SENIOR MATHEMATICS B

BOARD OF SENIOR SECONDARY SCHOOL STUDIES

PROUDLY PRINTED IN QUEENSLAND

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This subject is one of three Board subjects available to schools for students of Senior Mathematics. Attention is drawn to the following statement on compatibility and prerequisite knowledge.

In any one semester, the following combinations may appear on the Senior Certificate:

Mathematics A
Mathematics A and Mathematics B
Mathematics B
Mathematics B and Mathematics C.

In exceptional circumstances, Mathematics C may appear on the Senior Certificate without Mathematics B. However, it is strongly recommended that schools ensure that students of Mathematics C have an adequate mathematical background.

The Senior Syllabus in Mathematics A is incompatible with the Senior Syllabus in Mathematics in Society.

The Senior Syllabus in Mathematics B is incompatible with the Senior Syllabus in Mathematics.

The Senior Syllabus in Mathematics C is incompatible with the Senior Syllabus in Mathematics.

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SENIOR SYLLABUS IN MATHEMATICS B

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TO BE USED FOR THE FIRST TIME WITH YEAR 11 STUDENTS IN 1993

SENIOR SYLLABUS IN MATHEMATICS B

1 RATIONALE

Mathematics is an integral part of a general education. It can enhance people's understanding of the world and the quality of their participation in a rapidly changing society. In recent times the use of mathematics has developed substantially in response to changes in technology. At the same time, mathematical developments have contributed to this changing technology. The most obvious changes have been related to the development and application of computing power. In all its aspects mathematics is valuable to people individually and collectively, providing important tools which can be used at the personal, civic, professional and vocational levels.

The most obvious use of mathematics at a personal level is to assist in making informed decisions in areas as diverse as buying and selling, home maintenance, interpreting media presentations and forward planning. The mathematics involved in these activities includes financial analysis, data description, statistical inference, number, quantification and spatial measurement.

In recent years, the range of career opportunities requiring an advanced level of mathematical competence has expanded dramatically. No longer are careers in fields such as physical sciences, engineering and accounting, the only ones requiring a high level of training in mathematics. Advances in technology have resulted in an increased need for, and use of, mathematical skills as important tools in fields such as geography, biology, environmental science, art, economics, fashion design and management. Mathematics underpins most industry, trade and commerce, social and economic planning, and communication systems. It is predicted that demand in Australia for mathematically skilled people will rise, creating a significant labour market problem unless more people are willing to undertake further mathematical study.

There is anecdotal and research evidence to suggest that many people dislike mathematics and may even feel intimidated in situations in which it is used. The effect on individuals of having to deal with an increasingly mathematically oriented society while feeling inadequate or alienated from mathematics is of considerable concern. Consequently, the raising of levels of competence in and confidence with mathematics is critical. This is also essential for widespread scientific literacy and for the development of a more technologically skilled work force.

To raise the level of competence in mathematics, mathematics education should concern itself with modes of thinking which provide ways of modelling situations in order to explore, describe or control our social, biological and physical environments. Mathematics education must recognise the dynamic nature of mathematics and encourage an approach to its study through problem solving and applications in life-related contexts. Students must be given the opportunity to appreciate and experience the power which has been given to mathematics by computer technology.

Given the impossibility of identifying all the skills which students will need in the future, narrow vocational views of mathematics are likely to be unproductive. The intent of Mathematics B is to encourage students to develop positive attitudes towards mathematics by an approach involving problem solving and applications. Students will also be encouraged to work systematically and logically, and to communicate with and about mathematics. Mathematics B is designed to raise the level of competence in the mathematics required for intelligent citizenship, to increase students' confidence in using mathematics to solve problems and to provide the basis for further studies.

2 GLOBAL AIMS

Having completed the course of study, students of the subject Mathematics B should:

- be able to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- be aware of the uncertain nature of their world and be able to use mathematics to assist in making informed decisions in life-related situations
- be able to manage their financial affairs in an informed way
- be able to visualise and represent spatial relationships in both two and three dimensions
- be aware of the diverse applications of mathematics
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information which is presented in a variety of forms
- communicate mathematical information in a variety of forms
- be able to benefit from the availability of a wide range of mathematical instruments
- be able to use appropriately selected mathematical instruments
- be able to recognise functional dependencies
- be aware of the diverse and evolutionary nature of mathematics and the wide range of mathematics-based vocations.

3 GENERAL OBJECTIVES

3.1 INTRODUCTION

The general objectives of this course are organised into three categories which are described below.

I COMMUNICATION

Clear communication is paramount both within the school setting and also in the world for which students are being prepared. The process of developing and building up mathematical knowledge through describing, questioning, arguing, predicting and justifying almost always requires a sharing of ideas. The productive sharing of ideas depends upon the clarity with which one can express oneself. Communication skills are needed in order to understand, assess and convey ideas and arguments, including mathematical concepts, presented in both everyday and mathematical language.

II MATHEMATICAL TECHNIQUES

Both dimensions included in this category involve developing **familiarity** with a variety of techniques and life-related applications.

DIMENSIONS:

(a) LEARNED RESULTS AND MATHEMATICAL PROCEDURES

An important goal in mathematics education is for students to develop the capacity to deal with commonly occurring or familiar mathematical situations readily. Facility with certain techniques can strengthen one's capacity to inquire and to explore unfamiliar situations involving mathematics. In this dimension the ability to apply mathematics in **familiar** situations is developed.

(b) USE OF INSTRUMENTS IN MATHEMATICS

In order that the emphasis of the mathematics learned is on concepts and techniques rather than tedious and/or involved calculations, it is appropriate that students are confident and competent in using a calculator and a computer. Other instruments encountered by students may include graphics calculators, drawing compasses, protractors, geoliners, tape measures, tables and graph paper. It is important that students become competent in the appropriate selection and use of instruments.

III MATHEMATICAL APPLICATIONS

The three dimensions included in this category involve applying mathematics in **unfamiliar** situations.

DIMENSIONS:

(a) APPLYING MATHEMATICS IN LIFE-RELATED SITUATIONS (REAL OR SIMULATED)

Problems are usually generated from the real world. In such cases, they need to be extracted from their particular context, the essential relationships within the context represented mathematically and the problem stated in mathematical terms. Once this is done, mathematical techniques may be applied to the problem. The derived solution then needs to be interpreted in terms of the life-related situation.

(b) APPLYING MATHEMATICS IN PURELY MATHEMATICAL CONTEXTS

At times, problems investigated are generated within the fields of mathematics and thus relate directly to mathematical abstractions. In such cases, there is no immediate interest in whether the relationships derived relate to any social, physical or biological phenomenon. The purpose is explicitly mathematical.

(c) JUSTIFICATION

Justification deals with developing logical arguments and justifying conclusions leading to the notion of mathematical proof. In order to analyse and interpret certain material, students will need to understand the value of mathematical arguments and the relationship between mathematics and its applications.

3.2 THE OBJECTIVES

The general objectives for each of these categories are detailed below.

I COMMUNICATION

The student should be able to:

- communicate results concisely, clearly, precisely and appropriately
- use mathematical terms and symbols accurately and appropriately

- use accepted spelling, punctuation and grammar in written communication

- understand material presented in the following forms:
  - verbal: both spoken and written
  - visual: including mathematical symbols, pictorial form and graphical form
  - combination of both verbal and visual

- translate material from one form to another where appropriate

- transmit material in the following forms where appropriate:
  - verbal: both spoken and written
  - visual: including mathematical symbols, pictorial form and graphical form
  - combination of both verbal and visual

- communicate mathematical results in a variety of forms appropriate for different audiences

- recognise necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical context.

II MATHEMATICAL TECHNIQUES

Both dimensions included in this category involve developing **familiarity** with a variety of techniques and life-related applications.

DIMENSION

(a) LEARNED RESULTS AND MATHEMATICAL PROCEDURES

The student should be able to:

- recall definitions and results

- identify skills applicable to a familiar situation

- demonstrate facility in skills applicable to a familiar situation.

N.B. Situations must range from simple to complex and cover both life-related contexts and purely mathematical contexts.

DIMENSION

(b) USE OF INSTRUMENTS IN MATHEMATICS

The student should be able to:

- demonstrate an ability to select and use appropriate instruments such as calculators, computers and selected software, measuring instruments, geometrical drawing instruments and tables, to perform the following functions:
  - arithmetic calculation
  - practical measurement
  - drawing of geometrical figures
  - data exploration
  - drawing of graphs

- use estimation as a means of checking whether results obtained from instruments are reasonable.

III MATHEMATICAL APPLICATIONS

The three dimensions included in this category involve applying mathematics in **unfamiliar** situations.

DIMENSION

(a) APPLYING MATHEMATICS IN LIFE-RELATED SITUATIONS (REAL OR SIMULATED)

The student should be able to:

- understand that a mathematical model is any mathematical representation of a situation

- identify the assumptions and variables of a simple mathematical model of a situation

- form a mathematical model of a life-related situation, over a variety of areas of application

- derive results from consideration of the mathematical model chosen for the particular situation

- interpret results from the mathematical model in terms of the given situation

- explore the strengths and limitations of a mathematical model

- develop a set of procedures to be used in approaching modelling problems.

N.B. Situations mentioned above must range from simple to complex.

DIMENSION

(b) APPLYING MATHEMATICS IN PURELY MATHEMATICAL CONTEXTS

The student should be able to:

- interpret, clarify and analyse the problem
- use a range of problem solving skills including estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understand that there is often more than one way to solve a problem
- select the appropriate mathematical techniques required to solve a problem
- provide a solution consistent with the problem posed
- develop a set of procedures to be used in approaching problems set in purely mathematical contexts.

N.B. Problems mentioned above must range from simple to complex.

DIMENSION

(c) JUSTIFICATION

The student should be able to:

- develop logical arguments expressed in everyday language, mathematical language and a combination of both, where appropriate, to support conclusions and/or propositions
- evaluate the validity of arguments designed to convince others of the truth of propositions
- recognise when and why derived "solutions" to a given problem are clearly improbable or unreasonable
- recognise that one counter example is sufficient to disprove a generalisation
- recognise the effect of assumptions on the conclusions that can be reached
- decide whether it is valid to use a general result in a specific case
- recognise that a proof requires more than verification of a number of instances.

4 ORGANISATION

4.1 INTRODUCTION

The subject matter has been organised into the following topics, which are discussed in detail in Section 5.

- Applied Geometry
- Introduction to Functions
- Rates of Change
- Periodic Functions and Applications
- Exponential and Logarithmic Functions and Applications
- Optimisation

    (a) Optimisation Using Derivatives

    (b) Operations Research
    either Networks
    or Linear Programming

- Financial Mathematics
- Introduction to Integration
- Applied Statistical Analysis

Throughout the course, certain fundamental knowledge and skills are required; some of these have been identified and listed under the heading Basic Mathematics in Appendix 1. Time should be provided to revise these aspects within topics as they are required. Similarly, the maintenance of Mathematical Techniques is important in developing the student's capacity to deal readily with commonly occurring or familiar mathematical situations; some relevant learning experiences are listed in Appendix 1. This maintenance takes time and should be budgeted for in designing the course sequence.

4.2 TIME ALLOCATION

The minimum time-tabled school time, including assessment, for this subject is **55 hours per semester.**

Notional times are given for each topic. These times are included as a guide and are not to be seen as prescriptive.

4.3 SEQUENCING

After considering the subject matter and the appropriate range of learning experiences to enable the general objectives to be achieved, an **integrated sequence** should be developed which allows students to see links between the different topics of mathematics included in the course rather than seeing them as discrete. For example, investigating the optimum shape of a drink can, for given volume, could provide a link between the topics Applied Geometry, Introduction to Functions and Optimisation, and could include links with Applied Statistical Analysis, when considering the reliability of the stated volume.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence but some topics include subject matter which is developed and extended in later topics. The school's sequence should be designed so that the subject matter in the topics is spiralled to allow students to internalise their knowledge before developing it further. Hence it is generally not desirable to complete a topic in isolation.

The following guidelines for the sequencing of the topics should be referred to when developing a sequence for the course.

- No section of a topic should be studied before the appropriate prerequisite material contained in other topics.

- It may be appropriate to divide a topic into sub-topics.

- The sequencing of topics may depend on the importance placed by schools on students being able to make decisions about the mathematics appropriate to their needs, for example, the Year 11 sections of the sequences for Mathematics A and Mathematics B may need to be developed together.

- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered where appropriate.

- The course of study for a particular student **must not** include the topic Linear Programming in both Mathematics B and Mathematics C.

- Topics should be linked where possible.

- Sequencing may be constrained by a school's ability to provide physical resources.

- Within each topic, time will be needed for the maintenance of Basic Mathematics and Mathematical Techniques.

Regardless of the sequence of topics chosen, it is paramount that each topic be considered in relation to the development of student performance within the general objectives.

5 TOPICS

5.1 INTRODUCTION

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

FOCUS

This section describes broadly the concepts which students should be encouraged to develop during the study of the topic and highlights the intent of the syllabus with respect to the topic.

SUBJECT MATTER

This section outlines the mandatory subject matter to be included in each topic. The order in which the subject matter is listed within a topic is not intended to imply a teaching sequence.

LEARNING EXPERIENCES

This section suggests some learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. Included are experiences which involve life-related applications of mathematics involving both real and simulated situations, use of instruments and the opportunity for problem solving. The listed learning experiences may require students to work individually, in small groups or as a class. The more traditional learning experiences have generally not been included but this does not mean that they should not be used where appropriate.

The learning experiences are suggestions only and are not to be seen as being prescriptive. Schools are encouraged to develop further learning experiences, especially those which relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of likely learning experiences that students will encounter should be shown in the work program and, in their totality, should enhance a balanced course of study.

5.2 THE TOPICS

The order in which topics and items within topics are given should not be seen as implying a teaching sequence.

Applied Geometry (notional time 20 hours)

Focus: Students are encouraged to develop skills in applying geometry in the real world by using and extending their knowledge of spatial relationships in two and three dimensions. Geometry should **not** be developed axiomatically nor should there be a concentration on purely geometric results.

Subject Matter:

- interpretation and drawing of scale drawings and plans
- trigonometry including the definition and practical applications of the sine, cosine and tangent ratios
- simple practical applications of the sine and cosine rules
- definition of a radian and its relationship with degrees
- practical applications of circle geometry including lengths of arcs and areas of sectors
- visualisation of simple three dimensional figures
- practical applications of volume and surface area of regular shapes.

Learning Experiences:

- estimate the number of boxes of tiles needed to tile a bathroom
- design an office layout, given the staff and their functions, the furniture and the legal space requirements for each employee
- use shadow reckoning or a sight line to calculate a height
- find the minimum length of material needed to cut a dress pattern, with or without regard to pattern matching
- find the minimum number of rolls of wallpaper needed to paper a room, given a pattern to be matched
- draw a scale plan of a house

- working from a scale drawing, mark out a floor plan of a house on a large flat area such as a basketball or netball court
- given a scale plan of a weatherboard house, determine the cost of painting the house (i) internally and (ii) externally
- visit a department that uses Computer Aided Design (CAD) packages, for example, the Manual Arts Department at the school, a mining company, an architect's office, the Department of Transport
- use trigonometry and/or surveying instruments to find distances such as widths of rivers, roads and valleys
- stake out an athletic track on the school oval; find staggered starting positions for 200 metres and 400 metres
- find the length of a transmission belt that passes around two pulleys of different sizes
- investigate the optimal sprinkler system for the school oval or a home garden
- a goat is tethered near the corner of a fence; find the area of land on which it can graze; consider the effects if the grazing area is restricted by an obstacle such as another fence or a building
- visit the local pony club and investigate the distances horses must be tied apart to avoid kicking each other
- investigate how angles are used in the orientation behaviour of animals, for example, homing pigeons, migrating birds, bees and fish
- investigate Euler's formula
- use nets or model kits to construct three dimensional solids
- investigate why there are only 5 platonic solids
- find the volume of earth that must be removed to construct a triangular shaped canal of given width, depth and length
- research how survey datum points are located
- given the dimensions of a building with a gable roof, investigate the largest attic that could be constructed within the roof space
- consider the effect on the water level in a tank when a cube of known dimensions is dropped into the tank

- investigate how windscreen wipers on cars are designed to maximise the effective area swept

- investigate the shape of a three dimensional structure by considering a number of two dimensional plans taken from different perspectives

- by assuming that the shape of the trunk of a growing tree, for example, a hoop pine, represents a cone, calculate the volume of useable timber for given minimum and maximum log diameters, the heights at which these diameters occur and the total height of the tree

- find the shortest path from a point on one wall of a room to a point on another wall, for example, via walls, floor, ceiling

- use a globe of the Earth to find the shortest air route from one city to another; compare this to the routes taken and explain anomalies

- for a given rectangular bar of chocolate, investigate the minimum sized rectangular aluminium foil necessary for wrapping (consider different orientations of the chocolate to the foil).

Introduction to Functions (notional time 20 hours)

Focus: Students are encouraged to develop an understanding of relationships between variables, and of the concept of a function. It is intended that both algebraic and graphical techniques, and their inter-relationships, be stressed. No particular type of function needs to be treated in depth.

Subject Matter:

- concepts of function, domain and range

- mappings, tables and graphs as representations of functions and relations

- graphs as a representation of the points whose co-ordinates satisfy an equation

- distinction between functions and relations

- distinctions between continuous functions, discontinuous functions and discrete functions

- practical applications of linear functions, quadratic functions, absolute value functions and the reciprocal function

$$f(x) = \frac{1}{x}$$

- relationships between the graph of $f(x)$ and the graphs of $f(x)+a$, $f(x+a)$, $af(x)$, $f(ax)$, for both positive and negative values of the constant a

- concept of the inverse of a function

- general shapes of graphs of polynomial functions

- solution of two simultaneous equations in two variables

- composition of functions.

Learning Experiences:

- given an hourly rate and overtime loadings, construct a graph of wage against hours worked

- estimate likely domains and ranges of functions in life-related contexts, for example, height against mass for a particular person

- draw graphs of telephone STD charges against distance with constant time and charges against time with constant distance; consider the effect of different times of the day on the graphs

- make a chart which allows conversion from: miles per gallon to litres per 100 kilometres; °F to °C; pounds per square inch pressure to kilopascals

- investigate the general shapes of polynomial functions

- use computer software to investigate the shapes of different functions

- consider regression as an application of a linear function

- investigate the shapes of common relations which are not functions, for example, circles and ellipses

- use the vertical line test for functions and relations

- investigate the difficulties encountered in using computer software to draw graphs of relations which are not functions

- find approximate solutions of two simultaneous equations by using graphs

- locate the position of the highest point on a projectile path

- derive the quadratic formula

- investigate how closely a quadratic function approximates (a) the shape of a hanging chain and (b) the curve of the cables of a suspension bridge; suggest an explanation for any difference between the two results

- find the domain and range of a function, given its graph

- a company works on a profit function given by

  $$P(x) = \frac{26x - 25}{x}$$

  where x is the number of items sold and P is given in hundreds of dollars; explore the situation as the number of items sold increases without bound; develop other situations and cost functions for other students to explore

- graph the two functions $f(x) = x$ and $g(x) = x + 1$ using a computer or graphics calculator; from the shapes predict the shapes of the following functions and then check using the computer or graphics calculator:

  $$h(x) = 2x + 1, \quad k(x) = x^2 + x, \quad p(x) = \frac{x}{x + 1}$$

- write $(x + a)^n$ as a polynomial in x for several values of n; investigate the graphs of the polynomials and the relationships between the coefficients of powers of x

- investigate the power function relating the length of a quadruped from shoulder to hip, with its height from ground to shoulder; consider the effect of gravity on this relationship

- by approximating the surface area of a living being as a function of the square of its linear dimension, and the volume as a function of the cube of its linear dimension, investigate the limitations on the sizes of living beings in different environments

- research Piet Hein's "superellipses", that is, curves of the form $|x|^n + |y|^n = a^n$ where n can be fractional

- given graphs of two functions, $f(x)$ and $g(x)$, devise a procedure for producing the graph of the composite function $f[g(x)]$

- investigate how many times a straight line can intersect the graph of a polynomial of degree n

- research ways of approximating a given function by a polynomial

- find the dose of a chemical needed to obtain a 50% kill of insects, if the percentage of insects killed, p, is related to the dose level, x, by the equation, $p = a + bx + cx^2$ where a, b and c are constants with values such as 2, -1 and 3

- many species of animals have a distinct breeding season, and the population size can be regarded as a function of the year, $f(n)$; use a computer to investigate some simple models of non-exponential population growth for such species, such as

$$f(n+1) = \frac{a\,f(n)}{1 + b\,f(n)}$$

where n is the generation ($n = 1,2,\ldots$), a is a measure of the unrestricted reproductive rate and b is a constraint on reproduction due to environmental or other causes.

Rates of Change

(notional time 25 hours)

Focus: Students are encouraged to develop an understanding of the concepts of average and instantaneous rates of change. This understanding should be developed using both algebraic and graphical approaches, in life-related situations as well as in purely mathematical contexts.

Subject Matter:

- concept of rate of change
- calculation of average rates of change in both practical and purely mathematical situations
- interpretation of the average rate of change as the gradient of the secant
- intuitive understanding of a limit
- definition of the derivative of a function at a point
- derivative of simple algebraic functions from first principles
- derivative theorems including

$\frac{d}{dp}\,p^n$ for rational values of n

$$\frac{d}{dr}\,kf(r)$$

$$\frac{d}{ds}\,[f(s) + g(s)]$$

$$\frac{d}{dt}\,[f(t)g(t)]$$

$$\frac{d}{dx}\,f[g(x)]$$

- calculation of the derivative of a function at a point
- interpretation of the derivative as an instantaneous rate of change
- interpretation of the derivative as the gradient of the tangent
- practical applications of instantaneous rates of change.

Learning Experiences:

- investigate the change in Australia's population over time in terms of the increase and percentage increase
- sketch graphs of the Consumer Price Index (CPI) and the rate of inflation over time, and obtain a relationship between these two functions

$$\left[\text{rate of inflation} = \frac{k}{CPI}\frac{d(CPI)}{dt}\right]$$

  where k is an arbitrary constant
- plot the evaporation rate of water in open cylinders of different shapes
- calculate the effect of small measurement errors in the calculation of a volume; relate this to a graph
- determine average and instantaneous speeds from a distance-time graph
- determine average and instantaneous accelerations from a velocity-time graph
- research Zeno's paradox
- consider marginal tax rates in relation to average tax rates and the total tax paid
- use a numerical technique to estimate a limit or an average rate of change
- use a system of engaged cogs or an odometer/speedometer combination to illustrate the derivatives of functions of the form $f(ax + b)$
- investigate how the rate of change of air temperature varies during the daylight hours when the relationship is approximated by a quadratic
- investigate the concept of marginal costs related to the derivative of a cost function

- investigate practical uses of instantaneous rates of change in agriculture; consider the software used by Extension Officers in the Queensland Department of Primary Industries

- investigate how instantaneous rates of change are used to measure the sensitivity of the human body to various stimulants or sedatives.

Periodic Functions and Applications (notional time 15 hours)

Focus: Students are encouraged to develop an understanding of periodic functions, together with a working knowledge of their simple applications in life-related situations. Trigonometric identities need not be developed beyond the Pythagorean identity.

Subject Matter:

- definition of periodic function, period and amplitude

- definition of trigonometric functions of any angle in degrees and radians, including $\sin x$, $\cos x$ and $\tan x$

- graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$

- significance of the constants A, B and C on the graphs of $y = A \sin (Bx + C)$, $y = A \cos (Bx + C)$

- applications of periodic functions

- Pythagorean identity $\sin^2 x + \cos^2 x = 1$

- solution of simple trigonometric equations within a specified domain

- derivatives of $\sin x$ and $\cos x$

- applications of the derivatives of $\sin x$ and $\cos x$ in life-related situations.

Learning Experiences:

- illustrate the notion of periodicity with planetary motion, hormone cycles or a business cycle

- investigate the concept of biorhythms

- find the periods, amplitude and frequency of functions given their graphs, equations or recursive definition

- plot, as a function of the date, the elapsed time between sunrise and sunset for capital cities in Australia

- plot, as a function of the date, the difference between noon as indicated by a sundial and noon from time signals

- investigate the path of a point on a moving bicycle wheel

- use a video camera to capture the movement of a spring; graph its displacement against time by viewing the video, frame by frame

- investigate the superposition of two almost equal frequencies of sinusoidal waves to produce "beats" in sound

- plot the tide heights at a specified point over a twenty-four hour period

- plot the motion of a cork on a body of water as a wave passes across the water

- plot the high and low tide heights, at a common point, over a twelve month period

- graph the slopes of the tangent of a sine curve

- research the sizes of populations of a predator and its prey

- investigate daily electrical energy consumption over a period of time

- investigate the periodic functions traced by an ECG machine or other medical devices

- investigate the use of square waves for testing hi-fidelity systems

- use computer software or a graphics calculator to investigate more complicated periodic functions, for example:

  $$\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x$$

- research the concept of periodic extinctions in the fossil record

- by approximating the height of the tide as a sine function, calculate the speed at which water is moving up a sloping beach.

Exponential and Logarithmic Functions and Applications

(notional time 20 hours)

Focus: Students are encouraged to develop an understanding of exponential and logarithmic functions, and the relationships between them. Students should also develop a working knowledge of simple applications of these functions in life-related situations. It is **not** intended that logarithms be used for arithmetic calculation, **nor** is it intended that a great emphasis be placed on simplification of expressions involving indices or logarithms.

Subject Matter:

- definitions of a^x and $log_e\, x$, for $a > 1$
- index laws and definitions including

  $$a^m \times a^n = a^{m+n}$$

  $$a^m \div a^n = a^{m-n}$$

  $$(a^m)^n = a^{mn}$$

  $$a^0 = 1$$

  $$a^{-n} = \frac{1}{a^n}$$

  $$a^{\frac{1}{n}} = \sqrt[n]{a}$$

  where m and n are rational numbers

- logarithmic laws and definitions including

  $$\log_c(ab) = \log_c a + \log_c b$$

  $$\log_c \left(\frac{a}{b}\right) = \log_c a - \log_c b$$

  $$\log_c a^n = n \log_c a$$

  $$\log_c 1 = 0$$

  $$\log_c \frac{1}{a} = -\log_c a$$

  $$\log_a b = \frac{\log_x b}{\log_x a}$$

  where a, b, c and x are positive real numbers and n is a rational number

- definition of the irrational number e

- graphs of, and the relationships between, $y = a^x$, $y = \log_a x$ for $a = e$ and one other value of a

- graphs of $y = e^{kx}$ for $k \neq 0$

- solution of equations involving indices

- use of logarithms to solve equations involving indices

- derivatives of exponential and logarithmic functions for base e

- significance of the number e in situations where the rate of change of a function is proportional to the value of the function

- applications of exponential and logarithmic functions.

Learning Experiences:

- investigate the existence of life-related situations that can be modelled by simple exponential functions, for example, radioactive decay, cooling curves, concentration against time in chemistry, applications in electric circuits, carbon dating in archaeology, growth of bacteria, and decrease of atmospheric pressure with altitude

- use computer software or a graphics calculator to investigate exponential and logarithmic functions and their derivatives

- plot the logarithms of some apparent growth functions, for example, car registrations over time, to produce a near-linear graph

- simulate the growth of a population and explain the effects of changes of constraints or parameters

- investigate the frequencies that generate musical scales

- investigate the changes in time to repay a loan caused by varying the amount of the repayments or the rate of interest

- investigate the combined effects of exponential and trigonometric functions in damped oscillations

- investigate how closely the shape of a hanging chain can be approximated by using exponential functions

- graph the derivative of a growth function or a decay function and interpret the result

- given $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$

  investigate the derivative of e^x; using this series, investigate e^{ax}

- in a simple two-step radioactive decay, that is, the original substance decays to an intermediate substance which in turn decays to an inert substance, investigate the time at which the quantity of the intermediate substance reaches a maximum

- the proportion of a radioactive material remaining after time t has elapsed is e^{-kt}, where k is a positive constant; investigate the relationship between k and the half-life of the material

- show that the derivative of the logarithm of the CPI is proportional to the rate of inflation

- in a bay, the variation in the height of a tide is given by

  $$h = 4\sin\left(\frac{t}{2}\right) + 2 \qquad \text{for } t \geq 0$$

  where h is the height in metres and t is the time in hours after midnight; at 12 midnight, when the depth of water in the bay is 20 metres, a piece of timber lying at the bottom of the bay works itself free and drifts to the surface following a path described by

  $$h = 2^t - 19 \qquad \text{for } t \geq 0;$$

  use computer software or a graphics calculator to investigate the range of these functions and to determine when the timber reaches the surface of the water in the bay

- by graphing the logarithm of the distance of planets from the sun against the logarithm of the time of revolution about the sun, investigate the relationship between the variables

- investigate various logarithmic scales, for example, decibels, Richter scale and pH

- plot, on three separate graphs, the logarithm of the population of Australia at censuses (a) from 1891 to 1933, (b) from 1947 to 1971 and (c) from 1971 to 1991; fit straight lines (by eye) to these graphs and use these to estimate the population of Australia in 2001, 2101 and 2201

- research the economic theory of Malthus

- explain the workings of a slide rule.

Optimisation (notional time 30 hours)

Focus: Students are encouraged to develop an understanding of the concept of optimisation using both Calculus and Operations Research (**either** Networks **or** Linear Programming).

(a) Optimisation Using Derivatives

Focus: Students are encouraged to develop an understanding of the use of differentiation as a tool in situations which involve the optimisation of continuous functions.

Subject Matter:

- positive and negative values of the derivative as an indication of the points at which the function is increasing or decreasing
- zero values of the derivative as an indication of stationary points
- concept of relative and absolute maxima and minima of functions
- relationships between the graph of a differentiable function, the derivative of that function and its maxima and minima
- methods of determining the nature of stationary points
- greatest and least values of a function in a given interval
- recognition of the problem to be optimised (maximised or minimised)
- identification of variables and construction of the function to be optimised
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem.

Learning Experiences:

- consider a simple optimisation problem, for example, given 100 metres of fencing, find the maximum rectangular area that can be fenced; use a spreadsheet to generate a table of values of areas for increasing side lengths and identify the apparent maximum area; graph this information and observe the relationship between the apparent optimum value and the graph; construct the function and consider the gradients of the tangents at points on either side of, and at, the optimum point
- given sections of linear fence, perhaps modelled by matches, find the greatest area which can be enclosed (with or without assuming a rectangular shape)

- investigate some standard cans and assess their volume to surface area efficiency

- combine graphs of cost and revenue functions to produce a profit function; determine the optimum number of units to maximise the profit

- investigate the difference between the number of maxima and the number of minima for polynomials of different degrees

- given a rectangular sheet of cardboard, find the maximum volume for a box of breakfast cereal

- find the smallest rectangular sheet of cardboard needed to construct a box of a given volume; compare the dimensions of an actual cereal box with those of the optimal solution and discuss possible reasons for any variation

- use computer graphing software or a graphics calculator to determine the relative maxima and minima of functions

- investigate the method used by motoring organisations to determine the optimum speed of a vehicle given certain fixed and variable costs

- investigate why it is more efficient for modern aircraft to fly at high altitudes. For an aircraft in steady flight the lift must be equal to the weight, and the engine thrust must be equal to the drag of the air. Both the lift L and the drag D depend on the airspeed V, but can be written in terms of nondimensional lift and drag coefficients C_L and C_D, as,

  $$L = A\,C_L \rho V^2 \text{ and } D = A\,C_D \rho V^2,$$

  where A is a constant relating to the geometry of the aircraft and ρ is the density of the air (thus flying at a lower speed requires a larger value of C_L). There is a relationship between C_D and C_L given by

  $$C_D = C_{D_o} + KC_L^2,$$

  where C_{D_o} is the (constant) drag coefficient measured under zero lift conditions and K is another constant related to the geometry of the aircraft. The power required to propel the aircraft is equal to the drag multiplied by the speed and the rate of fuel consumption is proportional to the engine power; show that, if the fuel used on a given journey is to be minimised, the aircraft should be flown so that the ratio of lift to drag is as high as possible. Since air density decreases with altitude, show that the cruising speed can be increased (and hence the travel time decreased) if the cruising altitude is also increased.

(b) Operations Research

EITHER

Networks

Focus: Students are encouraged to develop an understanding of the procedures involved in Networks. Students should become familiar with the shortest path and minimum spanning tree algorithms, and with the applications of these techniques to modelling and solving life-related problems which involve directed networks. Students should also develop an intuitive understanding of simple critical path analysis.

Subject Matter:

- networks including node, branch, path and tree
- shortest path algorithm
- minimum spanning tree algorithm
- effect of a critical step in critical path analysis from an intuitive point of view
- slack time in a critical path analysis
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem.

Learning Experiences:

- take a life-related problem given in English, formulate it into a network problem, solve and interpret the solutions in terms of the given problem
- discuss the shortest path algorithm and apply it to a real problem
- consider a road map, for example, from a street directory, showing two places connected by a network of roads and find the shortest path between them
- a sales representative is to service a series of stores; determine the shortest paths the sales representative can take from home base to each of the stores
- a complicated rail system connecting a number of towns is to be rationalised; investigate which lines could be removed so that a minimum amount of line will be left to service all the towns

- consider the times taken for different activities involved in constructing an entertainment centre; determine the critical steps and consider the effects of changing the time taken for certain activities on the critical path

- a road repair team has to inspect every section of road in the vicinity of the repair depot and it must return to the depot after inspection; investigate which types of networks of roads enable each section of road to be inspected without travelling over any section more than once and investigate ways of determining a possible inspection route; if the network is such that this cannot be done, investigate methods of finding a route which minimises the length of duplicated travel

- examine the way critical path analysis could be used by surveyors, architects, engineers, chefs and physiotherapists

- use critical path analysis to find the most efficient procedure and the shortest time needed for preparing a meal

- in a contest between groups, find an efficient procedure to pitch a tent, or clean and wash a car.

OR

Linear Programming

N.B. A student who studies both Mathematics B and Mathematics C must not study Linear Programming in both subjects.

Focus: Students are encouraged to develop an understanding of the methodology of Linear Programming and to see how it is used to solve problems in life-related situations. Problems with non-integer solutions are not appropriate for this topic.

Subject Matter:

- recognition of the problem to be optimised (maximised or minimised)

- identification of variables, parameters and constraints

- construction of the linear objective function and constraints with associated parameters

- graphing linear functions associated with the constraints and identification of the regions defined by the constraints

- recognition that the area bounded by the constraints gives the feasible (possible) solutions

- recognition that different values of the objective function can be represented by a series of parallel lines

- use of a series of parallel lines to find the optimal value of the objective function (parallel or rolling ruler, graphical method)

- observation that the feasible region is always a convex polygon and thus the optimal solutions occur at an edge or a corner point of the feasible region

- interpretation of mathematical solutions and their communication in a form appropriate to the given problem.

Learning Experiences:

- take a life-related problem given in English, formulate it into a linear programming problem, solve by the graphical method and interpret the solutions in terms of the original English problem

- investigate how linear programming is used to assist management decisions in areas such as manufacturing, transport, primary industries and environmental management

- consider optimal solutions of simple problems such as balancing diets

- use a computer software package to graph linear functions

- use parallel rulers to identify optimal solutions

- change parameters or constraints in a given problem and investigate the effect on optimal solutions

- consider the composition of a fleet of vehicles necessary to do a particular job at minimum cost

- solve a problem involving the allocation of two crops to the areas available on a farm in order to optimise profit when there are constraints on the labour and finances available

- investigate the design of an optimal sized solar powered home which is to be competitive in the market place; constraints will include the size of solar cells, cost of storage batteries, living area and total cost of construction

- research the history of linear programming.

Financial Mathematics (notional time 25 hours)

Focus: Students are encouraged to develop a working knowledge of the mathematics involved in the application of simple interest, compound interest and annuity formulae in fixed interest borrowings and investment. This knowledge will allow students to examine the role of interest rates in the more general context of consumer spending and earnings, and to make informed, personal financial decisions. Note that arithmetic and geometric progressions are to be developed only for their applications in the context of financial mathematics.

Subject Matter:

- simple interest and compound interest
- effective and nominal rates of interest
- past, present and future values
- arithmetic and geometric progressions as applied to interest rates; general term and finite sum of arithmetic and geometric progressions
- annuities and amortising a loan as applications of geometric series
- effect of personal taxation on investment returns
- budgeting.

Learning Experiences:

- use a spreadsheet or a calculator to compare interest accrued and yearly balances for an investment over five years where (a) a flat rate applies, and (b) compound interest applies
- show that the interest accrued under a simple interest investment can be represented as an arithmetic progression; compare this with the interest accrued with a compound interest plan
- develop the formula for compound interest
- prepare a list of charges, interest rates and conditions for investment in different commercial institutions; justify the selection of one of these to match a given financial scenario
- construct a loan repayment schedule showing principal, interest charged and balance owing

- compare the effects of various conditions quoted on bankcards from different banks and investigate what is meant by the "interest free period"

- derive the formula for the amount (future value) and present value of an annuity using geometric series:

  $$\text{Amount} = \frac{R[(1+i)^n - 1]}{i}$$

  $$\text{Present value} = \frac{R[1 - (1+i)^{-n}]}{i}$$

  where R is the repayment (Rent), n is the number of time periods and i is the interest rate expressed as a decimal

- investigate the use of tables by financial institutions in annuity calculations

- investigate monetary returns and real returns of assurance and superannuation packages

- use a spreadsheet or a financial calculator to examine the effect of changing the interest rate, term or repayment on a housing loan

- discuss responses of newspaper financial columnists to financial questions

- convert the bankcard daily interest rate to a yearly one and compare with the quoted annual rate

- investigate how organisations such as the local council develop sinking funds for the repayment of borrowing for plant and equipment

- prepare a personal budget

- investigate the changes in time to repay a loan caused by varying the amount of the repayments or the rate of interest.

Introduction to Integration (notional time 25 hours)

Focus: Students are encouraged to develop an understanding of the concept of integration as a process by which a "whole" can be obtained from the summation of a large number of parts. This understanding should be developed using both numerical and analytical techniques, in life-related situations as well as in purely mathematical contexts. The emphasis in the topic should be on the applications of integration rather than on developing a large repertoire of techniques.

Subject Matter:

- definitions of the indefinite integral and definite integral
- definition of the integral as the limit of a sum
- integrals including

  $$\int af(x)\,dx$$

  $$\int [f(x) \pm g(x)]\,dx$$

  $$\int f(ax + b)\,dx$$

  where $f(x)$ and $g(x)$ are simple polynomial functions, simple exponential functions, $\sin x$ or $\cos x$
- use of integration to find area
- practical applications of the integral
- trapezoidal rule as a method for evaluating an integral numerically.

Learning Experiences:

- investigate the motion (displacement and velocity) of a falling body neglecting air resistance
- use a numerical technique of integration to approximate the value of a, by finding values of a such that

  $$\int_1^a \frac{1}{t}\,dt \approx 1$$
- determine the volume of water that could be contained in a trough of given length and parabolic cross-section

- find an expression for the sum of the nth powers of the first k integers, for example, the sum of the 4th powers, $1^4 + 2^4 + \dots + k^4$; use this to evaluate a Riemann sum for $f(x) = x^n$

- investigate the varying accuracy of the trapezoidal rule applied to various integrable functions over a given range, with varying number of strips; consider using software to facilitate this task

- calculate an area bounded by two intersecting curves

- find as many ways as you can of calculating the volume of objects such as a touch football or a loaf of bread

- investigate the Monte Carlo technique for finding area

- investigate the growth of entities given the appropriate growth function, for example, given the growth rate for the radius of a slick of spilt oil, determine the size of the slick at a specified time. A person learns N items at a rate approximated by

  $$N'(t) = \frac{25}{\sqrt{t}} \qquad \text{for } 1 \le t \le 9$$

  where t is hours of continuous study, find the total number of items learned from t=1 to t=9 hours of study; the rate of change of an average person's pulse rate with respect to height is approximated by

  $$P'(h) = -1194h^{-3/2} \qquad \text{for } 76 \le h \le 190,$$

  where h is the height in centimetres, find the total change in pulse rate for a child growing from 121 to 169 centimetres

- investigate what is represented by the area under a graph of velocity against time and a graph of acceleration against time

- investigate consumer surplus and producer surplus in relation to supply and demand functions

- investigate the method for finding the volume of timber in a tree trunk by using several functions to describe the shape of the trunk; a typical taper equation to describe a tree could consist of a quadratic equation for the base, a linear equation for the main part of the trunk, and a second quadratic equation for the tip of the tree

- find the volume of the planet Earth, given the polar and equatorial radii

- research the methods used by sailmakers to find the amount of material in a spinnaker

- investigate the difference in the population of moths obtained if it is modelled by (a) an unlimited growth curve ($y = e^x$), and (b) a limited growth curve ($y = \ln x$)

- research the use of Lorenz Curves to measure income inequality in a society
- calculate the distance travelled in a car by taking speedometer readings at regular intervals and then using a numerical method for calculation, check the accuracy of the result by using a tripmeter
- calculate the approximate volume of fill to be removed in the construction of a road cutting by approximating the cross-sectional area using a numerical method.

Applied Statistical Analysis (notional time 25 hours)

Focus: Students are encouraged to develop a working knowledge of the concepts involved in describing and summarising data, and of the role played by probability in drawing conclusions from empirical data. The emphasis throughout should be on description and interpretation **from the point of view of the recipient** rather than the presenter. It is expected that calculators and/or computer software will be used routinely for calculations.

Subject Matter:

- interpretation of graphical displays of data including stem-and-leaf plots, histograms and other frequency plots
- identification and use of five-number summaries and their display using a box-and-whisker plot
- population parameters and sample statistics including measures of central tendency and dispersion, their estimates and roles as descriptors of large data sets; the notion of expectation
- probability as a measure of chance and likelihood; as a relative frequency of equally likely outcomes in an experiment; as a proportion between zero and unity
- random sampling and bias; the role of sample size
- discrete probability distributions, uniform (rectangular) and binomial, as data models; the table of binomial probabilities
- formulation and testing of statistical hypotheses for a life-related problem where the Sign test is appropriate
- continuous random variables; the normal distribution; the standard normal table, its definition and use; random normal numbers
- defining and testing the statistical hypotheses for a life-related problem involving a normally distributed random variable

- uses and misuses of statistics and probability.

Learning Experiences:

- examine the use of summary statistics in, for example, newspapers, TV programs such as weather reports and advertisements, government reports and research reports

- examine reports by the Real Estate Institute and explain their choice of measure of central tendency

- compare the effects of an outlier on a variety of summary statistics

- given a set of data, prepare a statistical presentation to answer a specific question

- examine a statistical report from, for example, a government agency or a research project; identify the summary statistics used and how residuals (or variation) are incorporated

- organise a relatively large set of real data into an understandable form using a variety of approaches such as summary statistics, graphical displays, and/or frequency distributions

- use a computer database to store and sort data

- summarise a given set of data to produce a concise report explaining the main information contained in the data

- examine material presented in Government publications such as year books and reports from the Australian Bureau of Statistics

- examine how box-and-whisker plots can be used to give an effective visual comparison between two or more data sets; for example, compare the heights of girls and boys in the class, compare the rainfall data over a twelve month period for different parts of Australia, compare the prices in a number of different shops of a variety of items such as compact discs, cans of soft drink, text books and chocolate bars

- research the daily newspapers for examples of graphical presentations which may be designed to convey a particular viewpoint to the reader

- examine media reports and everyday expressions for the use of probability language and discuss the meaning and accuracy of terms used

- use a tree diagram to find the probabilities associated with successive births to a couple who are known to carry a specific genetic disorder, for example, the condition of haemophilia

- use the tabled information given in the newspaper about previous Gold Lotto draws to determine whether the numbers are drawn at random, that is, whether the numbers follow a uniform probability distribution model

- research the effects of seat belts in car accidents; define the statistical hypothesis, for example, as many people wearing seat belts are killed in road accidents as people not wearing seat belts, that is, a person killed in a road accident is just as likely to have been wearing a seat belt as not wearing a seat belt; identify the appropriate binomial model, for example, the probability of a road fatality being a seat belt wearer is 50%, $p = 0.50$; obtain a sample of empirical data, for example, out of 362 road fatalities last year, 50 were wearing seat belts and 312 were not wearing seat belts; write a conclusion based on the comparison with the theoretical probabilities of the chosen binomial model (the theoretical probabilities should be provided as a table for sample sizes greater than five)

- have each student select a random sample of specified size from a physical model of a population, for example, a population of paper frogs on each of which is marked a relevant body length and gender, a population of paper waves each with a given height or a population of paper houses with specified purchase price, and calculate the sample statistics; collate the class results and compare them with the population parameters, explore the effect of increasing the sample size

- examine the use of statistics in a range of research areas such as the environment, health, athletics and forensic science

- for each month of the year compare the proportions of days in the month where the temperature exceeds the average (or where the UV rating is extreme)

- research the proposition that blue cars are more likely to be involved in accidents than are cars of other colours (information from insurance companies, research institutes and panel beaters)

- research the probabilities associated with cyclones originating from different locations and their subsequent crossing or not crossing of the Queensland coast

- review the basis on which scientific hypotheses are tested, and the role of random errors; compare this situation with the Australian legal system

- working in groups, select a current issue that has involved the collection of binary data; identify the statistical hypothesis; use the given data and prepare a report using the results and relevant probability statements, for example, the majority of the Queensland public agree with the establishment of nude bathing beaches; Year 12 girls are more likely to play sport than Year 12 boys; owning a pet such as a cat or a dog gives old people a more contented lifestyle; more people are now waiting until they are in their thirties to marry

- suggest a method that could be used to obtain data to test the hypothesis that butterflies cannot fly if the air temperature is less than 12°C

- research the data collection methods used by the local authority to monitor water, air and/or noise pollution, for example, methods used by the Brisbane City Council to monitor the water in the Brisbane River (or the air pollution levels throughout Brisbane)

- use a statistical model with known mean, standard deviation and a normal distribution, to simulate a sample of data, for example, create a sample of "DDT levels in cows' milk" by assuming that the DDT level, in standard concentration units, is a random normal variable with a mean of 2.3 and a standard deviation of 0.23; calculate the sample statistics and compare them with the parameters used to create the data (the normally distributed random components could be generated using a table of random normal numbers or a computer package)

- graph various distributions from data collected and discuss whether they are approximately normal

- compare probabilities given in normal tables with those obtained experimentally for a nearly normal distribution

- investigate the role played by statistics in everyday life, for example, forecasting weather patterns, forecasting economic trends, predicting public feelings and attitudes, determining market preferences

- investigate how the binomial distribution can be approximated by the normal distribution; consider the minimum sample size for which this approach is reasonable

- investigate how the binomial table is derived by using a sample no larger than six.

6 ASSESSMENT

6.1 INTRODUCTION

FORMATIVE ASSESSMENT

Formative assessment is a process which provides teachers, students and parents with information on the performance of students within the criteria and their dimensions with a view to assisting students to improve their understanding and achievement. The formative assessment should be both informal and formal in nature.

The formal techniques used should be similar to the summative assessment techniques which students will meet later in the course so that they have experience in responding to particular types of tasks under the appropriate conditions. The information gathered from this type of assessment is used when making decisions for reporting and monitoring purposes and therefore should be included in monitoring submissions.

SUMMATIVE ASSESSMENT

Summative assessment is intended to assist teachers in determining the exit Levels of Achievement of students. It is designed to supply information for reporting and certification at the end of a course of study. Accordingly, work programs must specify the range of summative assessment instruments which will be used, when they will be administered and how they will contribute to determining exit Levels of Achievement.

The following underlying principles should be addressed when planning assessment in this subject.

> Assessment is concerned with determining the extent to which students meet the general objectives of Communication, Mathematical Techniques and Mathematical Applications.
>
> Assessment of student achievement should not be seen as a separate entity, but as an integral part of the developmental learning process. An effective course of study includes a variety of learning experiences. Therefore, a range of assessment techniques which reflect the learning experiences of the students needs to be employed in gathering assessment data.
>
> The overall assessment plan should be devised so that the fullest and latest information on a student's achievement in the course of study may be obtained. Information should be gathered through a process of continuous assessment.
>
> Assessment tasks and subsequent allocation of exit Levels of Achievement should address the significant aspects of the course of study identified in both the syllabus and the school's work program. Time spent on topics should be reflected in the scope and depth of the assessment.

The assessment should reflect a balance over the course of study. This balance need not necessarily be achieved within every semester but should be discernible over the summative assessment.

6.2 CRITERIA FOR AWARDING EXIT LEVELS OF ACHIEVEMENT

At exit from the course of study, each student will be awarded one of five Levels of Achievement:

Very High Achievement (VHA)
High Achievement (HA)
Sound Achievement (SA)
Limited Achievement (LA)
Very Limited Achievement (VLA).

Student performance will be judged on the following three criteria which match the categories defined in section 3.1.

CRITERION I: COMMUNICATION

CRITERION II: MATHEMATICAL TECHNIQUES

which incorporates the dimensions:

(a) Learned Results and Mathematical Procedures
(b) Use of Instruments in Mathematics.

CRITERION III: MATHEMATICAL APPLICATIONS

which incorporates the dimensions:

(a) Applying Mathematics in Life-related Situations (Real or Simulated)
(b) Applying Mathematics in Purely Mathematical Contexts
(c) Justification.

6.3 ASSESSMENT WITHIN THE CRITERIA

Assessment tasks should be designed to allow students to demonstrate their ability in all the criteria including all their dimensions. It is not necessary for each assessment task to address every criterion, but all three criteria must be represented in assessment data that contribute to the awarding of exit Levels of Achievement.

Some points to consider when assessing student performance on the criteria are given below.

CRITERION I: COMMUNICATION

As the two-way processes of communication are an integral part of everything we do in mathematics, it is difficult, and indeed not desirable, to assess student performance on this criterion independently. Information on student achievement in this criterion should be collected by a global consideration of the Communication skills evident in responding to tasks used to assess student performance in Mathematical Techniques and Mathematical Applications. The types of assessment instruments used to assess student performance on Criteria II and III must, therefore, be considered very carefully to ensure that students have the opportunity to demonstrate that they have achieved the Communication objectives.

CRITERION II: MATHEMATICAL TECHNIQUES

This criterion focuses on the recall of results and procedures, use of instruments and the application of all of these in **familiar situations.** It is not necessary to record information in each of the associated dimensions, but the assessment package must enable students to demonstrate their ability in the general objectives of both dimensions.

The assessment package must include:

- items which range from simple to complex
- items which require the appropriate use of a variety of instruments in mathematics
- items which involve applying mathematics in familiar, life-related situations as well as items which are presented in a purely mathematical context
- items which allow students to demonstrate that they have achieved the Communication objectives.

CRITERION III: MATHEMATICAL APPLICATIONS

This criterion involves applying mathematics in **unfamiliar situations.** It is neither necessary nor desirable to have separate items to assess Justification - the objectives of this dimension should be achieved through the applications in both life-related and purely mathematical situations. Recording information in each of the associated dimensions is not necessary, but the assessment package must enable students to demonstrate their ability in the general objectives of all three dimensions.

The assessment package must include:

- items which range from simple to complex
- items which allow students to demonstrate their ability in the objectives of all dimensions; it is not necessary that all dimensions appear in an individual item
- items which allow students to demonstrate that they have achieved the Communication objectives.

In summary, work programs will need to outline a balanced assessment program including a variety of assessment techniques which have validity in assessing achievement of the General Objectives.

6.4 ASSESSMENT TECHNIQUES

Assessment techniques other than traditional written tests or examinations are to be included in the assessment program at least twice each year.

A variety of assessment techniques is to be used and might include:

- written traditional tasks
  - multiple choice
  - short answer
  - extended response (mainly symbolic)
- other written tasks
  - extended response (prose)
  - annotations of collections of media articles
  - reports on :
    - projects
    - investigations
    - group work
    - modelling tasks
    - research topics
- oral tasks
  - short answer
  - seminar presentation
  - debates
  - hypotheticals
- practical tasks using instruments.

6.5 RECORDING INFORMATION

Information on student achievement in each criterion may be recorded in various ways. Teachers may use marks, standards derived from verbal descriptors or marks, or a combination of these. However, the methods of recording and the frequency with which records will be updated must be clearly outlined in the work program.

6.6 STANDARDS FOR DETERMINING EXIT LEVELS OF ACHIEVEMENT

In awarding an exit Level of Achievement for each criterion, a student's performance over the course of study is to be compared with the standards expressed in the following tables.

CRITERION I: COMMUNICATION

STANDARD	DESCRIPTOR
A	The student has communicated clearly and concisely using appropriate terminology and appropriate forms of presentation, while consistently adhering to the conventions of language and mathematics.
B	The student has communicated clearly using appropriate terminology and appropriate forms of presentation, while generally adhering to the conventions of language and mathematics.
C	The student has generally communicated satisfactorily, while generally adhering to the basic conventions of language and mathematics.
D	The student has communicated inappropriately and inadequately.

CRITERION II: MATHEMATICAL TECHNIQUES

STANDARD	DESCRIPTOR
A	The student has an extensive knowledge of Mathematical Techniques and is consistently accurate and proficient when applying them to familiar situations.
B	The student has a substantial knowledge of Mathematical Techniques and is generally accurate and proficient when applying them to familiar situations.
C	The student has a working knowledge of Mathematical Techniques and is generally accurate and proficient when applying them to familiar situations.
D	The student has some knowledge of Mathematical Techniques and is occasionally accurate and proficient when applying them to familiar situations.
E	The student has little knowledge of Mathematical Techniques and is rarely accurate and proficient when applying them to familiar situations.
N.B. The situations mentioned above must range from simple to complex in both life-related and purely mathematical contexts.	

CRITERION III: MATHEMATICAL APPLICATIONS

STANDARD	DESCRIPTOR
A	The student has consistently provided solutions in simple, unfamiliar situations, and generally provided solutions in complex, unfamiliar situations. The student has, within the above situations, consistently determined the appropriateness of results and consistently developed logical arguments. The student has consistently tested the validity of arguments which use mathematics.
B	The student has generally provided solutions in simple, unfamiliar situations, and occasionally provided solutions in complex, unfamiliar situations. The student has, within the above situations, generally determined the appropriateness of results and generally developed logical arguments. The student has generally tested the validity of arguments which use mathematics.
C	The student has occasionally provided solutions in simple, unfamiliar situations. The student has, within the above situations, generally determined the appropriateness of results and attempted to develop logical arguments. The student has considered the validity of arguments which use mathematics.
D	The student has rarely provided solutions in unfamiliar situations.
N.B. The situations mentioned above must be set in both life-related and purely mathematical contexts.	

6.7 DETERMINATION OF EXIT LEVELS OF ACHIEVEMENT

On completion of the course of study, the school must award, to each student, a standard for each of the three criteria using the tables in section 6.6. These three standards are then used to determine a student's exit Level of Achievement.

The table below gives the **minimum** performance composite of standards required for each Level of Achievement.

Minimum Standards Table

LEVEL OF ACHIEVEMENT	COMMUNICATION	MATHEMATICAL TECHNIQUES	MATHEMATICAL APPLICATIONS
VERY HIGH	B	A	A
HIGH	C	B	B
SOUND	C	C	D
LIMITED	D	D	D
VERY LIMITED	D	E	D

A given Level of Achievement can only be awarded to a student who achieves the required **minimum standard in all three criteria.** Hence it is imperative that students are continually made aware of any need to improve in the dimensions of a particular criterion where inconsistencies across the criteria are developing.

7 DEVELOPING A WORK PROGRAM

The work program is a formal expression of the school's interpretation of this syllabus. It has three primary functions. Firstly, it provides guidance to the teachers of the subject as to the nature and requirements of the Mathematics B program at the school. Secondly, it provides similar guidance to the school's students, and their parents, in relation to the subject matter to be studied and how achievement of the program's objectives will be assessed. Thirdly, it provides a basis for accreditation by the Board of Senior Secondary School Studies for the purposes of including students' results for the subject on their Senior Certificates.

The school's work program should be a document which does not require reference to other documents in order to be understood. The work program must contain the following components.

- **Table of Contents** - to increase the readability of the document. This will mean that pages must be numbered.

- **Rationale** - which provides justification for including the subject in the school curriculum. The rationale may be derived principally from the syllabus statement but should also include information on the school's philosophy, student population, resources and any other factors which may influence the decisions made in designing a program of study to cater for the special characteristics of the school and its students.

- **Global Aims** - which are statements of the long term achievements, attitudes and values that are to be developed by the students in studying the subject but which are not directly assessed by the school. These should be the Global Aims listed in this syllabus.

- **General Objectives** - as indicated in this syllabus.

- **Course Organisation** - which shows:

  - a summary of the integrated sequence developed by the school to give an overview of the course to be offered; time allocations should be included

  - details of the integrated sequence indicating the scope and depth of the subject matter to be taught.

  The sequence must be developed in accordance with Section 4 of this syllabus..

  If the school wishes to take the opportunity to offer Linear Programming to one group of students and Networks to another, this should be addressed in this section.

A statement should be included by the school indicating that due consideration has been given to the issues associated with the Ministerial statement on Educational Equity included in all syllabuses. The Ministerial statement has implications for the course organisation including the selection of learning experiences and assessment techniques.

- **Learning Experiences** - which are appropriate to the age group concerned and consistent with the objectives of the work program. A variety of learning experiences should be included which, in their totality, should enhance a balanced course of study. The learning experiences may be integrated into the Course Organisation section.

- **Assessment** - including:

  - an assessment plan which:

    - indicates the **formal formative** and the **summative techniques** used to gather data on student performance, the criteria associated with tasks and the conditions of implementation of tasks

    - provides a balance with respect to techniques used and conditions applied and must ensure that sufficient information is gathered on achievement in each criterion to make valid judgments

    - clearly indicates the summative tasks which will be included in the review folio - data gathered as a result of implementing the assessment plan should allow fullest and latest information to determine exit Levels of Achievement

    - allows sufficient information to be available for the recommendation of interim Levels of Achievement, for example, for Monitoring purposes

  - the standards (either numerical or verbal) used in the school to record assessment data and how that data will be recorded

  - an example of the individual student profile to be included in the review folio

  - how assessment data are combined to reach an overall standard in each criterion (a completed student profile may clarify the explanation). Fullest and latest information is not obtained by an arbitrary 'weighting' of semesters or by using semester 1 assessment instruments as summative in a well sequenced course

- the procedure for awarding exit Levels of Achievement which are consistent with the criteria and standards of this syllabus (Section 6)

- sample assessment items including:

  · at least one Criterion II task which must allow the assessment of student performance on a range of objectives from each of the dimensions of the criterion as well as some of the objectives from the Communication criterion

  · at least one Criterion III task which must allow the assessment of student performance on a range of objectives from each of the dimensions (a) and (c) of the criterion as well as some of the objectives from the Communication criterion

  · at least one Criterion III task which must allow the assessment of student performance on a range of objectives from each of the dimensions (b) and (c) of the criterion as well as some of the objectives from the Communication criterion.

The Minister for Education has supplied the following statement and requested it be inserted in this section of the syllabus.

EDUCATIONAL EQUITY

Schools should provide a curriculum which in subject matter, language, methodology, learning experience and assessment instruments meets the educational needs and entitlements of all students and, in particular, of girls as well as boys, of Aboriginal and Torres Strait Islander students, of students who are from Non-English Speaking Backgrounds and of students who have an intellectual and/or physical impairment. Care should be taken by schools to ensure that the curriculum content recognises the contributions of women to society, that it is culturally inclusive of the activities and achievements of Aboriginal, Torres Strait Islander and Non-English Speaking Background people and that it takes every opportunity to draw students' attention to positive examples of people with intellectual and/or physical impairments.

In order to achieve more equitable educational outcomes by meeting these requirements, educators are asked to consider the following when constructing work programs:

1. Gender issues to be considered

- gender inclusive curriculum;
- the contributions of women;
- the life experiences of women and girls;
- non-stereotyped non-sexist print and non-print resources;
- teaching approaches and learning experiences that suit girls as well as boys;
- equitable access to and experience in practical exercises in co-educational classes;
- assessment plans that suit girls as well as boys;
- the utilisation of a range of single-sex and co-educational settings; and
- a school environment which supports and encourages girls' active participation and achievements in all aspects of the educational program available.

2. Aboriginal and Torres Strait Islander issues to be considered

- curriculum inclusive of Aboriginal and Torres Strait Islander people;
- the contributions of Aboriginal and Torres Strait Islander people;
- the life experiences of Aboriginal and Torres Strait Islander people, including the impact of the non-Aboriginal settlement of Australia;
- non-stereotyped and non-racist print and non-print resources;
- teaching approaches and learning experiences that suit Aboriginal and Torres Strait Islander students;
- equitable access to curriculum choices;
- curriculum content which reflects the diversity amongst Aboriginal and Torres Strait Islander people and, in particular, the differences between traditional and non-traditional communities;

- assessment plans which suit Aboriginal and Torres Strait Islander students; and
- a school environment which supports and encourages the active participation and achievements of Aboriginal and Torres Strait Islander students in all aspects of the educational program available.

3. Non-English Speaking Background issues to be considered

- culturally inclusive curriculum;
- a school environment which supports and encourages the active participation and achievements of students from Non-English Speaking Backgrounds in all aspects of the educational program available;
- teaching approaches and learning experiences which suit students from Non-English Speaking Backgrounds and which meet their needs for English language development support;
- non-stereotyped non-racist print and non-print resources;
- recognition of bicultural and bilingual skills and experiences of people from Non-English Speaking Backgrounds, including the impact of immigration; and
- assessment plans which suit students from Non-English Speaking Backgrounds.

4. Issues to be considered with respect to intellectual and/or physical impairments

- curriculum inclusive of those with intellectual and/or physical impairment;
- curriculum appropriate to the needs of intellectually and/or physically impaired learners within the context of local schools;
- positive examples of activities and achievements of people with intellectual and/or physical impairments;
- non-stereotyped print and non-print resources which avoid imposing notions of physical perfection;
- recognition of the skills and abilities of people with intellectual and/or physical impairments;
- teaching approaches and learning experiences which suit students with intellectual and/or physical impairments and which cater for any resource and support needs they may have;
- a school environment which supports and encourages the active participation and achievements of students with intellectual and/or physical impairments in all aspects of the educational program available; and
- utilisation of special needs settings as well as integrated settings.

Educators may find the following publications useful for devising an inclusive work program.

1. A Fair Deal, Equity Guidelines for developing and reviewing educational resources. Department of Education, Queensland, 1991.
2. Anti-racism: a handbook for adult educators. Human Rights Commission Education Series No. 1. AGPS. Canberra 1986.

8 REVIEW FOLIOS

A Review Folio is a collection of a student's responses to assessment items on which the Level of Achievement is based. Each folio should contain a variety of assessment techniques demonstrating performance in the three criteria, Communication, Mathematical Techniques and Mathematical Applications over the range of topics. This variety of assessment techniques is necessary to provide information which is not solely dependent on the student being familiar or comfortable with a single assessment technique.

It is necessary that a student's achievement in the three criteria is monitored throughout the course so that appropriate feedback in terms of the criteria and the dimensions is provided to the student. The review folio is an ideal medium for students and teachers to monitor progress throughout the course.

For Certification purposes, schools must submit student folios which contain:

- student achievement data profiled in the three exit criteria
- the student responses to **all** summative assessment instruments (In the case of non-written responses, the minimum requirement will be a completed student criterion sheet(s) for that assessment instrument.)
- a minimum of four instruments from year 12 with at least one of these being non-traditional (for example, project, oral presentation, seminar, group work).

A review folio must consist of a minimum of 4 to a maximum of 14 pieces of summative work.

APPENDIX 1

Maintenance of Basic Skills and Mathematical Techniques

Basic Mathematics

The following knowledge and skills will be required throughout the course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- basic algebraic manipulations
- identities, linear equations and inequalities
- gradient of a straight line
- equation of a straight line
- plotting points using Cartesian co-ordinates
- solutions of a quadratic equation
- graphs of quadratic functions
- tree diagrams as a tool for defining sample spaces and estimating probabilities
- the summation notation: $\sum_{i=1}^{n} x_i$.

Maintenance of Mathematical Techniques

The following learning experiences are included as suggestions.

Learning Experiences:

- prepare a seminar revising prerequisite material, for example, revising differentiation before integration
- complete an entrance test for the Public Service or a major employer
- solve a new problem requiring previously learnt skills
- help other students in a lower year with their mathematics
- work without a calculator for a time
- complete a test given to a lower year, have another student mark it and then discuss anomalies
- design some questions to help others revise a topic
- each member of a group writes a summary of a topic, then students share the summaries.

APPENDIX 2

Explanation of Some Terms

amortising a loan

The repayment of a debt with constant repayments at fixed intervals.

annuities

The accumulation of fixed payments at fixed intervals over a period of time.

box-and-whisker plot

A graphical presentation of the five-number summary formed by drawing a *box* around the lower and upper quartiles, locating the median within that box, and identifying the extreme values in the data by drawing lines, *whiskers*, out from the box. The technique is illustrated in the following example.

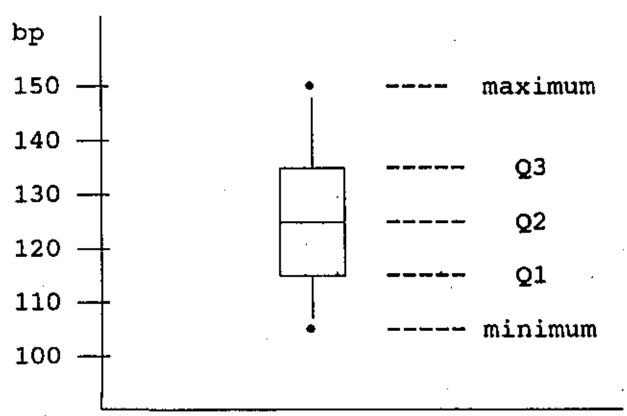
Example: Consider the following 20 systolic blood pressures (bp)

139, 110, 130, 108, 125, 111, 151, 122, 126, 119
114, 134, 120, 132, 134, 130, 107, 137, 120, 136

These data are tabulated below.

X	f	cum f	%
107	1	1	5
108	1	2	10
110	1	3	15
111	1	4	20
114	1	5	25(Q1)
119	1	6	30
120	2	8	40
122	1	9	45
125	1	10	50(Q2)
126	1	11	55
130	2	13	65
132	1	14	70
134	2	16	80(Q3)
136	1	17	85
137	1	18	90
139	1	19	95
151	1	20	100

box-and-whisker plot for 20 systolic blood pressures (bp).



Note : the box-and-whisker plot can be presented vertically or horizontally.

five-number summary

The combination of the *median*, Q_2, the *lower* and *upper quartiles*, Q_1 and Q_3, and the two extreme values (that is, the *minimum* and the *maximum*).

Minimum value	25% of obs.	Q_1	25% of obs.	Q_2	25% of obs.	Q_3	25% of obs.	Maximum value
↑		↑		↑		↑		↑

Observations arranged in ascending order

frequency plot

A diagrammatic presentation of the frequency distribution of the observations, for example, a bar chart, a pie chart, a histogram, a frequency polygon or an ogive.

fullest and latest

Exit Levels of Achievement must be judged on the **latest** information from a **full** coverage of all the significant aspects of the course. The fullest and latest information consists of the most recent data on developmental aspects together with data on components that will not be further developed or assessed.

geoliner

A protractor-like device.

linear objective function

The function to be optimised in linear programming.

mathematical model

Any representation of a situation which is expressed in mathematical terms. It should be noted that models may be as simple as expressing simple interest as

$$I = \frac{Prt}{100}$$

or showing the relationship between two variables as a scattergram.

minimum spanning tree

The minimum spanning tree of a network is a collection of arcs which:
- is connected
- has no loops
- covers all nodes
- is of total minimum length.

operations research

A systematic application of quantitative methods, techniques and tools to solve problems.

outlier

An extreme value in the observations, for example, an observation which lies beyond the box in the box-and-whisker plot, or a point which is well away from the line of best fit.

random normal numbers

Numbers which are selected at random from a population of numbers which have a standard normal distribution; that is, a population with a mean of zero, a standard deviation of 1 and the characteristics of a normal distribution.

Sign test

A simple statistical test which can be used as a quick test whenever the data can be regarded as consisting of positives, negatives or zeros.

The Sign test is a special case of the Binomial distribution where the probability is always equal to 0.5. Its null distribution can be obtained from a Binomial table, from special Sign tables or from first principles for small sample sizes. When used for small samples it is a very useful way to introduce students to the statistical inference of hypothesis testing.

When data consist of paired samples, the **sign** of the difference between the values of the variable for each member of a pair is either positive, negative or zero. If there is no difference in the distributions of the populations from which the paired samples are taken then, for any one pair, a "plus" sign will be just as likely to occur as a "minus" sign. The Sign test can then be used to test the hypothesis that the probability of a "plus" is equal to 0.5.

For example, if observations of height are taken on a sample of individuals at two different times, then the sign of the difference between the two height measurements for an individual could be regarded as a plus if the second value is greater than the first value. That is, individuals who have grown over the time period are regarded as positives; individuals who have become shorter are regarded as negatives; individuals whose height has not changed are regarded as zeros. If the two populations of heights are the same, the difference between the two heights of an individual selected at random will be just as likely to be positive as negative, that is, the probability of a positive sign will be 0.5.

The data need not be quantitative, provided that the difference between the two observations in the pair can be classified as positive, negative or zero. For example, suppose a teacher has been trying a new method of teaching; after a specified period of time the teacher can determine whether the students in the class have learned more (+), less (-), or stayed the same (zero).

The Sign test can be used for a single sample of data if the concept of a plus, minus or zero can be identified. For example, suppose data on a pollutant are to be assessed against a specified minimum standard; each observation is compared with the standard and assigned a 'plus' if it exceeds the standard, a 'minus' if it lies below the standard and a zero if it equals the standard. To test the hypothesis that, on average, the pollutant does not exceed the minimum, the number of 'plus' signs is treated as a Binomial variable with the probability of success (a plus) being 0.5 and the sample size being equal to the sum of the number of plus signs and number of minus signs.

stem-and-leaf plot

An exploratory technique that simultaneously ranks the data and gives an idea of the distribution.

Example: The following 16 average daily temperatures have been recorded to the nearest degree Celsius:

31	21	35	30	22	23	9	24
13	41	30	21	29	24	18	28

Preliminary stem-and-leaf plot of the temperatures:

```
Unit  =  1

   2 | 1   represents 21

   0 | 9
   1 | 3, 8
   2 | 1, 2, 3, 4, 1, 9, 4, 8
   3 | 1, 5, 0, 0
   4 | 1
```

Final stem-and-leaf plot of the temperatures:

```
Unit  =  1

   2 |  1   represents 21
     |
   0 |  9
   1 |  3, 8
   2 |  1, 1, 2, 3, 4, 4, 8, 9
   3 |  0, 0, 1, 5
   4 |  1
     |
```

summary statistics

Characteristics which describe the sample of observations, for example, the mean, median or standard deviation.

trapezoidal rule

The area under the curve above the x-axis between the limits $x = a$ and $x = b$, can be approximated by dividing the interval $[a,b]$ into n sub-intervals of equal length, $l = \frac{|b - a|}{n}$, and using the formula:

$$A \approx l[\tfrac{1}{2}f(a) + f(a+l) + (f(a+2l) + \ldots + f(a+(n-1)l) + \tfrac{1}{2}f(b)]$$

variation

The way in which the observations differ (vary) from each other, often measured by the standard deviation or range.

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