CSE230: Discrete Mathematics

Combinatorics (Additional Selected Problems)

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Sum Rule & Product Rule

- One can travel from Dhaka to Sylhet by: 3 airlines, 5 bus services, 2 train services. How many total ways to travel?
- You can go from A to B in 3 ways and from B to C in 5 ways. In how many ways can you go from A to C via B?
- Group A consists of 5 persons and group B consists of 7 persons.
  - In how many ways can you interview 1 person from them?
  - In how many ways can you interview 1 person from each group?

Applications of the Product Rule

- Suppose a passport number contains 2 letters followed by 7 digits, e.g., AB1234567. How many passport numbers are possible?
- How many 5-letter words/strings are possible such that no 2 consecutive letters are the same?
- Given $|A| = 7$. What is the size of $P(\mathrm{A})$?
- How many factors/divisors does 108 have?

Permutations

- **STANDARD PROBLEM:** Ways to rearrange n objects:

$$\boldsymbol{n}\textbf{–factorial:}\ \ n! = n \times (n-1) \times \cdots \times 1.$$

- In how many ways can 6 persons be seated in a row in a table?
- **STANDARD PROBLEM:** Ways to arrange r out of n objects:

$$\boldsymbol{n_{P_r}} = n \times (n-1) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

- How many 4-letter words/strings can be formed such that no letter is repeated?

Permutation with Repeated Elements

- How many rearrangements of the following words are possible?
  - **carrier, mississippi**
  - **betterment**, so that the vowels stay together.
  - **endeavour,** so that not all the vowels are together.

Combinations

- In how many ways can you form a team of 4 people, if 11 people are available?
- **STANDARD PROBLEM:** Ways to choose r out of n objects:

$$\binom{\boldsymbol{n}}{\boldsymbol{r}} = \boldsymbol{n}_{\boldsymbol{C}_r} = \frac{n \times (n-1) \times \cdots \times (n-r+1)}{r!} = \frac{n!}{r!\,(n-r)!}$$

  Note that $\binom{n}{n-r} = \binom{n}{r}$

  Combinatorially, choosing r people is the same as not choosing $n - r$ people.

- How many committees of 4 members (2 men, 2 women) are possible if 5 men and 6 women are available?

Application of Combinations

- In how many ways can you divide 12 people into 3 teams A, B, C of equal members?
- In how many ways can you arrange 5 boys and 8 girls in a line so that no two boys are next to each other?
- How many 4 letter words can be formed from **betterment**?