PHY 111 Principles of Physics-I

Newtonian Mechanics and Calorimetry

→ vector

→ Equations of motion

→ Newton's Law

→ Conservation Law's of Energy

→ Momentum, conservation of momentum

→ Work- Energy Theorem

→ Rotational Motion

→ Gravitation

→ Elasticity and Properties of Matter

→ Oscillation → SHM
→ Damping Oscillation

→ Waves → Sound

→ Fluid Mechanics

→ Calorimetry

Marks Distribution:

Attendance - 5%

Quiz - 15% [4 Quizes → N-1 Avg]

Lab - 10% [4 or 3 → Avg]

Assignment - 15%

Mid - 20%

Final - 35%

Attendance

90% and above - 5
85% - 89% - 4
80% - 84% - 3
75% - 79% - 2
70% - 74% - 1
Less than 70% - 0

Grade points:

97 - 100	-	A^+	- 4.00
90 - <97	-	A	- 4.00
85 - <90	-	A^-	- 3.70
80 - <85	-	B^+	- 3.30
75 - <80	-	B	- 3.00
70 - <75	-	B^-	- 2.70
65 - <70	-	C^+	- 2.30
60 - <65	-	C	- 2.00
57 - <60	-	D^+	- 1.70
55 - <57	-	D	- 1.30

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Final Question Pattern

Full Marks (35)

You have to answer 3 questions out of 4

→ One mandatory (15 marks; problem solving)

→ 2 out of 3 (10 marks each; problem solving, but one of the questions could be derivation/theoretical)

Books - Fundamentals of Physics - 10th, Resnick, Halliday, Walker

- University physics with Modern Physics - Young

Consultation hour: Saturday - Thursday (11:00am - 12:20pm)

'Vectors' (Chapter - 2)

Physical Quantities: Physical quantities are obtained by ~~numbers~~ measurements and are expressed in numbers.

There are two types of physical Quantities.

1. Scalars: → Scalars are specified by number with unit

→ Follow simple algebra

Example: Temperature (T), mass (m), distance d, speed s

2. Vectors: → Vectors are specified by magnitude and direction

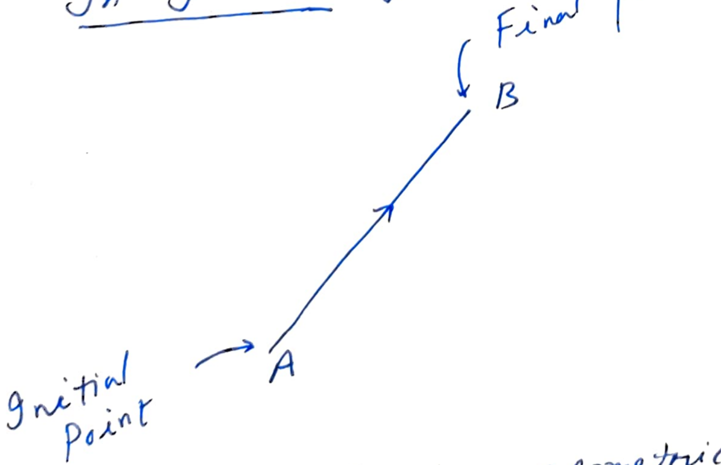
→ Follow vector algebra

Example: Displacement $\Delta\vec{r}$, velocity $\vec{v}$, acceleration, $\vec{a}$

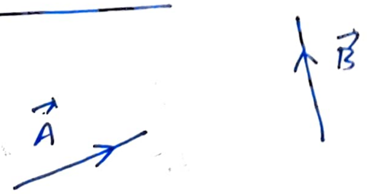
Vector can be represented as $\vec{AB}$, $\overline{AB}$ or $\underline{AB}$

$|\vec{AB}|$ → Magnitude of vector

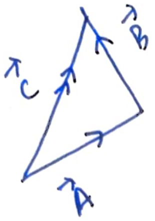
<u>In geometrically</u>



<u>vector addition</u>: (Follows geometrical process)



$\vec{A} + \vec{B} = ?$



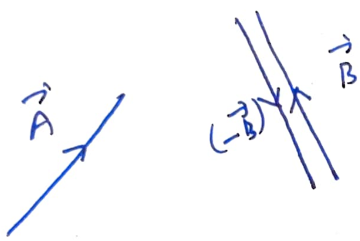
$\vec{C} = \vec{A} + \vec{B}$

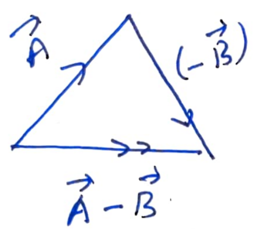
Resultant

vector component

Propert

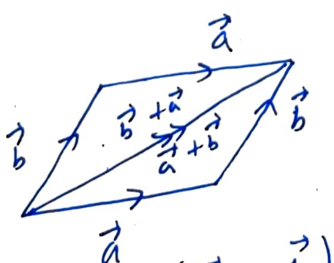
Vector substraction:





Properties of vector addition:

Property - 1: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative rule)



Property - 2: $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

(Associative rule)

Properties of vector (scalar) multiplication:

Let, m, n are scalars and $\vec{A}, \vec{B}$ are vectors,

$m\vec{A} = \vec{A}m$ (commutative rule (multiplication))

$m(n\vec{A}) = n(m\vec{A})$ (Associative rule ("))

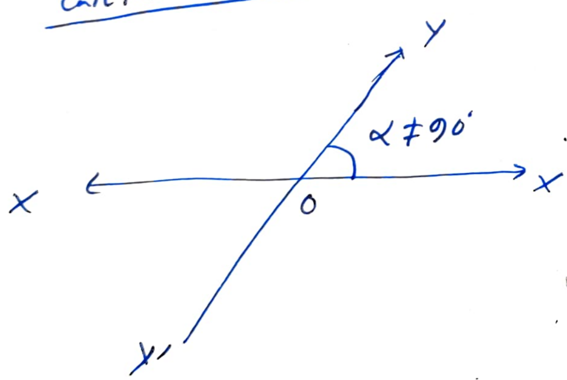
$(m+n)\vec{A} = m\vec{A} + n\vec{A}$ (Distributive rule ("))

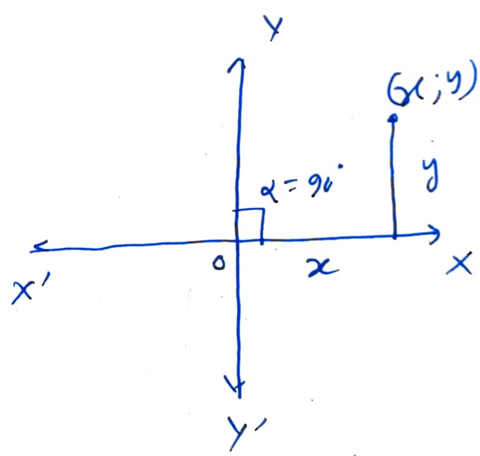
$m(\vec{A}+\vec{B}) = m\vec{A} + m\vec{B}$ (" ("))

Co-ordinate system :

- Cartesian
- Polar

Cartesian co-ordinate:



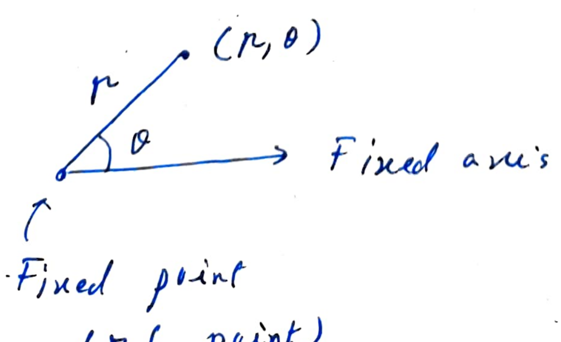


axis are perpendicular

$-\infty < x < \infty+$

$-\infty < y < \infty+$

Polar Co-ordinate:



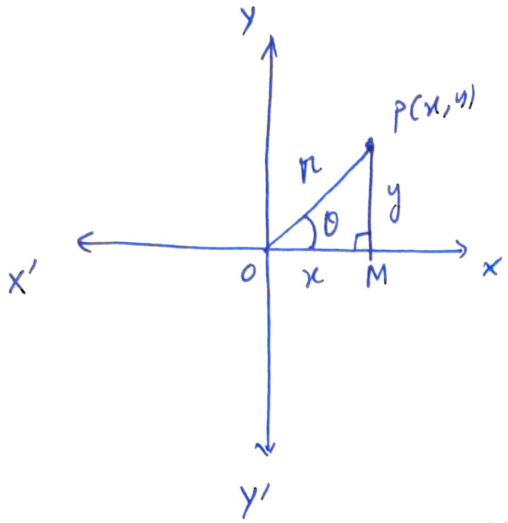
$0 \leq r < \infty+$,

$0 \leq \theta \leq 2\pi$

$\theta \rightarrow$ (+ve) $\rightarrow$ clockwise

$\theta \rightarrow$ (−ve) $\rightarrow$ anti-clockwise

Relation betⁿ Polar and Cartesian Co-ordinate:



$\angle PMO = \frac{\pi}{2}$

ΔPOM, $x^2 + y^2 = r^2$

$\Rightarrow \boxed{r = \sqrt{x^2+y^2}}$

$\cos\theta = \frac{x}{r}$

$\boxed{x = r\cos\theta}$

$\sin\theta = \frac{y}{r} \Rightarrow \boxed{y = r\sin\theta}$

$\boxed{\tan\theta = \frac{y}{x}}$

** $(1, \sqrt{3}) \xrightarrow{(r,\theta)}$

$r = \sqrt{x^2+y^2}$

$= \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$(-1, \sqrt{3}) \xrightarrow{(r,\theta)}$

$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

H.W

$(-1, -\sqrt{3})$

$(+1, -\sqrt{3})$

Spherical Polar Co-ordinate

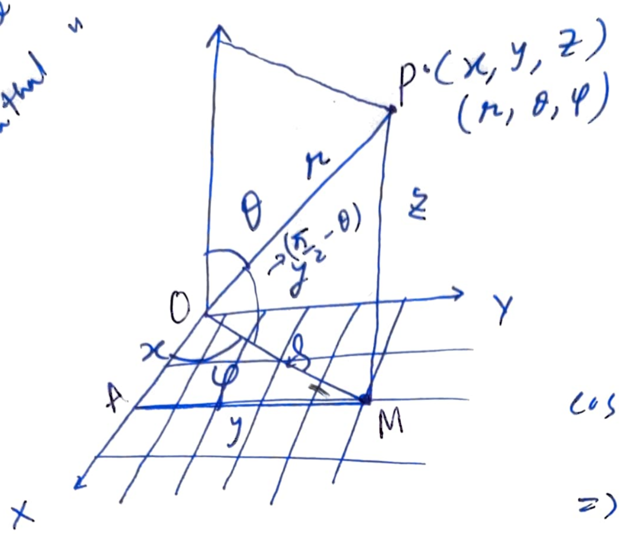
$0 \le r < \infty$ +

$0 \le \theta \le \pi$

$0 \le \varphi \le 2\pi$

$\theta \rightarrow$ polar angle

$\varphi \rightarrow$ Azimuthal "



ΔPOM

$$\sin\left(\frac{\pi}{2}-\theta\right) = \frac{z}{r}$$

$$\Rightarrow z = r\cos\theta \quad -(1)$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{\rho}{r}$$

$$\Rightarrow \rho = r\sin\theta$$

ΔOAM,

$$\cos\varphi = \frac{x}{\rho} \Rightarrow x = \rho\cos\varphi$$

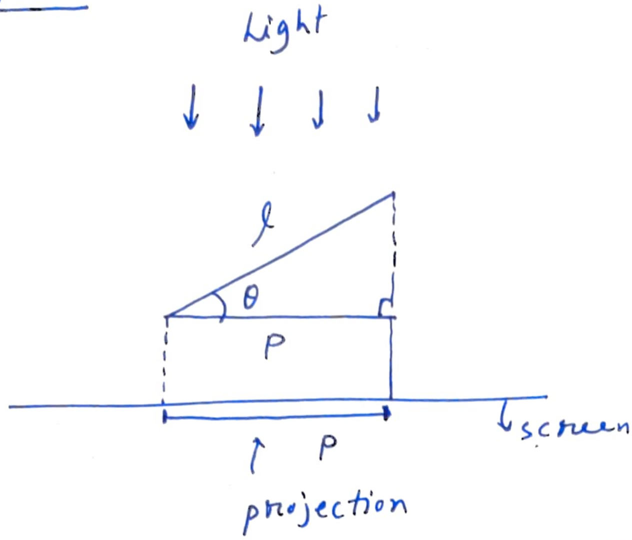
$$\Rightarrow x = r\sin\theta\cos\varphi \quad -(2)$$

$$\sin\varphi = \frac{y}{\rho} \Rightarrow y = \rho\sin\varphi$$

$$y = r\sin\theta\sin\varphi \quad -(3)$$

$$\left.\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned}\right\} \qquad \left\{\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \cos^{-1}\left(\frac{z}{r}\right) \\ \varphi &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}\right.$$

Projection:

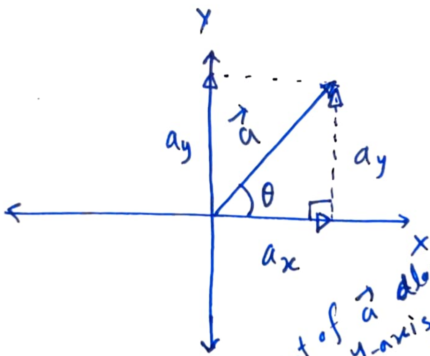


$p \sim \cos\theta$

$p \sim l$

$p \sim l\cos\theta$

$$\cos\theta = \frac{p}{l} \Rightarrow \boxed{p = l\cos\theta}$$



component of $\vec{a}$ along the direction of x

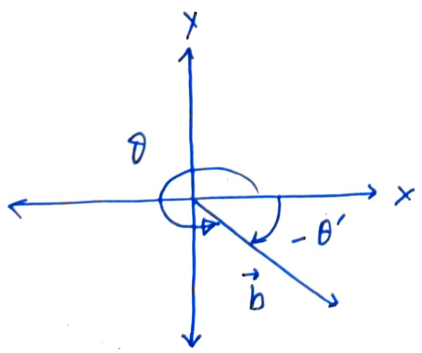
$a_x = |\vec{a}| \cos\theta$

$a_y = |\vec{a}| \cos(90° - \theta)$

component of $\vec{a}$ along y-axis

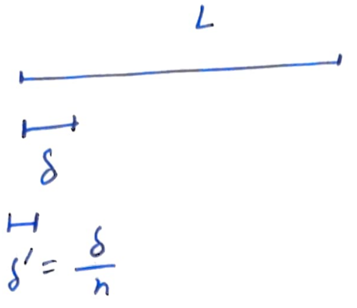
$a_y = |\vec{a}| \sin\theta$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$ $\quad \tan\theta = \frac{a_y}{a_x}$



$b_x = |\vec{b}| \cos\theta$ $\quad$ or $\quad b_x = |\vec{b}| \cos(-\theta')$

$b_y = |\vec{b}| \sin\theta$ $\quad\quad\quad\quad b_y = |\vec{b}| \sin(-\theta')$



$$N(\delta) = \frac{L}{\delta}$$

$$N(\delta') = \frac{L}{\delta'}$$

$$= \frac{L}{\frac{\delta}{n}} = n\frac{L}{\delta}$$

$$= nN(\delta)$$

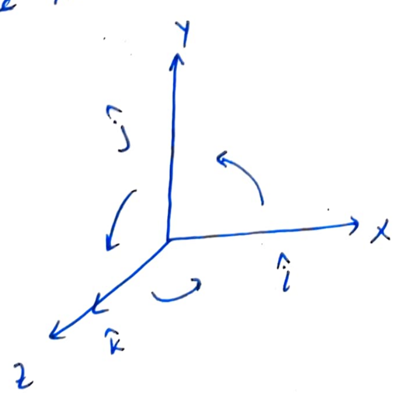
Unit vector:

$$\vec{A} = |\vec{A}|\,\hat{a}$$

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$ ← unit vector in the direction of $\vec{A}$

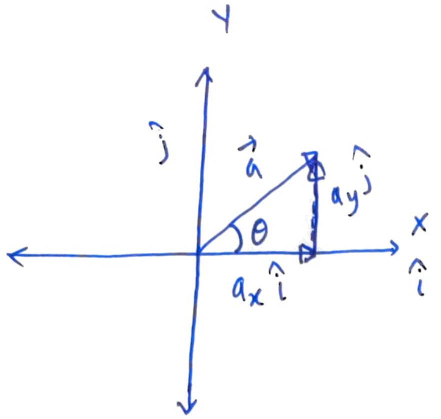
Unit vector → has magnitude exactly 1
→ has a particular direction
→ has no ~~b~~unit

In Cartesian co-ordinate unit vectors are $\hat{i}$, $\hat{j}$, $\hat{k}$ ~~respec~~ in the direction of x, y and z-axis respectively.



$\hat{i}$
$\hat{j}$
$\hat{k}$

$\hat{r}$
$\hat{\theta}$
$\hat{\phi}$



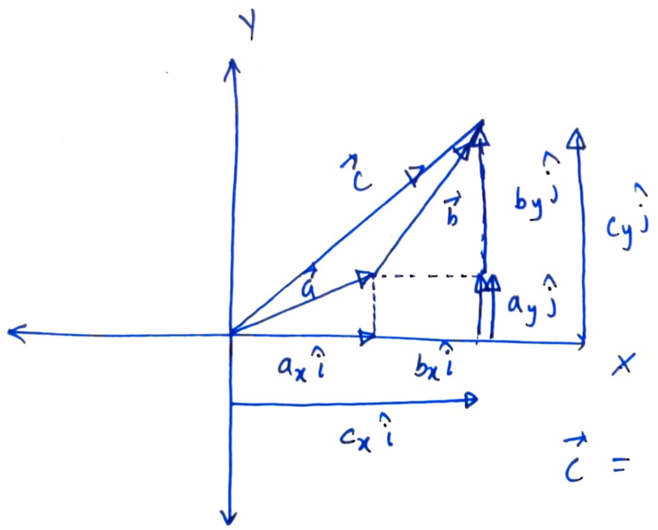
$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x\hat{i},\ a_y\hat{j}$ → vector components of $\vec{a}$

$a_x,\ a_y$ → scalar component of $\vec{a}$ or component of $\vec{a}$

$\boxed{|\vec{a}| = \sqrt{a_x^2 + a_y^2}}$, $\boxed{\tan\theta = \frac{a_y}{a_x}}$

In 3D - $\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$



$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{b} = b_x\hat{i} + b_y\hat{j}$

$\vec{c} = c_x\hat{i} + c_y\hat{j}$

$= a_x\hat{i} + b_x\hat{i} + a_y\hat{j} + b_y\hat{j}$

$= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$

In 3D $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

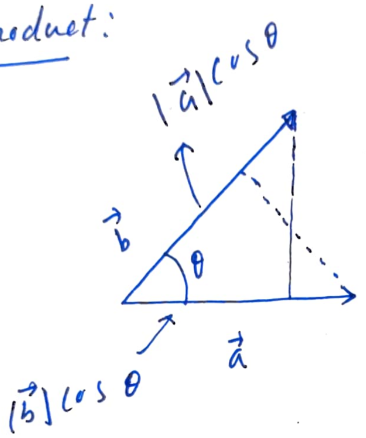
$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z + B_z)\hat{k}$

Multiplying vectors

(1) Multiplying a vector by a scalar

(2) Multiplying a vector by a vector
- → Scalar product / dot product
- → vector product / cross product

Scalar Product:



$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$= |\vec{a}|\,(|\vec{b}|\cos\theta)$

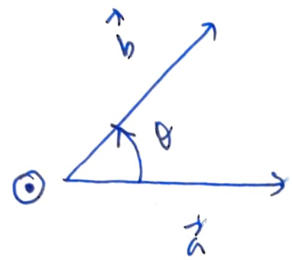
= (magnitude of vector $\vec{a}$) (Component of $\vec{b}$ along the direction of $\vec{a}$)

$= |\vec{b}|\,(|\vec{a}|\cos\theta)$

= (magnitude of vector $\vec{b}$) (component of $\vec{a}$ along the direction of $\vec{a}$)

so, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

Vector product:

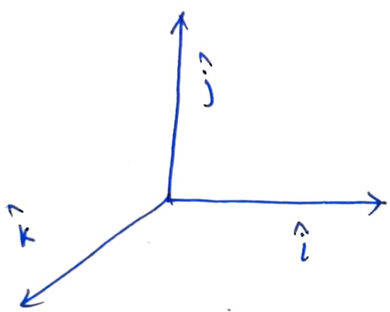


$$\vec{a} \times \vec{b} = \hat{n}\, |\vec{a}|\, |\vec{b}| \sin\theta$$

$$\vec{b} \times \vec{a} = -\hat{n}\, |\vec{a}|\, |\vec{b}| \sin\theta$$

$\odot \rightarrow$ Outward through the board

$\otimes \rightarrow$ Inward through the board



$$\hat{i} \cdot \hat{i} = |\hat{i}|\, |\hat{i}| \cos 0^\circ = 1$$

$$\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = |\hat{i}|\, |\hat{j}| \cos 90^\circ = 0$$

$$\hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$\hat{i} \times \hat{j} = \hat{k}\, |\hat{i}|\, |\hat{j}| \sin 90^\circ = \hat{k}$$

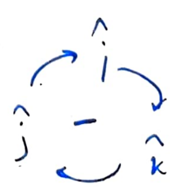
$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$= (a_x b_x)\, \hat{i} \cdot \hat{i} + (a_x b_y)\, \hat{i} \cdot \hat{j} + (a_x b_z)\, \hat{i} \cdot \hat{k}$$
$$+ (a_y b_x)\, \hat{j} \cdot \hat{i} + (a_y b_y)\, \hat{j} \cdot \hat{j} + (a_y b_z)\, \hat{j} \cdot \hat{k}$$
$$+ (a_z b_x)\, \hat{k} \cdot \hat{i} + (a_z b_y)\, \hat{k} \cdot \hat{j} + (a_z b_z)\, \hat{k} \cdot \hat{k}$$

$$\boxed{\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

$\vec{a} \cdot \vec{a} = |\vec{a}|\, |\vec{a}| \cos 0°$

$\Rightarrow |\vec{a}|^2 = a_x^2 + a_y^2 + a_z^2$

$$\boxed{|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

$\vec{a} \cdot \vec{b} = |\vec{a}|\, |\vec{b}| \cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\, |\vec{b}|}$$

$$\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$= (a_x b_x)(\hat{i} \times \hat{i}) + (a_x b_y)(\hat{i} \times \hat{j}) + (a_x b_z)(\hat{i} \times \hat{k})$$

+ +

+ +

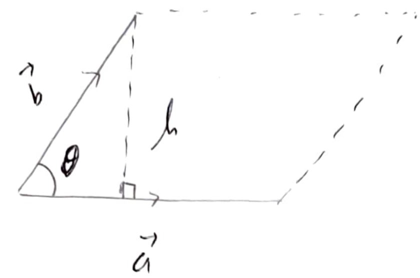
+

+

$$= (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & a_z \end{vmatrix}$$

Geometrical interpretation of vector product (14)

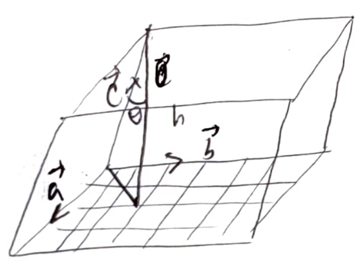


Area of parallelogram, $\square = |\vec{a}|\, h$

$= |\vec{a}|\, |\vec{b}| \sin\theta$ $\qquad \sin\theta = \dfrac{h}{|\vec{b}|}$

$= |\vec{a} \times \vec{b}|$

Area vector, $\vec{a} \times \vec{b}$



Volume of parallelopiped,

$V = |\vec{a} \times \vec{b}|\, h$

$\cos\theta = \dfrac{h}{|\vec{c}|}$ $\qquad$ ~~$\cos\theta = \dfrac{|\vec{c}|}{h} \Rightarrow |\vec{c}| = h\cos\theta$~~

$\Rightarrow h = |\vec{c}| \cos\theta$

$V = |\vec{a} \times \vec{b}|\, |\vec{c}| \cos\theta$

$= \vec{a} \times \vec{b} \cdot \vec{c}$

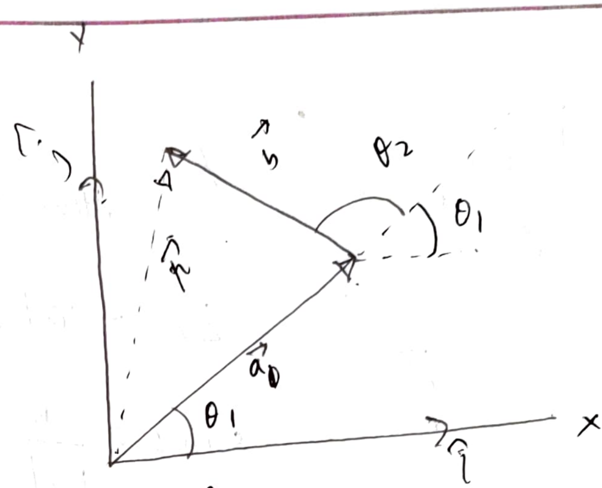
vector tripple product

H.W $\quad \vec{a} \times \vec{b} \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$

(21)

18 $|\vec{a}| = |\vec{b}| = 10.0\ m$

15 $\theta_1 = 30°, \quad \theta_2 = 105°,$



$$\vec{a} = |\vec{a}| \cos\theta_1 \ \hat{i} + |\vec{a}| \sin\theta_1 \ \hat{j}$$
$$= (10 \cos 30°)(m)\ \hat{i} + (10 \sin 30°)(m)\ \hat{j}$$
$$= \left(10 \times \frac{\sqrt{3}}{2}\right) m\ \hat{i} + \left(10 \times \frac{1}{2}\right)(m)\ \hat{j}$$
$$+ 5\ (m)\ \hat{j}$$
$$= 5\sqrt{3}\ (m)\ \hat{i}$$

$$\vec{b} = |\vec{b}| \cos(\theta_1 + \theta_2)\ \hat{i} + |\vec{b}| \sin(\theta_1 + \theta_2)\ \hat{j}$$
$$= 10 \cos(30° + 105°)\ (m)\ \hat{i} + 10 \sin(30° + 105°)(m)\ \hat{j}$$
$$= 10 \cos(90° + 45°)\ (m)\ \hat{i} + 10 \sin(90° + 45°)(m)\ \hat{j}$$
$$= 10\ (-\sin 45°)\ (m)\ \hat{i} + 10\ (\cos 45°)\ (m)\ \hat{j}$$
$$= -10 \times \frac{1}{\sqrt{2}}\ (m)\ \hat{i} + 10 \times \frac{1}{\sqrt{2}}\ (m)\ \hat{j}$$
$$= -5\sqrt{2}\ (m)\ \hat{i} + 5\sqrt{2}\ (m)\ \hat{j}$$

(22)

$\vec{r} = r_x \hat{i} + r_y \hat{j}$

$\vec{r} = \vec{a} + \vec{b}$

$= \left(5\sqrt{3}\,(m)\,\hat{i} + 5\,(m)\,\hat{j}\right) + \left(-5\sqrt{2}\,(m)\,\hat{i} + 5\sqrt{2}\,\hat{j}\,(m)\right)$

$= 5(\sqrt{3}-\sqrt{2})\,(m)\,\hat{i} + 5(1+\sqrt{2})\,(m)\,\hat{j}$

(a) x-component of $\vec{r}$,

$r_x = 5(\sqrt{3}-\sqrt{2})\,(m)$

(b) y-component of $r_y = 5(1+\sqrt{2})\,(m)$

(c) Magnitude of $|\vec{r}| = \sqrt{\left(5(\sqrt{3}-\sqrt{2})\right)^2 + \left(5(1+\sqrt{2})\right)^2}\,(m)$

(d) $\vec{r}\cdot\hat{i} = \cancel{5(\sqrt{3}-\sqrt{2})\,(m)\,\hat{i}}\ |\vec{r}|\cdot|\hat{i}|\cos\theta_x$

$$\cos\theta_x = \frac{\vec{r}\cdot\hat{i}}{|\vec{r}|}$$

$$\Rightarrow \cos\theta_x = \frac{5(\sqrt{3}-\sqrt{2})\,(m)}{\sqrt{\left(5(\sqrt{3}-\sqrt{2})\right)^2 + \left(5(1+\sqrt{2})\right)^2}\ m}$$

$$\Rightarrow \theta_x = \cos^{-1}\left(\quad\right)$$

H.W, $\vec{a}\times\vec{b}\cdot\vec{c} = \vec{c}\times\vec{a}\cdot\vec{b} = \vec{b}\times\vec{c}\cdot\vec{a} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$

If $\vec{a}, \vec{b}, \vec{c}$ are co-planar, then,

$$\boxed{\vec{a}\times\vec{b}\cdot\vec{c} = \vec{c}\times\vec{a}\cdot\vec{b} = \vec{b}\times\vec{c}\cdot\vec{a} = 0}$$

61 (a) $\vec{R} = \vec{a} - \vec{b} + \vec{c} = ?$

where $\vec{a} = 5\hat{i} + 4\hat{j} - 6\hat{k}$

$\vec{b} = -2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{c} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

(a) $\vec{R} = \vec{a} - \vec{b} + \vec{c}$

$= (5\hat{i} + 4\hat{j} - 6\hat{k}) - (-2\hat{i} + 2\hat{j} + 3\hat{k}) + (4\hat{i} + 3\hat{j} + 2\hat{k})$

$= (5+2+4)\hat{i} + (4-2+3)\hat{j} + (-6-3+2)\hat{k}$

$= \boxed{11\hat{i} + 5\hat{j} - 7\hat{k}}$

(b) $\vec{R}\cdot\hat{k} = |\vec{R}||\hat{k}| \cos\theta_z$

$\Rightarrow \cos\theta_z = \dfrac{\vec{R}\cdot\hat{k}}{|\vec{R}|\,|\hat{k}|}$

$= \dfrac{(-7)}{\sqrt{(11)^2 + (5)^2 + (-7)^2}}$

$\Rightarrow \theta_z = \cos^{-1}(\quad) = \boxed{120^\circ}$

(c) $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$ (16)

$$\Rightarrow |\vec{a}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \vec{a}\cdot\hat{b}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{(-2)^2+(2)^2+(3)^2}}$$

$$= \frac{-2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{17}}$$

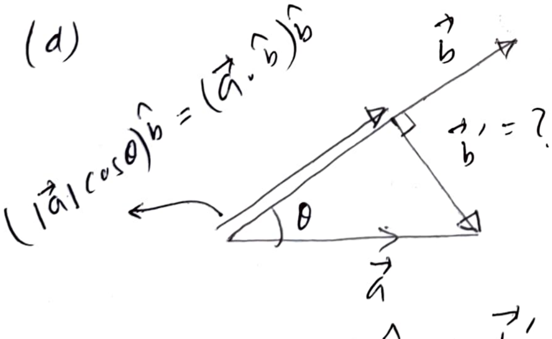
$$= -\frac{2}{\sqrt{17}}\hat{i} + \frac{2}{\sqrt{17}}\hat{j} + \frac{3}{\sqrt{17}}\hat{k}$$

$$\vec{a}\cdot\hat{b} = (5\hat{i}+4\hat{j}-6\hat{k})\cdot\left(-\frac{2}{\sqrt{17}}\hat{i}+\frac{2}{\sqrt{17}}\hat{j}+\frac{3}{\sqrt{17}}\hat{k}\right)$$

$$= \frac{-10}{\sqrt{17}} + \frac{8}{\sqrt{17}} - \frac{8}{\sqrt{17}}$$

$$= \boxed{-4.85}$$

(d)



$$\vec{a} = (\vec{a}\cdot\hat{b})\hat{b} + \vec{b}'$$

$$\Rightarrow \vec{b}' = \vec{a} - (\vec{a}\cdot\hat{b})\hat{b}$$

$$= (5\hat{i}+4\hat{j}-6\hat{k}) - (-4.85)\left(-\frac{2}{\sqrt{17}}\hat{i}+\frac{2}{\sqrt{17}}\hat{j}+\frac{3}{\sqrt{17}}\hat{k}\right)$$

$$= (5\hat{i}+4\hat{j}-6\hat{k}) + (-2.35\hat{i}+2.35\hat{j}+3.53\hat{k})$$

$\vec{b}' = (5-2.35)\hat{i} + (4+2.35)\hat{j} + (-6+3.53)\hat{k}$

$= 2.65\,\hat{i} + 6.35\,\hat{j} - 2.47\,\hat{k}$

$|\vec{b}'| = \sqrt{(2.65)^2 + (6.35)^2 + (-2.47)^2}$

$= \boxed{7.31}$

Chapter-3

SP : 3.02, 3.03, 3.04, 3.05

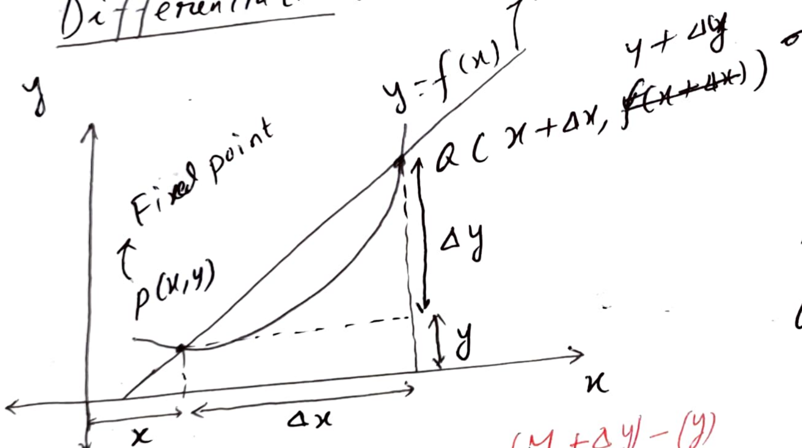
CP : 1, 2, 3, 4, 5

PR : $\boxed{9}$, $\boxed{15}$, 17, $\boxed{39}$, 36, $\boxed{37}$, 50, $\boxed{61}$, 63

Review of Calculus:

Differentiation:

secant line



$p(x,y) = p(x, f(x))$

$Q(x+\Delta x, y+\Delta x)$

$= Q(x+\Delta x, f(x+\Delta x))$

Slope, $m_{sec} = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{(y+\Delta y)-(y)}{x+\Delta x - x}$

$= \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$

$= \dfrac{\Delta f}{\Delta x}$

If $Q \to P$, then $\Delta x \to 0$

$m_{tangent} = \lim_{\Delta x \to 0} \dfrac{\Delta f}{\Delta x}$

$\Rightarrow \boxed{\dfrac{d}{dx} f(x) = \cancel{\dfrac{df}{}} \lim_{\Delta x \to 0} \dfrac{\Delta f}{dx}}$

(18)

$f(x) = x^2$

$f(x+\Delta x) = (x+\Delta x)^2$

$= x^2 + 2\Delta x\, x + (\Delta x)^2$

$\Delta f = f(x+\Delta x) - f(x)$

$= x^2 + 2\Delta x\, x + (\Delta x)^2 - x^2$

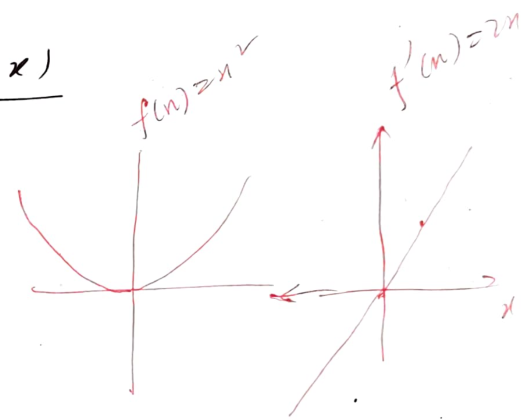
$= 2\Delta x\, x + (\Delta x)^2$

$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2\Delta x\, x + (\Delta x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}}$

$= \lim_{\Delta x \to 0} (2x + \Delta x)$



$\boxed{\frac{d}{dx}(x^2) = 2x}$

$\frac{d}{dx}(x^n) = n x^{n-1}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\sin x) = \cos x$

chain rule $\quad \frac{d}{dx}(\sin mx) = ?$

$\frac{d}{dx}(\sin mx) = \cos(mx)\, \frac{d}{dx}(mx)$

$= \boxed{m \cos(mx)}$

Fundamental Theorem of Calculus:

$$\frac{d}{dx} f(x) = g(x) \longleftrightarrow \int g(x)\,dx = f(x) + c$$

Indefinite Integral

$$\frac{d}{dx}(x^n) = n x^{n-1} \qquad \int n x^{n-1}\,dx = x^n + c$$

Let, $n-1 = n'$

$$\int (n'+1)\, x^{n'}\,dx = x^{n'+1}$$

$$\Rightarrow \int x^{n'}\,dx = \frac{x^{n'+1}}{n'+1}$$

$$\boxed{n' = n} \Rightarrow \boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$$

$n \neq -1$,

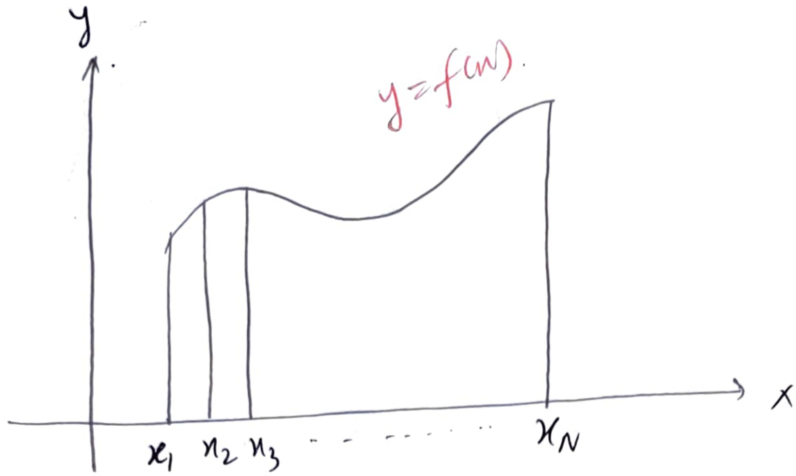
$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln x + c$$

$\frac{d}{dx}$

Definite Integral:

$$\int_a^b f(x)\,dx = [g(x)]_a^b$$

$$= g(b) - g(a)$$



$$\Delta x = x_2 - x_1 = x_3 - x_2 = \cdots = x_N - x_{N-1}$$

$$\text{Area} = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_{N-1})\Delta x.$$

$$= \sum_{i=1}^{N-1} f(x_i)\,\Delta x$$

If $N \to \infty$, $\Delta x \to dx$

$$\boxed{\text{Area} = \int_{x_1}^{x_N} f(x)\,dx}$$

* $\displaystyle\int_{-5}^{+5} |x|\,dx = ?$

BRAC University
Department of Mathematics & Natural Sciences
Principles of Physics-I (PHY 111)

Solution of Suggested Problems (vector) Fall, 2024

Formulas

Vector addition:

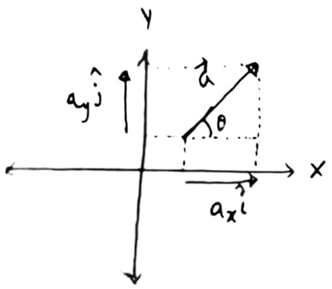
Component

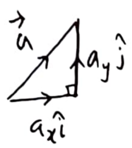
$\vec{c} = \vec{a} + \vec{b}$

Resultant

Unit vector:

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$





$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = |\vec{a}|\cos\theta$ = X-component of $\vec{a}$

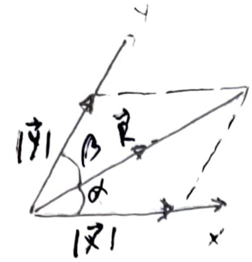
$a_y = |\vec{a}|\sin\theta$ = Y- component of $\vec{a}$

$a_x\hat{i}$ = vector component of $\vec{a}$ along the direction of +X-axis

$a_y\hat{j}$ = vector component of $\vec{a}$ along the direction of +Y-axis

θ = angle between $\vec{a}$ and +X-axis

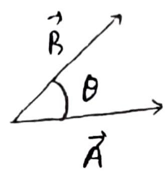
$\tan\theta = \frac{a_y}{a_x}$; magnitude of $\vec{a}$, $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$



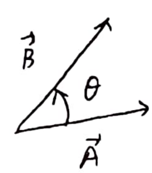
$$|\vec{X}| = \frac{|\vec{R}|\sin\beta}{\sin(\alpha+\beta)}$$

$$|\vec{Y}| = \frac{|\vec{R}|\sin\alpha}{\sin(\alpha+\beta)}$$

<u>Scalar product</u> $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$

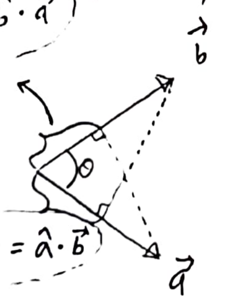


<u>Vector product</u>: $\vec{A}\times\vec{B} = \hat{\eta}\,|\vec{A}|\,|\vec{B}|\sin\theta$



$\hat{\eta} \rightarrow$ unit vector

$|\vec{a}|\cos\theta = \hat{b}\cdot\vec{a}$ $\longrightarrow$ component of $\vec{a}$ along the direction of $\vec{b}$

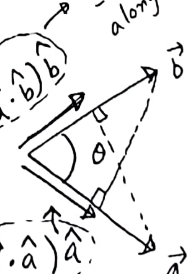


$|\vec{b}|\cos\theta = \hat{a}\cdot\vec{b}$

$\downarrow$

component of $\vec{b}$ along the direction of $\vec{a}$

$(\vec{a}\cdot\hat{b})\hat{b}$ $\rightarrow$ vector component of $\vec{a}$ along the direction of $\vec{b}$



$(\vec{b}\cdot\hat{a})\hat{a}$

$\downarrow$

vector component of $\vec{b}$ along the direction of $\vec{a}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

- $$\vec{a} + \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) + (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$
$$= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$$
$$\vec{a} - \vec{b} = (a_x - b_x)\hat{i} + (a_y - b_y)\hat{j} + (a_z - b_z)\hat{k}$$
$$\vec{b} - \vec{a} = (b_x - a_x)\hat{i} + (b_y - a_y)\hat{j} + (b_z - a_z)\hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

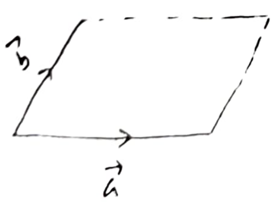
$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$\uparrow$
angle between $\vec{a}$ and $\vec{b}$

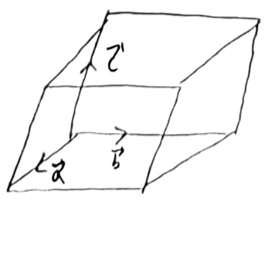
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\left.\begin{aligned} \theta_x &= \cos^{-1}\left(\frac{a_x}{|\vec{a}|}\right) \\ \theta_y &= \cos^{-1}\left(\frac{a_y}{|\vec{a}|}\right) \\ \theta_z &= \cos^{-1}\left(\frac{a_z}{|\vec{a}|}\right) \end{aligned}\right\}$$

$\theta_x, \theta_y, \theta_z \rightarrow$ angle between vector $\vec{a}$ and $+X, +Y, +Z$ axes respectively



$\longrightarrow$ Area of parallelogram $= |\vec{a} \times \vec{b}|$



- Volume of parallelopiped
$$= \vec{a} \cdot \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

(Problem No: 17)

1 Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30°, 195°, and 315°, respectively. What are (a) the magnitude and (b) the angle of the vector $\vec{a}+\vec{b}+\vec{c}$, and (c) the magnitude and (d) the angle of $\vec{a}-\vec{b}+\vec{c}$? (e) What are the magnitude and (f) angle of a fourth vector $\vec{d}$ such that $(\vec{a}+\vec{b})-(\vec{c}+\vec{d})=0$?

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 50\,m, \ \theta_1 = 30°, \ \theta_2 = 195°, \ \theta_3 = 315°$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = |\vec{a}|\cos\theta_1\,\hat{i} + |\vec{a}|\sin\theta_1\,\hat{j}$$

$$= (50)m(\cos 30°)\,\hat{i} + (50\,m)\sin(30°)\,\hat{j}$$

$$= (50\,m)\frac{\sqrt{3}}{2}\,\hat{i} + (50\,m)\frac{1}{2}\,\hat{j}$$

$$= (25\sqrt{3})\,m\,\hat{i} + (25\,m)\,\hat{j}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} = |\vec{b}|\cos\theta_2\,\hat{i} + |\vec{b}|\sin\theta_2\,\hat{j}$$

$$= (50\,m)\cos(195°)\,\hat{i} + (50\,m)\sin(195°)\,\hat{j}$$

$$= (-48.29\,m)\,\hat{i} + (-12.94\,m)\,\hat{j}$$

$$\vec{c} = c_x\hat{i} + c_y\hat{j} = |\vec{c}|\cos\theta_3\,\hat{i} + |\vec{c}|\sin\theta_3\,\hat{j}$$

$$= (50\,m)\cos(315°)\,\hat{i} + (50\,m)\sin(315°)\,\hat{j}$$

$$= (25\sqrt{2}\,m)\,\hat{i} + (-25\sqrt{2})\,m\,\hat{j}$$

(a) $$\vec{a}+\vec{b}+\vec{c} = (25\sqrt{3} - 48.29 + 25\sqrt{2})\,m\,\hat{i} + (25 - 12.94 - 25\sqrt{2})\,m\,\hat{j}$$

$$= \boxed{(30.4\,m)\,\hat{i} + (-23.3\,m)\,\hat{j}}$$

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{(30.4\,m)^2 + (-23.3\,m)^2} = \boxed{38.3\,m}$$

(b) $(\vec{a}+\vec{b}+\vec{c})\cdot\hat{i} = |\vec{a}+\vec{b}+\vec{c}||\hat{i}|\cos\theta_x$

$$\Rightarrow \cos\theta_x = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\hat{i}}{|\vec{a}+\vec{b}+\vec{c}|(1)} = \frac{30.4\text{ m}}{38.3\text{ m}}$$

$$\Rightarrow \theta_x = \cos^{-1}\left(\frac{30.4}{38.3}\right) = 37.5^\circ$$

(c) $\vec{a}-\vec{b}+\vec{c} = (25\sqrt{3}+48.29+25\sqrt{2})\text{m}\hat{i} + (25+12.94-\cancel{25.94}\ 25\sqrt{2})\text{m}\hat{j}$

$$= (126.95\text{ m})\hat{i} + (2.6\text{ m})\hat{j}$$

$$|\vec{a}-\vec{b}+\vec{c}| = \sqrt{(126.95)^2+(2.6)^2}\ \text{m}$$

$$= \boxed{126.98\text{ m}}$$

(d) $(\vec{a}-\vec{b}+\vec{c})\cdot\hat{i} = |\vec{a}-\vec{b}+\vec{c}||\hat{i}|\cos\theta_x'$

$$\Rightarrow \cos\theta_x' = \frac{126.95\text{ m}}{126.98\text{ m}}$$

$$\therefore \theta_x' = \cos^{-1}\left(\frac{126.95}{126.98}\right) = \boxed{1.25^\circ}$$

(e) $\vec{a}+\vec{b}-(\vec{c}+\vec{d}) = 0$

$$\Rightarrow \vec{c}+\vec{d} = \vec{a}+\vec{b}$$

$$\Rightarrow \vec{d} = \vec{a}+\vec{b}-\vec{c}$$

$$= (25\sqrt{3}+(-48.29)-25\sqrt{2})\text{ m}\hat{i} + (25+(-12.94)-(-25\sqrt{2}))\text{ m}\hat{j}$$

$$= (-40.3)\text{ m}\hat{i} + (47.4)\text{ m}\hat{j}$$

$|\vec{d}| = \sqrt{(-40.3\text{ m})^2+(47.4\text{ m})^2} = 62.28\text{ m}$; (f) $\theta_x'' = \cos^{-1}\left(\frac{-40.3}{62.28}\right)$

$= \boxed{[illegible]}$

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(Problem - 35)

2. Two vectors $\vec{r}$ and $\vec{s}$ lie in the xy plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0°, respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r} \cdot \vec{s}$ and (b) $\vec{r} \times \vec{s}$?

$$|\vec{r}| = 4.5 , \quad \theta' = 320^\circ$$

$$|\vec{s}| = 7.3 , \quad \theta'' = 85^\circ$$

$$\vec{r} = |\vec{r}| \cos\theta' \,\hat{i} + |\vec{r}| \sin\theta' \,\hat{j}$$

$$= (4.5) \cos(320^\circ) \,\hat{i} + (4.5) \sin(320^\circ) \,\hat{j}$$

$$= 3.45 \,\hat{i} - 2.89 \,\hat{j}$$

$$\vec{s} = |\vec{s}| \cos\theta'' \,\hat{i} + |\vec{s}| \sin\theta''$$

$$= (7.3) \cos 85^\circ \,\hat{i} + (7.3) \sin(85^\circ)$$

$$= 0.64 \,\hat{i} + 7.27 \,\hat{j}$$

(a) $$\vec{r} \cdot \vec{s} = (3.45)(0.64) + (-2.89)(7.27)$$

$$= 2.01 + (-21.01)$$

$$= \boxed{-19}$$

(b) $$\vec{r} \times \vec{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.45 & -2.89 & 0 \\ 0.64 & 7.27 & 0 \end{vmatrix}$$

$$= \hat{i}\,(-2.89 \times 0 - 7.27 \times 0) \cancel{\hat{j}\,(3.45 \times 0)}$$

$$- \hat{j}\,(3.45 \times 0 - 0.64 \times 0)$$

$$+ \hat{k}\,(3.45 \times 7.27 + (2.89) \times (.64))$$

[redacted]

$$\vec{r} \times \vec{s} = \hat{k}\, 26.94$$

$$|\vec{r} \times \vec{s}| = |\hat{k}|\, 26.94 = \boxed{26.94}$$

(Problem 37)

Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a}\cdot(\vec{b}\times\vec{c})$, (b) $\vec{a}\cdot(\vec{b}+\vec{c})$, and (c) $\vec{a}\times(\vec{b}+\vec{c})$.

$\vec{a} = 3\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{b} = -\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{c} = 2\hat{i} + 2\hat{j} + \hat{k}$

(a) $\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 3 & 3 & -2 \\ -1 & -4 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$\cancel{= 3(-4-4) - \ldots - 2(-2+8)}$

$\cancel{= 24 - 18 - 12}$

$= 3(-4-4) - 3(-1-4) - 2(-2+8)$

$= -18 + 15 - 12 = \boxed{-21}$

(b) $\vec{b} + \vec{c} = (-1+2)\hat{i} + (-4+2)\hat{j} + (2+1)\hat{k}$

$= \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a}\cdot(\vec{b}+\vec{c}) = (3)(1) + (3)(-2) + (-2)(3)$

$= 3 - 6 - 6 = \boxed{-9}$

(c) $\vec{a}\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix}$

$= \hat{i}(9-4) - \hat{j}(9+2) + \hat{k}(-6-3)$

$= \boxed{5\hat{i} - 11\hat{j} - 9\hat{k}}$

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(Problem 63)

40 Here are three vectors in meters: $\vec{d_1} = -3.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$, $\vec{d_2} = -2.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, $\vec{d_3} = 2.0\hat{i} + 3.0\hat{j} + 1.0\hat{k}$ What results from: (a) $\vec{d_1} \cdot (\vec{d_2} + \vec{d_3})$ (b) $\vec{d_1} \cdot (\vec{d_2} \times \vec{d_3})$ (c) $\vec{d_1} \times (\vec{d_2} + \vec{d_3})$?

$\vec{d_1} = (-3\hat{i} + 3\hat{j} + 2\hat{k})\,m$

$\vec{d_2} = (-2\hat{i} - 4\hat{j} + 2\hat{k})\,m$

$\vec{d_3} = (2\hat{i} + 3\hat{j} + \hat{k})\,m$

(a) $$(\vec{d_2} + \vec{d_3}) = [(-2+2)\hat{i} + (-4+3)\hat{j} + (2+1)\hat{k}]\,m$$

$$= [-\hat{j} + 3\hat{k}]\,m$$

$$\vec{d_1} \cdot (\vec{d_2} + \vec{d_3}) = [(-3)(0) + (+3)(-1) + (2)(3)]\,m^2$$

$$= m^2[-3+6] = \cancel{8}\ \boxed{3\,m^2}$$

(b) $$\vec{d_1} \cdot \vec{d_2} \times \vec{d_3} = \begin{vmatrix} -3 & 3 & 2 \\ -2 & -4 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= (-3)(-4-6) - 3(-2-4)$$
$$+ 2(-6+8)$$

$$= (-3)(-10) - 3(-6) + 2(2)$$

$$= 30 + 18 + 4 = \boxed{52\,m^3}$$

(c) $$\vec{d_1} \times (\vec{d_1} + \vec{d_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 2 \\ 0 & -1 & 3 \end{vmatrix}$$

$$\vec{d}_1 \times (\vec{d}_2 \times \vec{d}_3) = \hat{i}\,(9+2) - \hat{j}\,(-9-0) + \hat{k}\,(3-0)$$

$$= \boxed{(11\,\hat{i} + 9\,\hat{j} + 3\,\hat{k})\ m^2}$$

Solve Problem no : $\boxed{9,\ 30,\ 36,\ 50}$