

Senior Syllabus

Mathematics C

2008 (amended 2014)



Statement on choice of Mathematics subjects

There are three Authority subjects available to schools for students of senior Mathematics. Choice of subject may be helped by considering the following statements on prior study and future pathways.

The *Senior Syllabus in Mathematics A* is a recommended precursor to further study and training in the technical trades such as toolmaking, sheet-metal working, fitting and turning, carpentry and plumbing, auto mechanics, tourism and hospitality, and administrative and managerial employment in a wide range of industries. It is also suitable as a precursor to tertiary studies in subjects with moderate demand in mathematics.

The *Senior Syllabus in Mathematics B* is a recommended precursor to tertiary studies in subjects with high demand in mathematics, especially in the areas of science, medicine, mining and engineering, information technology, mathematics, finance, and business and economics.

The *Senior Syllabus in Mathematics C* is a recommended companion subject to Mathematics B. It provides additional preparation for tertiary studies in subjects with high demand in mathematics, especially in the areas of science, medicine, mining and engineering, information technology, mathematics, finance, and business and economics.

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Mathematics C Senior Syllabus 2008 (amended 2014)

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Summary of syllabus amendments January 2014

The following table outlines the amendments made to Mathematics C Senior Syllabus 2008. These amendments are a consequence of the directions of the Minister as outlined in the *Queensland Government Response to the Education and Innovation Committee Report No. 25: The assessment methods used in senior mathematics, chemistry and physics in Queensland schools*.

<i>Syllabus section</i>	<i>2014 update</i>
Section 1: Rationale	<p>Mathematics C has been designed to be taken in conjunction with Mathematics B. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematics B and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Mathematics C is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.</p> <p>Mathematics C is recommended for students wishing to pursue further study and training at tertiary level in areas such as:</p> <ul style="list-style-type: none"> • mathematics and statistics • mathematics and science education • natural and physical sciences • medical and health sciences, including human biology, biomedical, nanoscience and forensics • engineering sciences, including avionics, chemical, civil, communications, electrical, mechanical and mining • information technology and computer science, including electronic and software • mathematical applications in: <ul style="list-style-type: none"> – energy and resources — management and conservation – climatology – design and built environment – industry, manufacturing and trades – business and tourism – economics and commerce – statistics and data analysis. • pure mathematics.
Section 6.3.4: Authentication of student work	<p>It is essential that judgments of student achievement be made on genuine student assessment responses. Teachers must take reasonable steps to ensure that each student's work is their own, particularly where students have access to electronic resources or when they are preparing collaborative responses to tasks.</p> <p>The QSA's A–Z of Senior Moderation contains a strategy for authenticating student work <www.qsa.qld.edu.au/10773.html>. This provides information about various methods teachers can use to monitor that students' work is their own. Particular methods outlined include:</p> <ul style="list-style-type: none"> • teachers seeing plans and drafts of student work • student production and maintenance of evidence for the development of responses • student acknowledgment of resources used. <p>Teachers must ensure students use consistent, accepted conventions of in-text citation and referencing, where appropriate.</p>
Section 6.8: Requirements for a verification folio	<ul style="list-style-type: none"> • a student profile completed to date.

1. Rationale

Mathematics is an integral part of a general education. It enhances an understanding of the world and the quality of participation in a rapidly changing society. It is a truly international system for the communication of ideas and concepts, and has developed over many thousands of years through contributions by scholars of ancient and present-day cultures around the world.

Mathematics is a:

- unique and powerful way of viewing the world to investigate patterns, order, generality and uncertainty
- way of thinking in which problems are explored through observation, reflection and logical, inductive or deductive reasoning
- powerful, concise and unambiguous symbolic system with written, spoken and visual components
- creative activity with its own intrinsic value, involving invention, intuition and exploration.

Mathematics C involves the study of mathematical concepts such as groups, Real and complex number systems, matrices, vectors, calculus, mathematical structures, linear programming, conics, dynamics, and advanced periodic and exponential functions. These are used to develop:

- knowledge and skills in advanced computation and algebraic methods and procedures
- mathematical modelling and problem-solving strategies and skills
- the capacity to justify mathematical arguments and make decisions
- the capacity to communicate about mathematics in a variety of forms.

Why study this subject?

Mathematics C has been designed to be taken in conjunction with Mathematics B. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematics B and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Mathematics C is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

Mathematics C is recommended for students wishing to pursue further study and training at tertiary level in areas such as:

- mathematics and statistics
- mathematics and science education
- natural and physical sciences
- medical and health sciences, including human biology, biomedical, nanoscience and forensics
- engineering sciences, including avionics, chemical, civil, communications, electrical, mechanical and mining
- information technology and computer science, including electronic and software
- mathematical applications in:
 - energy and resources — management and conservation
 - climatology
 - design and built environment

- industry, manufacturing and trades
 - business and tourism
 - economics and commerce
 - statistics and data analysis.
- pure mathematics.

Key competencies

Mathematics C provides opportunities for the development of the key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The seven key competencies are:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems
- using technology.

2. Global aims

The global aims are statements of the long-term achievements, attitudes and values that are developed by students through studying Mathematics C but which are not directly assessed by the school.

By the end of this course, students should develop:

- broad mathematical knowledge and skills
- the ability to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis and solve problems with confidence
- an awareness of the uncertain nature of their world and be able to use mathematics to help make informed decisions in life-related situations
- an understanding of the diverse applications of mathematics
- an ability to comprehend mathematical information which is presented in a variety of forms
- an ability to communicate mathematical information in a variety of forms
- an ability to use mathematical procedures to justify conclusions
- an ability to benefit from the availability of a wide range of technologies
- an ability to choose and use mathematical instruments appropriately
- positive attitudes to the learning and practice of mathematics.

3. General objectives

3.1 Introduction

The general objectives of this course are organised into four categories:

- *Knowledge and procedures*
- *Modelling and problem solving*
- *Communication and justification*
- *Affective.*

3.2 Objectives

The general objectives for each of the categories are detailed below. These general objectives incorporate several key competencies. The first three categories of objectives, *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, are linked to the exit criteria in Section 6.6.

Knowledge and procedures

The objectives of this category involve recalling and using results and procedures across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- recall, access, select and apply mathematical definitions, rules and procedures
- demonstrate number and spatial sense
- demonstrate algebraic facility
- select and use mathematical technology
- demonstrate knowledge and use of the nature of mathematical proof.

Modelling and problem solving

The objectives of this category involve the uses of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- apply problem-solving strategies and procedures to identify problems to be solved and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- modify mathematical models as appropriate.

Communication and justification

The objectives of this category involve presentation, communication (mathematical and everyday language), logical arguments, interpretation and justification of mathematics across the range of subject matter in this syllabus.

By the end of the course students should be able to:

- interpret and use appropriate mathematical terminology, symbols and conventions
- organise and present information for different purposes and audiences, in a variety of representations (such as written, symbolic, pictorial and graphical)
- analyse information displayed in a variety of representations (such as written, symbolic, pictorial and graphical) and translate information from one representation to another
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- justify the reasonableness of results obtained through technology or other means using everyday language, mathematical language or a combination of both
- provide supporting arguments in the form of a proof and recognise that a proof may require more than a verification of a number of instances.

Affective

Affective objectives refer to attitudes, values and feelings which this subject aims to develop in students. Affective objectives are not assessed for the award of exit levels of achievement.

By the end of the course, students should appreciate the:

- diverse applications of mathematics
- precise language and structure of mathematics
- nature of mathematical proof
- diverse evolutionary nature of mathematics and the wide range of mathematics-based vocations
- contribution of mathematics to human culture and progress
- power and value of mathematics.

3.3 Principles of a balanced course

The categories of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* incorporate principles of application, technology, initiative and complexity. Each of the principles has a continuum for the particular aspects of mathematics it represents. A balanced course of study developed from this syllabus must give expression to these principles over the two years. It is expected that all students are provided with the opportunity to experience mathematics along the continuum within each of the principles outlined below.

Application

Students must have the opportunity to recognise the usefulness of mathematics through its application, and the power of mathematics that comes from the capacity to abstract and

generalise. Thus students' learning experiences and assessment programs must include mathematical tasks that demonstrate a balance across the range from life-related to pure abstraction.

Technology

A range of technological tools must be used in the learning and assessment experiences offered in this course. This ranges from pen and paper, measuring instruments and tables, through to higher technologies such as computers and graphing calculators, including those that allow for algebraic manipulations. The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator.

Initiative

Learning experiences and the corresponding assessment must provide students with the opportunity to demonstrate their capability when dealing with tasks that range from well-rehearsed (routine) through to those that require demonstration of insight and creativity (non-routine).

Complexity

Students must be provided with the opportunity to work on simple, single-step tasks through to tasks that are complex in nature. Complexity may derive from either the nature of the concepts involved or from the number of ideas or procedures that must be sequenced in order to produce an appropriate conclusion.

4. Course organisation

4.1 Introduction

The syllabus contains both core and option topics. A course of study in Mathematics C must contain *all* core topics and a minimum of *two* complete option topics. Although some topics contain material which is required in other topics, the order in which they are presented does not imply a teaching sequence.

Core topics

- Introduction to groups
- Real and complex number systems
- Matrices and applications
- Vectors and applications
- Calculus
- Structures and patterns

Option topics

- Linear programming
- Conics
- Dynamics
- Introduction to number theory
- Introductory modelling with probability
- Advanced periodic and exponential functions
- School option(s).

The core and option topics are discussed in detail in Section 5.

Throughout the course, certain previously learned mathematical knowledge and procedures will be required. Whilst some of these have been identified and listed in Section 8, “Quantitative concepts and skills”, others have been developed in Mathematics B. In designing the course sequence, provision should be made to allow these aspects to be revised within topics as they are required throughout the course of study.

4.2 Selection of topics

The choice of option topics should be made so that they best suit the interests and needs of the particular cohort of students, the expertise and interests of the teaching staff and the resources of the school. This might mean that different choices are offered for different classes within the one cohort, or that the choices differ from year to year. If the school wishes to allow for this flexibility, the possibilities should be outlined in the work program.

4.3 Time allocation

The minimum number of hours of timetabled school time, including assessment, for a course of study developed from this syllabus is 55 hours per semester. A course of study will usually be completed over two years (220 hours).

Notional times are given for each core topic. These times are included as a guide, and minor variations may occur. Approximately *30 hours* should be spent on each option. All options should be seen as equivalent with respect to level of difficulty.

4.4 Sequencing

After considering the subject matter and the appropriate range of learning experiences to enable the general objectives to be achieved, a *spiralling* and *integrated* sequence should be developed which allows students to see links between the different topics rather than seeing them as discrete units. For example, Introduction to groups can be used to provide a link between most of the core topics. It provides a thread which runs through the Real and complex number systems, Matrices and applications, Vectors and applications, and Structures and patterns.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence, but some topics include subject matter which is developed and extended in the subject matter of other topics. The school's sequence should be designed so that the subject matter is revisited and spiralled to allow students to internalise their knowledge before developing it further.

The following guidelines for the sequencing of subject matter should be referred to when developing a sequence for the course:

- No subject matter should be studied before the relevant prerequisite material.
- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered appropriately.
- Subject matter across topics should be linked when possible.
- Sequencing may be constrained by a school's ability to provide physical resources.
- Time will be needed for maintaining quantitative concepts and skills.

4.5 Technology

The advantage of mathematics-enabled technology in the mathematics classroom is that it allows for the exploration of the concepts and processes of mathematics. Graphing calculators and spreadsheeting, for example, let students explore and investigate; they help students understand concepts and they complement traditional approaches to teaching.

More specifically, the mathematics-enabled technology allows students to tackle more diverse, life-related problems. Real-life matrix application problems are more easily solved with this technology. This technology may be used in statistics to investigate larger datasets and rapidly produce a variety of graphical displays and summary statistics, thus freeing students to look for patterns, to detect anomalies in the data and to make informed comments. It must be used where numerical techniques are involved.

The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator. While student ownership of graphing calculators is not a requirement, *regular and frequent student access* to appropriate technology is necessary to enable students to develop the full range of skills required for successful problem solving during their course of study. Use of graphing calculators or computers will significantly enhance the learning outcomes of this syllabus.

To meet the requirements of this syllabus, schools should consider the use of:

- general purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheeting software
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- hand held (calculator) technologies designed for mathematics teaching and learning, e.g. graphics calculators with and without algebraic manipulation or dynamic geometry facilities.

Complete dependence on calculator and computer technologies at the expense of students demonstrating algebraic facility may not satisfy syllabus requirements for *Knowledge and procedures*.

4.6 Composite classes

In some schools, it may be necessary to combine students into a composite Year 11 and 12 class. This syllabus provides teachers with an opportunity to develop a course of study that caters for a variety of circumstances such as combined Year 11 and 12 classes, combined campuses, or modes of delivery involving periods of student-directed study.

The multilevel nature of such classes can prove advantageous to the teaching and learning process because:

- it provides opportunities for peer teaching
- it allows teachers to maximise the flexibility of the syllabus
- it provides opportunities for a mix of multilevel group work, and for independent work on appropriate occasions
- learning experiences and assessment can be structured to allow students to consider the key concepts and ideas at the level appropriate to their needs, in both Year 11 and Year 12.

The following guidelines may prove helpful in designing a course of study for a composite class:

- The course of study could be written in a Year A/Year B format, if the school intends to teach the same topic to both cohorts.
- Place a topic at the beginning of each year that will allow new Year 11 students easy entry into the course.
- Learning experiences and assessment items need to cater for both year levels throughout the course. Even though tasks may be similar for both year levels, it is recommended that more extended and/or complex tasks be used with Year 12 students.

4.7 Work program requirements

A work program is the school's plan of how the course will be delivered and assessed based on the school's interpretation of the syllabus. It allows for the special characteristics of the individual school and its students.

The school's work program must meet all syllabus requirements and must demonstrate that there will be sufficient scope and depth of student learning to meet the general objectives and the exit criteria.

The requirements for work program approval can be accessed on our website, <www.qsa.qld.edu.au>. This information should be consulted before writing a work program. Updates of the requirements for work program approval may occur periodically.

5. Topics

5.1 Introduction

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus


This section highlights the intent of the syllabus with respect to the topic and indicates how students should be encouraged to develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, but the order in which it is presented is not necessarily intended to imply a teaching sequence.

Learning experiences

This section provides some suggested learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. The numbers provided with the subject matter link to suggested learning experiences. Included are experiences which involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for *Modelling and problem solving*. The listed learning experiences may require students to work individually, in small groups or as a class.

The learning experiences are suggestions only and are not prescriptive. Schools are encouraged to develop further learning experiences, especially those which relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of learning experiences that students will encounter should be shown in the work program. Learning experiences which have a technology component beyond the use of a scientific calculator have been labelled by the use of an icon ().

NB The learning experiences must provide students with the opportunity to experience mathematics along the continuum within each of the principles of a balanced course (see Section 3.3).

Some of the key competencies, predominantly *Using mathematical ideas and techniques*, *Solving problems*, and *Using technology* are to be found in the learning experiences within the topic areas. Opportunities are provided for the development of key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The key competencies of: *Collecting, analysing and organising information*, *Planning and organising activities*, and *Working with others and in teams* also feature in some of the learning experiences.

5.2 Core topics

The order in which topics and items within topics are given do not imply a teaching sequence. Numbers listed after each item of subject matter refer to suggested learning experiences (SLEs).

Introduction to groups (notional time 7 hours)

Focus

Students are encouraged to investigate the structures and properties of groups. It is intended that this introduction to groups should provide a basis for identifying the common features which are found in systems such as real and complex numbers, matrices and vectors.

Subject matter

Concepts of:

- closure
- associativity
- identity
- inverse (suggested learning experiences (SLEs) 1–9)
- definition of a group (SLEs 1–9).

Suggested learning experiences (SLEs)

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Determine whether the elements of a set form a group under a binary operation.
2. Determine the identity element and inverses in a group table.
3. Use a small Cayley table to determine whether a set of elements under a binary operation forms a group.
4. Investigate when the integers modulo n form groups under addition or multiplication.
5. Investigate groups formed by geometric transformations such as the reflections of a rectangle in its axes of symmetry and rotations of an equilateral triangle.
6. Construct a Cayley table and use it to identify subgroups (if any) such as the rotations of a square about its centre.
7. Find the element(s) which generate(s) the group in a group table.
8. Investigate the group structure of friezes, wallpapers or simple crystals by studying their symmetries under translations, rotations and reflections.
9. Investigate commutativity and abelian groups.

Real and complex number systems (notional time 25 hours)

Focus

Students are encouraged to extend their knowledge of the real number system and to develop an understanding of the complex number system. Students should see the group structure within these systems as a link between complex numbers and real numbers.

Subject matter

- structure of the real number system including:
 - rational numbers
 - irrational numbers (SLEs 2, 9, 10)
- simple manipulation of surds
- definition of complex numbers including standard and trigonometrical (modulus-argument) form (SLEs 1, 2)
- algebraic representation of complex numbers in Cartesian, trigonometric and polar form (SLEs 3, 4)
- geometric representation of complex numbers — Argand diagrams (SLE 4)
- operations with complex numbers including addition, subtraction, scalar multiplication, multiplication of complex numbers, conjugation (SLEs 1–8, 12)
- roots of complex numbers (SLE 6)
- use of complex numbers in proving trigonometric identities
- powers of complex numbers including de Moivre's Theorem
- simple, purely mathematical applications of complex numbers (SLEs 6, 7, 8, 11, 12).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Solve quadratic equations whose discriminant is negative.
2. Solve simple inequality statements such as $|z - a| > b$ in both the real and complex systems, and be able to give a verbal description of the meaning of the mathematical symbolism.
3. Use polar forms to demonstrate multiplication and division of complex numbers.
4. Use geometry to demonstrate the effect of addition, subtraction and multiplication of complex numbers.
5. Solve simple equations of the form $z^n = w$ where n is an integer and w is a complex number.
6. Solve polynomial equations with real and complex coefficients (degree ≤ 3).
7. Investigate the use of complex conjugates in the solution of polynomial equations with real coefficients.
8. Use a proof by contradiction to show that $\sqrt{2}$ is irrational.
9. Investigate some of the approximations to π which have been used.
10. Research areas in which complex numbers are used in life-related applications such as electric circuit theory, vibrating systems and aerofoil designs.


11. Investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication.

12.  Investigate fractals; for example, the Mandelbrot Set, the Julia Set, using a CAS calculator.

Matrices and applications (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the algebraic structure of matrices including situations where they form groups. Students should apply matrices in a variety of situations and use technology to facilitate the solution of problems involving matrices.

Subject matter

- definition of a matrix as data storage and as a mathematical tool (SLEs 1–7)
- dimension of a matrix
- relationship between matrices and vectors (SLEs 1, 6, 7, 12)
- matrix operations
 - addition and subtraction
 - transpose
 - multiplication by a scalar
 - multiplication by a matrix (SLEs 1–7, 13, 14, 15)
- inverse of a matrix
- solution of simple matrix equations
- definition and properties of the identity matrix (SLEs 1, 3, 15)
- group properties of 2×2 matrices (SLE 5)
- determinant of a matrix (SLE 3)
- singular and non-singular matrices (SLE 1)
- solution of systems of homogeneous and non-homogeneous linear equations using matrices (SLEs 1, 6)
- applications of matrices in both life-related and purely mathematical situations (SLEs 1–12).

Suggested learning experiences

NB Many learning experiences in this topic are enhanced by the use of a calculator or computer software with matrix operations.

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. ■ Solve linear equations by using matrices and Gaussian elimination; solution of equations involving more than three variables will involve the use of graphing calculators.
2. ■ Investigate transition probability matrices (Markov) in situations such as recording over a period of time, the changes in major weather conditions as stated by newspaper or TV weather reports (fine, showers, cloudy, clearing); construct the matrix describing the probabilities that one condition will be followed by each different condition; given today's weather find the most probable sequence of weather conditions in the near future.
3. Use matrices to encode and decode messages.
4. Demonstrate the use of the transformation matrices (rotation, reflection, dilation) as an application of 2×2 matrices to geometric transformations in the plane.
5. Consider subsets of 2×2 matrices forming a group under addition or multiplication.
6. Show that the change of frame of reference used in Newtonian mechanics,

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}$$

$x' = x - vt$, $t' = t$, can be written using matrices.

7. ■ Use input-output (Leontief) matrices in economics.
8. Investigate the use of matrices in dietary problems in health.
9. ■ Research the use of Leslie matrices in ecology.
10. Investigate the use of matrices in dominance problems such as in predicting the next round results (rankings) for the national netball competition.
11. Investigate the use of matrices in game strategies.
12. ■ Investigate the application of matrices in formulating a mathematical model for a closed economic system.
13. Research nilpotent matrices where the matrix, \mathbf{A} , is nilpotent if it has the property $\mathbf{A}^2 = \mathbf{O}$, (\mathbf{O} is the zero matrix).
14. Research idempotent matrices where the matrix, \mathbf{A} , is idempotent if it has the property $\mathbf{A}^2 = \mathbf{A}$.
15. Investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication.

16.  Research the use of Eigenvalues and Eigenvectors.

Vectors and applications (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of vectors as entities which can be used to describe naturally occurring systems. They should also understand that the meaning of a vector comes from the situation and the model being considered. Students should become aware of the links between vectors and matrices. The emphasis should be on those vectors which describe situations involving magnitude and direction.

Subject matter

(a) For vectors as a single column or single row array

- definition of a vector
- relationship between vectors and matrices (SLE 1)
- operations on vectors including:
 - addition
 - multiplication by a scalar
- scalar product of two vectors (SLE 1)
- simple life-related applications of vectors (SLEs 1, 3)

(b) For vectors describing situations involving magnitude and direction

- definition of a vector, including standard unit vectors i, j and k
- relationship between vectors and matrices (SLE 2)
- two- and three-dimensional vectors and their algebraic and geometric representation (SLEs 3, 4, 6)
- operations on vectors including: (SLEs 3–5, 9, 10, 13)
 - addition
 - multiplication by a scalar
- unit vectors
- scalar product of two vectors (SLEs 2, 8)
- vector product of two vectors (SLE 7)
- resolution of vectors into components acting at right angles to each other (SLEs 3–5, 11, 12)
- calculation of the angle between two vectors
- applications of vectors in both life-related and purely mathematical situations (SLEs 1–13).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Show that the cost of weekly shopping is the scalar product of the shopping list vector and the unit cost vector.
2. Show that if \mathbf{x} is a column vector of order n

$$\mathbf{x}^T \mathbf{x} = \sum_{i=1}^n x_i^2$$

3. Use addition and subtraction in life-related situations such as the effect of current flow on a boat; consider the concept of relative velocity.
4. Use resolution of vectors to consider the equilibrium of a body subject to a number of coplanar forces acting at a point.
5. Use vectors in three dimensions such as the placement of a TV aerial mast and its wire supports on a roof of a building.
6. Solve problems from geometry using vectors: for example; prove that the angle of a semicircle is a right angle; Pythagoras' theorem; the concurrency of (a) the medians and (b) the bisectors of the internal angles of a triangle.
7. Use the vector product to calculate areas in situations such as the calculation of the area of a suspended triangular shade canopy.
8. Use scalar product to solve problems in situations such as the evaluation of work as a product of force and displacement.
9. Investigate the use of vectors in surveying.
10. Investigate the effect of wind on wind-propelled craft.
11. Investigate the way medical staff use vectors to put a broken bone in suitable traction; consider the weights and angles of the ropes that are needed.
12. Investigate the forces exerted by the hip, knee and ankle joints in pushing and pulling a bicycle pedal.
13. Use addition of vectors to see how the apparent motion of planets in the solar system depends on the frame of reference chosen.

Calculus (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of analytical and numerical techniques of integration. Students should also gain further experience in applying differentiation and integration to both life-related and purely mathematical situations. They should appreciate the importance of differential equations in representing problems involving rates of change.

Subject matter

- integrals of the form

$$\int \frac{f'(x)}{f(x)} dx$$

$$\int f[g(x)] \cdot g'(x) dx$$

- simple integration by parts (SLE 4)
- development and use of Simpson's rule (SLEs 5, 6, 7, 9, 10, 11, 12, 15)
- approximating small changes in functions using derivatives (SLEs 1, 2)
- life-related applications of simple, linear, first-order differential equations with constant coefficients (SLEs 3, 8, 13, 14)
- solution of simple, linear, first-order differential equations with constant coefficients (SLEs 3, 6, 8, 10, 13, 14, 15).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Investigate life-related situations where small changes in calculated quantities due to small errors in measurements can be approximated using derivatives, such as the tolerance in the volume of a soft-drink can, produced by small errors in the diameter or height.
2. Find approximate solutions to equations by using small changes from equations with exact solutions such as an approximate value of $\sqrt[3]{28}$
3. Investigate life-related situations that can be modelled by simple differential equations such as growth of bacteria, cooling of a substance.
4. Use integration by parts to evaluate expressions such as

$$\int e^{ax} \sin bx \, dx \quad \text{and} \quad \int t^2 e^t \, dt$$

5. Use Simpson's rule to evaluate definite integrals where the indefinite integral cannot be found, such as


$$\int_0^{\pi} \frac{\sin x}{x} \, dx$$

6. Compare the values of areas of simple shapes determined from known rules with approximations determined from Simpson's rule such as the area of a quadrant of a circle of radius 4 compared with the area represented by

$$\int_0^4 \sqrt{16 - x^2} \, dx$$

evaluated using Simpson's rule.

7. Use Simpson's rule with discrete data in situations such as the volume of fill to be removed in the construction of a road cutting.
8. Investigate the motion of falling objects, where resistance is proportional to the velocity, by considering the differential equation

$$m \frac{dv}{dt} = mg - kv$$
9. Compare the accuracy of numerical techniques with analytical results for selected integrals.
10. Verify integrals in integral tables by differentiation of the result.
11. Show that Simpson's Rule is exact for polynomials of degree three or less.
12. Investigate the varying volumes for the earth obtained when its shape is assumed to be (a) a sphere, and (b) an ellipsoid.
13. Find an expression for the pressure, P , as a function of altitude in an isothermal atmosphere where the rate of decrease of atmospheric pressure with increasing altitude is proportional to the density of the air, ρ , pressure, density and temperature, T , are related by $P = R\rho T$, and R is a constant.
14. Find an expression for the amount of the desired product Pu_{239} present, as a function of time after start up in a breeder reactor where U_{238} is converted to Pu_{239} at a constant rate and Pu_{239} is converted to Pu_{240} at a rate proportional to the amount of Pu_{239} present.
15.  Use tables of integrals or computer software to evaluate a given integral.
16. Illustrate how the definite integral can be defined using Riemann sums.

Structures and patterns (notional time 30 hours)

Focus

Students are encouraged to develop their ability to recognise and use structures and patterns in a wide variety of situations. They should appreciate the value of symmetries and patterns in making generalisations to explain, simplify or extend their mathematical understanding. Justification of results is important and, where appropriate, results should be validated inductively or deductively.


It is not intended that a great emphasis be placed on repetitive calculations in arithmetic progressions, geometric progressions, permutations or combinations.

Subject matter

- sum to infinity of a geometric progression (SLEs 1, 2)
- purely mathematical and life-related applications of arithmetic and geometric progressions (SLEs 10, 11, 12)
- sequences and series other than arithmetic and geometric (SLEs 3, 4, 16)
- permutations and combinations and their use in purely mathematical and life-related situations (SLEs 7, 8, 9, 14, 15)
- recognition of patterns in well-known structures including Pascal's Triangle and Fibonacci sequence (SLEs 5, 6, 13)
- applications of patterns (SLEs 1–11, 17)
- use of the method of finite differences (SLEs 4, 18)
- proof by induction (SLE 4)
- use mathematical induction to prove de Moivre's Theorem.

Suggested learning experiences



The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Establish the formula for the sum to n terms of a geometric progression, and hence the formula for the sum to infinity of a geometric progression; verify the formula by mathematical induction.
2. Recognise geometric progressions in many different algebraic forms such as $2, 4p^2, 8p^4, 16p^6$; determine the general term, sum of n terms, sum to infinity of such sequences.
3. Use finite differences in determining polynomial coefficients for polynomials of degree ≤ 3
4.  Verify the outcomes of finite differences by using graphing calculators.
5. Use finite difference methods to establish the formula in situations such as the sum of the first n positive integers, the sum of the first n squares, the sum of the first n cubes; use the principle of mathematical induction to prove the formula obtained.
6. Search for patterns in Pascal's Triangle and verify any claims algebraically or otherwise, such as ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
7. Investigate patterns in Fibonacci numbers such as:

$$f_1 + f_2 + f_3 + \dots + f_n = f_{n+2} - 1$$

$$f_1 + f_3 + f_5 + \dots + f_{2n-1} = f_{2n} - 1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix} \text{ for } n \geq 1 \text{ given } f_0 = 0$$

8. Investigate the way in which a sequence can be defined recursively.
9. Investigate permutations and combinations which arise in games of chance such as, in poker, the number of hands containing two aces, the number of hands that are a full house.
10. Apply counting techniques to investigate problems in situations such as:
 - the amount of wool eaten by the offspring of one female moth who lays 300 eggs if each larva eats 15 milligrams of wool, two-thirds of eggs die, 50% of remaining eggs are female and there are five generations per year
 - the number of different sequences possible in 4 months, when a patient must receive H-insulin for 2 months and P-insulin for the other 2 months in a medical study.
11. Investigate the use of the inclusion–exclusion principle in counting cases in situations such as determining how many individuals were interviewed in a survey researching the eating habits of teenagers if 10 ate pizzas, 5 ate pies, 2 ate both and 7 ate neither; consider the effect of adding hamburgers as a third option.
12. Investigate the use of an arithmetic progression in situations such as:
 - the calculation of the length of batten material required for tiling a hip roof
 - the calculation of the total number of potatoes required for a “potato race” over a given distance if the distance between potatoes is a specified constant.
13. Use geometric progressions in situations involving the sum to infinity.
14.  Investigate logarithmic spirals and polar curves which occur in nature such as in nautilus shells.
15.  Investigate the occurrence of Fibonacci numbers in nature such as in spirals in sunflowers, pineapples and other plants.
16. Given a cube and six different colours, determine how many different ways the cube can be painted so that each face is a different colour; extend to other regular solids.
17. Apply the pigeonhole principle to solve problems in situations such as showing there are at least 2 in a group of 8 people whose birthdays fall on the same day of the week in any given year.
18. Use series expansions for e^x , $\sin x$ and $\cos x$ to illustrate derivatives of e^x , $\sin x$, $\cos x$,
 $e^{ix} = \cos x + i \sin x$
19. Use symmetries to find the order of groups of rotations such as rotations of squares, equilateral triangles.
20. Use polar forms to demonstrate multiplication and division of complex numbers.

5.3 Option topics

Linear programming (notional time 30 hours)

Focus




Students are encouraged to develop an understanding of the methodology of linear programming and to see how it is used to solve problems in life-related situations. They should appreciate that graphical techniques of solution have limited applicability, but that other techniques can be used for the solution of problems with a large number of variables. The use of technology is expected to assist students in these processes.

Subject matter (SLEs 1–14)

- recognition of the problem to be optimised (maximised or minimised)
- identification of variables, parameters and constraints
- construction of the linear objective function and constraints with associated parameters
- graphing linear functions associated with the constraints and identification of the regions defined by the constraints
- recognition that the region bounded by the constraints gives the feasible (possible) solutions
- recognition that different values of the objective function in two variables can be represented by a series of parallel lines
- use of a series of parallel lines to find the optimal value of the objective function in two variables (parallel or rolling ruler, graphical method)
- observation that the feasible region is always convex, and thus the optimal solutions occur at an edge or a corner point of the feasible region
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem
- relationship between algebraic and geometric aspects of problems with constraints in two and three dimensions
- use of the simplex algorithm to solve simple life-related problems in which optimal solutions are required.

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Take a life-related problem given in English, formulate it into a linear programming problem, solve by the graphical method, and interpret the solutions in terms of the original English problem.
2. Investigate how linear programming is used to assist management decisions in areas such as manufacturing, transport, primary industries and environmental management.
3. Consider optimal solutions of simple problems such as balancing diets.
4.  Use a computer graphing package to graph linear functions.
5. Use parallel rulers to identify optimal solutions.
6. Change parameters or constraints in a given problem and investigate the effect on optimal solutions.
7. Consider the composition of a fleet of vehicles necessary to do a particular job at minimum cost.
8. Solve a problem involving the allocation of two crops to the areas available on a farm in order to optimise profit when there are constraints on the labour and finances available; consider the allocation of three crops.
9.  Investigate the design of an optimal-sized solar-powered home which is to be competitive in the marketplace; constraints will include the size of solar cells, living area, cost of storage batteries and total cost of construction.
10.  Use a computer to find optimal solutions when the variables must have integer values.
11. Research the history of linear programming.

12. Formulate a two-dimensional linear programming problem and write it in matrix form.
13. Research techniques used for solving linear optimisation problems when the variables are restricted to integer values (integer programming).
14. Explain how the simplex algorithm systematically examines the vertices of the feasible region to determine the optimal solution.

Conics (notional time 30 hours)

Focus



Students are encouraged to extend their knowledge of coordinate geometry in two dimensions. They are encouraged to appreciate the interrelationships that exist between areas of mathematics. These relationships should be illustrated by applying coordinate geometry and complex numbers to conics.

Subject matter

- definitions of circles, ellipses, parabolas and hyperbolas in terms of cones, and their use in the development of conic equations
- concept of a locus, directrix and focal point (SLEs 1–17)
- circle as a locus in:
 - Cartesian form $x^2 + y^2 = a^2$
 - parametric form $x = a \cos \theta$, $y = a \sin \theta$
 - complex number form $|z| = s$ (SLEs 1–6, 8, 11, 17)
- definition of eccentricity e
- ellipse as a locus in:
 - Cartesian form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 - parametric form $x = a \cos \theta$, $y = b \sin \theta$
 - complex number form $|z - p| + |z - q| = s$ where $s > |p - q|$
 - polar form $r = \frac{d}{1 + e \cos \theta}$, $0 < e < 1$ (SLEs 3, 5, 7, 9, 10, 11, 14, 16, 17)
- hyperbola as a locus in:
 - Cartesian form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $xy = c$
 - parametric form $x = \frac{a}{\cos \theta}$, $y = b \tan \theta$; $x = t$, $y = \frac{c}{t}$
 - complex number form $||z - p| - |z - q|| = s$; where $0 < s < |p - q|$
 - polar form $r = \frac{d}{1 + e \cos \theta}$, $e > 1$ (SLEs 3, 5, 10, 11, 15, 17)
- parabola as a locus in:
 - Cartesian form $y^2 = 4ax$
 - parametric form $x = at^2$, $y = 2at$
 - polar form $r = \frac{d}{1 + e \cos \theta}$, $e = 1$
(SLEs 1, 2, 3, 5, 8, 10, 11, 12, 13, 17)
in all cases above a, b, c, d, p, q and s are parameters
- simple applications of conics (SLEs 1–17).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Introduce the concept of locus using situations such as the path followed by:
 - the handle on an opening door
 - a boat sailing equidistant from two fixed lights
 - a netballer moving to remain equidistant from a goal post and a sideline.
2. Derive the Cartesian forms for the circle and parabola.
3. Find the Cartesian equations of conics after a given translation has been applied.
4. Find the centre and radius of a circle given its Cartesian equation.
5. Sketch conics given the Cartesian form showing directrices, focuses, asymptotes, and axes of symmetry as appropriate.
6. Sketch conics given the complex number form on an Argand diagram.
7. Convert the complex number form for a circle to the corresponding Cartesian form using both algebraic and geometric methods.
8. Convert the complex number form for an ellipse to the corresponding Cartesian form using both algebraic and geometric methods; situations should be limited to those where both focuses lie on the same coordinate axis.
9. Find the equation of a tangent to a circle or a parabola given its parametric form.
10. Investigate how to construct the elliptical hole required to be cut out of a sloping roof to fit a vertical cylindrical vent.
11.  Use a graphing calculator to plot a conic whose form is given parametrically.
12. Find the equation of tangents, chords, and normals to conics whose equations are given in Cartesian form.
13. Investigate why parabolic reflectors are used in astronomical telescopes, hand-held torches and microwave repeater stations.
14. Investigate the shape on the ground of the leading edge of the sonic boom produced by a supersonic aircraft flying at high altitudes.
15. Research the use of elliptic reflectors in treating kidney stones.
16. Research how hyperbolas were used in the Omega navigation system.
17. Investigate how the properties of ellipses are used in whispering rooms.
18.  Use a dynamic geometry package to investigate locus behaviour.

Dynamics (notional time 30 hours)***Focus***

The approach used throughout this topic should bring together concepts *from both vectors and calculus*. Students should use *a vector and/or a calculus approach* to develop an understanding of the motion of objects that are subjected to forces.

Subject matter

- derivatives and integrals of vectors (SLEs 1, 2, 3, 13)
- Newton's laws of motion in vector form applied to objects of constant mass (SLEs 2–15)
- application of the above to: (SLEs 4–12, 14, 15)
 - straight line motion in a horizontal plane with variable force
 - vertical motion under gravity with and without air resistance
 - projectile motion without air resistance
 - simple harmonic motion (derivation of the solutions to differential equations is not required)
 - circular motion with uniform angular velocity.


Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.


1. Given the position vector of a point as a function of time such as $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + \sin t \mathbf{k}$ determine the velocity and acceleration vectors.
2. Given the displacement vector of an object as a function of time, by the processes of differentiation, find the force which gives this motion.
3. Given the force on an object as a function of time and suitable prescribed conditions, such as velocity and displacement at certain times, use integration to find the position vector of the object.
4. Investigate the motion of falling objects such as in situations in which:
 - resistance is proportional to the velocity, by considering the differential equation

$$m \frac{dv}{dt} = mg - kv$$
 - resistance is proportional to the square of velocity, by considering the differential equation

$$m \frac{dv}{dt} = mg - kv^2$$

where k is a positive constant.
5. Model vertical motion under gravity alone; investigate the effects of the inclusion of drag on the motion.
6. Develop the equations of motion under Hooke's law; verify the solutions for displacement by substitution and differentiation; relate the solutions to simple harmonic motion and circular motion with uniform angular velocity.
7. From a table of vehicle stopping distances from various speeds, calculate (a) the reaction time of the driver and (b) the deceleration of the vehicle, which were assumed in the calculation of the table.
8. Model the path of a projectile without air resistance, using the vector form of the equations of motion starting with $\mathbf{a} = -g \mathbf{j}$ where upwards is positive.
9.  Use the parametric facility of a graphing calculator to model the flight of a projectile.
10. Investigate the flow of water from a hose held at varying angles, and model the path of the water.

11. Investigate the motion of a simple pendulum with varying amplitudes.
12. Investigate the angle of lean required by a motorcycle rider to negotiate a corner at various speeds.
13. Use the chain rule to show that the acceleration can be written as

$$a = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$
 if the velocity, v , of a particle moving in a straight line is given as a function of the distance, x .
14. Investigate the speed required for a projectile launched vertically to escape from the earth's gravitational field, ignoring air resistance but including the variation of gravitational attraction with distance.
15. Use detectors or sensors to investigate problems, e.g. rolling a ball down a plank.
16.  Use spreadsheets to investigate problems.

Introduction to number theory (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of the properties of integers, and to appreciate the usefulness of apparently abstract mathematics. They are also encouraged to gain an understanding of the power of congruence in solving problems involving integers.

Subject matter

- primes, composites and the Fundamental Theorem of Arithmetic (SLEs 3, 8, 12, 14, 16–18, 21–24, 27, 28)
- divisors, Euclidean algorithm, lowest common multiples (LCM) and greatest common divisors (GCD), (SLEs 4, 11, 15, 19–21, 25)
- modular arithmetic (SLEs 5, 7, 10, 13, 26)
- congruence including simple simultaneous congruence (SLE 9)
- simple Diophantine equations (SLE 6).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Prove that the sum of any two odd numbers is an even number.
2. Prove that the product of any two odd numbers is an odd number.
3. Consider conditions on the integers a , b and r under which $ax + by = r$ is solvable for the integers x and y .
4. Develop tests for divisibility by 2, 2^n , 3, 3^2 , 5, 5^n , 7, 11, 13, 1001.
5. Investigate the existence of inverses under multiplication modulo n .
6. Solve some simple Diophantine equations.
7. Use modular arithmetic to prove/disprove propositions about integers.
8. Research Fermat, Mersenne, perfect, abundant and/or amicable numbers.
9. Research the use of congruence in ciphering methods.
10. Investigate the period of the operation $x_{n+1} = ax_n + b \pmod{c}$ for generating random numbers.

11. Research interesting facts about numbers and create problems for others to solve, e.g. “The sum of the divisors of 7^3 is a square number — can you find any other numbers that fit this condition?” (400 is one such number).
12. Investigate facts about prime numbers.
13. Investigate the way modular arithmetic is used to calculate the date of Easter.
14. From a table of prime numbers, multiply the number of prime numbers less than an integer, n , by the logarithm of n , and compare the result with n ; do this for several values of n .
15. Investigate greatest common divisors among pairs of Fibonacci numbers.
16. Investigate for which integers (a, b) a number of the form $a^b - 1$ can be a prime.
17. Investigate for which integers (a, b) a number of the form $a^b + 1$ can be a prime.
18. Investigate prime numbers of the form $a^2 + b$, for various values of b .
19. Investigate Pythagorean triples.
20. Investigate the representation of numbers by sums of squares.
21. Explain Fermat’s Little Theorem and/or the Chinese Remainder Theorem.
22. Research Goldbach’s Conjecture.
23. Research Fermat’s Last Theorem.
24. Research different proofs that the number of primes is infinite.
25. In ancient Egypt calculations were done using only rational numbers with 1 as the numerator; show that any rational number p/q can be rewritten as the sum of not more than p distinct rational numbers, each of which has 1 as the numerator.
26. Investigate the use of modular arithmetic in the construction of Latin squares in the design of experiments.
27. Discuss the definition of even and odd numbers:
 - $E = \{x \mid x = 2k, k \in \mathbb{Z}\}$
 - $O = \{x \mid x = 2k + 1, k \in \mathbb{Z}\}$
28. Investigate proofs of properties of addition:
 - $E + E = E$
 - $O + E = O$ (SLEs 1, 2).

Introductory modelling with probability (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the key mathematical concepts and tools that underlie the areas of modelling with probability. Students are encouraged to develop interpretative and problem-solving skills in applying operations of a Boolean nature, the basic probability rules, and conditional probability, and in identifying and formulating stochastic models. The approach should include the role of data in estimating probabilities and parameters, and in investigating stochastic and deterministic models.

Subject matter



- events and set operations, including translation to and from word descriptions (SLEs 1, 2, 4)
- logical circuits and truth tables, including similarities and contrasts to set operations (SLEs 2, 3, 4)
- working with probabilities of events (SLEs 1, 4, 5, 6)

- independence; system/circuit reliability (SLEs 6, 7, 8)
- conditional probability and the law of total probability (SLEs 9, 10, 11)
- use of transition probability matrices in simple Markov chain models (SLEs 12, 13)
- estimating conditional probabilities in simple Markov chain models (SLEs 6, 9, 12, 13)
- random numbers — the uniform distribution (SLEs 14–18)
- random numbers versus the chaotic behaviour of a simple non-linear system such as the logistic map (SLEs 16, 17, 18)
- the exponential distribution (SLEs 19, 20)
- use data to compare and explore the suitability of an exponential model or a uniform model (SLEs 19, 20).

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. Explore the correspondence between word descriptions and set notations of events — for example, each queue has exactly 2 people; there are at least 4 people over the two queues.
2. Express the event that logical circuits involving series and parallel components are operational using “and”, “exclusive or” and “negation” operations.
3. Investigate truth tables for situations such as political statements.
4. Investigate the correspondences between set operations and the operations of “and”, “exclusive or” and “negation”.
5. From the three probability axioms establish the key probability rules.
6. Use data to estimate probabilities of simple and complex events.
7. Use independence to establish the reliability of a variety of communication configurations involving series and parallel components.
8. Investigate the role of independence in situations such as the Challenger disaster.
9. Use data from surveys and the percentages quoted in media outlets to explore and define conditional probability.
10. Use processes over time and processes ranging from manufacturing to medical to epidemiology, to establish and explore the law of total probability.
11. *Investigate the roles of independence and conditional probability in famous law cases and royal commissions in Australia and similar investigations in the USA — for example, those involving forensic evidence such as the Splatt case of South Australia.
12. ■ Investigate transition probability matrices in situations such as recording, over a period of time, changes in the weather as stated by newspaper or TV weather reports (fine, showers, cloudy, clearing); construct the matrix describing the probabilities that one condition will be followed by each different condition; given today’s weather, find the most probable sequence of the weather in the near future.
13. ■ *Use data and simple Markov chain modelling to develop models with estimated probabilities for a range of situations such as signals, weather, psychology, savings, traffic, or insurance.
14. Establish the model for choosing a point at random on a line of given length.

15. Collect and investigate human data on random number generation.
16.  *Collect and explore data from the random number generators of various computer packages.
17. *Research mathematical models for pseudo-random number generation.
18.  *Investigate graphical methods for comparing data from stochastic and deterministic sources; for example, compare (pseudo-) random numbers with the numbers produced by the sequence $x_n = kx_{n-1}(1 - x_{n-1})$ for different values of k .
19. *Collect data on such variables as the length of phone calls, the times between trucks, the response time on the internet; explore the key features of the data.
20. *Obtain the expected value and median of the uniform distribution and of the exponential distribution; use data to estimate parameters of the uniform and exponential models for data, and explore the suitability of the two models.

**Particularly suitable for assignment/project learning experiences*

Advanced periodic and exponential functions (notional time 30 hours)

Focus



Students are encouraged to extend their knowledge of trigonometric and exponential functions. They are also encouraged to investigate various ways these can be combined to model life-related situations.

Subject matter

- definitions of secant, cosecant, cotangent (SLE 12)
- expansions of $\sin(x \pm y)$, $\cos(x \pm y)$ and the double angle formulas (SLEs 1, 3, 5, 23)
- life-related applications of the sine and cosine functions (SLEs 2, 4–9, 14, 15, 16, 17, 19, 22)
- general shapes of graphs of $y = e^{ax}\sin bx$ and $y = e^{ax}\cos bx$ where a is both positive and negative
- applications of $e^{ax}\sin bx$ and $e^{ax}\cos bx$ (SLEs 5, 18, 20)
- logistic curve $y = \frac{A}{1 + Ce^{-kt}}$ (where A , C and k are constants) as a model of many natural systems (SLEs 10, 11, 12, 21).





Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1.  Express $y = a \cos x + b \sin x$ in the form $A \cos(x + C)$ or $A \sin(x + D)$ and graph the curve.
2.  The unmodulated waveform from a radio or TV transmitter can be described by $x = a \sin bt$; this is changed when broadcasting music. For AM broadcasting the constant a is modified to $a + c \sin ft$. for FM broadcasting the constant b is modified to $b + c \sin ft$. Use a computer package to plot these waveforms, and relate the parameters involved to the frequency of transmission, and the pitch and loudness of the sound.

3. Investigate the use of rotation matrices to show the identities for $\sin(x \pm y)$, $\cos(x \pm y)$.
4. Use expansions of $\sin(x + y)$, $\cos(x + y)$ and $\tan(x + y)$ to develop expressions for $\sin(2x)$, $\cos(2x)$ and $\tan(2x)$.
5. Use complex numbers to evaluate $\int e^{ax} \sin bx \, dx$ or $\int e^{ax} \cos bx \, dx$.
6. Investigate musical notes as a combination of sine functions.
7. Investigate the period of a pendulum for large angles.
8. Investigate the motion of a water wheel or ferris wheel as an example of a mathematical model based on a sinusoidal function.
9. Investigate the motion of an orbiting satellite as an example of a situation that can be modelled by a sinusoidal function.
10. Investigate systems which follow the logistic curve, e.g. biological growth with environmental constraints, the spread of disease or rumour, death of individuals with increasing dose rates of a toxic substance, concentration of end product in a chemical reaction.
11. Verify that the differential equation $\frac{dy}{dt} = \frac{k}{A} y(A - y)$ where $y = \frac{A}{1 + C}$ at $t = 0$ has the logistic equation $y = \frac{A}{1 + Ce^{-kt}}$ as its solution.
12. ■ Use Australian Census data for your city, town or shire to fit the logistic curve for population growth
$$p(t) = \frac{L}{1 + Ce^{-r(t-t_0)}}$$
 where $p(t)$ is the population at time t , t_0 is some convenient starting date, and L , C and r are parameters to be estimated.
13. ■ Use a computer or a graphing calculator to investigate the graphs of $\sec x$, $\operatorname{cosec} x$ and $\cot x$, and relate these to the graphs of $\sin x$, $\tan x$ and $\cos x$.
14. ■ Use a computer or a graphing calculator to draw the graphs of
$$y = \sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \dots$$
$$y = \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots$$
$$y = \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots$$
$$y = \sin x + \frac{1}{2^3} \sin 2x + \frac{1}{3^3} \sin 3x + \dots$$
15. ■ Large amplitude waves sometimes appear on the ocean with no apparent cause. They arise from the superposition of a number of waves of different amplitudes and wavelengths.

Use computer software or a graphing calculator to investigate the result of adding various sine waves with different wavelengths and amplitudes.

16. Investigate the periodic patterns which are traced out by a point on the circumference of a circle which rolls around the inside or outside of a larger circle; use polar coordinates to write an expression for the curve.
17.  Identify some oscillations with increasing amplitude in physical situations such as the Tacoma Bridge collapse.
18.  Use a video of a mass oscillating on the end of a spring to see how closely the position of the mass follows an expression of the form $e^{at} \sin bt$, where a is negative.
19.  Some surfers believe that ocean waves occur in “sets” of seven, with the seventh wave being “larger” than the preceding six waves. Use a computer or graphing calculator to find functions whose graphs represent these sets of waves.
20.  Study the vertical motion of a car on its springs; use a video to see whether the motion is the sum of two exponential functions, Ae^{at} and Be^{bt} , where a and b are negative, or whether it consists of a decaying oscillation of the form $e^{at} \sin bt$, where a is negative.
21. All the heavy elements were formed in a supernova explosion which preceded the formation of the solar system. Assuming that the two major isotopes of uranium, U_{238} and U_{235} , were then formed in equal amounts, use their present relative abundances and half-lives to estimate the time at which the supernova explosion took place (a suitable unit for time is a thousand million years).
22. Investigate how the description of periodic phenomena in electric circuits uses complex numbers.
23. Develop multiple angle trigonometric formulas using de Moivre’s Theorem.

School option(s) (notional time 30 hours per option)

Schools may develop option(s) of their own choice subject to the guidelines below:

- Each option must be consistent with the rationale and global aims of the course.
- Each option offered to a student must have subject matter substantially different from any other subject matter that the student will meet in Mathematics B and Mathematics C.
- Each option should contain subject matter ensuring a level of challenge and integration comparable to that provided by the other optional topics.
- In addition to specifying the subject matter, the work program will need to indicate a focus and state the range of likely learning experiences for each option.
- Each option should assist in providing the student with experiences in a balance of branches of mathematics.
- Although the use of computer packages is encouraged, the study of computer programming language(s) is not appropriate as a school option.

6. Assessment

The purposes of assessment are to provide feedback to students and parents about learning that has occurred, to provide feedback to teachers about the teaching and learning processes, and to provide information on which to base judgments about how well students meet the general objectives of the course. In designing an assessment program, it is important that the assessment tasks, conditions and criteria are compatible with the general objectives and the learning experiences. Assessment then is an integral aspect of a course of study. It can be formative or summative. The distinction between formative and summative assessment lies in the purpose for which that assessment is used.

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify the students' strengths and weaknesses so students may improve their achievement and better manage their own learning. The formative techniques used should be similar to summative assessment techniques, which students will meet later in the course. This provides students with experience in responding to particular types of tasks, under appropriate conditions. So that students can prepare, it may be that feedback on any early assessment tasks can be used in a formative sense also to assist students' preparation for later assessment tasks.

Summative assessment, while also providing feedback to students, parents and teachers, provides cumulative information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment techniques and instruments/tasks to be used, when they will be administered, and how they contribute to the determination of exit levels of achievement. Students' achievements are matched to the standards of exit criteria, which are derived from the general objectives of the course. Thus, summative assessment provides the information for certification at the end of the course.

6.1 Underlying principles of exit assessment

The policy on exit assessment requires consideration to be given to the following principles when devising an assessment program for the two-year course of study.

- Information is gathered through a process of continuous assessment.
- Balance of assessments is a balance over the course of study and not necessarily a balance over a semester or between semesters.
- Exit achievement levels are devised from student achievement in all areas identified in the syllabus as being mandatory.
- Assessment of a student's achievement is in the significant aspects of the course of study identified in the syllabus and the school's work program.
- Selective updating of a student's profile of achievement is undertaken over the course of study.
- Exit assessment is devised to provide the fullest and latest information on a student's achievement in the course of study.

These principles are to be considered together and not individually in the development of an assessment program. Exit assessment must satisfy concurrently the six principles associated with it.

Continuous assessment

The major operating principle is “continuous assessment”. The process of continuous assessment provides the framework in which all the other five principles of balance, mandatory aspects of the syllabus, significant aspects of the course, selective updating, and fullest and latest information exist and operate.

This is the means by which assessment instruments are administered at suitable intervals and by which information on student achievement is collected. It involves a continuous gathering of information and the making of judgments in terms of the stated criteria and standards throughout a two-year course of study.

Decisions about levels of achievement are based on information gathered, through the process of continuous assessment, at points in the course of study appropriate to the organisation of the learning experiences. Levels of achievement must not be based on students’ responses to a single assessment task at the end of a course or instruments set at arbitrary intervals that are unrelated to the developmental course of study.

Balance

Balance of assessments is a balance over the course of study and not necessarily a balance within a semester or between semesters.

Within the two-year course for Mathematics C it is necessary to establish a suitable balance in the general objectives, assessment techniques and instruments/tasks, conditions and across the criteria. The exit criteria are to have equal emphasis across the range of summative assessment. The exit assessment program must ensure an appropriate balance over the course of study as a whole.

Mandatory aspects of the syllabus

Judgment of student achievement at exit from a two-year course of study must be derived from information gathered about student achievement in those aspects stated in the syllabus as being mandatory, namely

- the general objectives of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*
- the six core topics, *Introduction to groups*, *Real and complex number systems*, *Matrices and applications*, *Vectors and applications*, *Calculus*, *Structures and patterns*, in addition to two elective topics.
- The exit criteria and standards stated in Sections 6.6 and 6.9 respectively, must be used to make the judgment of student achievement at exit from a two-year course of study.

Significant aspects of the course of study

Significant aspects refer to those areas in the school’s course of study selected from the choices permitted by the syllabus. Significant aspects can complement mandatory aspects or be in addition to them. They will be determined by the context of the school and the needs of students at that school to provide choice of learning experiences appropriate to the location of the school, the local environment and the resources available.

The significant aspects must be consistent with the general objectives of the syllabus and complement the developmental nature of learning in the course over two years.

Selective updating

In conjunction with the principle of fullest and latest information, information on student achievement should be selectively updated throughout the course.

Selective updating is related to the developmental nature of the course of study and operates within the context of continuous assessment. As subject matter is treated at increasing levels of complexity, assessment information gathered at earlier stages of the course may no longer be representative of student achievement. The information therefore should be selectively and continually updated (not averaged) to accurately reflect student achievement.

The following conceptions of the principle of selective updating apply:

- A systemic whole subject-group approach in which considerations about the whole group of students are made according to the developmental nature of the course and, in turn, the assessment program. In this conception, developmental aspects of the course are revisited so that later summative assessment replaces earlier formative information.
- An act of decision-making about individual students — deciding from a set of assessment results the subset which meets study area specification requirements and typically represents a student's achievements, thus forming the basis for a decision about a level of achievement. In the application of decisions about individual students, the set of assessment results does not have to be the same for all students. However, the subset which represents the typical achievement of a student must conform to the parameters outlined in the school's study plan for the strand.

Selective updating must not involve students reworking and resubmitting previously graded assessment tasks. Opportunities may be provided for students to complete and submit additional tasks. Such tasks may provide information for making judgments where achievement on an earlier task was unrepresentative or atypical, or there was insufficient information upon which to base a judgment.

Fullest and latest information

Judgments about student achievement made at exit from a school course of study must be based on the fullest and latest information available. This information is recorded on a student profile.

"Fullest" refers to information about student achievement gathered across the range of general objectives. "Latest" refers to information about student achievement gathered from the most recent period in which the general objectives are assessed. As the assessment program in Mathematics C is developmental, fullest and latest information will most likely come from Year 12.

Information recorded on a student profile will consist of the latest assessment data on mandatory and significant aspects of the course, which includes the data gathered in the summative assessment program that is not superseded.

6.2 Planning an assessment program

At the end of Year 12, judgments are made about how students have achieved in relation to the standards stated in the syllabus for each of the criteria. These summative judgments are based on achievement in each of the general objectives.

When planning an assessment program, schools must consider:

- general objectives (Section 3)
- principles of a balanced course (Section 3.3)

- the learning experiences (Section 5)
- the underlying principles of assessment (Section 6.1)
- a variety of assessment techniques and instruments over the two-year course (Section 6.4)
- conditions under which the assessment is implemented
- the exit criteria and standards (Sections 6.6 and 6.9)
- verification folio requirements, especially the number and the nature of student responses to assessment tasks to be included (Section 6.8)
- minimum assessment necessary to reach a valid judgment of the student's standard of achievement.

Students should be conversant with the assessment techniques and have knowledge of the criteria to be used in assessment instruments.

6.3 Implementing assessment

Assessment instruments are developed by the school to provide:

- information on which teachers may make judgments about student achievement of the general objectives
- a level of challenge suitable for the whole range of students.
- An assessment instrument is accompanied by:
 - a statement of the conditions of assessment that apply (Section 6.3.1)
 - a detailed description of the instrument (Section 6.3.2)
 - a detailed criteria sheet (Section 6.3.3)
 - details of procedures for authentication of student responses (Section 6.3.4).

6.3.1 Conditions of assessment

Across the whole assessment program, teachers should establish a range of conditions. This can be done by systematically varying the factors that are most significant in establishing the conditions for an instrument, namely:

- the time allowed to prepare and complete the response
- access to resources, both material and human, during the preparation for and completion of the instrument.

Every instrument description must include clear statements of the assessment conditions that apply. These may include:

- time available for the preparation and completion of the response
- resources accessible and available (both material and human) during the preparation for and completion of the response
- location for the preparation and completion of the response, e.g. in class, at home
- whether the response is to be an individual or group production
- the strategy used to ensure student authorship and ownership, e.g. the degree of teacher supervision and teacher monitoring that will apply.

6.3.2 Instrument descriptions

Instrument descriptions are to:

- state all instrument requirements, including the length and conditions
- be congruent with the general objectives of the syllabus, the standards associated with exit criteria and the school work program. This congruence ensures the essential relationship between learning, teaching and assessment practices.

6.3.3 Criteria sheets

Where criteria sheets specific to each instrument are developed, they should be provided to students before undertaking assessment.

An instrument-specific criteria sheet:

- should be derived from the exit criteria
- must describe/state standards consistent with those associated with exit criteria (see section 6.9)
- should provide a clear expectation of how standards will be demonstrated
- should inform teaching and learning practice.

Once the student has completed an assessment instrument, the criteria sheet:

- must indicate student achievement
- is used to inform teacher judgment about student achievement
- may provide students with the opportunity to develop self-evaluative expertise.

The extent to which the exit standards are reflected in the criteria sheet will vary according to the general objectives associated with the instrument and according to the stage in the course at which the instrument is undertaken.

6.3.4 Authentication of student work

It is essential that judgments of student achievement be made on genuine student assessment responses. Teachers must take reasonable steps to ensure that each student's work is their own, particularly where students have access to electronic resources or when they are preparing collaborative responses to tasks.

The QSA's A–Z of Senior Moderation contains a strategy for authenticating student work <www.qsa.qld.edu.au/10773.html>. This provides information about various methods teachers can use to monitor that students' work is their own. Particular methods outlined include:

- teachers seeing plans and drafts of student work
- student production and maintenance of evidence for the development of responses
- student acknowledgment of resources used.

Teachers must ensure students use consistent, accepted conventions of in-text citation and referencing, where appropriate.

6.4 Assessment techniques

Assessment techniques in this syllabus are grouped under categories. The following categories of assessment techniques may be considered:

- extended modelling and problem-solving tasks
- reports
- supervised tests.

Assessment of student achievement should not be seen as a separate activity, but as an integral part of the developmental learning process which reflects the learning experiences of students. There should be variety and balance in the types of assessment instruments used, thereby enabling students with different learning styles to demonstrate their understanding.

An extended modelling and problem-solving task or a report or similar must be included at least twice each year. These should contribute significantly to the decision-making process in each of the three exit criteria.

6.4.1 Category: Extended modelling and problem-solving tasks

What is an extended modelling and problem-solving task?
<p>An extended modelling and problem-solving task is an assessment instrument developed in response to a mathematical task. It may require a response that involves mathematical language, appropriate calculations, tables of data, graphs and diagrams, and could involve standard Australian English.</p> <p>Students may provide a response to a specific task or issue that could be set in a context that highlights a real-life application of Mathematics.</p> <p>Aspects of each of the three criteria should be evident in the task.</p>
What might a student do to complete an extended modelling and problem-solving task?
<ul style="list-style-type: none"> • Analyse information and data from a variety of sources • Process information to identify assumptions and parameters • Interpret and synthesise data • Explain relationships to develop and support mathematical arguments • Reflect on and evaluate data collected, propositions, results and conclusions • Communicate ideas.
What do teachers do when planning and implementing an extended modelling and problem-solving task?
<ul style="list-style-type: none"> • The teacher should provide the mathematical task. • Teachers must implement strategies to ensure authentication of student work. Some strategies are annotated notes in response to issues that emerged during the extended modelling and problem-solving task; teacher observation sheets; research checklists and referencing and reference lists. • Teachers may consult, negotiate and provide feedback prior to and during students' preparation of the report to provide ethical guidance and to monitor student work. Feedback and assistance should be provided judiciously. • When students undertake extended modelling and problem-solving tasks for the first time, scaffolding may be provided to help students complete the assessment. However, if the task is intended to demonstrate high initiative then the scaffolding provided should not specify the procedures, nor lead the student through a series of steps to reach a solution. <p>Scaffolding should be reduced from Year 11 to Year 12 to allow the student to better demonstrate the principle of initiative in the problem-solving process.</p>

6.4.2 Category: Reports

What is a report?

A report is typically an extended response to a practical or investigative task such as:

- an experiment in which data are collected, analysed and modelled
- a mathematical investigation
- a field activity
- a project.

A mathematical report could comprise such forms as:

- a scientific report
- a proposal to a company or organization
- a feasibility study

The report and the activities leading to a report could be done individually and/or in groups; in class time and/or in the students' own time. A report will typically be in written form, or a combination of written and oral multimedia forms.

The report will generally include an introduction, analysis of results and data, conclusions drawn, justification, and, when necessary, appendixes and a bibliography and/or reference list.

Aspects of each of the three criteria should be evident in the task.

What might a student do to complete a report?

- Gather and sort information and data from a variety of sources
- Process information to identify assumptions and parameters
- Interpret, analyse and synthesise data
- Explain relationships to develop and support mathematical arguments
- Reflect on and evaluate data collected, propositions, results and conclusions
- Communicate ideas.

What do teachers do when planning and implementing a report?

- The teacher suggests topics and provides some stimulus to trigger student interest.
- Teachers can provide the research question or it may be instigated by the student. In those instances teachers should negotiate with students to ensure the possibility of success. It is more likely that students will be able to generate their own research questions the further they progress in the course of study.
- Teachers may allow some class time for students to be able to effectively undertake each component of the report. Teachers may allow elements of the report to be conducted in small groups or pairs.
- Teachers must implement strategies to ensure authentication of student work. Some strategies are annotated notes in response to issues that emerged during the report; teacher observation sheets; checklists and referencing and reference lists.
- Teachers may consult, negotiate and give feedback before and during the report to provide ethical guidance and to monitor student work. Feedback and assistance should be provided judiciously.
- When students undertake reports for the first time, scaffolding may be provided to help students complete the assessment. However, if the task is intended to demonstrate high initiative then the scaffolding provided should not specify the procedures, nor lead the student through a series of steps to reach a solution.
- Scaffolding should be reduced from Year 11 to Year 12 to allow the student to better demonstrate the principle of initiative in the problem-solving process.

6.4.3 Category: Supervised tests

What is a supervised test?

A supervised test is an assessment instrument that is conducted under supervised conditions. The supervised test commonly includes tasks requiring quantitative and/or qualitative responses. The tasks could be done individually and/or in groups.

The supervised test must provide adequate opportunities for students to demonstrate their level of expertise in Mathematics across the full range of standards in the syllabus.

The supervised test may be constructed from the following four types of techniques:

1. **Short items**

- requiring multiple-choice, single word, sentence or short paragraph (up to 50 words) responses, written in mathematical language, symbols and/or standard Australian English.

2. **Practical exercises**

- using graphs, tables, diagrams, data or the application of algorithms

3. **Paragraph responses**

- these are used when explanation of a greater complexity is required and are written in mathematical language, symbols and/or standard Australian English. Responses should be 50–150 words.

4. **Responses to seen or unseen stimulus materials**

- this may take the form of a series of short items, practical exercises and paragraph responses (see above)
- the question or statement is not provided before the assessment (unseen) and should focus on asking the students to evaluate and justify.
- Stimulus materials should be succinct enough to allow students to engage with them in the time provided for the supervised test. Perusal times may be required or if the stimulus materials are lengthy they may need to be shared with students prior to the administration of the supervised test.

What do teachers do when planning a supervised test?

The teacher should:

- construct questions that are unambiguous
- format the paper to allow for ease of reading and responding
- consider the individual needs of the students
- ensure the questions allow students to demonstrate the full range of standards
- ensure that formula sheets, if used, are supplied by the school and are common and constant across the cohort.
- consider whether students will have access to information previously stored in their calculator.

6.5 Special consideration

Guidance about the nature and appropriateness of special consideration and special arrangements for particular students may be found in the Authority's *Policy on Special Consideration in School-based Assessments in Senior Certification* (2006), available from <www.qsa.qld.edu.au> under Assessment > Senior assessment > Special consideration. This statement also provides guidance on responsibilities, principles and strategies that schools may need to consider in their school settings.

To enable special consideration to be effective for students so identified, it is important that schools plan and implement strategies in the early stages of an assessment program and not at the point of deciding levels of achievement. The special consideration might involve alternative teaching approaches, assessment plans and learning experiences.

6.6 Exit criteria

The following exit criteria must be used in making judgments about a student's level of achievement at exit from this course. They reflect the three assessable categories of general objectives of the syllabus as defined in Section 3:

Knowledge and procedures

This criterion refers to the student's ability to recall, access, select and apply mathematical definitions, rules and procedures, to demonstrate numerical and spatial sense and algebraic facility, with and without the use of mathematical technology, and to demonstrate knowledge and application of the nature of mathematical proof.

Modelling and problem solving

This criterion refers to the student's ability to apply problem-solving strategies to investigate and model situations, generate and use data, analyse and interpret results in the context of problems to investigate the validity of mathematical arguments and models, and, when appropriate, modify mathematical models.

Communication and justification

This criterion refers to the student's ability to interpret, translate, communicate, present, justify and prove mathematical arguments and propositions, using mathematical and everyday language and symbols to provide supporting arguments in the form of proof.

6.7 Determining exit levels of achievement

On completion of the course of study, the school is required to award each student an exit level of achievement from one of the five categories:

Very High Achievement

High Achievement

Sound Achievement

Limited Achievement

Very Limited Achievement.

The school must award an exit standard for each of the criteria *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* based on the principles of

assessment described in Section 6.1. The criteria are derived from the general objectives and are described in Section 3. The *typical* standards associated with the three exit criteria are described in the matrix in Section 6.9. When teachers are determining a standard for each criterion, *it is not always necessary* for the student to have met each descriptor for a particular standard; the standard awarded should be informed by how the qualities of the work match the descriptors overall.

For Year 11, particular standards descriptors may be selected from the matrix and/or adapted to suit the task. These standards are used to inform the teaching and learning process. For Year 12 tasks, students should be provided with opportunities to understand and become familiar with the expectations for exit. The *typical* standards are applied to the summative body of work selected for exit.

The seven key competencies* referred to in the rationale are embedded in the descriptors in the standards matrix. The descriptors refer mainly to aspects of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*.

When standards have been determined in each of the criteria of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, the following table is used to award exit levels of achievement, where A represents the highest standard and E the lowest. The table indicates the *minimum combination of standards* across the criteria for each level.

Awarding exit levels of achievement

VHA	Standard A in any two criteria and no less than a B in the remaining criterion
HA	Standard B in any two criteria and no less than a C in the remaining criterion
SA	Standard C in any two criteria, <i>one of which must be the Knowledge and procedures criterion</i> , and no less than a D in the remaining criterion
LA	At least Standard D in any two criteria, <i>one of which must be the Knowledge and procedures criterion</i>
VLA	Standard E in the three criteria

6.8 Requirements for verification folio

A verification folio is a collection of a student's responses to assessment instruments on which the level of achievement is based. Each folio should contain a variety of assessment techniques demonstrating achievement in the criteria *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, over a range of topics. The variety of assessment techniques is necessary to provide a range of opportunities from which students may demonstrate achievement.

For information about preparing monitoring and verification submissions schools should refer to *Moderation Processes for Senior Certification* available at <www.qsa.qld.edu.au> under Assessment > Senior Assessment > Forms and procedures.

Students' verification folios for Mathematics C must contain:

- a minimum of four assessment instruments from Year 12, with at least one of these being an extended modelling and problem-solving task or a report or similar
- student responses to a minimum of four and a maximum of 10 summative assessment instruments

* KC1: collecting, analysing and organising information; KC2: communicating ideas and information; KC3: planning and organising activities; KC4: working with others and in teams; KC5: using mathematical ideas and techniques; KC6: solving problems; KC7: using technology

- where used, a criteria sheet for each assessment instrument which provides evidence of how students meet standards associated with the assessment criterion involved in that instrument
- formula sheets or other allowable materials used, where appropriate
- a student profile completed to date.

6.8.1 Post-verification assessment

In addition to the contents of the verification folio, there must be subsequent summative assessment in the exit folio. In Mathematics C at least one instrument must be completed after verification. It is desirable for the assessment instrument to include all criteria.

6.9 Standards associated with exit criteria

Criterion	Standard A	Standard B	Standard C	Standard D	Standard E
Knowledge and procedures	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> use of stated rules and procedures in simple situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> statements of relevant mathematical facts
	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations 	<ul style="list-style-type: none"> application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations 		
	<ul style="list-style-type: none"> numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations 	<ul style="list-style-type: none"> numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations 	<ul style="list-style-type: none"> numerical sense, spatial sense and algebraic facility in routine, simple life-related or abstract situations 	<ul style="list-style-type: none"> numerical sense, spatial sense and/or algebraic facility in routine or simple tasks 	
	<ul style="list-style-type: none"> appropriate selection and accurate use of technology 	<ul style="list-style-type: none"> appropriate selection and accurate use of technology 	<ul style="list-style-type: none"> selection and use of technology 	<ul style="list-style-type: none"> use of technology 	<ul style="list-style-type: none"> use of technology
	<ul style="list-style-type: none"> knowledge of the nature of and use of mathematical proof 				

Modelling and problem solving	Standard A	Standard B	Standard C	Standard D	Standard E
	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple tasks through to routine complex tasks in life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations 	<p>The student work has the following characteristics:</p> <ul style="list-style-type: none"> • evidence of simple problem-solving strategies in the context of problems 	<p>The student work has the following characteristic:</p> <ul style="list-style-type: none"> • evidence of simple mathematical procedures
	<ul style="list-style-type: none"> • identification of assumptions and their associated effects, parameters and/or variables 	<ul style="list-style-type: none"> • identification of assumptions, parameters and/or variables 			
	<ul style="list-style-type: none"> • use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations 	<ul style="list-style-type: none"> • use of data to synthesise mathematical models in simple situations and generation of data from mathematical models in simple through to complex situations 	<ul style="list-style-type: none"> • use of mathematical models to represent routine, simple situations and generate data 	<ul style="list-style-type: none"> • use of given simple mathematical models to generate data 	
	<ul style="list-style-type: none"> • investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems, the strengths and limitations of models, both given and developed 	<ul style="list-style-type: none"> • interpretation of results in the context of simple through to complex problems and mathematical models 	<ul style="list-style-type: none"> • interpretation of results in the context of routine, simple problems 		
	<ul style="list-style-type: none"> • refinement of mathematical models 				

Communication and justification	Standard A	Standard B	Standard C	Standard D	Standard E
	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> use of mathematical terminology, symbols or conventions in simple or routine situations 	<p>The student's work has the following characteristics:</p> <ul style="list-style-type: none"> use of mathematical terminology, symbols or conventions
	<ul style="list-style-type: none"> organisation and presentation of information in a variety of representations 	<ul style="list-style-type: none"> organisation and presentation of information in a variety of representations 	<ul style="list-style-type: none"> organisation and presentation of information 	<ul style="list-style-type: none"> presentation of information 	<ul style="list-style-type: none"> presentation of information
	<ul style="list-style-type: none"> analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine 	<ul style="list-style-type: none"> analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine 	<ul style="list-style-type: none"> translation of information from one representation to another in simple routine situations 		
	<ul style="list-style-type: none"> use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language 	<ul style="list-style-type: none"> use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language 	<ul style="list-style-type: none"> use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language 		
	<ul style="list-style-type: none"> coherent, concise and logical justification of procedures, decisions and results 	<ul style="list-style-type: none"> coherent and logical justification of procedures, decisions and results 	<ul style="list-style-type: none"> justification of procedures, decisions or results 		
	<ul style="list-style-type: none"> justification of the reasonableness of results 				
	<ul style="list-style-type: none"> provision of supporting arguments in the form of proof 				

7. Language education

Teachers of Senior English have a special responsibility for language education. However, it is the responsibility of all teachers to develop and monitor students' abilities to use the forms of language appropriate to their own subject areas. Their responsibility entails developing the following skills:

- ability in the selection and sequencing of information required in the various forms (such as reports, essays, interviews and seminar presentations)
- the use of technical terms and their definitions
- the use of correct grammar, spelling, punctuation and layout.

Mathematics C requires students to use language in a variety of ways — mathematical, spoken, written, graphical, symbolic. The responsibility for developing and monitoring students' abilities to use effectively the forms of language demanded by this course rests with the teachers of mathematics. This responsibility includes developing students' abilities to:

- select and sequence information
- manage the conventions related to the forms of communication used in Mathematics C (such as short responses, reports, multimedia presentations, seminars)
- use the specialised vocabulary and terminology related to Mathematics C
- use language conventions with reference to standard Australian English, related to grammar, spelling, punctuation and presentation.
- communicate effectively and efficiently.

Thus, when writing, reading, questioning, listening and talking about mathematics, teachers and students should use the specialised vocabulary related to the subject. Students should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions.

Assessment in Mathematics C needs to take into consideration appropriate use of language. Assessment instruments should use format and language that are familiar to students. Attention to language education within Mathematics C should assist students to meet the language components of the exit criteria, especially the criterion *Communication and justification*.

8. Quantitative concepts and skills

Success in dealing with issues and situations in life and work depends on the development and integration of a range of abilities, such as being able to:

- comprehend basic concepts and terms underpinning the areas of number, space, probability and statistics, measurement and algebra
- extract, convert or translate information given in numerical or algebraic forms, diagrams, maps, graphs or tables
- calculate, apply algebraic procedures, implement algorithms
- make use of calculators and computers
- use skills or apply concepts from one problem or one subject domain to another.

In all subjects students are to be encouraged to develop their understanding and to learn through the incorporation — to varying degrees — of mathematical strategies and approaches to tasks. Similarly, students should be presented with experiences that stimulate their mathematical interest and hone those quantitative skills that contribute to operating successfully within each of their subject domains.

Mathematics C focuses on the development and application of numerical and other mathematical concepts and skills. It provides a basis for the general development of such quantitative skills to prepare students to cope with the quantitative demands of their personal lives or to participate in a specific workplace environment.

The distinctive nature of Mathematics C will require that new mathematical concepts be introduced and new skills be developed. Within appropriate learning contexts and experiences in the subject, opportunities are to be provided for revising, maintaining and extending such skills and understandings

The following quantitative knowledge and procedures will be required throughout the Mathematics C course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in
- practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- identities, linear equations and inequalities
- the gradient of a straight line
- plotting points using Cartesian coordinates
- basic algebraic manipulations
- the equation of a straight line
- solutions of a quadratic equation
- the formula for the zeros of a quadratic equation
- completing the square in a quadratic
- absolute value

- the summation notation: $\sum_{i=1}^n x_i$

9. Educational equity

Equity means fair treatment of all. In developing work programs from this syllabus, schools should incorporate the following concepts of equity.

All young people in Queensland have a right to gain an education that meets their needs, and prepares them for active participation in creating a socially just, equitable and democratic global society. Schools need to provide opportunities for all students to demonstrate what they know and can do. All students, therefore, should have equitable access to educational programs and human and physical resources. Teachers should ensure that particular needs of the following groups of students are met: female students; male students; Aboriginal students; Torres Strait Islander students; students from non-English-speaking backgrounds; students with disabilities; students with gifts and talents; geographically isolated students; and students from low socioeconomic backgrounds.

Subject matter chosen should include, whenever possible, the contributions and experiences of all groups of people. Learning contexts and community needs and aspirations should also be considered. In choosing appropriate learning experiences teachers can introduce and reinforce non-racist, non-sexist, culturally sensitive and unprejudiced attitudes and behaviour. Learning experiences should encourage the participation of students with disabilities and accommodate different learning styles.

Resource materials used should recognise and value the contributions of both females and males to society and include social experiences of both genders. Resource materials should also reflect cultural diversity within the community and draw from the experiences of the range of cultural groups in the community.

To allow students to demonstrate achievement, barriers to equal opportunity need to be identified, investigated and removed. This may involve being proactive in finding the best ways to meet the diverse range of learning and assessment needs of students. The variety of assessment techniques in the work program should allow students of *all* backgrounds to demonstrate their knowledge and skills related to the criteria and standards stated in this syllabus. The syllabus criteria and standards should be applied in the same way to all students.

Teachers should consider equity policies of individual schools and schooling authorities, and may find the following resources useful for devising an inclusive work program:

ACACA 1996, *Guidelines for Assessment Quality and Equity*, available from <www.acaca.org.au>

ANTA 2004, *A Guide to Equity and the AQTF*, available from Australian Training Products Ltd <www.atpl.net.au>

QSA 2006, *Policy on Special Consideration in School-based Assessments in Senior Certification*, available from <www.qsa.qld.edu.au>

QSA 2006, *Policy Statement: Equity*, available from <www.qsa.qld.edu.au>

EQ 2005, *Inclusive education statement*, available from <<http://education.qld.gov.au/studentservices/>>

EQ 2006, *Education Policy and Procedures Register: Inclusive education*, available from <<http://education.qld.gov.au/strategic/eppr/>>

QSCC 2001, *Equity Considerations for the development of curriculum and test material*, available from <www.qsa.qld.edu.au>

QCEC 2001, *Inclusive Practices in Queensland Catholic Schools*, available from <www.qcec.qld.catholic.edu.au>

10. Resources

Text and reference books

A wide variety of textbooks and resource materials that could be used as sources of information about Mathematics C are available. Book suppliers provide information regarding current publications.

World Wide Web

Many interactive and static websites can be used to enhance a course in Mathematics C and often include useful resources. Some relevant sites can be sourced through the QSA website. Interactive websites can be found by searching for *applets*. Some particularly useful sites include:

- Department of Education, Training and the Arts Maps and Compasses Resource Page: <http://education.qld.gov.au/curriculum/area/maths/compass/html/resources.html>
- Online Teachers' Resource Network: http://www.otrnet.com.au/Weblinks/lesson_resources.html
- The Australian Association of Mathematics Teachers: <http://www.aamt.edu.au>
- The Queensland Association of Mathematics Teachers: <http://qamt.org>

Newspaper reports

Newspapers can be a source of useful data. The compilation of news files on particular topics can broaden students' knowledge and provide a valuable source of material for developing assessment instruments.

Periodicals

Journals and periodicals provide current, relevant information. Journals and periodicals relevant to Mathematics C may include:

- *Australian Senior Mathematics Journal* — journal of the Australian Association of Mathematics Teachers, Inc.
- *Mathematics Education Research Journal* — journal of Mathematics Education Research Group of Australasia, Inc.
- *Teaching Mathematics* — journal of the Queensland Association of Mathematics Teachers
- *The ANZIAM Journal* (The Australian and New Zealand Industrial and Applied Mathematics Journal) — journal of the Australian Mathematics Society
- *The Australian Mathematics Teacher* — journal of the Australian Association of Mathematics Teachers, Inc.

School librarians should be able to help identify and locate other useful periodicals.

Electronic media and learning technology

A wide range of videos, DVDs and television recordings are available on a variety of topics related to Mathematics C. A variety of computer software programs and CD-ROMs may be useful for a course in Mathematics C, as learning tools, to gain access to information presented in a variety of forms, and to help students gain ICT skills. Educational program distributors are able to supply updated resource lists.

Organisations and community resources

A variety of government and community organisations provide personnel, advice, resources and information to help construct and implement a course in Mathematics C. Some of these include:

- Australian Bureau of Statistics
- Bureau of Meteorology
- Queensland Association of Mathematics Teachers.

Glossary

Abstraction

The process of extracting the underlying essence of a mathematical concept, removing dependence on the real-world contexts which might have inspired it.

Algebraic facility

The ability to model, manipulate, simplify, substitute, factorise, expand and solve some symbolic representation.

Algorithm

Process or set of rules to be used; a list of well-defined instructions for completing a task; step-by-step approach.

Analyse

To break up a whole into its parts, to examine in detail to determine the nature of, to look more deeply into and to detect the relationships between parts.

Assessment instrument

Particular methods developed and used by a school to gather information about student achievement.

Assessment techniques

The methods (categories) identified in the syllabus (Section 6.4) to gather evidence about student achievement.

Axiom

An established rule, principle or law.

Conclusion

Final result or summing up; inference deduced from previous information; reasoned judgment.

Congruence

A congruence is any statement of the form $a \equiv b \pmod{n}$, read as “ a is congruent to b modulo n ”, and means that $a - b$ is divisible by n , where all of a , b and n are integers; for example, $13 \equiv -2 \pmod{5}$.

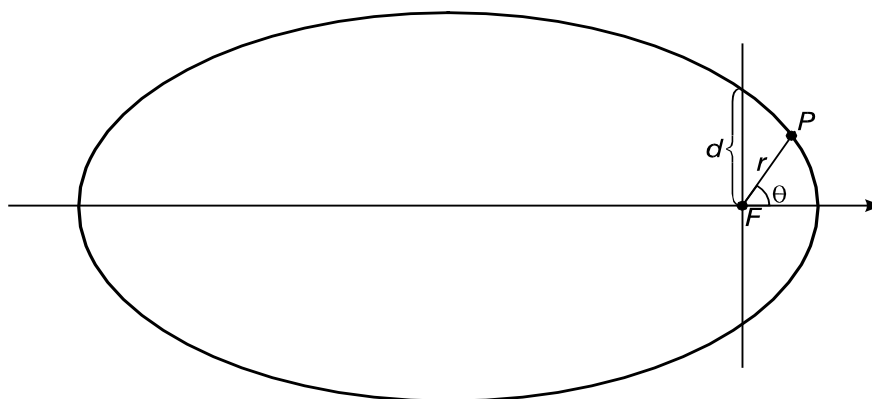
Conics, polar form

The position of a point P on a conic can be expressed using polar coordinates, (r, θ) with one focus at the origin, as

$$\frac{d}{r} = 1 + e \cos \theta$$

where e is the eccentricity of the conic, and d is as shown in the following diagram:

(for the classically minded, d is called the *semi latus rectum*).



Context

A context is a framework for linking concepts and learning experiences that enables students to identify and understand the application of mathematics to their world. A context is a group of related situations, phenomena, technical applications and social issues likely to be encountered by students. A context provides a meaningful application of concepts in real-world situations.

Criterion (pl. Criteria)

A property, dimension or characteristic by which something is judged or appraised. In senior syllabuses, the criteria are the significant dimensions of the subject, described in the Rationale and used to categorise the general objectives and exit criteria.

Criteria sheets

Criteria sheets are developed from the standards associated with exit criteria to describe the attributes of student work anticipated at each level of achievement for the particular assessment instrument.

Data

In the context of the Mathematics C syllabus, data are thought of as documented information or evidence of any kind that lends itself to mathematical interpretation. Data may be quantitative or qualitative. (See Primary data, Secondary data, Qualitative data, Quantitative data.)

Deduce

Infer; reach a conclusion which is necessarily true provided a set of assumptions is true.

Depth

The development of knowledge and understandings from simple through to complex.

Describe

To give an account of in speech or writing; to convey an idea or impression of; characterise; to represent pictorially; depict; to trace the form or outline of.

Determine

To come to a resolution or decide.

Diophantine equation

An equation in more than one variable, with integer coefficients, where integer solutions (normally non-negative) are required. A Diophantine equation may have no solutions, a finite number of solutions, or infinitely many solutions. For example:

$3x + 6y = 16$ has no solutions;

$3x + 5y = 23$ has two positive solutions, (1,4) and (6,1);

$x^2 + y^2 = z^2$ has infinitely many solutions.

Discuss

Consider a particular topic in speaking or writing; talk or write about a topic to reach a decision.

Estimate

Calculate an approximate amount or quantity.

Euclidean algorithm

A procedure for determining the greatest common divisor of two integers, x and y , without factorising them. Denote the quotient and remainder at each division by q_i and r_i respectively, then:

$$x = yq_1 + r_1$$

$$y = r_1q_2 + r_2$$

$$r_1 = r_2q_3 + r_3$$

$$r_2 = r_3q_4 + r_4$$

.....

$$r_{n-2} = r_{n-1}q_n$$

The last non-zero remainder is the greatest common divisor of x and y .

Evaluate

Establish the value, quality, importance, merit, relevance or appropriateness of information, data or arguments based in logic as opposed to subjective preference.

Exemplify

To show or illustrate using examples.

Exit level of achievement

The standard reached by students at exit judged by matching standards in student work with the exit criteria and standards stated in a syllabus. (Section 6.7)

Explain

Make clear or understandable, know in detail.

Exponential distribution

This is a continuous distribution whose probability density function is given by

$$f(x) = \eta e^{-\eta x} \quad 0 < x$$

where η is a non-negative parameter. It is the distribution of the time until the next event in a memoryless process, that is, a process in which events are occurring at random at an average rate of η per unit time.

Formative assessment

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify students' strengths and weaknesses so students may improve their achievement and better manage their own learning.

Fundamental theorem of arithmetic

Any positive integer greater than one can be factorised into the product of prime numbers in only one way, apart from the order in which the factors occur.

General objectives

General objectives are those which the school is intended to pursue directly and student achievement of these is assessed by the school.

Generalisation

Creation of a statement or formula by considering similar situations subject to the same constraints. Includes the nature of proof.

Example 1

By considering how a hot mass cools, students are able to generalise the statement *an exponential function* $y = ab^x + ta$ can be used to model the cooling of a hot mass to ambient temperature.

Example 2

Find the first and second derivatives of the function $y = \sin 2x$

Extend to find the third, fourth and fifth derivatives and generalise from your solutions to develop a rule that would help find the 100th derivative.

Solution

$$y = \sin 2x$$

$$y^i = 2 \cos 2x$$

$$y^{ii} = -4 \sin 2x$$

$$y^{iii} = -8 \cos 2x$$

$$y^{iv} = 16 \sin 2x$$

$$= 2^4 \sin 2x$$

$$y^v = 32 \cos 2x$$

$$2^5 \cos 2x$$

From the results, the original trigonometric function is regenerated every 4th derivative; the factor at the front of the function is the power of 2 raised to the same power as that of the derivative number; the sign of the derivative varies according to the sequence : +, −, −, +.

Thus the 100th derivative of $y = \sin 2x$ will be $y^{100} = 2^{100} \sin 2x$ since there are exactly 25 lots of 4.

Interpret

To give meaning to information presented in various forms, e.g. words, symbols, pictures, graphs.

Justify

Provide sound reasons based on logic or theory to support response; prove or show statements are just or reasonable; convince. Specifically the justification of procedures may include:

- providing evidence (words, diagrams, symbols etc.) to support processes used
- stating a generic formula before using specifically
- providing reasoned, well-formed, logical sequence within a response.

Key competencies

The key competencies define skills essential for effective participation in adult life, including further education and employment.

Law of total probability

This is the term often given to the following procedure for obtaining the probability of a complex event through knowledge of how it depends on each of a set of simple events: Consider an event B and consider a partition of the whole sample space into a set of events A_1, \dots, A_k , that are disjoint and whose union is the whole space, i.e. whose union covers every possibility. If we know the probabilities of all the events A_i , and the conditional probabilities of B given A_i , then we can find the probability of B by noting that:

$$\begin{aligned}\Pr(B) &= \Pr(BA_1) + \Pr(BA_2) + \dots + \Pr(BA_k) \\ &= \Pr(B|A_1)\Pr(A_1) + \dots + \Pr(B|A_k)\Pr(A_k)\end{aligned}$$

Leontief matrices

Row entries give the amount of input of different products required to contribute to the production of one unit of column product. Post-multiplication by a gross-product vector then gives the internal consumption vector — the amount of products of different types required to produce the given output amounts. That is, gross-product (output) vector (x) and input–output matrix (A) gives the internal consumption as Ax . Note that consumer demand equals $x - Ax$.

Linear first order differential equations with constant coefficients

These are equations of the form

$$p \frac{dy}{dx} = ky + b$$

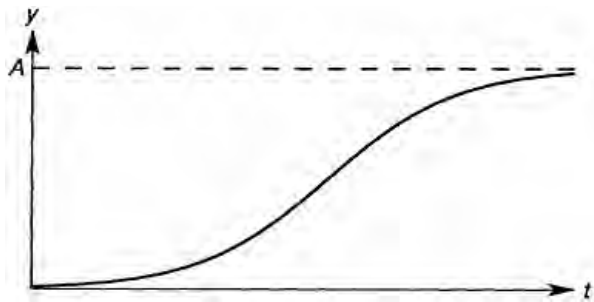
where p , k , and b are constants.

Linear objective function

The function to be optimised in linear programming.

Logistic curve, logistic growth

Many natural systems may be modelled by a sigmoidal curve in which the rate of change in y , with respect to t say, begins slowly and builds rapidly before finally tapering off.



The logistic growth equation

$$\frac{dy}{dt} = ry(a - y).$$

describes such a system where r is the growth rate and a may be thought of as the “carrying capacity” of the environment if $y(t)$ is taken to be a population.

$$y(t) = \frac{a}{1 + ke^{-rt}}$$

Note that a is the asymptotic limit of y and that the maximum rate of change occurs at

$$(t, y) = \left(\frac{\ln k}{ar}, \frac{a}{2} \right) \text{ the point of inflection in the logistic curve.}$$

Logistic map

This is a discrete recurrence relationship that corresponds to the differential equation for the logistic function. The discrete logistic recurrence relationship on $(0, 1)$ is given by $x_{n+1} = cx_n(1 - x_n)$, where c is some constant < 4 . For different values of c the sequence of values produced by this recurrence relationship exhibits different behaviour.

Mathematical argument

A discussion in mathematical terms or language of some or all of the propositions, procedures, results or conclusions.

Mathematical model

Any depiction of a situation expressing a relationship between ideas in mathematical terms.

Mathematical modelling

The act of creating a mathematical model, which may involve the following steps:

- identify assumptions, parameters and/or variables
- interpret, clarify and analyse the problem
- develop strategies or identify procedures required to develop the model and solve the problem
- investigate the validity of the mathematical model.

Moderation

Moderation is the name given to the quality assurance process for senior secondary studies used by the QSA to ensure that:

- Authority subjects taught in schools are of the highest possible standards
- student results in the same subject match the requirements of the syllabus and are comparable across the state
- the process used is transparent and publicly accountable.

Non-routine

Insight and creativity (refer to Section 3.3).

Number sense

An intuitive understanding of numbers, their magnitude, relationships and how they are affected by operations.

Parameter

The values that allow a model to define a particular situation, e.g. m and c in the equation $y = mx + c$

Primary or raw data

Data that have been collected first hand, but not yet processed.

Problem-solving strategies

May include estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems, and organising data.

Procedure

A list of sequential instructions that is to be used to solve a problem or perform a task.

Proof

In mathematics, a proof is a demonstration that, given certain axioms, some statement is necessarily true.

Qualitative data

Data concerned with quality; verbal analysis.

Quantitative data

Data concerned with measurement; mathematical analysis.

Representation

Refers both to process and to product, i.e. the act of capturing a mathematical concept or relationship in some form and the form itself.

Representation applies to processes and products that are observable externally as well as to those that occur “internally” in the minds of people doing mathematics. Some forms of representation include diagrams, graphical displays and symbolic expressions. Representations are essential elements in supporting understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings.

Routine

Well-rehearsed (refer to Section 3.3).

Scaffolding

The scaffolding analogy comes from the building industry, and refers to the process of supporting a student’s learning to solve a problem or perform a task that could not be accomplished by that student alone. The aim is to support the student as much as necessary while they build their understanding and ability to use the new learning; then gradually reduce the support until the student can use the new learning independently.

Secondary data

Data that have been collected by someone else, or data that have been processed.

Spatial sense

The sense of geometric space, particularly patterns and relationships that have to do with geometric figures.

Scalar product

There are two types of scalar product, corresponding to the two types of vectors discussed below. The scalar product of vectors of the data structure type is the real number obtained by taking the sum of the products of corresponding components of the two vectors. It is related to the product of matrices. For vectors expressed in terms of direction and magnitude, the scalar product, or “dot” product, is the product of the magnitudes of the two vectors and the cosine of the angle between them.

$$x \cdot y = |x||y|\cos \theta$$

It is also equal to the sum of the products of corresponding components of the two vectors.

Simplex algorithm

Most life-related applications involve more than two variables. The simplex algorithm is a general procedure used in linear programming to find the optimal solution for situations involving two or more variables by systematically examining the vertices of the feasible region and stopping when the optimum has been found.

Simpson's Rule

A formula for numerical integration.

$$\int_a^{a+2h} f(x)dx \approx \frac{1}{3}h[f(a) + 4f(a+h) + f(a+2h)]$$

By using Simpson's Rule on adjoining intervals a more general form can be obtained.

$$\int_a^{a+2nh} f(x)dx \approx \frac{1}{3}h[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + 2f(a+4h) + \dots + 4f(a+(2n-1)h) + f(a+2nh)]$$

Solution

Answers to problems, investigations, research or questions. A concise solution includes the following characteristics:

- succinct use of language that avoids repetition
- use of appropriate terminology and symbols

This does not preclude the exploration of additional aspects of a problem/situation.

Standard

A standard is a fixed reference point for use in assessing or describing the quality of something. In senior syllabuses, standards are usually described at five points within each exit criterion.

Strategy

A plan of action designed to achieve a goal (see Problem-solving strategies).

Student profile of achievement

This records information about student performance on instruments undertaken periodically throughout the course of study. Techniques are chosen to sample the significant aspects of a course across relevant exit criteria to ensure balance in assessment. In particular, it is important that the profile of achievement illustrates how assessment of significant aspects is selectively updated and eventually leads to summative assessment within each exit criterion.

Summative assessment

Summative assessment provides cumulative information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment instruments to be used, when they will be administered, and how they will contribute to the determination of exit levels of achievement.

Synthesise

Assembling constituent parts into a coherent entity. The term "entity" includes a system, theory, plan or set of operations.

Uniform distribution

This is the simplest of the continuous distributions and is sometimes also called the rectangular distribution because its probability density function is a rectangular shape. If a point is chosen at random from the interval (a, b), then the probability density function of the position of the point, X, is given by

$$f(x) = 1/(b-a), \quad a < x < b$$

$$0, \quad \text{elsewhere.}$$

Valid

Sound, reasonable, relevant, defensible, well grounded, able to be supported with logic or theory. May include the strengths and limitations of models and the analysis of the effect of assumptions.

Variable

A symbol that represents a value that is allowed to change.

Verification

Towards the end of Year 12, school submissions, one for each Authority subject, are sent to the relevant (usually district) review panels who review the material to confirm that the standards assigned to students' work are in line with the descriptors in the syllabus. These submissions comprise folios of the work of sample students about to exit from the course of study, together with the school's judgment.

Verification folio

This is the collection of documents (tests, reports, assignments, checklists and other assessment instruments) used to make the decision about a student's level of achievement. At October verification, it will contain a minimum of four and a maximum of ten pieces of work that conform to the underlying principles of assessment as outlined in Section 6.1.

Work program

The school's program of study in Authority and Authority-registered subjects for which the students' results may be recorded on QSA certificates (requirements are listed on the QSA website).