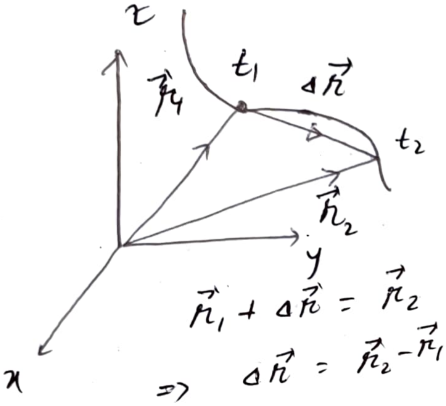
"Chapter - 4"

"Motion in "2D" and '3D':

Position vector, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Displacement, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$



$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$

$\Rightarrow \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Average velocity,

$$\vec{V}_{avg, t_1, t_2} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$= V_{avg,x}\hat{i} + V_{avg,y}\hat{j} + V_{avg,z}\hat{k}$$

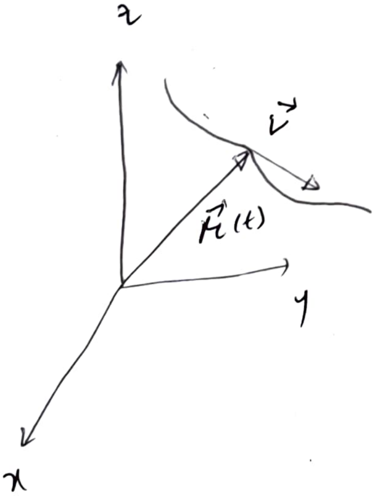
Instantaneous velocity/Velocity,

$$\vec{V} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t}$$

$$= \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$



Average acceleration,

$$\vec{a}_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \frac{\Delta\vec{V}}{\Delta t}$$

$$= \frac{\Delta V_x}{\Delta t}\hat{i} + \frac{\Delta V_y}{\Delta t}\hat{j} + \frac{\Delta V_z}{\Delta t}\hat{k}$$

Acceleration,

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

See SP: 4.01, 4.02, 4.03

Chp: 2
SP: 2.03, 2.05

CP: 2, 5

Pr: $\boxed{5}$, $\boxed{15}$, $\boxed{18}$, 20

Chap: 4

SP: 4.01, 4.02, 4.03, 4.04, 4.05

CP: 1, 2, 3, 4, 5

Pr: $\boxed{11}$, ~~1~~4, $\boxed{16}$, 26, $\boxed{27}$, $\boxed{28}$, 32, 43

Motion in 1D - constant, a

$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

"Motion in 3D" constant $\vec{a}$ (acceleration)

$$\vec{v} = \vec{v}_0 + \vec{a}t$$
$$\vec{v}\cdot\vec{v} = \vec{v}_0\cdot\vec{v}_0 + 2\vec{a}\cdot(\vec{r} - \vec{r}_0)$$
$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

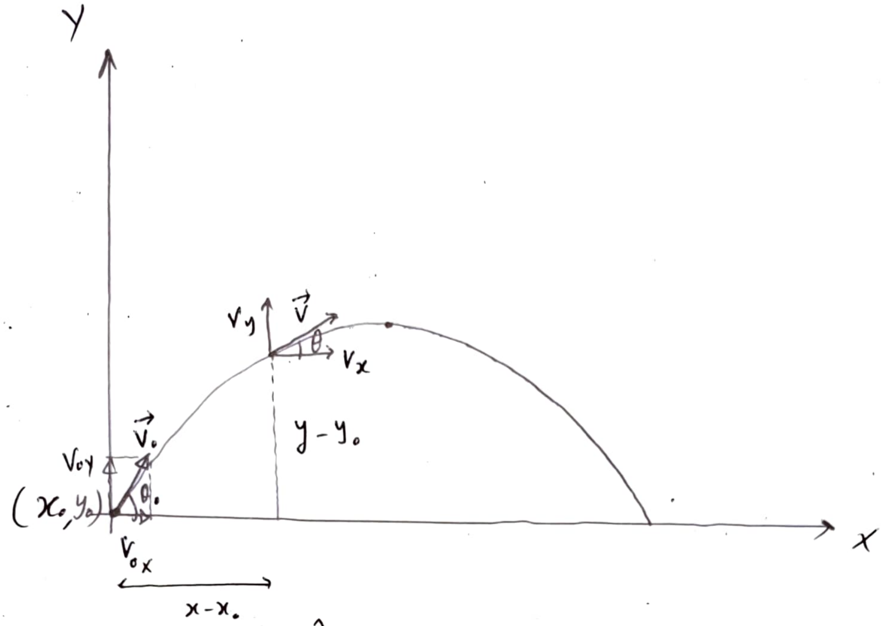
$$(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = (v_{0x}\hat{i} + v_{0y}\hat{j} + v_{0z}\hat{k}) + (a_x t\hat{i} + a_y t\hat{j} + a_z t\hat{k})$$

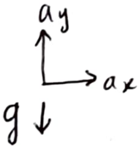
$$\left.\begin{aligned} v_x &= v_{0x} + a_x t \\ v_y &= v_{0y} + a_y t \\ v_z &= v_{0z} + a_z t \end{aligned}\right\}$$

Finally,
$$\left\{\begin{aligned} v_x &= v_{0x} + a_x t \\ v_x^2 &= v_{0x}^2 + 2a_x(x - x_0) \\ x - x_0 &= v_{0x}t + \frac{1}{2}a_x t^2 \end{aligned}\right.$$

$x \to y \to z$

"Projectile Motion" (2D Motion)





$$\vec{V_0} = \vec{V_{0x}}\,\hat{i} + \vec{V_{0y}}\,\hat{j} \quad \text{---} \quad (1)$$

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} \quad \text{------} \quad (2)$$

$$\left.\begin{aligned} a_x &= g\cos 90^\circ = 0 \\ a_y &= g\cos(180^\circ) = -g \end{aligned}\right\} \quad \text{---} \quad (2)\,a$$

$$\left.\begin{aligned} V_{0x} &= V_0 \cos\theta_0 \\ V_{0y} &= V_0 \sin\theta_0 \end{aligned}\right\} \quad \text{----} \quad (1)\ a$$

Horizontal Motion:

$$x - x_0 = V_{0x}\,t + \frac{1}{2} a_x t^2$$

$$= V_0 \cos\theta_0\, t + \frac{1}{2} \times 0 \times t^2$$

$$\boxed{x - x_0 = V_0 \cos\theta_0\, t} \quad \text{-----} \quad (3)$$

$$V_x = V_{0x} + a_x t$$

$$= V_0 \cos\theta_0 + (0)\, t$$

$$\boxed{V_x = V_0 \cos\theta_0} \quad \text{----} \quad (4)$$

Vertical Motion:

$$y - y_0 = V_{0y}\, t + \frac{1}{2} a_y t^2$$

$$\boxed{y - y_0 = V_0 \sin\theta_0\, t - \frac{1}{2} g t^2} \quad \text{---} \quad (5)$$

$$V_{0y} = V_{0y} + a_y t \implies \boxed{V_y = V_0 \sin\theta_0 - g t} \quad \text{----} \quad (6)$$

$$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$$

$$\boxed{v_y^2 = v_0^2\sin^2\theta_0 - 2g(y-y_0)} \quad \cdots (7)$$

Velocity:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\boxed{\vec{v} = (v_0\cos\theta_0)\hat{i} + (v_0\sin\theta_0 - gt)\hat{j}} \quad \cdots (8)$$

$$\boxed{\tan\theta = \frac{v_y}{v_x} = \frac{v_0\sin\theta_0 - gt}{v_0\cos\theta_0}} \quad \cdots (9)$$

Equation of Path:

For simplicity, $x_0 = 0$, $y_0 = 0$,

Equation (3), $x = v_0\cos\theta_0 . t \Rightarrow \boxed{t = \dfrac{x}{v_0\cos\theta_0}} \quad \cdots (10)$

From, equation (3) and (10), we get,

$$y = v_0\sin\theta_0 \frac{x}{v_0\cos\theta_0} - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\theta_0}$$

$$= x(\tan\theta_0) + \left(\frac{-g}{2v_0^2\cos^2\theta_0}\right)x^2$$

$$\boxed{y = ax + bx^2} \quad \cdots (11)$$

where, $a = \tan\theta_0$

$b = -\dfrac{g}{2v_0^2\cos^2\theta_0}$

Equation (11) is the equation of path of projectile, which is the equation of Parabola. So the path is parabolic.

Horizontal Range:

For Horizontal range $x - x_0 = R$, $y - y_0 = 0$,

air time, $t = T$.

From equation, (5);

$$0 = v_0\sin\theta_0 T - \frac{1}{2}gT^2$$

$$\Rightarrow T\left(v_0 \sin\theta_0 - \frac{gT}{2}\right) = 0$$

$$\therefore T = 0 \quad ; \quad v_0 \sin\theta_0 - \frac{g}{2}T = 0$$

$$\Rightarrow T = \frac{2 v_0 \sin\theta_0}{g} \quad \cdots (12)$$

(air time)

Equation (3), $R = v_0 \cos\theta_0 \dfrac{2 v_0 \sin\theta_0}{g}$

Horizontal Range

$$\boxed{R = \frac{v_0^2}{g} \sin(2\theta_0)} \quad \cdots (13)$$

Maximum Height:

At maximum height, $v_y = 0$; $y - y_0 = H$

From equation (6), $0 = v_0 \sin\theta_0 - gt$

$$\Rightarrow \boxed{t = \frac{v_0 \sin\theta_0}{g}} \quad \_ (14)$$

From equation (5), $H = (v_0 \sin\theta_0) \dfrac{v_0 \sin\theta_0}{g} - \dfrac{g}{2} \dfrac{v_0^2 \sin^2\theta_0}{g^2}$

$$= \frac{v_0^2 \sin^2\theta_0}{g} - \frac{1}{2}\frac{v_0^2 \sin^2\theta_0}{g}$$

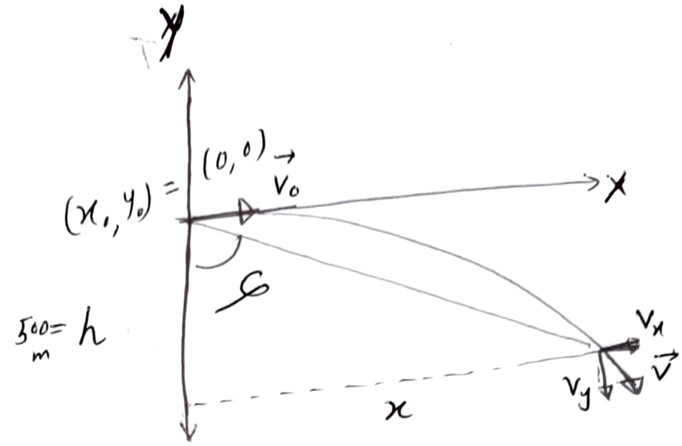
$$\boxed{H = \frac{v_0^2}{2g} \sin^2\theta_0} \quad \cdots (15)$$

See SP: 4.04, 4.05

Sample Problem 4.04

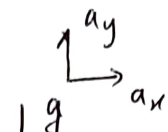
$|\vec{v}_0| = 198$ km/h

$= 55$ m/s



$v_{0x} = v_0 \cos 0^\circ = 55$ m/s

$v_{0y} = v_0 \sin 0^\circ = 0$ m/s



$a_x = g \cos 90^\circ = 0$ m/s²

$a_y = g \cos(180^\circ) = -g$ m/s² $= -9.8$ m/s²

(a) $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow x = 55t$ --- (1)

$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$

$\Rightarrow -500 = 0 \times t + \frac{1}{2}(-9.8) t^2$

$\Rightarrow t = \sqrt{\frac{2 \times 500}{9.8}}$ s

$\boxed{t = 10.1 \text{ s}}$

From equation, (1), $x = 55 \times 10.1$ m

$= \boxed{555.5 \text{ m}}$

$\tan\varphi = \frac{x}{h} \Rightarrow \varphi = \tan^{-1}\left(\frac{555.5}{500}\right) = 48^\circ$

(b) $\vec{v} = ?$ $\vec{v} = v_x \hat{i} + v_y \hat{j}$

$v_x = v_{0x} + a_x t$

$= (55 + 0 \times 10.1)$ m/s $= 55$ m/s

$v_y = v_{0y} + a_y t$

$= (0 + (-9.8) \times 10.1)$ m/s

$= -99$ m/s

$\vec{v} = (55 \text{ m/s})\,\hat{i} + (-99 \text{ m/s})\,\hat{j}$

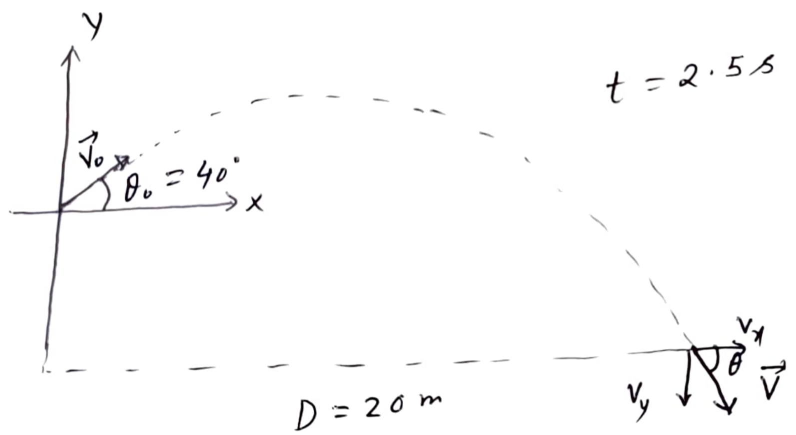
$|\vec{v}| = \sqrt{(55)^2 + (-99)^2}$ m/s

$= 113.25$ m/s

$\theta = \tan^{-1}\left(\frac{-99}{55}\right)$

$= \boxed{-60.9^\circ}$

Sample Problem 4.05



$v_{0x} = v_0 \cos\theta_0$

$v_{0y} = v_0 \sin\theta_0$

$v_0 = ?$ $\qquad v = ?$

a_y, a_x

$a_x = g\cos(90^\circ) = 0$ $\quad \downarrow g$

$a_y = g\cos(180^\circ) = -g = -9.8$ m/s²

$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow 20 = v_0 \cos\theta_0\, t + \frac{1}{2} \times 0 \times t^2$

$\Rightarrow v_0 = \frac{20}{(\cos\theta_0)\, t} = \frac{20}{(\cos 40^\circ) \times (2.5)}$ m/s $= \boxed{10.4 \text{ m/s}}$

$$V_x = V_{0x} \cos + a_x t$$

$$= V_0 \cos\theta_0 + 0 \times t$$

$$= (10.4) \cos(40^\circ) \text{ m/s}$$

$$= 7.97 \text{ m/s}$$

$$V_y = V_{0y} + a_y t$$

$$= V_0 \sin\theta_0 - 9.8 \times 2.5$$

$$= ((10.4) \sin(40^\circ) - 9.8 \times 2.5) \text{ m/s}$$

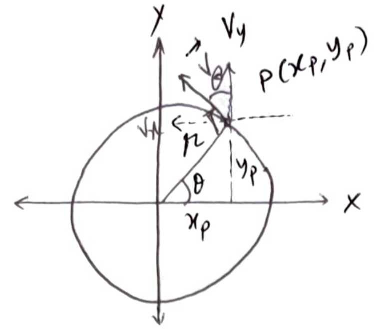
$$= -17.81 \text{ m/s}$$

$$V = \sqrt{(7.97)^2 + (-17.81)^2} \text{ m/s}$$

$$= \boxed{19.5 \text{ m/s}}$$

" Uniform Circular Motion " (2D

$|\vec{V}| = \text{constant}$



$$V_x = V\cos(90° + \theta)$$
$$= -V\sin\theta$$

$$V_y = V\cos\theta$$

Velocity, $\vec{V} = V_x\hat{i} + V_y\hat{j}$

$$= (-V\sin\theta)\hat{i} + (V\cos\theta)\hat{j}$$

$$x_p = r\cos\theta \Rightarrow \cos\theta = \frac{x_p}{r}$$
$$y_p = r\sin\theta \Rightarrow \sin\theta = \frac{y_p}{r}$$

$$\vec{V} = -\frac{V y_p}{r}\hat{i} + \frac{V x_p}{r}\hat{j}$$

acceleration, $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\left(-\frac{V y_p}{r}\hat{i} + \frac{V x_p}{r}\hat{j}\right)$

$$= -\frac{d}{dt}\left(\frac{V y_p}{r}\right)\hat{i} + \frac{d}{dt}\left(\frac{V x_p}{r}\right)\hat{j}$$

$$= -\frac{V}{r}\frac{d y_p}{dt}\hat{i} + \frac{V}{r}\frac{d x_p}{dt}\hat{j}$$

$$= -\frac{V}{r}V_y\hat{i} + \frac{V}{r}V_x\hat{j}$$

$$= -\frac{V}{r}V\cos\theta\,\hat{i} + \frac{V}{r}(-V\sin\theta)\hat{j}$$

$$\vec{a} = -\frac{V^2}{r}\cos\theta\,\hat{i} - \frac{V^2}{r}\sin\theta\,\hat{j}$$

Magnitude of acceleration,

$$|\vec{a}| = \sqrt{\left(\frac{v^2}{r}\right)^2 (\cos^2\theta + \sin^2\theta)}$$

$$\boxed{|\vec{a}| = \frac{v^2}{r}}$$

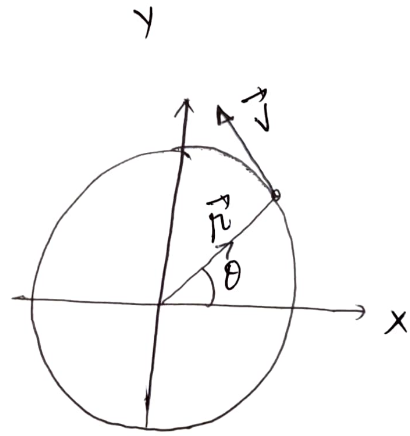
$$\varphi = \tan^{-1}\frac{a_y}{a_x}$$

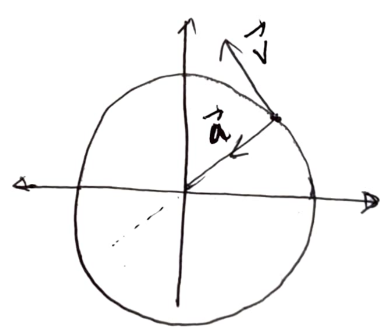
$$= \tan^{-1}\frac{-\frac{v^2}{r}\sin\theta}{-\frac{v^2}{r}\cos\theta}$$

$$= \pi + \tan^{-1}\frac{\frac{v^2}{r}\sin\theta}{\frac{v^2}{r}\cos\theta}$$

$$= \pi + \tan^{-1}\tan\theta$$

$$\boxed{\varphi = \pi + \theta}$$





$\boxed{11}$

$$\vec{r} = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$$

(a) at $t = 2s$, $\vec{r} = [(2 \times 2^3 - 5 \times 2)\hat{i} + (6 - 7 \times 2^4)\hat{j}]\ m$

$$= 6\ m\ \hat{i} - 106\ m\ \hat{j}$$

(b) $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}\right)$

$$= \frac{d}{dt}(2t^3 - 5t)\hat{i} + \frac{d}{dt}(6 - 7t^4)\hat{j}$$

$$= (6t - 5)\hat{i} - 28t^3\hat{j}$$

at, $t = 2s$, $\vec{v}|_{t=2s} = [(6 \times 2 - 5)\hat{i} - (28 \times 2^3)\hat{j}]\ m/s$

$$= (19\hat{i} - 224\hat{j})\ m/s$$

(c) $\vec{a} = \frac{d\vec{v}}{dt}$

$$= \frac{d}{dt}[(6t - 5)\hat{i} - 28t^3\hat{j}]$$

$$= 6\hat{i} - 28 \times 3t^2\hat{j}$$

$$= 6\hat{i} - 84t^2\hat{j}$$

$\vec{a}|_{t=2s} = [6\hat{i} - 84 \times 2^2\hat{j}]\ m/s^2$

$$= [6\hat{i} - 336\hat{j}]\ m/s^2$$

(d) Tangent of the path is the direction of the velocity, so, $\theta = \tan^{-1}\left(\frac{-224}{19}\right)$

$$= \boxed{-85.15}$$

14

16

(a) $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$

$= \left[\frac{(-2-4)\hat{i} + (-2+2)\hat{j} + (5-3)\hat{k}}{4}\right] m/s^2$

$= \left[-\frac{6}{4}\hat{i} + 0\hat{j} + \frac{2}{4}\hat{k}\right] m/s^2$

$= \left[-\frac{3}{2}\hat{i} + 0\hat{j} + \frac{1}{2}\hat{k}\right] m/s^2$

(b) $|\vec{a}|_{avg} = \sqrt{(-\frac{3}{2})^2 + (0)^2 + (\frac{1}{2})^2}\ m/s^2 = 1.58\ m/s^2$

(c) $\vec{a}_{avg} \cdot \hat{i} = -\frac{3}{2}$

$\Rightarrow \cos\theta_x = \frac{-\frac{3}{2}}{|\vec{a}_{avg}|\,|\hat{i}|}$

$\Rightarrow \cos\theta_x = \frac{-\frac{3}{2}}{1.58}$

$\Rightarrow \theta_x = \cos^{-1}\left(\frac{-1.5}{1.58}\right)$

$= \boxed{161.69^\circ}$

16

(a) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left((6t - 4t^2)\hat{i} + 8\hat{j}\right)$

$= [(6 - 8t)\hat{i} + 0\hat{j}]$

$\vec{a}\big|_{t=3s} = [(6 - 8\times 3)\hat{i} + 0\hat{j}]$ m/s²

$= (-18\hat{i} + 0\hat{j})$ m/s²

(b) $6 - 8t = 0$

$\Rightarrow t = \frac{6}{8}\,s = \boxed{\frac{3}{4}\,s}$, $|\vec{a}| = 0$

(c) $|\vec{v}| = \sqrt{(6t - 4t^2)^2 + (8)^2}$

Here, $(6t - 4t^2)^2$ and 8^2 are both positive, so, $|\vec{v}|$ can't be zero.

(d) $|\vec{v}| = 10 = \sqrt{((6t) - 4t^2)^2 + 8^2}$

$10^2 = 36t^2 - 48t^3 + 16t^4 + 64$

$\Rightarrow -100 + 36t^2 - 48t^3 + 16t^4 + 64 = 0$

$\Rightarrow 16t^4 - 48t^3 + 36t^2 - 36 = 0$

$\Rightarrow 8t^4 - 24t^3 + 18t^2 - 18 = 0$

$\Rightarrow 4t^4 - 6t^3 + 9t^2 - 9 = 0$

$\Rightarrow 4t^4 -$

$\Rightarrow ((6t) - 4t^2)^2 = 100 - 64$

$(6t - 4t^2)^2 = 36$

$\Rightarrow 6t - 4t^2 = \pm 6$

$\Rightarrow 6t - 4t^2$

$\Rightarrow 4t^2 - 6t \pm 6 = 0$

$\Rightarrow 2t^2 - 3t \pm 3 = 0$

$2t^2 - 3t + 3 = 0$

$\therefore t = \dfrac{+3 \pm \sqrt{9 - 4 \cdot 2 \cdot 3}}{2 \cdot 2}$ s

↑

which is not real

or, $2t^2 - 3t - 3 = 0$

$\Rightarrow t = \dfrac{+3 \pm \sqrt{9 + 4 \cdot 2 \cdot 3}}{2 \cdot 2}$ s

$= \dfrac{3 \pm \sqrt{9 + 24}}{4}$ s

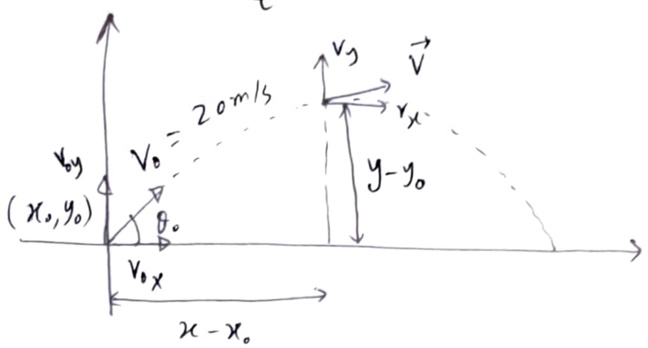
$= \dfrac{3 \pm \sqrt{33}}{4}$ s

$= \boxed{2 \cdot 2 \text{ s}}$

negative t is not accepted

$\boxed{26}$

$t = 1.10s$



$v_{ox} = v_o \cos\theta_o = 20 \times \cos(40^\circ) \ m/s = 15.3 \ m/s$

$v_{oy} = v_o \sin\theta_o = 20 \times \sin(40^\circ) \ m/s = 12.9 \ m/s$

$a_y \uparrow \ \ \rightarrow a_x$

$\vec{g} \downarrow$

$a_x = g\cos(90^\circ) = 0 \ m/s^2$

$a_y = g\cos(180^\circ) = -g \ m/s^2 = -9.8 \ m/s^2$

(a) at, $t = 1.10 s$,

$x - x_o = v_{ox} t + \frac{1}{2} a_x t^2$

$= [15.3 \times 1.10 + \frac{1}{2} \times 0 \times (1.10)^2] \ m$

$= \boxed{16.9 \ m}$

(b) at, $t = 1.10s$ $\quad y - y_o = v_{oy} t + \frac{1}{2} a_y t^2$

$= [12.9 \times 1.10 + \frac{1}{2} \times (-9.8) \times (1.10)^2] \ m$

$= \boxed{8.26 \ m}$

(c) at $t = 1.80s$, $\quad x - x_o = ?$

(d) at $t = 1.80s$ $\quad y - y_o = ?$

(e) at $t = 5s$, $\quad x - x_o = ?$

$y - y_o = ?$

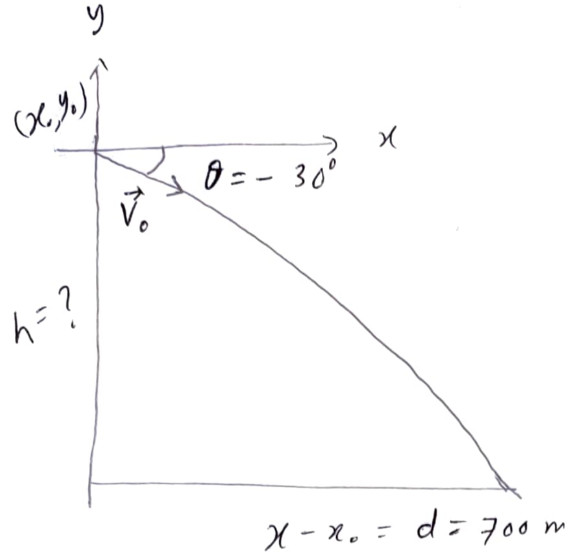
Here, $T = \frac{2 v_o \sin\theta_o}{g}$

$= 2.62 s < 5s$

So put the value of $t = 2.62s$ instead of $\boxed{5s}$.

~~Here~~

(27)



$V_0 = 290$ km/h

$= \dfrac{290 \times 10^3}{3600}$ m/s

$= 80.56$ m/s

$V_{0x} = V_0 \cos(-30°) = 80.56 \times \cos(-30°)$ m/s $= 69.77$ m/s

$V_{0y} = V_0 \sin(-30°) = 80.56 \times \sin(-30°)$ m/s $= -40.28$ m/s

$a_x = g \cos 90° = 0$ m/s²

$a_y = g \cos(180°) = -9.8$ m/s²

(a) $x - x_0 = V_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow 700 = 69.77 \times t + \frac{1}{2} \times 0 \times t^2$

$\Rightarrow t = \dfrac{700}{69.77}$ s $= \boxed{10.03\ s}$

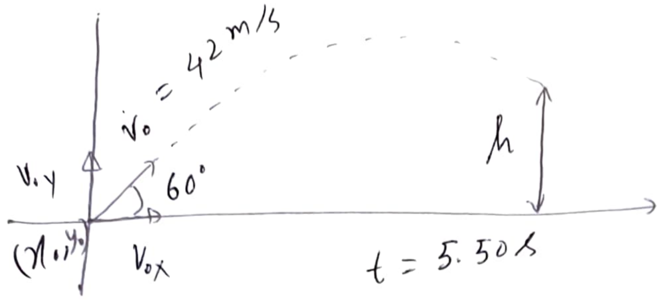
(b) $y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$\Rightarrow -h = -40.28 \times t - \frac{1}{2} \times 9.8 \times t^2$

$= -40.28 \times 10.03 - \frac{1}{2} \times 9.8 \times (10.03)^2$

$\Rightarrow h = \boxed{896.95\ m}$

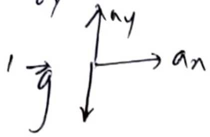
$\boxed{28}$



$t = 5.50$ s

$V_{ox} = V_0 \cos(60°) = 42 \times \cos(60°)$ m/s $= 21$ m/s

$V_{oy} = V_0 \sin(60°) = 42 \times \sin(60°)$ m/s $= 21\sqrt{3}$ m/s



$a_x = g \cos 90° = 0$ m/s^2

$a_y = g \cos(180°) = -9.8$ m/s^2

at, $t = 5.5$ s

(a) $y - y_0 = h = V_{oy} t + \frac{1}{2} a_y t^2$

$= [21\sqrt{3} \times 5.5 + \frac{1}{2} \times (-9.8)(5.5)^2]$ m

$= 51.83$ m

(b) $V_x = V_{ox} + a_x t = 21$ m/s $+ 0 \times t$ m/s

$= 21$ m/s

$V_y = V_{oy} + a_y t = [21\sqrt{3} - 9.8 \times 5.5]$ m/s

$= -17.53$ m/s

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21)^2 + (-17.53)^2}$

$= \boxed{27.36 \text{ m/s}}$

(c) $H = \dfrac{v_0^2 \sin^2\theta_0}{2g} = \dfrac{42^2 \times \sin^2(60°)}{2 \times 9.8}$ m $= \boxed{67.5 \text{ m}}$

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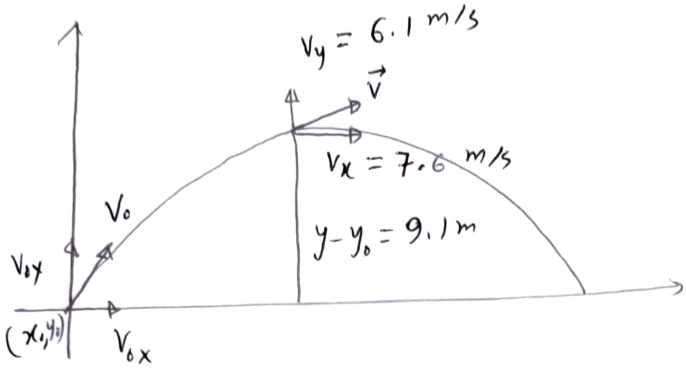

$V_{0x} = 7.6$ m/s $= V_0 \cos\theta_0$

$V_{0y} = V_0 \sin\theta_0$

$a_x = 0$ m/s^2

$a_y = -9.8$ m/s^2

$V_y = 6.1$ m/s, $y - y_0 = 9.1$ m, $H = ?$

(a) $$H = \frac{V_0^2 \sin^2\theta_0}{2g} \quad \text{—(1)}$$

$$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$$

$$\Rightarrow (6.1)^2 = V_{0y}^2 + 2 \times (-9.8) \times 9.1$$

$$\Rightarrow V_{0y}^2 = (6.1)^2 + 2 \times 9.8 \times 9.1$$

$$\Rightarrow V_{0y} = \sqrt{(6.1)^2 + 2 \times 9.8 \times 9.1} \text{ m/s}$$

$$\Rightarrow V_0 \sin\theta_0 = 14.67 \text{ m/s}$$

Putting in eq (1), $$H = \frac{(14.67)^2}{2 \times 9.8} \text{ m}$$

$$= \boxed{10.98 \text{ m}}$$

(b) $$R = \frac{V_0^2}{g} \sin 2\theta_0$$

$$= \frac{V_0^2}{g} 2\sin\theta_0 \cos\theta_0$$

$$= \frac{2 \; V_0 \sin\theta_0 \; V_0 \cos\theta_0}{g}$$

$$= \frac{2 \times 14.67 \times 7.6}{9.8} \text{ m} = \boxed{22.75 \text{ m}}$$

(c) at, $t = T = \frac{2 V_0 \sin\theta_0}{g} = \frac{2 \times 14.67}{9.8}$ s

$= 2.99$ s

$V_x = V_{0x} + a_x t = 7.6$ m/s

$V_y = V_{0y} + a_y t = [14.67 - 9.8 \times 2.99]$ m/s

$= -14.63$ m/s

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7.6^2 + (-14.63)^2}$ m/s

$= \boxed{16.49 \text{ m/s}}$

(d) $\theta = \tan^{-1}\left(\frac{-14.63}{7.6}\right) = \boxed{-63.55^\circ}$

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Here, initial velocity, $V_0 = 25.0 m/s$,

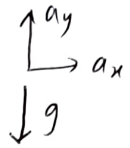
$\theta_0 = 40°$

$x - x_0 = d = 22 m$

(a) $y - y_0 = ?$

$V_{0x} = V_0 \cos\theta_0 = 25 \times \cos 40° \ m/s = 19.15 \ m/s$

$V_{0y} = V_0 \sin\theta_0 = 25 \times \sin 40° \ m/s = 16.1 \ m/s$



$y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$= 16.1 \times t - \frac{1}{2} \times 9.8 \times t^2 \quad -(1)$

$a_x = g \cos 90° = 0 \ m/s^2$

$a_y = g \cos(180°)$
$= -9.8 \ m/s^2$

$x - x_0 = V_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow 22 = 19.15 \times t + \frac{1}{2} \times 0 \times t^2$

$\Rightarrow t = \frac{22}{19.15} s = \boxed{1.15 \ s}$

From eq. (1), $y - y_0 = [16.1 \times 1.15 - \frac{1}{2} \times 9.8 \times (1.15)^2] m$
$= \boxed{12 \ (m)}$ Ans

(b) $V_x = ?$ $\quad V_x = V_{0x} + a_x t = (19.15 + 0 \times t) \ m/s$
$= \boxed{19.15 \ m/s}$ Ans - b

(c) $V_y = ?$

$V_y = V_{0y} + a_y t$
$= (16.1 - 9.8 \times 1.15) \ m/s$
$= \boxed{4.83 \ m/s}$ Ans - c

(d)

$\frac{T}{2} = \frac{2 V_0 \sin\theta_0}{2g} = \frac{2 \times 16.1}{2 \times 9.8} s = \frac{3.29 s}{2} = 1.645 > t$

so it hasn't passed the heighest point while it hits.