CSE230: Discrete Mathematics

Practice Sheet 2: **Proofs: Direct, Indirect, Contradiction, Induction**

Q1	Prove that, For all integers n, if $n^3 + 5$ is odd then n is even.
Q2	Prove the following: Suppose a, b $\in$ Z. If a + b ≥ 19, then a ≥ 10 or b ≥ 10.
Q3	Prove the following: Suppose a, b, and c are positive real numbers. If ab = c then a ≤ √c or b ≤ √c.
Q4	Prove the following: The sum of a rational number and an irrational number is irrational.
Q5	Prove the following: Every nonzero rational number can be expressed as a product of two irrational numbers.
Q6	Prove the following: Suppose a,b,c $\in$ Z. If $a^2 + b^2 = c^2$, then a or b is even.
Q7	Prove the following: If a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.
Q8	Prove by mathematical induction that if n is a positive integer then $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
Q9	Prove by mathematical induction that if n is a positive integer then $(1 \times 2) + (2 \times 3) + (3 \times 4) + \ldots + (n \times (n + 1)) = \frac{n(n+1)(n+2)}{3}$
Q10	Prove by mathematical induction that $\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n + 1)(2n + 1)$ for all n $\in$ N.
Q11	Prove by mathematical induction that if n is a positive integer then $1 + 5 + 9 + 13 + \ldots + (4n - 3) = \frac{n(4n-2)}{2}$

Q12	Prove by mathematical induction that, $\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5)$ where n > 0 and n ∈ N.
Q13	Prove by mathematical induction that, $\sum_{r=1}^{n} (r-1)(r+1) = \frac{1}{6}n(n-1)(2n+5)$ where n > 0 and n ∈ N.
Q14	Prove by mathematical induction that, $\sum_{r=2}^{n} r^2(r-1) = \frac{1}{12}n(n-1)(n+1)(3n+2)$ where n > 1 and n ∈ N.
Q15	Prove by mathematical induction that, $\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ where n > 0 and n ∈ N.
Q16	Prove by mathematical induction that, $\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$ where n > 0 and n ∈ N.
Q17	Prove by mathematical induction that, $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ where n > 0 and n ∈ N.
Q18	Prove by mathematical induction that, $\sum_{r=1}^{n} (3^{r-1}) = \frac{3^n-1}{2}$ where n > 0 and n ∈ N.
Q19	Prove by mathematical induction that, $\sum_{r=1}^{n} r.2^r = 2 + (n-1)2^{n+1}$ where n > 0 and n ∈ N.

Q20	Prove by mathematical induction that, $\sum_{r=1}^{n} (r + 1).2^{r} = n.2^{n}$ where n > 0 and n ∈ N.
Q21	Prove by mathematical induction that, $-1 + 1 + 5 + 11 + \ldots\ldots\ldots\ldots + \{n(n-1) - 1\} = \frac{1}{3}n(n+2)(n-2)$ where n > 0 and n ∈ N.
Q22	Prove by mathematical induction that, $4^{n} + 6n + 8$ is divisible by 18 where n > 0 and n ∈ N.
Q23	Prove by mathematical induction that, $5^{2n} + 3n - 1$ is divisible by 9 where n > 0 and n ∈ N.
Q24	Prove by mathematical induction that, $7^{n} + 5$ is divisible by 6 where n ∈ N.
Q25	Prove by mathematical induction that, $7^{2n-1} + 1$ is divisible by 8 where n ∈ N.
Q26	Prove by mathematical induction that, $4^{n} + 6n - 1$ is divisible by 9 where n ∈ N.
Q27	Prove by mathematical induction that, $5^{n} + 8n + 3$ is divisible by 4 where n ∈ N.
Q28	Prove by mathematical induction that, $3^{4n} + 2^{4n+2}$ is divisible by 5 where n ∈ N.
Q29	Prove by mathematical induction that, $9^{n} - 5^{n}$ is divisible by 4 where n ∈ N.

Q30	Prove by mathematical induction that, $(4n + 3)\times5^{n} - 3$ is divisible by 16 where n ∈ N.
Q31	Prove by mathematical induction that, the sum of the cubes of any three consecutive positive integers is always divisible by 9 .
Q32	Prove by mathematical induction that, $15^{n} - 8^{n-2}$ is divisible by 7 where n>1 and n ∈ N.
Q33	Prove by mathematical induction that, $(2n + 1)7^{n} + 11$ is divisible by 4 where n>1 and n ∈ N.
Q34	Prove by mathematical induction that, $24 \times 2^{4n} + 3^{4n}$ is divisible by 5 where n>1 and n ∈ N.
Q35	Prove by mathematical induction that, $4\times7^{n} + 3\times5^{n} + 5$ is divisible by 12 where n ∈ N.
Q36	Prove by mathematical induction that, $(2n + 1)7^{n} - 1$ is divisible by 4 where n>1 and n ∈ N.
Q37	Prove by mathematical induction that, $4^{n+1} + 5^{2n-1}$ is divisible by 21 where n ∈ N.
Q38	Prove by mathematical induction that, $2^{n} + 6^{n}$ is divisible by 8 where n ∈ N.
Q39	Prove by mathematical induction that, $5^{n-1} + 11^{n}$ is divisible by 6 where n is a positive integer.

Q40	Prove by mathematical induction that, $5^{n+1} - 4n - 5$ is divisible by 16 where n $\in$ N.