Elements of Statistics and Probability

STA 201

S M Rajib Hossain

MNS, BRAC University

Lecture-14

Introduction to Hypothesis Testing

- ✓ Hypothesis testing is a statistical method used to make inferences about a population parameter based on sample data.
- ✓ It involves formulating null and alternative hypotheses and using statistical tests to determine the likelihood of observing the sample results if the null hypothesis is true.
- ✓ Hypothesis testing is widely used in scientific research, quality control, and decision-making processes.

Null and Alternative Hypotheses

- ✓ The null hypothesis (H_0) is a statement that there is no effect or no difference in the population parameter.
- ✓ The alternative hypothesis (H_1 or H_A) is a statement that contradicts the null hypothesis and suggests that there is an effect or difference in the population parameter.

Simple hypothesis and composite hypothesis

- ✓ The hypothesis in which all the parameters are completely specified is called simple hypothesis. Ex: $H_0: \mu = 27 \; vs \; H_1: \mu = 29$
- ✓ The hypothesis in which all the parameters are not completely specified is called composite hypothesis.

  Example: $H_0: \mu = 27 \; vs \; H_1: \mu \neq 27$

  Or $H_0: \mu = 27 \; vs \; H_1: \mu > 27$

  Or $H_0: \mu = 27 \; vs \; H_1: \mu < 27$

One-tail Test vs. Two-tail Test

- ✓ In a one-tail test, the alternative hypothesis specifies the direction of the effect (e.g., greater than or less than).

  Example: $H_0: \mu = 27 \; vs \; H_1: \mu > 27$

Or $H_0: \mu = 27\ vs\ H_1: \mu < 27$

- ✓ In a two-tail test, the alternative hypothesis does not specify the direction of the effect, only that there is a difference.
  Example: $H_0: \mu = 27\ vs\ H_1: \mu \neq 27$

Type I and Type II Errors

- ✓ Type I error occurs when the null hypothesis is rejected when it is actually true.
- ✓ Type II error occurs when the null hypothesis is not rejected when it is actually false.

Level of significance $= \boldsymbol{\alpha = prob(type\ I\ error)}$

Test statistic: A statistic which is calculated from sample data during a hypothesis test to determine whether the null hypothesis is false or true is called test statistic.

Critical region/Rejection region: From the sample data, one computes a test statistic. If the test statistic falls within a specified region of the sampling distribution of test statistic for which null hypothesis is rejected, then that specified region is called critical region.

There are 3 ways to determine whether the null hypothesis is true or not.

i. Critical value.
ii. P-value.
iii. Confidence interval.

Procedures of test:

i. State the null and alternative hypothesis.
ii. State the assumptions such as the normality of the population, equality of variances etc.
iii. Choose the level of significance (α).

iv. Collect data and find the estimate of statistic.

v. Choose the appropriate test statistic and calculate the value of the test statistic.

vi. Construct acceptance/rejection region.

vii. Comparing (v) and (vi), one can take the decision of hypothesis.

Single mean test:

We compute Z test for single mean when,

- ✓ Mean and variance is known or
- ✓ Mean and variance is unknown but sample size is large (≥ 30)

T test when

- ✓ Mean and variance is unknown and sample size is small (< 30)

Practice problems:

1. A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated with the result that the average daily water uses of these 20 homes were as follows: 340, 344, 362, 375, 356, 386, 354, 364, 332, 402, 340, 355, 362, 322, 372, 324, 318, 360, 338, and 370.

a. Determine the null and the alternative hypothesis.
b. Which test would you prefer? Explain.
c. Is it one-tail test or two-tail? Explain.
d. State the type I and type II error from this scenario.

Solution:

a. The null and alternative hypotheses can be written as

Null Hypothesis: The mean daily home water use is 350 gallons, as claimed by the public health official. $H_0: \mu = 350$

Alternative Hypothesis: The mean daily home water use differs from 350 gallons.

$H_1: \mu \neq 350$

b. I would prefer t test. As the population mean and standard deviation is unknown and the sample size is small (< 30).
c. This is a two-tail test. As the alternative hypothesis does not specify the direction of the test.
d. Type I Error: Rejecting the null hypothesis when it is true. In this context, it would mean concluding that the mean daily home water use differs from 350 gallons when it actually does not.

   Type II Error: Failing to reject the null hypothesis when the alternative hypothesis is true. In this context, it would mean failing to conclude that the mean daily home water use differs from 350 gallons when it actually does.

2. Robin usually took 16.5 seconds on average for swimming the 30-yeard freestyle with standard deviation 0.8 seconds. His father thought that Robin could swim faster by using goggles. Robin took an average 16 seconds for 15 swims.

   a. Determine the null and the alternative hypothesis.
   b. Which test would you prefer? Explain.
   c. Is it one-tail test or two-tail? Explain.
   d. State the type I and type II error from this scenario.

Solution:

a. The null and alternative hypotheses can be written as

   Null Hypothesis: The goggles do not improve Robin's swimming time. The mean swimming time remains the same as 16.5 seconds. $H_0: \mu = 16.5$

   Alternative Hypothesis: The goggles improve Robin's swimming time, meaning the mean swimming time is less than 16.5 seconds. $H_1: \mu < 16.5$

b. I would prefer Z test. As the population mean and standard deviation is known.
c. This is a one-tail test. As the alternative hypothesis specifies the direction of the test (Less than the population mean).
d. Type I Error: Rejecting the null hypothesis when it is true. In this context, it would mean concluding that the goggles improved Robin's swimming time when, in reality, they did not.

   Type II Error: Failing to reject the null hypothesis when the alternative hypothesis is true. In this context, it would mean failing to conclude that the goggles improved Robin's swimming time when, in fact, they did.