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section-26

1. Bisection method:

$f(x) = e^{\sin(x)} - x$

let, $a = 2$, $b = 3$, and the root is between a and b

error $= |b - a|$

$f(2) = 0.4826 = +ve$

$f(3) = -1.8484 = -ve$

$c = \frac{2+3}{2} = 2.5$

$f(c) = -0.6807 = -ve$, error $= 1$

So, we set $b := c = 2.5$

$f(2.5) = -0.6807$

$f(2) = 0.4826$

$c = 2.25$

$f(c) = -0.0727 = -ve$, error $= 0.5$

So, we set $b := c = 2.25$

$f(2) = 0.4826$

$f(2.25) = -0.0727$

$c = \frac{2+2.25}{2} = 2.125$, error $= 0.25$

$f(c) = 0.2154 = +ve$

so, we set $a := c = 2.125$

$f(2.125) = 0.2154$

$f(2.25) = -0.0727$

$$c = \frac{2.125 + 2.25}{2} = 2.1875$$

, error = 0.125

$f(c) = 0.0735$

So, we set $a := c = 2.1875$

$f(2.1875) = 0.0735$

$f(2.25) = -0.0727$

$$c = \frac{2.25 + 2.1875}{2}$$

$$= 2.21875$$

$f(c) = 0.0008$, error = 0.0625

So, for next term, $a := c = 2.21875$

~~So, tolerance / absolute error $= b - a$~~
~~$= 0.0625$~~

$f(2.21875) = 0.0008$

$f(2.25) = -0.0727$

$c = \frac{a+b}{2} = 2.234375$, $f(c) = -0.035836$

So, error $= |b - a|$

$= 0.03125$

1. Newton-Raphson Method

$$f(x) = e^{\sin(x)} - x$$

$$f'(x) = e^{\sin(x)} \cdot \cos(x) - 1$$

Let, initial guess $x_0 = 3$. ~~error~~

So, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 + \frac{-1.84844}{-2.14004}$

$= 2.13626$, error $= |x_1 - x_0|$
$= 0.86374$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.13626 - \frac{0.19018}{-.24652}$

$= 2.22092$, error $=$ 0.08466

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.22092 - \frac{-0.00423}{-2.34172}$

$= 2.21911$, error $= 0.00181$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.21911 - \frac{-1.56141 \times 10^{-6}}{-2.33999}$

$= 2.21911$

error $= 0$ (not exactly 0)

$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$

~~= 2.21911 - 4.44089 × 10~~

$$= 2.21911 - \frac{-2.14051 \times 10^{-13}}{-2.33999}$$

$$= 2.21911$$

So, error, $(2.21911 - 2.219911) = 0$ (not exactly 0)

from the errors and considering other terms, we can say that newton raphson method converges faster as after 5 iteration, ~~bisection method is giving~~ it is giving smaller error than bisection method

2.

x	0	1	2	3	4	5	6
$f(x)$	3.65	3.98	4.43	4.04	3.88	3.71	3.59

here, $h=1$, $x=3$, accuracy $O(h^2)$

for forward ~~difference~~ Derivative:

$$f'(x) = \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h} + O(h^2)$$

$$= \frac{4f(4) - f(5) - 3f(3)}{2}$$

$$= -0.155$$

for backward Derivative:

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h} + O(h^2)$$

$$= \frac{3f(3) - 4f(2) + f(1)}{2}$$

$$= -0.81$$

for central Derivative:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

$$= \frac{f(4) - f(2)}{2}$$

$$= -0.275$$

3.

x	0	1	2	3	4	5	6
$f(x)$	3.65	3.98	4.43	4.04	3.88	3.71	3.59

here, $n = 6$

$$h = x_6 - x_5 = x_5 - x_4 = x_4 - x_3 = x_3 - x_2 = x_2 - x_1 = x_1 - x_0 = 1$$

We know,

$$\text{Area} = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

$$= \frac{1}{2} \times \left\{3.65 + 2 \times (3.98 + 4.43 + 4.04 + 3.88 + 3.71) + 3.59\right\}$$

$$= \frac{1}{2} \times \left\{3.65 + 40.08 + 3.59\right\}$$

$$= 23.66$$