

2. x	0	1	2	3	4	5	6
$f(x)$	3.85	3.98	4.43	4.04	3.88	3.71	3.50

here, $h=1$, $n=3$, accuracy $O(h^2)$

for forward ~~difference~~ Derivative:

$$\begin{aligned}
 f'(x) &= \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h} + O(h^2) \\
 &= \frac{4f(4) - f(5) - 3f(3)}{2} \\
 &= -0.155
 \end{aligned}$$

for backward Derivative:

$$\begin{aligned}
 f'(x) &= \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h} + O(h^2) \\
 &= \frac{3f(3) - 4f(2) + f(1)}{2} \\
 &= -0.81
 \end{aligned}$$

for central Derivative:

$$\begin{aligned}
 f'(x) &= \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \\
 &= \frac{f(4) - f(2)}{2} \\
 &= -0.275
 \end{aligned}$$

3.

x	0	1	2	3	4	5	6
$f(x)$	3.65	3.98	4.43	4.04	3.88	3.71	3.59

here, $n=6$

$$h = x_6 - x_5 = x_5 - x_4 = x_4 - x_3 = x_3 - x_2 = x_2 - x_1 = x_1 - x_0 = 1$$

We know,

$$\text{Area} = \frac{h}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

$$= \frac{1}{2} \times \left\{ 3.65 + 2 \times (3.98 + 4.43 + 4.04 + 3.88 + 3.71) + 3.59 \right\}$$

$$= \frac{1}{2} \times \{ 3.65 + 40.08 + 3.59 \}$$

$$= 23.56$$

1. Bisection method

$$f(x) = e^{\sin(x)} - x$$

let, $a = 2$, $b = 3$, and the root is between a and b

$$f(2) = 0.4826 = +ve$$

$$f(3) = -1.8484 = -ve$$

$$c = \frac{2+3}{2} = 2.5$$

$$f(c) = -0.6807 = -ve$$

So, we set $b := c = 2.5$

$$f(2.5) = -0.6807$$

$$f(2) = 0.4826$$

$$c = 2.25$$

$$f(c) = -0.0727 = -ve$$

So, we set $b := c = 2.25$

$$f(2) = 0.4826$$

$$f(2.25) = -0.0727$$

$$c = \frac{2+2.25}{2} = 2.125$$

$$f(c) = 0.2154 = +ve$$

So, we set $a := c = 2.125$

$$f(2.125) = \cancel{2.125} \cdot 0.2154$$

$$f(2.25) = -0.0727$$

$$c = \frac{2.125 + 2.25}{2} = 2.1875$$

$$f(c) = 0.0735$$

$$\text{So, we set } a := c = 2.1875$$

$$f(2.1875) = \cancel{2.1875} \cdot 0.0735$$

$$f(2.25) = -0.0727$$

$$c = \frac{2.25 + 2.1875}{2}$$

$$= \cancel{2.21875} \cdot 2.21875$$

$$f(c) = 0.0008$$

$$\text{So, for next term, } a := c = 2.21875$$

$$\text{So, tolerance / absolute error} = b - a$$

$$= \cancel{0.0625}$$

$$f(2.21875) = 0.0008$$

$$f(2.25) = -0.0727$$

$$c = \frac{a+b}{2} = 2.234375$$

$$\text{So, error} = |b - a|$$

$$= 0.03125$$

1. Newton-Raphson Method

$$f(u) = e^{\sin(u)} - u$$

$$f'(u) = e^{\sin(u)} \cos(u) - 1$$

Let, initial guess $u_0 = 3$

$$\text{So, } u_1 = u_0 - \frac{f(u_0)}{f'(u_0)}$$

$$= 3 + \frac{-1.84844}{-2.14004}$$

$$= 2.13626$$

$$u_2 = u_1 - \frac{f(u_1)}{f'(u_1)}$$

$$= 2.13626 - \frac{0.19018}{-2.24652}$$

$$= 2.22092$$

$$u_3 = u_2 - \frac{f(u_2)}{f'(u_2)}$$

$$= 2.22092 - \frac{-0.00423}{-2.34172}$$

$$= 2.21911$$

$$u_4 = u_3 - \frac{f(u_3)}{f'(u_3)}$$

$$= 2.21911 - \frac{-1.56141 \times 10^{-6}}{-2.33999}$$

$$= 2.21911$$

$$u_5 = u_4 - \frac{f(u_4)}{f'(u_4)}$$

$$= 2.21911 - \frac{4.44089 \times 10^{-16}}{-2.33999}$$

$$= 2.21911 - \frac{-2.14051 \times 10^{-13}}{-2.33999}$$

$$= 2.21911$$

So, error, $(2.21911 - 2.21911) = 0$

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