Sample Questions on Root Finding

1. Bisection Method

Conceptual Questions

Q1. Which of the following is a necessary condition for the bisection method to work on the continuous function $f(x)$ on the interval $[a, b]$?

(1) $f(a) \cdot f(b) > 0$

(2) $f(a) \cdot f(b) < 0$

(3) $f'(x)$ is continuous in $[a, b]$

(4) $f(x)$ is quadratic

Q2. What happens to the width of the interval after each iteration of the bisection method?

(1) It remains the same.

(2) It doubles.

(3) It reduces by a factor of 4.

(4) It halves.

Numerical Questions

Q3. Use the bisection method to find the root of $f(x) = x^3 - 4x - 9$ in the interval $[2, 3]$ after 2 iterations.

(1) 2.25

(2) 2.375

(3) 2.5

(4) 2.625

Q4. The error in the bisection method is given by $(b - a)/2^n$, where n is the number of iterations. How many iterations are required to achieve an error less than 10^{-3} for $[1, 2]$?

(1) 10

(2) 11

(3) 12

(4) 13

2. Fixed Point Iteration

Conceptual Questions

Q5. Fixed-point iteration $x_{n+1} = g(x_n)$ will converge if:

(1) $|g'(x)| > 1$

(2) $|g'(x)| < 1$

(3) $g(x)$ is quadratic

(4) $f(x) = 0$

Q6. If $g(x) = \cos(x)$, the fixed-point iteration will:

(1) Always converge for any initial guess.

(2) Converge only if $|g'(x)| > 1$.

(3) Converge in certain intervals based on $|g'(x)| < 1$.

(4) Never converge.

Numerical Questions

Q7. Perform one iteration of fixed-point iteration for $g(x) = 1 + \sin(x)$ with an initial guess $x_0 = 0.5$. What is x_1?

(1) 1.4794

(2) 0.9794

(3) 1.0994

(4) 0.5994

Q8. Given $g(x) = x - 0.2(x^3 - 2)$, perform two iterations of fixed-point iteration starting with $x_0 = 1.5$. What is x_2?

(1) 1.25

(2) 1.45

(3) 1.35

(4) 1.5

3. Newton-Raphson Method

Conceptual Questions

Q9. Newton-Raphson is derived from the Taylor series by:

(1) Ignoring higher-order terms.

(2) Using all terms up to the second order.

(3) Assuming $f'(x) = 0$.

(4) Linearizing $f(x)$ around any point.

Q10. Newton-Raphson may fail to converge when:

(1) The initial guess is close to the root.

(2) The derivative $f'(x)$ at the root is zero.

(3) $f(x)$ is differentiable.

(4) The function has a single root.

Numerical Questions

Q11. Using Newton-Raphson, find the root of $f(x) = x^2 - 2$ starting from $x_0 = 1$. Perform one iteration.

(1) 1.2

(2) 1.5

(3) 1.4142

(4) 1.3333

Q12. Solve $f(x) = x^3 - 4x + 1 = 0$ using Newton-Raphson with $x_0 = 1$. Perform one iteration and find x_1.

(1) 0.5

(2) 1.5

(3) 0.75

(4) 1.25