1. a) success rate 70%
= 0.7

total case 6
failure rate $(1-0.7) = 0.3$

fewer than 3 patients will be successfully treated, $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

we know, for binomial distribution

$$P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(x=0) = \binom{6}{0} (0.7)^0 (0.3)^6 = 7.29 \times 10^{-4}$$

$$P(x=1) = \binom{6}{1} (0.7)^1 (0.3)^5 = 0.010206$$

$$P(x=2) = \binom{6}{2} (0.7)^2 (0.3)^4 = 0.059535$$

So, $P(x<3) = 0.07047$

(b) none of patients or 0 patients will be successfully treated,

So, $P(x=0) = \binom{6}{0} (0.7)^0 (0.3)^6 = 7.29 \times 10^{-4}$

2.a) average number of event, $\lambda = 3$

We know, for poisson distributn, $P(x=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

for exactly 5 emails, $P(x=5)$

$$P(x=5) = \frac{3^5 e^{-3}}{5!}$$

$$= 0.100819$$

b) The company receive at least 3 emails per hour, $P(x \geq 3)$

$$P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) + \cdots$$

So, $P(x \geq 3) = 1 - P(x<3)$

$$= 1 - P(x=0) - P(x=1) - P(x=2)$$

$$= 1 - 0.049787 - 0.149361 - 0.224042$$

$$= 0.577416$$

3.(a) The probability of not resolving on the first

$(1-p) = 20\% \Rightarrow 0.2$

The probability of resolving on the first attempt

$p = 80\% \Rightarrow 0.8$

We know, for geometric distribution,

$p(x=k) = (1-p)^{k-1} p$

~~$B = 0.8$~~

it takes at most 2 attempts, $p(x \leq 2) = P(x=1) + p(x=2)$

$p(x=1) = 0.8$

$p(x=2) = 0.16$

$\therefore p(x \leq 2) = 0.8 + 0.16 = 0.96$

(b) after failing ~~at least once~~ first time, $p(x=1) = 0.8$

The issue is resolved on the second attempt or later. So, $p(x \geq 2) = 1 - p(x=1)$

$= 1 - 0.8$

$= 0.2$

3.(a) The probability of not resolving on the first

$(1-p) = 20\% \Rightarrow 0.2$

The probability of resolving on the first attempt

$P = 80\% \Rightarrow 0.8$

We know, for geometric distribution,

$P(x=k) = (1-p)^{k-1} P$

~~P = 0.8~~

it takes at most 2 attempts, $P(x \leq 2) = P(x=1) + P(x=2)$

$P(x=1) = 0.8$

$P(x=2) = 0.16$

$\therefore P(x \leq 2) = 0.8 + 0.16 = 0.96$

(b) after failing ~~at least once~~ first time, $P(x=1) = 0.8$

The issue is resolved on the second attempt or later. So, $P(x \geq 2) = 1 - P(x=1)$

$= 1 - 0.8$

$= 0.2$

4. $f(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \quad \lambda > 0$, rate parameter of the distribution

we know,

$$E(x) = \int_0^\infty x f(x)\, dx$$

$$= \int_0^\infty x \lambda e^{-\lambda x} dx$$

$$= \left[-x e^{-\lambda x}\right]_0^\infty + \int_0^\infty e^{-\lambda x} dx$$

$$= 0 + \left[\frac{-1}{\lambda} e^{-\lambda x}\right]_0^\infty$$

$$= 0 - \left(-\frac{1}{\lambda}\right)$$

$$= \frac{1}{\lambda}$$

$$\therefore E(x) = \frac{1}{\lambda}$$

again we know,

$$Var(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^\infty x^2 f(x)\, dx$$

$$= \int_0^\infty x^2 \lambda e^{-\lambda x} dx$$

$$= \left[-x^2 e^{-\lambda x}\right]_0^\infty + \int_0^\infty 2x e^{-\lambda x} dx$$

$$= \left[-\frac{2x}{\lambda} e^{-\lambda x}\right]_0^\infty + \int_0^\infty \frac{2}{\lambda} e^{-\lambda x} dx$$

now, $\int_0^{\infty} \frac{2}{\lambda} e^{-\lambda x} dx = \frac{2}{\lambda} \times \frac{1}{\lambda} = \frac{2}{\lambda^2}$

$\therefore E(x^2) = \frac{2}{\lambda^2}$

$\therefore Var(x) = E(x^2) - (E(x))^2$

$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$

$= \frac{1}{\lambda^2}$

$\therefore$ Standard Deviation $S(x) = \sqrt{var(x)}$

$= \sqrt{\frac{1}{\lambda^2}}$

$= \frac{1}{\lambda}$

So, mean $\frac{1}{\lambda}$, standard deviation $\frac{1}{\lambda}$