requirements of the syllabus and are comparable across the state
- the process used is transparent and publicly accountable.

Non-routine

Insight and creativity (refer to Section 3.3)

Number sense

An intuitive understanding of numbers, their magnitude, relationships and how they are affected by operations.

Outlier

An extreme value in the observations, for example, a point which is well away from the line of best fit.

Parameter

The values that allow a model to define a particular situation, i.e. m and c in the equation $y = mx + c$

Primary or raw data

Data that have been collected first hand, but not yet processed.

Problem-solving strategies

May include: estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data.

Procedure

A list of sequential instructions that is to be used to solve a problem or perform a task.

Qualitative data

Data concerned with quality; verbal analysis.

Quantitative data

Data concerned with measurement; mathematical analysis.

Representation

Refers both to process and to product, i.e.

- the act of capturing a mathematical concept or relationship in some form, and
- the form itself.

Representation applies to processes and products that are observable externally as well as to those that occur “internally” in the minds of people doing mathematics. Some forms of representation include diagrams, graphical displays and symbolic expressions. Representations are essential elements in supporting understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings.

Routine

Well-rehearsed (refer to Section 3.3).

Scaffolding

The scaffolding analogy comes from the building industry, and refers to the process of supporting a student's learning to solve a problem or perform a task that could not be accomplished by that student alone. The aim is to support the student as much as necessary while they build their understanding and ability to use the new learning; then gradually reduce the support until the student can use the new learning independently.

Secondary data

Data that have been collected by someone else, or data that have been processed.

Solution

Answers to problems, investigations, research or questions. A concise solution includes the following characteristics:

- succinct use of language that avoids repetition
- use of appropriate terminology and symbols.

This does not preclude the exploration of additional aspects of a problem/situation.

Spatial sense

The sense of geometric space, particularly patterns and relationships that have to do with geometric figures.

Standard

A standard is a fixed reference point for use in assessing or describing the quality of something. In senior syllabuses, standards are usually described at five points within each exit criterion.

Stemplot (stem-and-leaf plot)

An exploratory technique that simultaneously ranks the data and gives an idea of the distribution. Example: The following 16 average daily temperatures have been recorded to the nearest degree Celsius:

31	21	35	30	22	23	9	24
13	41	30	21	29	24	18	28

2 | 1 represents 21

Preliminary stem-and-leaf plot of the temperatures:

0		9
1		3 8
2		1 2 3 4 1 9 4 8
3		0 0 1 5
4		1

Completed stem-and-leaf plot of the temperatures:

0		9
1		3 8
2		1 1 2 3 4 4 8 9
3		0 0 1 5
4		1

Strategy

A plan of action designed to achieve a goal (see problem-solving strategies).

Student profile of achievement

This records information about student performance on assessment instruments undertaken periodically throughout the course of study. Techniques are chosen to sample the significant aspects of a course across relevant exit criteria to ensure balance in assessment. In particular, it is important that the profile of achievement illustrates how assessment of significant aspects is selectively updated and eventually leads to summative assessment within each exit criterion.

Summary statistics

In descriptive statistics, summary statistics are used to summarise a set of observations, in order to communicate as much as possible as simply as possible. Statisticians commonly try to describe the observations in:

- a measure of location, or central tendency, such as the arithmetic mean, median, mode, or interquartile mean
- a measure of statistical dispersion like the standard deviation, variance, range, or interquartile range, or absolute deviation.
- a measure of the shape of the distribution like skewness or kurtosis.

Summative assessment

Summative assessment provides cumulative information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment instruments to be used, when they will be administered, and how they will contribute to the determination of exit levels of achievement.

Synthesise

Assembling constituent parts into a coherent entity. The term “entity” includes a system, theory, plan, or set of operations.

Trapezoidal rule

The area under the curve above the x -axis between the limits $x = a$ and $x = b$, can be approximated by dividing the interval $[a, b]$ into n sub-intervals of equal length,

$$w = \frac{|b - a|}{n}, \text{ and using the formula:}$$

$$A \approx \frac{w}{2} [f(a) + 2\{f(a + w) + f(a + 2w) + \dots + f(a + (n - 1)w)\} + f(b)]$$

Valid

Sound, reasonable, relevant, defensible, well grounded, able to be supported with logic or theory. May include the strengths and limitations of models and the analysis of the effect of assumptions.

Variable

A symbol that represents a value that is allowed to change.

Verification

Towards the end of Year 12, school submissions, one for each Authority subject, are sent to the relevant (usually district) review panels who review the material to confirm that the standards assigned to students' work are in line with the descriptors in the syllabus. These submissions comprise folios of the work of sample students about to exit from the course of study, together with the school's judgment.

Verification folio

This is the collection of documents (tests, reports, assignments, checklists and other assessment instruments) used to make the decision about a student's level of achievement. At October verification, it will contain a minimum of four and a maximum of ten pieces of work that conform to the underlying principles of assessment as outlined in Section 6.1.

Work program

The school's program of study in Authority and Authority-registered subjects for which the students' results may be recorded on Queensland Studies Authority certificates (requirements are listed on the QSA website).