Section 3.4 Summary

Truth Tables for the Conditional and Biconditional

The Definitions of Symbolic Logic

p	q	**Negation** $\sim p$	**Conjunction** $p \wedge q$	**Disjunction** $p \vee q$	**Conditional** $p \rightarrow q$	**Biconditional** $p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Example 1

Construct a truth table for the statement: $(p \vee q) \rightarrow \sim r$

Solution:

There are 3 simple statements, so we need 8 rows to write the 8 different true-false possibilities.

p	q	r	$\sim r$	$p \vee q$	$(p \vee q) \rightarrow \sim r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

Notice that our truth table has a column for $\sim r$, but not for $\sim p$ or $\sim q$. We only need to include a column for the negations that appear in the given compound statement. We also include any necessary columns that allow us to build up the given compound statement.

p	q	r	$\sim r$	$p \vee q$	$(p \vee q) \rightarrow \sim r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

Example 2

Determine if the compound statement is a tautology: $p \leftrightarrow \sim q$

Solution:
Construct the truth table.

p	q	$\sim q$	$p \leftrightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

Next, determine if the statement is a tautology. There are false values in the rightmost column of the truth table, which tells us that the given statement is false under certain circumstances. Therefore, $p \leftrightarrow \sim q$ is NOT a tautology.

Example 3

Determine if the compound statement is a tautology: $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$

Solution:
Construct the truth table.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$	$\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

Next, determine if the statement is a tautology. There are all true values in the rightmost column of the truth table, which tells us that the given statement is true under all circumstances. Therefore, $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$ is a tautology.

Example 4

Determine the truth value for each statement when p is true, q is false and r is false.
$(\sim q \rightarrow p) \vee (r \wedge \sim p)$

Solution:
Begin with the given compound statement.
$(\sim q \rightarrow p) \vee (r \wedge \sim p)$
Then substitute the given truth values and simplify.
$(\sim q \rightarrow p) \vee (r \wedge \sim p)$
$(\sim \mathrm{F} \rightarrow \mathrm{T}) \vee (\mathrm{F} \wedge \sim \mathrm{T})$
$(\mathrm{T} \rightarrow \mathrm{T}) \vee (\mathrm{F} \wedge \mathrm{F})$
$\mathrm{T} \vee \mathrm{F}$
T
Therefore the statement is TRUE when p is true, q is false and r is false.

Section 3.4 Worksheet

Name Key

For exercises 1–5, construct a truth table for the given statement.

1. $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

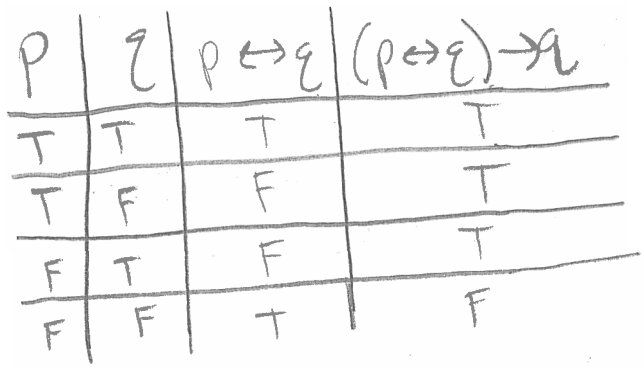
2. $(p \rightarrow q) \wedge \sim p$

p	q	$\sim p$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim p$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

3. $\sim (q \leftrightarrow p)$

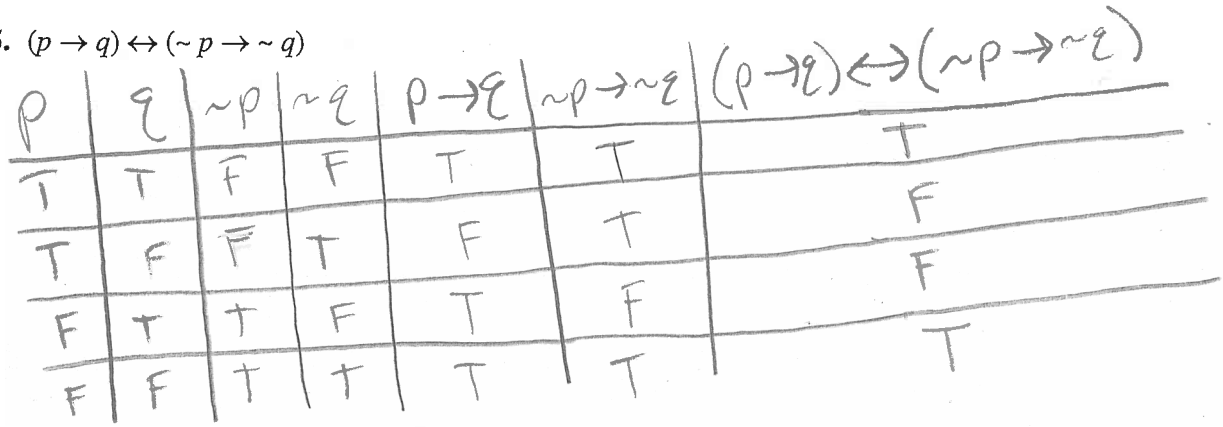
p	q	$q \leftrightarrow p$	$\sim (q \leftrightarrow p)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

4. $(p \leftrightarrow q) \rightarrow q$



p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	T	F

5. $(p \rightarrow q) \leftrightarrow (\sim p \rightarrow \sim q)$



p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \rightarrow \sim q$	$(p \rightarrow q) \leftrightarrow (\sim p \rightarrow \sim q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

6. Complete the truth table by filling in the required columns.
$(p \vee \sim q) \wedge r$

p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

Name_____

For exercises 7 and 8, construct a truth table for the given statement. Then determine whether the statement is a tautology.

7. $(p \to q) \leftrightarrow (p \vee \sim q)$

p	q	$\sim q$	$p \to q$	$p \vee \sim q$	$(p \to q) \leftrightarrow (p \vee \sim q)$
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Not a tautology.

8. $(q \leftrightarrow p) \leftrightarrow [(p \to q) \wedge (q \to p)]$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$q \leftrightarrow p$	$(q \leftrightarrow p) \leftrightarrow [(p \to q) \wedge (q \to p)]$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

Is a tautology.

For exercises 9–11, determine the truth value for each statement when p is false, q is true, and r is false.

9. $\sim (\overset{F}{p} \leftrightarrow \overset{T}{q})$

True

10. $\overset{F}{(p} \wedge \overset{F}{r)} \rightarrow \overset{T}{q}$

True

11. $\sim \overset{T}{p} \leftrightarrow (\sim \overset{F}{q} \wedge \overset{F}{r})$

False