SENIOR MATHEMATICS C

BOARD OF SENIOR SECONDARY SCHOOL STUDIES

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This subject is one of three Board subjects available to schools for students of Senior Mathematics. Attention is drawn to the following statement on compatibility and prerequisite knowledge.

In any one semester, the following combinations may appear on the Senior Certificate:

Mathematics A
Mathematics A and Mathematics B
Mathematics B
Mathematics B and Mathematics C

In exceptional circumstances, Mathematics C may appear on the Senior Certificate without Mathematics B. However, it is strongly recommended that schools ensure that students of Mathematics C have an adequate mathematical background.

The Senior Syllabus in Mathematics A is incompatible with the Senior Syllabus in Mathematics in Society.

The Senior Syllabus in Mathematics B is incompatible with the Senior Syllabus in Mathematics.

The Senior Syllabus in Mathematics C is incompatible with the Senior Syllabus in Mathematics.

ACKNOWLEDGEMENT

The Board of Senior Secondary School Studies wishes to acknowledge the contribution of the following to the syllabus:

A National Statement on Mathematics for Australian Schools (1991)
Published by the Curriculum Corporation for the Australian Education Council
St Nicholas Place
141 Rathdowne Street
CARLTON VIC 3053

SENIOR SYLLABUS IN MATHEMATICS C

CONTENTS **PAGE**

TO BE USED FOR THE FIRST TIME WITH YEAR 11 STUDENTS IN 1993

SENIOR SYLLABUS IN MATHEMATICS C

1 RATIONALE

Mathematics is an integral part of a general education. It can enhance people's understanding of the world and the quality of their participation in a rapidly changing, technological society.

In recent years, the range of career opportunities requiring an advanced level of mathematical competence has expanded dramatically. Mathematics has been central to nearly all major scientific and technological advances. Many of the developments and decisions made in industry and commerce, in the provision of social and community services, and in government policy and planning, rely on the use of mathematics. It is predicted that demand in Australia for mathematically skilled people will rise, creating a significant labour market problem unless more people are willing to undertake further mathematical study.

To encourage students to realise their full mathematical potential, it is imperative that they become confident with, and gain an appreciation of, mathematics. In order to achieve this, mathematics education must recognise the dynamic nature of mathematics and encourage an approach to its study through problem solving and applications in life-related contexts. Students must be given the opportunity to appreciate and experience the power which has been given to mathematics by computer technology.

Given the impossibility of identifying all the skills which students will need in the future, narrow vocational views of mathematics are likely to be unproductive. The intent of Mathematics C is to encourage students to develop positive attitudes towards mathematics by an approach involving problem solving and applications. Students will be required to work systematically and logically, and to communicate with and about mathematics.

Students should understand that mathematics and its applications are constantly being developed. They will be encouraged to appreciate the power and diversity of mathematics by investigating a range of its modern applications, even if the technical demands of the mathematics are beyond them at this stage. The study of Mathematics C will give students the opportunity to extend their mathematical knowledge into new areas, and hence will provide an excellent preparation for the further study of mathematics in a wide variety of fields. The additional rigour and structure of the mathematics required in this subject will equip students with valuable thinking skills which will serve them in more general contexts. Mathematics C is therefore a highly desirable choice for many students.

2 GLOBAL AIMS

Having completed the course of study, students of the subject Mathematics C should:

- be able to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- be aware of the uncertain nature of the world, and be able to use mathematics to assist in making informed decisions in life-related situations
- be able to visualise and represent spatial relationships in both two and three dimensions
- be aware of the diverse applications of mathematics
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information which is presented in a variety of forms
- communicate mathematical information in a variety of forms
- be able to benefit from the availability of a wide range of mathematical instruments
- be able to use appropriately selected mathematical instruments
- be able to recognise functional dependencies
- be aware of the diverse and evolutionary nature of mathematics through an understanding of its history
- be aware of the wide range of mathematics-based vocations
- be aware of the contribution of mathematics to society
- have an appreciation of the power and beauty of mathematics.

3 GENERAL OBJECTIVES

3.1 INTRODUCTION

The general objectives of this course are organised into three categories which are described below.

I COMMUNICATION

Clear communication is paramount both within the school setting and also in the world for which students are being prepared. The process of developing and building up mathematical knowledge through describing, arguing, predicting, and justifying almost always requires a sharing of ideas. The productive sharing of ideas depends upon the clarity with which one can express oneself. Communication skills are needed in order to understand, assess, and convey ideas and arguments including mathematical concepts presented in both everyday and mathematical language.

II MATHEMATICAL TECHNIQUES

Both dimensions included in this category involve developing **familiarity** with a variety of techniques and life-related applications.

DIMENSIONS:

(a) LEARNED RESULTS AND MATHEMATICAL PROCEDURES

An important goal in mathematics education is for students to develop the capacity to deal with commonly occurring or familiar mathematical situations readily. Facility with certain techniques can strengthen one's capacity to inquire and to explore unfamiliar situations involving mathematics. In this dimension the ability to apply mathematics in **familiar** situations is developed.

(b) USE OF INSTRUMENTS IN MATHEMATICS

In order that the emphasis of the mathematics learned is on concepts and techniques rather than tedious and/or involved calculations, it is appropriate that students are confident and competent in using a calculator and a computer. Other instruments encountered by students may include graphics calculators, drawing instruments, protractors, geoliners, tape measures, tables and graph paper. It is important that students become competent in the appropriate selection and use of instruments.

III MATHEMATICAL APPLICATIONS

The three dimensions included in this category involve applying mathematics in **unfamiliar** situations.

DIMENSIONS

(a) APPLYING MATHEMATICS IN LIFE RELATED-SITUATIONS (REAL OR SIMULATED)

Problems are usually generated from the real world. In such cases, they need to be extracted from their particular context, the essential relationships within the context represented mathematically, and the problem stated in mathematical terms. Once this is done, mathematical techniques may be applied to the problem. The derived solution then needs to be interpreted in terms of the life related situation.

(b) APPLYING MATHEMATICS IN PURELY MATHEMATICAL CONTEXTS

At times, the problems investigated are generated within the fields of mathematics and thus relate to mathematical abstractions. In such cases, there is no immediate interest in whether the relationships derived relate to any social, physical or biological phenomenon. The purpose is explicitly mathematical.

(c) JUSTIFICATION

Justification deals with developing logical arguments and justifying conclusions, leading to the notion of mathematical proof. In order to analyse and interpret certain material, students will need to understand the value of mathematical arguments and the relationship between mathematics and its applications.

3.2 THE OBJECTIVES

The general objectives for each of these categories are detailed below.

I COMMUNICATION

The student should be able to:

- communicate results concisely, clearly, precisely and appropriately
- use mathematical terms and symbols accurately and appropriately
- use accepted spelling, punctuation and grammar in written communication
- understand material presented in the following forms:
  - verbal: both spoken and written
  - visual: including mathematical symbols, pictorial form and graphical form
  - combination of verbal and visual
- translate material from one form to another where appropriate
- transmit material in the following forms where appropriate:
  - verbal: both spoken and written
  - visual: including mathematical symbols, pictorial form and graphical form
  - combination of verbal and visual
- communicate mathematical results in a variety of forms appropriate for different audiences
- recognise the necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical context.

II MATHEMATICAL TECHNIQUES

Both dimensions included in this category involve developing **familiarity** with a variety of techniques and life-related applications.

DIMENSION

(a) LEARNED RESULTS AND MATHEMATICAL PROCEDURES

The student should be able to:

- recall definitions and results
- identify skills applicable to a familiar situation
- demonstrate facility in skills applicable to a familiar situation.

N.B. Situations must range from simple to complex and cover both life-related contexts and purely mathematical contexts.

DIMENSION

(b) USE OF INSTRUMENTS IN MATHEMATICS

The student should be able to:

- demonstrate an ability to select and use appropriate instruments such as scientific and graphics calculators, computers and selected software, measuring instruments, geometrical drawing instruments and tables
- use estimation as a means of checking whether results obtained from instruments are reasonable.

III MATHEMATICAL APPLICATIONS

The three dimensions included in this category involve applying mathematics in **unfamiliar** situations.

DIMENSION

(a) APPLYING MATHEMATICS IN LIFE-RELATED SITUATIONS (REAL OR SIMULATED)

The student should be able to:

- understand that a mathematical model is any mathematical representation of a situation
- identify the assumptions and variables of a simple mathematical model of a situation
- form a mathematical model of a life-related situation, over a variety of areas of application
- derive results from consideration of the mathematical model chosen for the particular situation
- interpret results from the mathematical model in terms of the given situation
- explore the strengths and limitations of a mathematical model
- develop a set of procedures to be used in approaching modelling problems
- make judgments on the validity of a mathematical model
- modify a model as a result of changed assumptions or of inconsistency with empirical data
- find approximate solutions to problems which cannot be solved analytically.

N.B. Situations mentioned above must range from simple to complex.

DIMENSION

(b) APPLYING MATHEMATICS IN PURELY MATHEMATICAL CONTEXTS

The student should be able to:

- interpret, clarify and analyse the problem
- use a range of problem solving skills including estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understand that there is often more than one way to solve a problem
- select the appropriate mathematical techniques required to solve a problem
- provide a solution consistent with the problem posed
- develop a set of procedures to be used in approaching problems set in purely mathematical contexts
- generalise a solution to a problem.

N.B. Problems mentioned above must range from simple to complex.

DIMENSION

(c) JUSTIFICATION

The student should be able to:

- develop logical arguments expressed in everyday language, mathematical language and a combination of both, where appropriate, to support conclusions and/or propositions
- evaluate the validity of arguments designed to convince others of the truth of propositions
- recognise when and why derived "solutions" to a given problem are clearly improbable or unreasonable
- recognise that one counter example is sufficient to disprove a generalisation

- recognise the effect of assumptions on the conclusions that can be reached

- decide whether it is valid to use a general result in a specific case

- recognise that a proof requires more than verification of a number of instances unless the instances are exhaustive

- use different methods of proof including mathematical induction and proof by contradiction.

4 ORGANISATION

4.1 INTRODUCTION

The syllabus contains both Core and Option topics. A course of study in Mathematics C must contain **all** Core topics and a minimum of **two** complete Option topics. Although some topics contain material which is required in other topics, the order in which they are presented **does not imply** a teaching sequence.

The **Core Topics** are:

- Introduction to Groups
- Real and Complex Number Systems
- Matrices and Applications
- Vectors and Applications
- Calculus
- Structures and Patterns

The **Option Topics** are:

- Linear Programming
- Plane Geometry
- Dynamics
- Introduction to Number Theory
- Probability and Statistics
- Advanced Periodic and Exponential Functions
- School Option

The Core and Option topics are discussed in detail in Section 5.

At different stages throughout the course, certain previously learned mathematical knowledge and skills will be required. While some have been identified and are listed under the heading Basic Mathematics in Appendix 1, others have been developed in Mathematics B. Time should be provided to revise these aspects within topics as they are required. Similarly, the maintenance of Mathematical Techniques is important in developing the student's capacity to deal readily with commonly occurring or familiar mathematical situations. This maintenance takes time and should be budgeted for in designing the sequence.

4.2 SELECTING THE TOPICS

Some choices of topics need to be made when a course of study is developed. The Option topics should be chosen so that they best suit the interests and needs of the particular cohort of students, the expertise and interests of the teaching staff and the resources of the school. This might mean that different topics are chosen for different classes within the one cohort, or that the topics chosen differ from year to year. If the school wishes to allow for this flexibility, the possibilities should be addressed within the work program.

All students completing four semesters of Mathematics C must study **all** Core topics and **two** complete Option topics. Additional work may be undertaken if desired; this could be chosen from a variety of areas including the other Option topics.

4.3 TIME ALLOCATION

The **minimum** time-tabled school time, including assessment, for this subject is **55 hours per semester.**

Notional times are given for each Core topic. These times are included as a guide and are not to be seen as prescriptive. **Approximately 30 hours** should be spent on a complete Option topic. All Option topics should be seen as equivalent with respect to level of difficulty.

4.4 SEQUENCING

Having chosen the topics to be included in the course of study, an **integrated sequence** should be developed which allows students to see links between the different topics rather than seeing them as discrete units. For example, Introduction to Groups can be used to provide a link between most of the Core topics. It provides a thread which runs through the Real and Complex Number Systems, Matrices and Applications, Vectors and Applications, and Structures and Patterns.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence but some topics include subject matter which is developed and extended in other topics. The school's sequence should be designed so that the subject matter in the topics is spiralled to allow students to internalise their knowledge before developing it further. Hence, it is generally not desirable to complete a topic in isolation.

The following guidelines for the sequencing of the topics should be referred to when developing a sequence for the course.

- No section of a topic should be studied before the appropriate prerequisite material contained in other topics.
- It may be appropriate to divide a topic into sub-topics.
- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered where appropriate.
- The course of study for a particular student **must not** include the topic Linear Programming in both Mathematics B and Mathematics C.
- Topics should be linked where possible.
- Sequencing may be constrained by a school's ability to provide physical resources.
- Within each topic, time will be needed for maintenance of previously learned mathematical knowledge and skills, and for the maintenance of Mathematical Techniques.

Regardless of the Option topics or sequence of topics chosen, it is paramount that each topic be considered in relation to the development of student performance within the general objectives.

5 TOPICS

5.1 INTRODUCTION

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

FOCUS

This section describes broadly the concepts which students should be encouraged to develop during the study of the topic and highlights the intent of the syllabus with respect to the topic.

SUBJECT MATTER

This section outlines the subject matter to be included in the topic. The subject matter is mandatory if the topic is included in the course of study. The order in which the subject matter is listed within a topic is not intended to imply a teaching sequence.

LEARNING EXPERIENCES

This section suggests some learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. Included are experiences which involve life-related applications of mathematics involving both real and simulated situations, use of instruments and the opportunity for problem solving. The listed learning experiences may require students to work individually, in small groups or as a class. The more traditional learning experiences have generally not been included but this does not mean that they should not be used where appropriate.

The learning experiences are suggestions only and are not to be seen as being prescriptive. Schools are encouraged to develop further learning experiences, especially those which relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of likely learning experiences that students will encounter should be shown in the work program and, in their totality, should enhance a balanced course of study.

5.2 THE CORE TOPICS

The order in which topics and items within topics are given should not be seen as implying a teaching sequence.

Introduction to Groups (notional time 7 hours)

Focus: Students are encouraged to investigate the structures and properties of groups. It is **intended** that this introduction to groups should provide a **basis** for identifying the **common features** which are found in systems such as real and complex numbers, matrices and vectors.

Subject Matter:

- concepts of:
  - closure
  - associativity
  - identity
  - inverse

- definition of a group.

Learning Experiences:

- find all groups of a given small order

- investigate the group structure of friezes, wall-papers or simple crystals by studying their symmetries under translations, rotations and reflections

- given a group table, find the element(s) which generate(s) the groups

- investigate when the integers modulo n form groups under addition or multiplication

- use a small Cayley table to determine whether a binary operation forms a group

- investigate subgroups of rotations of a square.

Real and Complex Number Systems (notional time 25 hours)

Focus: Students are encouraged to extend their knowledge of the real number system and to develop an understanding of the complex number system. Students should see the group structure within these systems as a link between the unfamiliar complex numbers and the familiar real numbers. Although most of the life-related applications of complex numbers are beyond the scope of this topic, students should be aware of the existence of these applications.

Subject Matter:

- structure of the real number system including:
  - rational numbers
  - irrational numbers

- simple manipulation of surds

- structure and representation of complex numbers including:
  - algebraic definitions and interpretations
  - geometric definitions (Argand diagram) and interpretations
  - polar form
  - conjugates

- de Moivre's Theorem

- simple, purely mathematical applications of complex numbers.

Learning Experiences:

- investigate some of the approximations to π which have been used

- use induction to prove De Moivre's Theorem

- use a proof by contradiction to show that $\sqrt{2}$ is irrational

- solve quadratic equations whose discriminant is negative

- investigate the use of complex conjugates in the solution of polynomial equations with real coefficients

- solve simple inequality statements, for example, $|z - a| > b$ in both the real and complex systems, and be able to give a verbal description of the meaning of the mathematical symbolism

- research transcendental numbers

- use polar forms to demonstrate multiplication and division of complex numbers

- use geometry to demonstrate the effect of addition, subtraction and multiplication of complex numbers

- solve simple equations involving powers of complex numbers

- most modern electronic keyboard instruments obtain the various notes by dividing the frequency of a master oscillator by some integer; this means that the ratios of the frequencies of successive notes is a rational number, rather than the value $2^{1/12}$ that it should be for an equally tempered scale; investigate the divisors used, and see how close the ratios of frequencies are to the desired value of $2^{1/12}$

- research areas in which complex numbers are used in life-related applications, for example, electric circuit theory, vibrating systems and aerofoil designs

- investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication

- research the Mandelbrot set

- find an appropriate part of the Mandelbrot set to use as the logo for the Mathematics department in the school

- use a computer to calculate the sequence of complex numbers z_n, where

$$z_n = \sum_{k=1}^{n} \exp\left(\frac{2\pi i k^3}{M}\right)$$

and n ranges from 1 to M; plot these complex numbers; join the origin to z_1, z_1 to z_2, z_2 to z_3, and so on, and finally join z_{M-1} to z_M; investigate the patterns for different integer values of M; after seeing what happens for some small values of M (up to 20, say); try using the year 1066 for M, then this year, then last year, then next year

- research quaternions.

Matrices and Applications (notional time 30 hours)

Focus: Students are encouraged to develop an understanding of the algebraic structure of matrices including situations where they form groups. Students should apply matrices in a variety of situations and be aware that advances in technology have facilitated the solution of large-scale problems involving matrices.

Subject Matter:

- definition of a matrix
- definition and properties of the identity matrix
- matrix operations
  - addition
  - transpose
  - multiplication by a scalar
  - multiplication by a matrix
- group properties of matrices
- determinant of a matrix
- singular and non-singular matrices
- solution of systems of homogeneous and non-homogeneous linear equations using matrices
- applications of matrices in both life-related and purely mathematical situations
- relationship between matrices and vectors.

Learning Experiences:

N.B. Many learning experiences in this topic are enhanced by the use of computer software or a calculator with matrix operations.

- solve linear equations by using matrices, Gaussian elimination or other methods
- investigate the use of matrices in dietary problems in health
- investigate the application of matrices to simple linear economic and statistical models, for example, using design analysis or regression, trade-off matrices

- investigate transition probability matrices, for example, over a period of time record the changes in major weather conditions as stated by newspaper or television weather reports (fine, showers, cloudy, clearing); construct the matrix describing the probabilities that one condition will be followed by each different condition; given today's weather and assuming there will be a change tomorrow, find the most probable sequence of weather changes in the near future

- research the use of Leslie matrices in ecology

- investigate the ways of ranking players in a chess tournament where each player meets every other player (a) once or (b) more than once; establish a matrix **A** which consists of the wins of each player against each of the other players; produce the vectors $\mathbf{w} = \mathbf{A}.\mathbf{1}$ and $\mathbf{l} = \mathbf{A}^T.\mathbf{1}$ where **1** is a column vector of 1's; explain the meaning of the terms in **w** and **l**; consider ways in which the quality of the opponents could be taken into account in order to break ties arising in the simple combination of **w** minus **l**; explore the meaning of the terms in the products $\mathbf{A}.\mathbf{w}$ and $\mathbf{A}^T.\mathbf{l}$

- investigate the use of rotation matrices as an application of 2 x 2 matrices to geometric transformations in the plane

- investigate the use of matrices in dominance problems, for example, in predicting the next round results (rankings) for the national netball competition

- investigate the use of matrices in game strategies

- investigate the use of matrices to code messages

- investigate the use of the Leontief Inverse in input-output analysis in economics

- investigate the application of matrices in formulating a mathematical model for a closed economic system

- research the use of matrices in describing stresses and deformations in solids

- research the use of matrices in linear programming

- research nilpotent matrices where the matrix, **A**, is nilpotent if it has the property $\mathbf{A}^2 = \mathbf{O}$, (**O** is the zero matrix)

- research idempotent matrices where the matrix, **A**, is idempotent if it has the property $\mathbf{A}^2 = \mathbf{A}$

- consider subsets of matrices forming a group under addition or multiplication

- show that the change of frame of reference used in Newtonian mechanics, $x'=x-vt$, $t'=t$, can be written using matrices

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}$$

- investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication.

Vectors and Applications (notional time 30 hours)

Focus: Students are encouraged to develop an understanding of vectors as entities which can be used to describe naturally occurring systems. They should also understand that the meaning of a vector comes from the situation and the model being considered. Students should become aware of the links between vectors and matrices. The emphasis should be on those vectors which describe situations involving magnitude and direction.

Subject Matter:

(a) for vectors as structures which are used for the storage of data

- definition of a vector (see Appendix 2)
- relationship between vectors and matrices
- operations on vectors including:
  - addition
  - multiplication by a scalar
- scalar product of two storage vectors (see Appendix 2)
- simple life-related applications of vectors

(b) for vectors describing situations involving magnitude and direction

- definition of a vector (see Appendix 2)

- relationship between vectors and matrices

- two and three dimensional vectors and their algebraic and geometric representation

- operations on vectors including:
  - addition
  - multiplication by a scalar

- scalar product of two vectors

- vector product of two vectors

- unit vectors

- resolution of vectors into components acting at right angles to each other

- calculation of the angle between two vectors

- applications of vectors in both life-related and purely mathematical situations.

Learning Experiences:

- show that the cost of your weekly shopping is the scalar product of your shopping list vector and the unit cost vector

- if **x** is a column vector of observed values, $x_1, x_2, \ldots, x_n$, show that

$$\mathbf{x}^{\mathrm{T}}\mathbf{x} = \sum_{i=1}^{n} x_i^2$$

- investigate the ways of ranking players in a chess tournament where each player meets every other player (a) once or (b) more than once; establish a matrix **A** which consists of the wins of each player against each of the other players; produce the vectors $\mathbf{w} = \mathbf{A}.\mathbf{1}$ and $\mathbf{l} = \mathbf{A}^{\mathrm{T}}.\mathbf{1}$ where **1** is a column vector of 1's; explain the meaning of the terms in **w** and **l**; consider ways in which the quality of the opponents could be taken into account in order to break ties arising in the simple combination of **w** minus **l**; explore the meaning of the terms in the products **A.w** and $\mathbf{A}^{\mathrm{T}}.\mathbf{l}$

- investigate the effect of current flow on steering a boat or a surf board

- investigate the use of vectors in surveying
- investigate the effect of wind on wind propelled craft
- consider the equilibrium of a body subject to a number of forces acting at a point
- investigate the placement of a television aerial mast and its wire supports on a roof of a building
- investigate the way medical staff use vectors to put a broken bone in suitable traction; consider the weights and angles of the ropes that are needed
- investigate the forces exerted by the hip, knee and ankle joints in pushing and pulling a bicycle pedal
- investigate the forces used in animal locomotion, for example, the difference between: a trotting horse and a pacing horse; a locust leaping and flying; the ground movements of birds which walk and those which hop; step aerobics, low impact aerobics and high impact aerobics
- use addition of vectors to see how the apparent motion of planets in the solar system depends on the frame of reference chosen
- research the effects of winds at different altitudes on the movement of weather frontal systems
- solve problems from geometry using vectors
- compare the use of Euclidean and vector methods in proving the concurrency of (a) the medians and (b) the bisectors of the internal angles of a triangle.

Calculus (notional time 30 hours)

Focus: Students are encouraged to extend their knowledge of analytical and numerical techniques of integration. Students should also gain further experience in applying differentiation and integration to both life-related and purely mathematical situations. They should appreciate the importance of differential equations in representing problems involving rates of change.

Subject Matter:

- integrals of the form

$$\int \frac{f'(x)}{f(x)}\,dx$$

$$\int f[g(x)].g'(x)\,dx$$

- simple integration by parts
- Simpson's rule
- approximating small changes in functions using derivatives
- life-related applications of simple, linear, first order differential equations with constant coefficients
- solution of simple, linear, first order differential equations with constant coefficients.

Learning Experiences:

- investigate the tolerance (small errors) in the volume of a soft drink can, produced by small errors in the diameter or height that could arise during manufacture
- use parameters to find the shortest distance from a given point to a curve
- investigate the existence of life-related situations that can be modelled by simple differential equations, for example, radioactive decay, cooling curves, concentration against time in chemistry, applications in electric circuits, carbon dating in archaeology, growth of bacteria, decrease of atmospheric pressure with altitude
- verify integrals in integral tables by differentiation of the result
- compare the accuracy of numerical techniques with analytical results for selected integrals

- use tables of integrals to evaluate an unknown integral
- use integration by parts to reduce an integral to one given in a table of integrals
- use numerical techniques to evaluate definite integrals where the indefinite integral cannot be found,for example,

$$\int_0^{\pi}\frac{\sin x}{x}dx$$

- research Stirling's formula
- investigate the varying volumes for the earth obtained when its shape is assumed to be (a) a sphere, and (b) an ellipsoid
- prove that Simpson's Rule is exact for polynomials of degree three or less
- investigate the motion of a falling dust particle, where air resistance is proportional to the velocity, by considering the differential equation

$$m\frac{dv}{dt} = mg - kv$$

- obtain a formula which can be used to calculate the area inside a closed curve described by polar coordinates, $r = f(\theta)$
- find an expression for the pressure, P, as a function of altitude in an isothermal atmosphere where the rate of decrease of atmospheric pressure with increasing altitude is proportional to the density of the air, ρ; pressure, density and temperature, T, are related by $P = R\rho T$, and R is a constant
- find an expression for the amount of the desired product Pu-239 present, as a function of time after start up in a breeder reactor where U-238 is converted to Pu-239 at a constant rate and Pu-239 is converted to Pu-240 at a rate proportional to the amount of Pu-239 present
- research formulae used for numerical integration
- research situations in which Monte Carlo methods have been used.

Structures and Patterns (notional time 30 hours)

Focus: Students are encouraged to develop their ability to recognise and use symmetries and patterns in a wide variety of situations. They should appreciate the value of symmetries and patterns in making generalisations to explain, simplify or extend their mathematical understanding. Justification of results is important and, where appropriate, results should be validated inductively or deductively. It is **not** intended that a great emphasis be placed on the algebraic manipulation of arithmetic progressions, geometric progressions, permutations or combinations.

Subject Matter:

- sum to infinity of a geometric progression
- life-related applications of arithmetic and geometric progressions
- sequences and series other than arithmetic and geometric
- permutations and combinations and their use in life-related situations
- recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence
- applications of patterns
- use of finite differences.

Learning Experiences:

- use finite difference methods to establish the formula for a sum, for example, of the first n positive integers, of the first n squares; use the principle of mathematical induction to prove the formula obtained
- search for patterns in Pascal's Triangle and verify any claims algebraically or otherwise
- investigate patterns in Fibonacci numbers
- investigate logarithmic spirals and polar curves which occur in nature, for example, in nautilus shells
- investigate permutations and combinations which arise in games of chance, for example, poker or Gold Lotto
- investigate the link between the binomial expansion, Pascal's Triangle and combinations

- investigate the possible number of recipients of a chain letter

- investigate the symmetry properties of some Escher drawings

- investigate the use of pattern analysis in the interpretation of remote sensing data, for example, grouping of frequency intensities in data from LANDSAT images

- investigate the use of patterns in creating experimental designs, for example, an experiment to compare six different cheeses which must be presented to a taste panel of six different people so that each person tastes all cheeses and all cheeses are being tasted at the same time; consider the variations necessary if the cheeses have an after-taste

- given a cube and six different colours, determine how many different ways the cube can be painted so that each face is a different colour; extend to other regular solids

- given a cube and six different colours, determine how many different ways the cube can be painted if more than one face can be painted the same colour; extend to other regular solids

- investigate the growth of a population and its impact on the environment, for example, how much wool is eaten by the offspring of one female moth who lays 300 eggs if each larva eats 15 milligrams of wool, two-thirds of eggs die, fifty per cent of remaining eggs are female and there are five generations per year

- the number of constants required to describe the behaviour of a crystal when heated or deformed depends on the symmetry of the crystal under rotations and/or reflections; investigate the way these symmetries are used to classify crystals

- use a calculator to investigate finite differences in determining polynomial coefficients

- use a computer to calculate the sequence of complex numbers z_n, where

$$z_n = \sum_{k=1}^{n} \exp\left(\frac{2\pi i k^3}{M}\right)$$

and n ranges from 1 to M; plot these complex numbers; join the origin to z_1, z_1 to z_2, z_2 to z_3, and so on, and finally join z_{M-1} to z_M; investigate the patterns for different integer values of M; after seeing what happens for some small values of M (up to 20, say); try using the year 1066 for M, then this year, then last year, then next year

- investigate the use of the inclusion-exclusion principle in counting cases; for example, determine how many individuals were interviewed in a survey of the eating habits of teenagers if 10 ate pizzas, 5 ate pies, 2 ate both and 7 ate neither; consider the effect of adding hamburgers as a third option

- apply the pigeonhole principle to solve problems, for example, in a group of 8 people, show that there are at least 2 whose birthdays fall on the same day of the week in any given year

- use symmetries to find the order of the groups of rotations of the five platonic solids

- prove Euler's Formula by induction

- investigate the occurrence of Fibonacci numbers in nature, for example, in spirals in sunflowers, pineapples and other plants

- research Penrose tiles

- research the four colour map colouring problem

- research the use of patterns in classifying like groups of individuals, for example, in DNA finger printing, in the construction of phylogenetic trees, in forensic science

- investigate the use of an arithmetic progression in the calculation of the length of batten material required for tiling a hip roof

- investigate the use of arithmetic progressions in the calculation of the total number of potatoes required for a "potato race" over a given distance if the distance between potatoes is a specified constant

- investigate the stacking of oranges in a supermarket

- use induction to prove formulae for the sums of series such as:

  $$1 + 2a + 3a^2 + \dots + na^{n-1} = \frac{1 - na^n}{1 - a} + \frac{a(1 - a^{n-1})}{(1 - a)^2}$$

  or

  $$\sin\theta + \sin 2\theta + \dots + \sin n\theta = \frac{\sin\frac{1}{2}n\theta . \sin\frac{1}{2}(n + 1)\theta}{\sin\frac{1}{2}\theta}$$

- research other algebraic systems such as semigroups, for example, the natural numbers under addition; or rings, for example, the integers under addition and multiplication.

5.3 THE OPTION TOPICS

Linear Programming

N.B. A student who studies both Mathematics B and Mathematics C must not study Linear Programming in both subjects.

Focus: Students are encouraged to develop an understanding of the methodology of Linear Programming and to see how it is used to solve problems in life-related situations. They should appreciate that graphical techniques of solution have limited applicability, but that other techniques can be used for the solution of problems with a large number of variables. Problems with non-integer solutions are not appropriate for this topic.

Subject Matter:

- recognition of the problem to be optimised (maximised or minimised)
- identification of variables, parameters and constraints
- construction of the linear objective function and constraints with associated parameters
- graphing linear functions associated with the constraints and identification of the regions defined by the constraints
- recognition that the region bounded by the constraints gives the feasible (possible) solutions
- recognition that different values of the objective function in two variables can be represented by a series of parallel lines
- use of a series of parallel lines to find the optimal value of the objective function in two variables (parallel or rolling ruler, graphical method)
- observation that the feasible region is always convex, and thus the optimal solutions occur at an edge or a corner point of the feasible region
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem
- relationship between algebraic and geometric aspects of problems with constraints in two and three dimensions
- use of the simplex algorithm to solve life-related problems in which maximal solutions are required, and all constraints place positive, upper bounds on the variables.

Learning Experiences:

- take a life-related problem given in English, formulate it into a linear programming problem, solve by the graphical method and interpret the solutions in terms of the original English problem

- investigate how linear programming is used to assist management decisions in areas such as manufacturing, transport, primary industries and environmental management

- consider optimal solutions of simple problems such as balancing diets

- use a computer graphing package to graph linear functions

- use parallel rulers to identify optimal solutions

- change parameters or constraints in a given problem and investigate the effect on optimal solutions

- consider the composition of a fleet of vehicles necessary to do a particular job at minimum cost

- solve a problem involving the allocation of two crops to the areas available on a farm in order to optimise profit when there are constraints on the labour and finances available; consider the allocation of three crops

- investigate the design of an optimal sized solar powered home which is to be competitive in the market place; constraints will include the size of solar cells, living area, cost of storage batteries and total cost of construction

- use a computer to find optimal solutions when the variables must have integer values

- research the history of linear programming

- formulate a two dimensional linear programming problem and write it in matrix form

- research techniques used for solving linear optimisation problems when the variables are restricted to integer values (integer programming)

- explain how the simplex algorithm systematically examines the vertices of the feasible region to determine the optimal solution.

Plane Geometry

Focus: Students are encouraged to extend their knowledge of geometry in two dimensions. They are encouraged to appreciate the interrelationships that exist between areas of mathematics. These relationships should be illustrated by applying Cartesian geometry and complex numbers to conics.

Subject Matter:

- concept of a locus

- loci defined by distance properties in Cartesian form including

  $$x^2 + y^2 = a^2$$

  $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

  $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

  $$xy = c$$

  $$y^2 = 4ax$$

  where a, b, and c are constants

- loci defined by distance properties in complex number form including

  $$|z - b| = c$$

  $$|z - a| \pm |z - b| = c$$

  where a, b, and c are constants

- polar co-ordinate form of conics (see Appendix 2)

- parametric form of conics including

  $$x = a\cos\theta,\ y = a\sin\theta$$

  $$x = a\cos\theta,\ y = b\sin\theta$$

  $$x = \frac{a}{\cos\theta},\ y = b\tan\theta$$

  $$x = t,\ y = \frac{c}{t}$$

  $$x = at^2,\ y = 2at$$

  where a, b, and c are constants

- simple applications of conics.

Learning Experiences:

- investigate how the properties of ellipses are used in whispering rooms
- investigate the shape on the ground of the leading edge of the sonic boom produced by a supersonic aircraft flying at high altitudes
- discuss which form of a conic is best suited for investigating a particular situation
- use a computer package or a graphics calculator to plot a given curve parametrically
- plot polar curves using a graphics calculator or a computer package
- investigate why parabolic reflectors are used in astronomical telescopes, hand held torches and microwave repeater stations
- use the polar form of conics to investigate Kepler's Laws, for example, in determining the orbit of Halley's Comet
- examine ways in which a Cartesian co-ordinate system can be chosen to simplify problems without loss of generality, for example, using translation of the origin
- investigate how to construct the elliptical hole required to be cut out of a sloping roof to fit a vertical cylindrical vent
- research the use of elliptic reflectors in the treatment for kidney stones
- research how hyperbolas are used in the Omega navigation system.

Dynamics

Focus: Students are encouraged to develop an understanding of the motion of objects which are subjected to forces. The approach used throughout this topic should bring together concepts from both vectors and calculus.

Subject Matter:

- derivatives of vectors
- Newton's Laws of Motion in vector form

- application of the above to:
  - straight line motion in a horizontal plane with variable force
  - vertical motion under gravity with and without air resistance
  - projectile motion with and without air resistance
  - simple harmonic motion
  - circular motion with uniform angular velocity.

Learning Experiences:

- model vertical motion under gravity, and investigate the effects of drag on the motion, for example, linear drag

- investigate motion under Hooke's Law

- investigate the flow of water from a hose held at varying angles, and model the path of the water

- prove Kepler's Laws for a planet in a circular orbit

- investigate the motion of a simple pendulum with varying amplitudes

- use a computer program to investigate the motion of a spacecraft in the gravitational field of the earth and the moon

- from a table of vehicle stopping distances from various speeds, calculate (a) the reaction time of the driver and (b) the deceleration of the vehicle, which were assumed in the calculation of the table

- model the path of a projectile with and without air resistance, for example, use a video to study the actual path of a shot put, table tennis ball or netball

- investigate the angle of lean required by a motor cycle rider to negotiate a corner at various speeds; consider the implication of camber of the road

- investigate the motion of a falling dust particle, where air resistance is proportional to the velocity, by considering the differential equation

  $$m\frac{dv}{dt} = mg - kv$$

- if the velocity, v, of a particle moving in a straight line is given as a function of the distance, x, use the chain rule to show that the acceleration can be written as

  $$a = v\frac{dv}{dx} = \frac{d}{dx}(\frac{1}{2}v^2)$$

- a skyrocket is fired vertically upwards with initial velocity, V; if it experiences air resistance proportional to the square of its velocity, find the maximum height reached where the velocity, v, satisfies the equation

$$ma = -mg - kv^2 \quad \text{where } a = v\frac{dv}{dx} = \frac{d}{dx}(\frac{1}{2}v^2)$$

and x is the vertical displacement

- investigate the speed required for a projectile launched vertically to escape from the earth's gravitational field, ignoring air resistance but including the variation of gravitational attraction with distance

- research the viewpoints on motion of Aristotle, Newton and Einstein; identify the assumptions of each.

Introduction to Number Theory

Focus: Students are encouraged to extend their knowledge of the properties of integers, and to appreciate the usefulness of apparently abstract mathematics. They are also encouraged to gain an understanding of the power of congruence in solving problems involving integers.

Subject Matter:

- primes, composites and the Fundamental Theorem of Arithmetic

- divisors, Euclidean algorithm, lowest common multiples (LCM) and greatest common divisors (GCD)

- modular arithmetic

- congruence including simple simultaneous congruence

- simple Diophantine equations.

Learning Experiences:

- consider conditions on the integers a, b and r under which $ax + by = r$ is solvable for the integers x and y

- develop tests for divisibility by 2, 2^n, 3, 3^2, 5, 5^n, 7, 11, 13, 1001

- investigate the existence of inverses under multiplication modulo n

- solve some simple Diophantine equations

- use modular arithmetic to prove/disprove propositions about integers

- research Fermat, Mersenne, perfect, abundant and/or amicable numbers

- research the use of congruence in ciphering methods

- investigate the period of the operation $x_{n+1} = ax_n + b \bmod c$ for generating random numbers

- research interesting facts about numbers and create problems for others to solve, for example, "The sum of the divisors of 7^3 is a square number. Can you find any other numbers that fit this condition?" (400 is one such number)

- investigate facts about prime numbers

- investigate the way modular arithmetic is used to calculate the date of Easter

- from a table of prime numbers, multiply the number of prime numbers less than an integer, n, by the logarithm of n, and compare the result with n; do this for several values of n

- investigate greatest common divisors among pairs of Fibonacci numbers

- investigate for which integers (a, b) a number of the form $a^b - 1$ can be a prime

- investigate for which integers (a, b) a number of the form $a^b + 1$ can be a prime

- investigate prime numbers of the form $a^2 + b$, for various values of b

- investigate Pythagorean triples

- investigate the representation of numbers by sums of squares

- explain Fermat's Little Theorem and/or the Chinese Remainder Theorem

- research Goldbach's conjecture

- research Fermat's Last Theorem

- research different proofs that the number of primes is infinite

- in ancient Egypt calculations were done using only rational numbers with 1 as the numerator; show that any rational number p/q can be rewritten as the sum of not more than p distinct rational numbers, each of which has 1 as the numerator

- investigate the use of modular arithmetic in the construction of Latin squares in the design of experiments.

Probability and Statistics

Focus: Students are encouraged to develop an understanding of the basic concepts of expectation, deviation and probability distributions. They should develop a working knowledge of the way in which theoretical probability is used for decision making in a variety of life-related situations. It is **not** intended that a great emphasis should be placed on simple substitution into formulae.

Subject Matter:

- dependent and independent events
- expected value of a random variable
- conditional probabilities
- concept of degrees of freedom
- t-distribution
- testing of hypotheses using the t-test
- estimation and confidence intervals
- Poisson distribution.

Learning Experiences:

- investigate whether a given distribution is likely to be rectangular, binomial or Poisson; consider situations where the decision will be based on theoretical considerations and also situations which rely on the assessment of empirical data

- in medical diagnosis a doctor must determine the most likely illness of a patient, given a number of observable symptoms; explain how conditional probability is used by the doctor in reaching a decision

- consider whether the "home ground advantage" exists as a significant factor influencing the performance of an individual or a team; follow the weekend results given in the newspaper or on TV for some sport over a period of a number of weeks and justify your ideas

- investigate the use of probabilities by airlines in making bookings for flights

- calculate the probability that two or more events will happen if (a) they are independent and (b) they are not independent, for example, getting two flat tyres on the same car trip, a family having four children of the same gender, being late for an appointment on three successive days, being late for three consecutive appointments on the one day, that it will rain on four consecutive days

- examine the probabilities associated with different methods of determining the winning team in a sports contest

- in a round robin tournament where every player plays every other player, the winner is often identified by adding together the number of "wins" which may result in a tied outcome; investigate ways in which ties between players resulting from this method of adding wins can be resolved

- use contingency table data to explore conditional probability empirically, for example, construct a table of hair colour by eye colour for the students in the class, relate number of hours spent studying to the level of achievement in a number of subjects

- use contingency table data to explore conditional probability to relate stock market rises with other world events and/or weather conditions

- investigate the bias in using the population formula for calculating the standard deviation when the data represent a sample; have each student in the class select a sample from a population which you have found or constructed through simulation and which has a known mean and standard deviation; pool the sample estimates from each student to calculate the expected value of the population parameters; compare the expectations with the known population parameters; consider the effect on the estimations for different sample sizes

- examine opinion poll figures and interpretations in the newspapers to see how conditional probability is used

- explain the concept of degrees of freedom in a number of ways, for example, as a limitation on how many numbers can be chosen at random when the mean of all the numbers is fixed, the number of independent comparisons which can be made between the ages of a number of people, the frequencies in the cells of a contingency table

- use empirical data to demonstrate the rules of expected values: $E(c) = c$, $E(cx) = cE(x)$ and $E(a + cx) = a + cE(x)$, where a and c are constants

- investigate the expected returns (or losses) on the various lotteries available to the public from newsagencies

- compare the t-distribution with the normal distribution; consider the effect of varying degrees of freedom

- investigate the use of conditional probabilities in management strategies; for example, consider how farmers use the information from the computer software package RAINMAN developed by the Queensland Department of Primary Industries

- investigate the use of conditional probabilities in the use of "odds" in investments

- create a simulated population of data from a normal distribution with known mean; have each student select a random sample and calculate a 95% confidence interval for the true mean using the t-distribution; compare the estimated ranges with the known mean; vary the level of confidence to 90% and discuss the meaning of any differences in the results

- investigate the use of confidence intervals by NASA in its space exploration program, particularly with respect to moon landings

- as a class exercise have students measure some aspect and compare their estimated mean with a known standard, for example, the resting pulse rate for healthy teenagers is 60 pulses per minute, the air temperature is usually greater than the temperature of water

- investigate the occurrence in life-related situations of data which have a Poisson distribution, for example, the occurrence of bacteria in export products such as butter and beef, the distribution of cane toads around a water source, the occurrence of ciguatera in fish fillets sold at fish shops

- investigate how confidence intervals are used by manufacturers to determine "use by" dates

- formulate the statistical hypotheses appropriate for various situations, for example, to test whether or not people can distinguish between the tastes of different colas, to determine the time taken for a fly to die after it is exposed to fly spray, to test the effects of a new drug

- collect (either by direct measurement or from a written source) a sample of data to estimate a true mean, for example, the pulse rates of students after ten minutes of activity, the half hourly temperatures from sunrise to sunset, the daily rainfalls at a particular location over a month; use a statistical software package to calculate a 95% confidence interval within which the true mean will be expected to lie; investigate the effects of varying the level of confidence, for example, try 90%, 75% and 50%, on the length of the interval

- use a statistical software package to explore a sample of empirical data collected to test an hypothesis about a mean and to carry out a *t*-test, for example, use data from the class to test the hypothesis that the mean weekly part-time earnings of students in their final year of school is $20, collect daily temperatures (or rainfall) from the newspaper to test the hypothesis that the mean daily maximum temperature (or rainfall) in a particular month in your local area is 22°C (15mm)

- use a statistical software package to create samples of simulated data with different distributions, for example, a uniform distribution, a normal distribution, or a Poisson distribution.

Advanced Periodic and Exponential Functions

Focus: Students are encouraged to extend their knowledge of trigonometric and exponential functions. They are also encouraged to investigate various ways these can be combined to model life-related situations.

Subject Matter:

- definitions of secant, cosecant, cotangent

- expansions of $\sin(x \pm y)$, $\cos(x \pm y)$

- life-related applications of the sine and cosine functions

- general shapes of graphs of $y = e^{ax}\sin bx$ and $y = e^{ax}\cos bx$ where a is both positive and negative

- applications of $e^{ax}\sin bx$ and $e^{ax}\cos bx$

- logistic curve $y = \dfrac{A}{1 + Ce^{-kt}}$ (where A, C and k are constants) as a model of many natural systems.

Learning Experiences:

- express $y = a \cos x + b \sin x$ in the form $A \cos(x + C)$ or $A \sin(x + D)$ and graph the curve

- the unmodulated waveform from a radio or TV transmitter can be described by $x = a \sin bt$; this is changed when broadcasting music; for AM broadcasting the constant a is modified to $a + c \sin ft$; for FM broadcasting the constant b is modified to $b + c \sin ft$; use a computer package to plot these waveforms, and relate the parameters involved to the frequency of transmission, and the pitch and loudness of the sound

- investigate the use of rotation matrices to show the identities for $\sin(x \pm y)$, $\cos(x \pm y)$

- use complex numbers to evaluate

$$\int e^{ax} \sin bx \, dx \text{ or } \int e^{ax} \cos bx \, dx$$

- investigate musical notes as a combination of sine functions

- investigate the period of a pendulum for large angles

- investigate the motion of a water wheel, or a ferris wheel, as an example of a mathematical model based on a sinusoidal function

- investigate the motion of an orbiting satellite as an example of a situation that can be modelled by a sinusoidal function

- investigate systems which follow the logistic curve, for example, biological growth with environmental constraints, the spread of disease or rumour, death of individuals with increasing dose rates of a toxic substance, concentration of end product in a chemical reaction

- verify that the differential equation

$$\frac{dy}{dt} = \frac{k}{A}y(A - y) \text{ where } y = \frac{A}{1 + C} \text{ at } t = 0$$

has the logistic equation $y = \dfrac{A}{1 + Ce^{-kt}}$ as its solution

- use Australian Census data for your city, town or shire to fit the logistic curve for population growth $p(t) = \dfrac{L}{1 + Ce^{-r(t - t_0)}}$ where $p(t)$ is the population at time t, t_0 is some convenient starting date, and L, C and r are parameters to be estimated

- use a computer or a graphics calculator to investigate the graphs of sec x, cosec x and cot x, and relate these to the graphs of $\sin x$, $\tan x$ and $\cos x$

- use a computer or a graphics calculator to draw the graphs of

$$y = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3}$$
$$y = \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5}$$
$$y = \cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2}$$
$$y = \sin x + \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3}$$

- large amplitude waves sometimes appear on the ocean with no apparent cause; they arise from the superposition of a number of waves of different amplitudes and wavelengths; use computer software or a graphics calculator to investigate the result of adding various sine waves with different wavelengths and amplitudes

- investigate the periodic patterns which are traced out by a point on the circumference of a circle which rolls around the inside or outside of a larger circle; use polar coordinates to write an expression for the curve

- watch a video of the collapse of the Tacoma Narrows Bridge and identify some oscillations with increasing amplitude (see PSSC Physics film)

- use a video of a mass oscillating on the end of a spring to see how closely the position of the mass follows an expression of the form $e^{at} \sin bt$, where a is negative

- surfers believe that ocean waves occur in "sets" of seven, with the seventh wave being "larger" than the preceding six waves; use a computer or graphics calculator to find functions whose graphs represent these sets of waves

- study the vertical motion of a car on its springs; use a video to see whether the motion is the sum of two exponential functions, Ae^{at} and Be^{bt}, where a and b are negative, or whether it consists of a decaying oscillation of the form $e^{at} \sin bt$, where a is negative

- all the heavy elements were formed in a supernova explosion which preceded the formation of the solar system; assuming that the two major isotopes of uranium, U-238 and U-235, were then formed in equal amounts, use their present relative abundances and half-lives to estimate the time at which the supernova explosion took place (a suitable unit for time is a thousand million years)

- research Fourier series

- investigate how the description of periodic phenomena in electric circuits uses complex numbers

- develop multiple angle trigonometric formulae using de Moivre's Theorem.

SCHOOL OPTION

Schools may develop a single option topic of their own choice subject to the guidelines below:

- the school option must be consistent with the Rationale and Global Aims of the course

- the school option offered to a student must have subject matter substantially different from any other subject matter that the student will meet in Mathematics B and Mathematics C

- the school option should contain subject matter ensuring a level of challenge comparable to that provided by the other extension topics

- the school option must **not** consist of a combination of subject matter from other topics within this syllabus

- in addition to specifying the subject matter, the work program will need to indicate a focus and state the range of likely learning experiences

- the school option should assist in providing the student with experiences in a balance of branches of mathematics

- while the use of computer packages is encouraged, the study of computer programming language(s) is not appropriate in a school option.

6 ASSESSMENT

6.1 INTRODUCTION

FORMATIVE ASSESSMENT

Formative assessment is a process which provides teachers, students and parents with information on the performance of students within the criteria and their dimensions with a view to assisting students to improve their understanding and achievement. The formative assessment should be both informal and formal in nature.

The formal techniques used should be similar to the summative assessment techniques which students will meet later in the course so that they have experience in responding to particular types of tasks under the appropriate conditions. The information gathered from this type of assessment is used when making decisions for reporting and monitoring purposes and therefore should be included in monitoring submissions.

SUMMATIVE ASSESSMENT

Summative assessment is intended to assist teachers in determining the exit Levels of Achievement of students. It is designed to supply information for reporting and certification at the end of a course of study. Accordingly, work programs must specify the range of summative assessment instruments which will be used, when they will be administered and how they will contribute to determining exit Levels of Achievement.

The following underlying principles should be addressed when planning assessment in this subject.

- Assessment is concerned with determining the extent to which students meet the general objectives of Communication, Mathematical Techniques and Mathematical Applications.

- Assessment of student achievement should not be seen as a separate entity, but as an integral part of the developmental learning process. An effective course of study includes a variety of learning experiences. Therefore, a range of assessment techniques which reflect the learning experiences of the students needs to be employed in gathering assessment data.

- The overall assessment plan should be devised so that the fullest and latest information on a student's achievement in the course of study may be obtained. Information should be gathered through a process of continuous assessment.

- Assessment tasks and subsequent allocation of exit Levels of Achievement should address the significant aspects of the course of study identified in both the syllabus and the school's work program. Time spent on topics should be reflected in the scope and depth of the assessment.

The assessment should reflect a balance over the course of study. This balance need not necessarily be achieved within every semester but should be discernible over the summative assessment.

6.2 CRITERIA FOR AWARDING EXIT LEVELS OF ACHIEVEMENT

At exit from the course of study, each student will be awarded one of five Levels of Achievement:

Very High Achievement (VHA)
High Achievement (HA)
Sound Achievement (SA)
Limited Achievement (LA)
Very Limited Achievement (VLA).

Student performance will be judged on the following three criteria which match the categories defined in section 3.1.

CRITERION I: COMMUNICATION

CRITERION II: MATHEMATICAL TECHNIQUES

which incorporates the dimensions:

(a) Learned Results and Mathematical Procedures
(b) Use of Instruments in Mathematics.

CRITERION III: MATHEMATICAL APPLICATIONS

which incorporates the dimensions:

(a) Applying Mathematics in Life-related Situations (Real or Simulated)
(b) Applying Mathematics in Purely Mathematical Contexts
(c) Justification.

6.3 ASSESSMENT WITHIN THE CRITERIA

Assessment tasks should be designed to allow students to demonstrate their ability in all the criteria including all their dimensions. It is not necessary for each assessment task to address every criterion, but all three criteria must be represented in assessment data that contribute to the awarding of exit Levels of Achievement.

Some points to consider when assessing student performance on the criteria are given below.

CRITERION I: COMMUNICATION

As the two-way processes of communication are an integral part of everything we do in mathematics, it is difficult, and indeed not desirable, to assess student performance on this criterion independently. Information on student achievement in this criterion should be collected by a global consideration of the Communication skills evident in responding to tasks used to assess student performance in Mathematical Techniques and Mathematical Applications. The types of assessment instruments used to assess student performance on Criteria II and III must, therefore, be considered very carefully to ensure that students have the opportunity to demonstrate that they have achieved the Communication objectives.

CRITERION II: MATHEMATICAL TECHNIQUES

This criterion focuses on the recall of results and procedures, use of instruments and the application of all of these in **familiar situations.** It is not necessary to record information in each of the associated dimensions, but the assessment package must enable students to demonstrate their ability in the general objectives of both dimensions.

The assessment package must include:

- items which range from simple to complex
- items which require the appropriate use of a variety of instruments in mathematics
- items which involve applying mathematics in familiar, life-related situations as well as items which are presented in a purely mathematical context
- items which allow students to demonstrate that they have achieved the Communication objectives.

CRITERION III: MATHEMATICAL APPLICATIONS

This criterion involves applying mathematics in **unfamiliar situations.** It is not necessary to have separate items to assess Justification - the objectives of this dimension should be achieved through the applications in both life-related situations and purely mathematical situations. Recording information in each of the associated dimensions is not necessary, but the assessment package must enable students to demonstrate their ability in the general objectives of all three dimensions.

The assessment package must include:

- items which range from simple to complex

- items which allow students to demonstrate their ability in the objectives of all dimensions; it is not necessary that all dimensions appear in an individual item

- items which allow students to demonstrate that they have achieved the Communication objectives.

In summary, work programs will need to outline a balanced assessment program including a variety of assessment techniques which have validity in assessing achievement of the General Objectives.

6.4 ASSESSMENT TECHNIQUES

Assessment techniques other than traditional written tests or examinations are to be included in the assessment program at least twice each year.

A variety of assessment techniques is to be used and might include:

- written traditional tasks
    - multiple choice
    - short answer
    - extended response (mainly symbolic)

- other written tasks
    - extended response (prose)
    - reports on :
        - projects
        - investigations
        - group work
        - modelling tasks
        - research topics

- oral tasks
    - short answer
    - seminar presentation
    - debates
    - hypotheticals

- practical tasks using instruments.

6.5 RECORDING INFORMATION

Information on student achievement in each criterion may be recorded in various ways. Teachers may use marks, standards derived from verbal descriptors or marks, or a combination of these. However, the methods of recording and the frequency with which records will be updated must be clearly outlined in the work program.

6.6 STANDARDS FOR DETERMINING EXIT LEVELS OF ACHIEVEMENT

In awarding an exit Level of Achievement for each criterion, a student's performance over the course of study is to be compared with the standards expressed in the following tables.

CRITERION I: COMMUNICATION

STANDARD	DESCRIPTOR
A	The student has communicated clearly and concisely using appropriate terminology and appropriate forms of presentation, while consistently adhering to the conventions of language and mathematics.
B	The student has communicated clearly using appropriate terminology and appropriate forms of presentation, while generally adhering to the conventions of language and mathematics.
C	The student has generally communicated satisfactorily, while generally adhering to the **basic** conventions of language and mathematics.
D	The student has communicated inappropriately and inadequately.

CRITERION II: MATHEMATICAL TECHNIQUES

STANDARD	DESCRIPTOR
A	The student has an extensive knowledge of Mathematical Techniques and is consistently accurate and proficient when applying them to familiar situations.
B	The student has a substantial knowledge of Mathematical Techniques and is generally accurate and proficient when applying them to familiar situations.
C	The student has a working knowledge of Mathematical Techniques and is generally accurate and proficient when applying them to familiar situations.
D	The student has some knowledge of Mathematical Techniques and is occasionally accurate and proficient when applying them to familiar situations.
E	The student has little knowledge of Mathematical Techniques and is rarely accurate and proficient when applying them to familiar situations.
N.B. The situations mentioned above must range from simple to complex in both life-related and purely mathematical contexts.	

CRITERION III: MATHEMATICAL APPLICATIONS

STANDARD	DESCRIPTOR
A	The student has consistently provided solutions in simple, unfamiliar situations, and generally provided solutions in complex, unfamiliar situations. The student has, within the above situations, consistently determined the appropriateness of results and consistently developed logical arguments. The student has consistently tested the validity of arguments which use mathematics.
B	The student has generally provided solutions in simple, unfamiliar situations, and occasionally provided solutions in complex, unfamiliar situations. The student has, within the above situations, generally determined the appropriateness of results and generally developed logical arguments. The student has generally tested the validity of arguments which use mathematics.
C	The student has occasionally provided solutions in simple, unfamiliar situations. The student has, within the above situations, generally determined the appropriateness of results and attempted to develop logical arguments. The student has considered the validity of arguments which use mathematics.
D	The student has rarely provided solutions in unfamiliar situations.
N.B. The situations mentioned above must be set in both life-related and purely mathematical contexts.	

6.7 DETERMINATION OF EXIT LEVELS OF ACHIEVEMENT

On completion of the course of study, the school must award, to each student, a standard for each of the three criteria using the tables in section 6.6. These three standards are then used to determine a student's exit Level of Achievement.

The table below gives the **minimum** performance composite of standards required for each Level of Achievement.

Minimum Standards Table

LEVEL OF ACHIEVEMENT	COMMUNICATION	MATHEMATICAL TECHNIQUES	MATHEMATICAL APPLICATIONS
VERY HIGH	B	A	A
HIGH	C	B	B
SOUND	C	C	D
LIMITED	D	D	D
VERY LIMITED	D	E	D

A given Level of Achievement can only be awarded to a student who achieves the required **minimum standard in all three criteria.** Hence it is imperative that students are continually made aware of any need to improve in the dimensions of a particular criterion where inconsistencies across the criteria are developing.

7 DEVELOPING A WORK PROGRAM

The work program is a formal expression of the school's interpretation of this syllabus. It has three primary functions. Firstly, it provides guidance to the teachers of the subject as to the nature and requirements of the Mathematics C program at the school. Secondly, it provides similar guidance to the school's students, and their parents, in relation to the subject matter to be studied and how achievement of the program's objectives will be assessed. Thirdly, it provides a basis for accreditation by the Board of Senior Secondary School Studies for the purposes of including students' results for the subject on their Senior Certificates.

The school's work program should be a document which does not require reference to other documents in order to be understood. The work program must contain the following components.

- **Table of Contents** - to increase the readability of the document. This will mean that pages must be numbered.

- **Rationale** - which provides justification for including the subject in the school curriculum. The rationale may be derived principally from the syllabus statement but should also include information on the school's philosophy, student population, resources and any other factors which may influence the decisions made in designing a program of study to cater for the special characteristics of the school and its students.

- **Global Aims** - which are statements of the long term achievements, attitudes and values that are to be developed by the students in studying the subject but which are not directly assessed by the school. These should be the Global Aims listed in this syllabus.

- **General Objectives** - as indicated in this syllabus.

- **Course Organisation** - which shows:

  - a summary of the integrated sequence developed by the school to give an overview of the course to be offered; time allocations should be included

  - details of the integrated sequence indicating the scope and depth of the subject matter to be taught.

  The sequence must be developed in accordance with Section 4 of this syllabus.

  If the school wishes to take the opportunity to offer different topics to different groups of students, the possibilities should be addressed in this section.

A statement should be included by the school indicating that due consideration has been given to the issues associated with the Ministerial statement on Educational Equity included in all syllabuses. The Ministerial statement has implications for the course organisation including the selection of learning experiences and assessment techniques.

- **Learning Experiences** - which are appropriate to the age group concerned and consistent with the objectives of the work program. A variety of learning experiences should be included which, in their totality, should enhance a balanced course of study. The learning experiences may be integrated into the Course Organisation section.

- **Assessment** - including:

  - an assessment plan which:

    - indicates the **formal formative** and the **summative techniques** used to gather data on student performance, the criteria associated with tasks and the conditions of implementation of tasks

    - provides a balance with respect to techniques used and conditions applied and must ensure that sufficient information is gathered on achievement in each criterion to make valid judgments

    - clearly indicates the summative tasks which will be included in the review folio; data gathered as a result of implementing the assessment plan should allow fullest and latest information to determine exit Levels of Achievement

    - allows sufficient information to be available for the recommendation of interim Levels of Achievement, for example, for Monitoring purposes

  - the standards (either numerical or verbal) used in the school to record assessment data and how that data will be recorded

  - an example of the individual student profile to be included in the review folio

  - how assessment data are combined to reach an overall standard in each criterion (a completed student profile may clarify the explanation). Fullest and latest information is not obtained by an arbitrary "weighting" of semesters or by using semester 1 assessment instruments as summative in a well sequenced course

- the procedure for awarding exit Levels of Achievement which are consistent with the criteria and standards of this syllabus (Section 6)

- sample assessment items including:

  - at least one Criterion II task which must allow the assessment of student performance on a range of objectives from each of the dimensions of the criterion as well as some of the objectives from the Communication criterion

  - at least one Criterion III task which must allow the assessment of student performance on a range of objectives from each of the dimensions (a) and (c) of the criterion as well as some of the objectives from the Communication criterion

  - at least one Criterion III task which must allow the assessment of student performance on a range of objectives from each of the dimensions (b) and (c) of the criterion as well as some of the objectives from the Communication criterion.

The Minister for Education has supplied the following statement and requested it be inserted in this section of the syllabus.

EDUCATIONAL EQUITY

Schools should provide a curriculum which in subject matter, language, methodology, learning experience and assessment instruments meets the educational needs and entitlements of all students and, in particular, of girls as well as boys, of Aboriginal and Torres Strait Islander students, of students who are from Non-English Speaking Backgrounds and of students who have an intellectual and/or physical impairment. Care should be taken by schools to ensure that the curriculum content recognises the contributions of women to society, that it is culturally inclusive of the activities and achievements of Aboriginal, Torres Strait Islander and Non-English Speaking Background people and that it takes every opportunity to draw students' attention to positive examples of people with intellectual and/or physical impairments.

In order to achieve more equitable educational outcomes by meeting these requirements, educators are asked to consider the following when constructing work programs:

1. Gender issues to be considered

- gender inclusive curriculum;
- the contributions of women;
- the life experiences of women and girls;
- non-stereotyped non-sexist print and non-print resources;
- teaching approaches and learning experiences that suit girls as well as boys;
- equitable access to and experience in practical exercises in co-educational classes;
- assessment plans that suit girls as well as boys;
- the utilisation of a range of single-sex and co-educational settings; and
- a school environment which supports and encourages girls' active participation and achievements in all aspects of the educational program available.

2. Aboriginal and Torres Strait Islander issues to be considered

- curriculum inclusive of Aboriginal and Torres Strait Islander people;
- the contributions of Aboriginal and Torres Strait Islander people;
- the life experiences of Aboriginal and Torres Strait Islander people, including the impact of the non-Aboriginal settlement of Australia;
- non-stereotyped and non-racist print and non-print resources;
- teaching approaches and learning experiences that suit Aboriginal and Torres Strait Islander students;
- equitable access to curriculum choices;
- curriculum content which reflects the diversity amongst Aboriginal and Torres Strait Islander people and, in particular, the differences between traditional and non-traditional communities;
- assessment plans which suit Aboriginal and Torres Strait Islander students; and

- a school environment which supports and encourages the active participation and achievements of Aboriginal and Torres Strait Islander students in all aspects of the educational program available.

3. **Non-English Speaking Background issues to be considered**

   - culturally inclusive curriculum;
   - a school environment which supports and encourages the active participation and achievements of students from Non-English Speaking Backgrounds in all aspects of the educational program available;
   - teaching approaches and learning experiences which suit students from Non-English Speaking Backgrounds and which meet their needs for English language development support;
   - non-stereotyped non-racist print and non-print resources;
   - recognition of bicultural and bilingual skills and experiences of people from Non-English Speaking Backgrounds, including the impact of immigration; and
   - assessment plans which suit students from Non-English Speaking Backgrounds.

4. **Issues to be considered with respect to intellectual and/or physical impairments**

   - curriculum inclusive of those with intellectual and/or physical impairment;
   - curriculum appropriate to the needs of intellectually and/or physically impaired learners within the context of local schools;
   - positive examples of activities and achievements of people with intellectual and/or physical impairments;
   - non-stereotyped print and non-print resources which avoid imposing notions of physical perfection;
   - recognition of the skills and abilities of people with intellectual and/or physical impairments;
   - teaching approaches and learning experiences which suit students with intellectual and/or physical impairments and which cater for any resource and support needs they may have;
   - a school environment which supports and encourages the active participation and achievements of students with intellectual and/or physical impairments in all aspects of the educational program available; and
   - utilisation of special needs settings as well as integrated settings.

Educators may find the following publications useful for devising an inclusive work program.

1. A Fair Deal, Equity Guidelines for developing and reviewing educational resources. Department of Education, Queensland, 1991.
2. Anti-racism: a handbook for adult educators. Human Rights Commission Education Series No. 1. AGPS. Canberra 1986.

8 REVIEW FOLIOS

A Review folio is a collection of student's responses to assessment items on which a student's Level of Achievement is based. Each folio should contain a variety of assessment techniques demonstrating performance in the three criteria, Communication, Mathematical Techniques and Mathematical Applications over the range of topics including the options. This variety of assessment techniques is necessary to provide information which is not solely dependent on the student's being familiar or comfortable with a single assessment technique.

It is necessary that a student's achievement in the three criteria is monitored throughout the course so that appropriate feedback in terms of the criteria and the dimensions is provided to the student. The review folio is an ideal medium for students and teachers to monitor progress throughout the course.

For Certification purposes, schools must submit student folios which contain:

- student achievement data profiled in the three exit criteria
- the student responses to **all** summative assessment instruments (In the case of non-written responses, the minimum requirement will be a completed student criterion sheet(s) for that assessment.)
- a minimum of four instruments from year 12 with at least one of these being non-traditional (for example, project, oral presentation, seminar, group work).

A review folio must consist of a minimum of 4 to a maximum of 14 pieces of summative work.

APPENDIX 1 Basic Mathematics

The following knowledge and skills will be required throughout the course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- identities, linear equations and inequations
- the gradient of a straight line
- plotting points using Cartesian co-ordinates
- basic algebraic manipulations
- the equation of a straight line
- the formula for the zeros of a quadratic equation
- tree diagrams as a tool for defining sample spaces and estimating probabilities
- absolute value
- summation notation: $\sum_{i=1}^{n} x_i$.

APPENDIX 2

Explanation of some terms

congruence

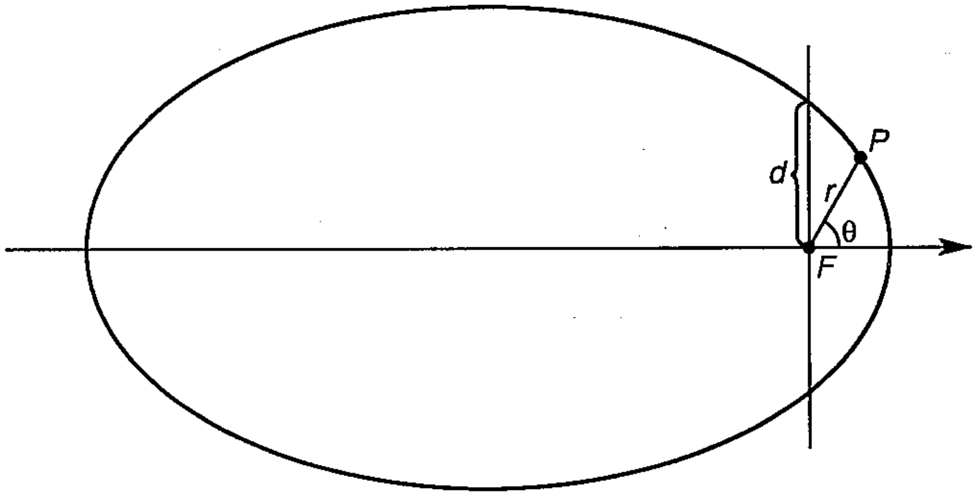
A congruence is any statement of the form $a \equiv b \bmod n$, read as "a is congruent to b modulo n", and means that $a - b$ is divisible by n, where all of a, b and n are integers; for example, $13 \equiv -2 \bmod 5$.

conics, polar form

The position of a point P on a conic can be expressed using polar coordinates, (r,θ), with one focus at the origin, as

$$\frac{d}{r} = 1 + e\cos\theta$$

where e is the eccentricity of the conic, and d is as shown in the following diagram (for the classically minded, d is called the *semi latus rectum*).



degrees of freedom

The **degrees of freedom** specifies the number of independent or "freely available" observations used to calculate a statistical estimate. When a summary statistic such as a sample mean or a sample standard deviation is calculated from a sample, its accuracy as an estimate of the population parameter depends on the number of individual observations which have contributed independently to its value. If the calculation requires some measure which itself must be estimated from the same data set, then some independence is lost.

For example, a person asked to select ten numbers at random has ten free choices to make; a person asked to select ten numbers whose mean is specified has only nine free choices to make - the tenth number must be that number which makes the correct mean for the sample of ten.

The most common use of the degrees of freedom concept occurs in the calculation of the standard deviation of a sample. The formula for calculating standard deviation requires that the sum of the squared deviations of the observations from their population mean be divided by the appropriate degrees of freedom. If the population mean must first be estimated by calculating the sample mean, then the calculation of the standard deviation has its degrees of freedom reduced by one. Thus, the denominator used must be one less than the number in the sample.

In general, one degree of freedom is lost for each sample estimate which is obtained from a set of observations.

Diophantine equation

An equation in more than one variable, with integer coefficients, where integer solutions (normally non-negative) are required. A Diophantine equation may have no solutions, a finite number of solutions, or infinitely many solutions. For example,

$3x + 6y = 16$ has no solutions;
$3x + 5y = 23$ has two positive solutions, (1,4) and (6,1);
$x^2 + y^2 = z^2$ has infinitely many solutions.

Euclidean algorithm

A procedure for determining the greatest common divisor of two integers, x and y, without factorising them. Denote the quotient and remainder at each division by q_i and r_i respectively, then

$$\begin{aligned} x &= yq_1 + r_1 \\ y &= r_1q_2 + r_2 \\ r_1 &= r_2q_3 + r_3 \\ r_2 &= r_3q_4 + r_4 \\ &\cdots\cdots \\ r_{n-2} &= r_{n-1}q_n. \end{aligned}$$

The last non-zero remainder is the greatest common divisor of x and y.

fullest and latest

Exit Levels of Achievement must be judged on the **latest** information from a **full** coverage of all the significant aspects of the course. The fullest and latest information consists of the most recent data on developmental aspects together with data on components that will not be further developed or assessed.

Fundamental Theorem of Arithmetic

Any positive integer can be factorised into the product of prime numbers in only one way, apart from the order in which the factors occur.

geoliner

A protractor-like device.

linear first order differential equations with constant coefficients

These are equations of the form

$$p\frac{dy}{dx} = ky + b$$

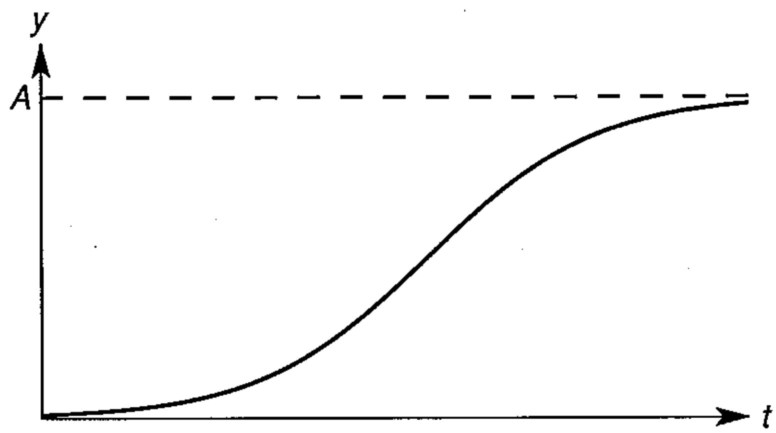
where p, k, and b are constants.

linear objective function

The function to be optimised in linear programming.

logistic curve

Many natural systems are modelled by a curve in which the change in y begins slowly, then becomes faster, and finally tapers off, thus producing the typical sigmoidal curve.



In the first stage of the process the rate of change of y is a function of y itself, that is,

$$\frac{dy}{dt} = f(y) \ .$$

In the second stage the rate of change is proportional to the remaining resources available to support the variable described by y, that is,

$$\frac{dy}{dt} = f(A-y)$$

where A is the asymptotic limit of y.

The two stages can be combined and solved for y, to give the general logistic function:

$$y = \frac{A}{1 + Be^{-kt}}$$

where A is the asymptotic limit of y,
B and k are coefficients whose meanings depend on the given situation.

The maximum rate of change arises at the point of inflection where $y = \frac{A}{2}$.

mathematical model

Any representation of a situation which is expressed in mathematical terms. It should be noted that models may be as simple as expressing simple interest as $I = \frac{Prt}{100}$ or showing the relationship between two variables as a scattergram.

Poisson distribution

The Poisson Distribution is appropriate when the variable represents the number of occurrences of a rare event (or phenomenon) over some specified number of units of space or time. Events must occur at random, be independent of each other, be independent of the starting time, and be uniformly distributed over a given interval.

The formula for the Poisson distribution is

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots$$

where the parameter m is both the mean, μ, and the variance, σ^2.

For example, the number of cyclones in a given area can be approximated by a Poisson variable in which m is the mean number of cyclones that occur in a specified period of time.

scalar product

There are two types of scalar product, corresponding to the two types of vectors discussed below.

The scalar product of vectors of the data structure type is the real number obtained by taking the sum of the products of corresponding components of the two vectors. It is related to the product of matrices.

For vectors expressed in terms of direction and magnitude, the scalar product, or "dot" product, is the product of the magnitudes of the two vectors and the cosine of the angle between them.

$$x \cdot y = | x || y | \cos \theta$$

It is also equal to the sum of the products of corresponding components of the two vectors.

simplex algorithm

The vast majority of life-related applications involve more than two variables. The simplex algorithm is a general procedure used in linear programming to find the optimal solution for situations involving two or more variables by systematically examining the vertices of the feasible region and stopping when the optimum has been found.

Simpson's Rule

A formula for numerical integration.

$$\int_a^{a+2h} f(x)dx = \frac{1}{3}h[f(a) + 4f(a+h) + f(a+2h)]$$

By using Simpson's Rule on adjoining intervals a more general form can be obtained.

$$\int_a^{a+2nh} f(x)dx = \frac{1}{3}h[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + \cdots + 4f(a+(2n-1)h) + f(a+2nh)]$$

***t*-distribution**

When the standard deviation of the population is unknown and must be estimated from the sample data, the Standard Normal Distribution should be replaced by the *t*-distribution with the appropriate degrees of freedom. The degrees of freedom are equal to the divisor of the standard deviation estimate, and for a single sample situation will be equal to one less than the sample size. For sample sizes greater than 40, the *t*-distribution is closely approximated by the Standard Normal Distribution.

Vector

Two types of vectors occur in Mathematics C. One type contains data, such as costs in linear programming or data for statistical analysis, and is represented by a one-dimensional array of elements in either a row or column. The other type represents quantities which have magnitude and direction, such as force or velocity, and can be represented by a directed line segment.

SEPTEMBER 1992

BOARD OF SENIOR SECONDARY SCHOOL STUDIES