CSE230: Discrete Mathematics
Practice Sheet 1: **Propositional Logic Basics**

Q1	Let p, q and r be the propositions "The student has submitted the assignment", "The professor has graded the assignment", "The student receives a passing grade" respectively. Express each of these compound propositions as an English sentence. a. $p \wedge \neg q$ b. $p \vee r$ c. $p \rightarrow q \wedge r$ d. $p \leftrightarrow q$ e. $\neg r \wedge (\neg p \vee \neg q)$
Q2	Let p, q, r and s be the propositions p: The student has submitted the research paper. q: The student has attended all workshops. r: The student has participated in group projects. s: The student will receive a degree with honors. Express each of these compound propositions as an English sentence. a. $p \vee (q \wedge \neg s)$ b. $\neg p \vee (q \wedge r)$ c. $(p \wedge r) \rightarrow s$ d. $(p \wedge q) \leftrightarrow r$ e. $(p \rightarrow q) \wedge (r \rightarrow s)$ f. $\neg(p \wedge s) \vee (q \wedge r)$

Q3	Let p and q be the propositions:
	     p: The weather is sunny.
	     q: We will go to the beach.
	
	 Write these propositions using p and q and logical connectives (including negations and exclusive OR):
	
	 a) It is sunny and we go to the beach.
	 b) It is sunny but we do not go to the beach.
	 c) It is not sunny and we do not go to the beach.
	 d) Either it is sunny, or we go to the beach (or both).
	 e) If it is sunny, then we go to the beach.
	 f) Either it is sunny or we go to the beach, but not both, and if it is sunny, then we do not go to the beach
	 g) That it is sunny is necessary and sufficient for going to the beach.

Q4	Let p, q, and r be the propositions:
	
	     p: The campground is open for visitors.
	
	     q: Campfires are allowed in the campground.
	
	     r: The river nearby is safe for swimming.

Write these propositions using p, q, and r and logical connectives (including negations).

a) The campground is open for visitors, but campfires are not allowed.
b) Campfires are allowed, and the river is safe for swimming, but the campground is not open for visitors.
c) If the campground is open, then campfires are allowed if and only if the river is safe for swimming.
d) Campfires are not allowed, but the campground is open and the river is safe for swimming.
e) For campfires to be allowed, it is necessary and sufficient that the campground be open and that the river is not safe for swimming.
f) Campfires are not allowed whenever the campground is open and the river is safe for swimming.

Q5	Let p and q be the propositions:
	
	    p: The project is complete.
	
	    q: The team has been informed.
	
	Express each of these compound propositions as an **English sentence.**
	
	a) $\neg p$
	b) $p \lor q$
	c) $\neg p \land q$
	d) $q \to p$
	e) $\neg q \to \neg p$
	f) $\neg p \to \neg q$
	g) $p \leftrightarrow q$
	h) $\neg q \lor (\neg p \land q)$

Q6	Let p, q, and r be the propositions:
	
	    p: You solve the practice sheet.
	
	    q: You pass the test.
	
	    r: You get a good grade.
	
	Express each of these compound propositions as an **English sentence.**
	
	a) $\neg q \leftrightarrow r$
	b) $q \to \neg r$
	c) $p \lor q \lor r$
	d) $(p \to \neg r) \lor (q \to \neg r)$
	e) $(p \land q) \lor (\neg q \land r)$

	f) $(q \wedge r) \leftrightarrow (p \oplus \neg r)$
Q7	Determine the **converse, contrapositive, and inverse** of each of these conditional statements (Apply De Morgan's Laws for conjunction and disjunction where necessary): a) If it rains tomorrow, the outdoor concert will be canceled. b) An angle is a right angle only if it measures 90 degrees. c) A polygon is a triangle only if it has three sides. d) You are eligible for the scholarship if your CGPA is at least 3.7 and you have completed 30 credits. e) A figure is a square only if it has four equal sides and four right angles. f) The machine will shut down if the temperature exceeds 100°C and the system pressure drops below the minimum threshold.
Q8	How many **rows** appear in a truth table for each of these compound propositions? a) $p \to \neg p(p \vee \neg r) \wedge (q \vee \neg s)$ b) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$ c) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$
Q9	Construct **truth tables** for each of the following compound propositions: a) $p \wedge \neg p$ b) $p \vee \neg p$ c) $(p \vee \neg q) \to q$ d) $(p \vee q) \wedge r$ e) $(p \wedge q) \vee r$ f) $(p \to q) \leftrightarrow (\neg q \to \neg p)$ g) $(p \vee (q \wedge r)) \to (r \vee p)$ h) $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ i) $(p \to (q \vee r)) \wedge (q \to r)$ j) $((p \vee q) \wedge (r \vee s)) \to (p \wedge s)$
Q10	Show that each of the following compound propositions is a **tautology** using the truth table. a) $p \vee \neg p$ b) $(p \to q) \wedge (q \to r) \to (p \to r)$ c) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$ d) $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ e) $(p \to q) \to ((\neg q \to p) \vee r)$ f) $\neg(p \wedge \neg p) \vee (q \wedge r)$

Q11	Show that each of these pairs of propositions are **logically equivalent** using truth tables. a) $p \rightarrow q \ and \ \neg q \rightarrow \neg p$ b) $p \leftrightarrow q \ and \ \neg p \leftrightarrow \neg q$ c) $(p \wedge q) \vee (\neg p \wedge \neg q) \ and \ p \leftrightarrow q$ d) $p \rightarrow (q \wedge r) \ and \ (p \rightarrow q) \wedge (p \rightarrow r)$ e) $\neg(p \leftrightarrow q) \ and \ p \leftrightarrow \neg q$ f) $p \wedge (q \vee r) \ and \ (p \wedge q) \vee (p \wedge r)$ g) $\neg(p \vee q) \ and \ \neg p \wedge \neg q$ h) $p \vee (q \wedge r) \ and \ (p \vee q) \wedge (p \vee r)$ i) $p \rightarrow (q \rightarrow r) \ and \ (p \wedge q) \rightarrow r$ j) $(p \vee q) \rightarrow r \ and \ \neg r \rightarrow (\neg p \wedge \neg q)$
Q12	Show that, "In the Olympic Games, either the USA will win the gold medal or both Canada and Australia will win medals" and "The USA will win a medal or Canada will win a medal, and the USA will win a medal or Australia will win a medal" are **logically equivalent** using truth tables.
Q13	Let D(x) denote "x donates to charity regularly," where the domain consists of **all people** in a city. Express each of these statements in **English**: a) $\exists x \, D(x)$ b) $\forall x \, D(x)$ c) $\exists x \, \neg D(x)$ d) $\forall x \, \neg D(x)$
Q14	Translate these statements into English, where T(x) means "x is a teacher" and R(x) means "x reads every day." The domain consists of all individuals in a neighborhood. a) $\forall x \, (T(x) \rightarrow R(x))$ b) $\exists x \, (T(x) \wedge R(x))$ c) $\forall x \, (T(x) \wedge R(x))$ d) $\exists x \, (T(x) \rightarrow R(x))$
Q15	Let E(x) represent "x is enrolled in an art class," B(x) means "x is in a beginner's course," G(x) means "x has a good grade," and S(x) represent "x likes studying." The domain consists of some students in an art class. Translate each of these statements into English. a) $\forall x \, (E(x) \rightarrow G(x))$ b) $\exists x \, (B(x) \wedge S(x))$

	c) $\neg\forall x\,(B(x)\rightarrow(G(x)\vee S(x)))$
Q16	Translate each of these statements **into logical expressions** using predicates, quantifiers, and logical connectives. Let F(x) denote "x is a family member," and K(x) denote "x is kind." a) At least one family member is kind. b) All of your family members are kind. c) Not every family member is kind. d) Everyone is either a family member or is kind. e) Not everyone is a family member or there is someone who is not kind.
Q17	Let L(x) represent "x knows how to play an instrument," and let M(x) represent "x is a member of a band." Express each of these statements in terms of L(x), M(x), quantifiers, and logical connectives. The domain for quantifiers consists of **all students at a school.** a) There is a student at your school who knows how to play an instrument and is a member of a band. b) There is a student at your school who knows how to play an instrument but is not a member of a band. c) Every student at your school either knows how to play an instrument or is a member of a band. d) No student at your school knows how to play an instrument or is a member of a band.
Q18	Let P(x), Q(x), and R(x) be the statements "x is a scientist," "x is curious," and "x likes to experiment," respectively. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), and R(x), where the domain consists of all people. a) All scientists are curious. b) All curious people like to experiment. c) No scientists like to experiment.

Q19	In a university, we define the following predicates:
	
	    S(x): x is a student.
	
	    P(x): x is a professor.
	
	    C(y): y is a course.
	
	    T(x,y): x takes course y.
	
	    E(x,y): x teaches course y.
	
	Express each of these statements using quantifiers, logical connectives.
	
	a) Every student takes at least one course.

	b) For every professor, there exists a course that they teach. c) There exists a student who takes every course. d) For every course, there exists a student who is enrolled in that course. e) Every professor teaches at least one student in their course. f) For every student, there exists a professor who teaches at least one course they are taking.
Q20	State the converse, contrapositive, and inverse of each of these conditional statements: a. The ground will be wet unless it is not raining. b. Having fuel is necessary for running the car. c. Being sunny is sufficient for the picnic to be enjoyable. d. I wake up whenever the alarm rings. e. The streets get wet when it rains.

Q21	Let p and q be the propositions:
	
	    p: The security system is activated.
	
	    q: The alarm will sound.
	
	    r: There is a breach.
	
	Write these propositions using p, q, and logical connectives (including negations).
	
	a) The security system is activated, but the alarm does not sound.
	b) The alarm sounds, and the security system is not activated.
	c) If the security system is activated, then the alarm will sound if and only if there is a breach.
	d) For the alarm to sound, it is necessary and sufficient that the security system is activated and there is a breach.

Q22	Let r and s be the propositions: r: I studied for the exam. s: I passed the exam. Express each of these propositions as an English sentence: a) $\neg r$ b) $r \leftrightarrow s$ c) $\neg r \oplus s$ d) $(r \leftrightarrow s) \wedge (\neg r \oplus s)$
Q23	Let p and q be the propositions p: You drive over 65 miles per hour.

q : You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

  a) You do not drive over 65 miles per hour.
  b) You drive over 65 miles per hour, but you do not get a speeding ticket.
  c) You will get a speeding ticket if you drive over 65 miles per hour.
  d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
  e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
  f) You get a speeding ticket, but you do not drive over 65 miles per hour.
  g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Q24	State the converse, contrapositive, and inverse of each of these conditional statements. a) If it snows today, I will ski tomorrow. b) I come to class whenever there is going to be a quiz. c) A positive integer is a prime only if it has no divisors other than 1 and itself.
Q25	Show that $\neg(p \wedge q) \equiv \neg p \vee \neg q$.
Q26	Show that the proposition is tautology $p \rightarrow (p \wedge (q \rightarrow p))$
Q27	Construct the truth table for $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
Q28	Construct the truth table for $(p \oplus q) \rightarrow (p \oplus \neg q)$
Q29	Show that, "In the FIFA World Cup either Germany will reach the final or England and Argentina will reach the final" and "Germany or England will reach the final, and Germany or Argentina will reach the final" are logically equivalent.
Q30	Let P(x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English. a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$ d) $\forall x \neg P(x)$

Q31	Translate these statements into English, where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people. a) $\forall x(C(x) \rightarrow F(x))$ b) $\forall x(C(x) \wedge F(x)$ c) $\exists x(C(x) \rightarrow F(x))$ d) $\exists x(C(x) \wedge F(x)$
Q32	Translate these statements into English, where A(x): x teaches CSE230 T(x): x likes STA201 F(x): x has a Facebook page C(x): x likes to cook The domain consists of all faculties in a university. a) $\forall x(T(x) \rightarrow F(x))$ b) $\exists x(T(x) \wedge A(x)$ c) $\neg \forall x(T(x) \rightarrow (F(x) \vee C(x))$
Q33	Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. a) No one is perfect. b) Not everyone is perfect. c) All your friends are perfect. d) At least one of your friends is perfect. e) Everyone is your friend and is perfect. f) Not everybody is your friend or someone is not perfect.
Q34	Let P(x) be the statement "x can speak English" and let Q(x) be the statement "x knows programming language Python." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school. (a) There is a student at your school who can speak English and who knows Python. (b) There is a student at your school who can speak English but who doesn't know Python. (c) Every student at your school either can speak English or knows Python. (d) No student at your school can speak English or knows Python.

Q35	Let P (x), Q(x), and R(x) be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and P (x), Q(x), and R(x), where the domain consists of all people. a) No professors are ignorant. b) All ignorant people are vain. c) No professors are vain.
Q36	Let C(x) means "x is a core course," T(x,y) means "Student y has taken course x," and G(y) means "Student y has met graduation requirements." The domain consists of all students and courses. Formulate the following statements using quantifiers: a) For every student, if they have taken all core courses, they meet the graduation requirements. b) There exists a student who has not taken every core course but still meets the graduation requirements. c) For every core course, there is at least one student who has not taken it.
Q37	Let S(x) mean "x submitted a research paper," R(x,y) mean "Reviewer y read x's paper," and A(x,y) mean "Reviewer y approved x's paper." The domain consists of all researchers and reviewers. Express the following statements using nested quantifiers: a) Every researcher has at least one reviewer who has read and approved their paper. b) There exists a reviewer who has read every paper but has not approved at least one of them. c) For every research paper, if it was read by a reviewer, then there exists another reviewer who did not read it.
Q38	Let P(x,y)mean "Person x can work on a project with Person y," Q(x) means "Person x knows Python," and R(x) means "Person x is a project manager." The domain consists of all people in a software development team. Express the following statements using quantifiers and logical connectives: a) There exists a project manager who can work with everyone on the team who knows Python. b) For every person who knows Python, there is at least one other person they can work with who is not a project manager. c) For every person, if they are a project manager, there exists at least one person on the team with whom they cannot work.
Q39	Let S(x) represent "x is skilled in software development," M(x) represent "x has managerial experience," and Q(x,y) represent "Person x qualifies for Job y." The domain consists of all

	applicants for the job, **and** there are multiple types of jobs.
	
	    a) Describe, using quantifiers, the condition where there exists a job such that only applicants with both software skills and managerial experience qualify.
	    b) Write the condition, using quantifiers, for a job that requires at least one applicant without software development skills but with managerial experience to qualify.

Q40	Let A(x) represent "x is an applicant," S(x) represent "x has strong academic scores," and E(x) represent "x participates in extracurricular activities." The domain consists of all college applicants.
	
	    a) Express with quantifiers the requirement that all applicants participate in extracurricular activities only if they have strong academic scores, but there is at least one applicant who has strong academic scores and does not participate in extracurricular activities.
	    b) Write the condition that for an applicant to be admitted, they must either have strong academic scores or participate in extracurricular activities, but not necessarily both.