Assignment

MAT: 120

Deadline: January 02, 2025

Instructions

- Answer all questions.
- Show all your calculations and steps clearly.
- Use appropriate tables where necessary.
- Ensure that your work is neat and well-organized.

1 Root Finding

Consider the nonlinear function:

$$f(x) = e^{\sin(x)} - x$$

Task

1. Apply both the Bisection Method and the Newton-Raphson Method to find the root of $f(x)$.
2. Perform **5 iterations** for each method.
3. Compare the errors obtained from both methods.

Instructions

Bisection Method

1. Select an initial interval $[a, b]$ where $f(a)$ and $f(b)$ have opposite signs.
2. Perform the Bisection Method for 5 iterations using the following steps:
   (a) Compute the midpoint:
   $$c = \frac{a+b}{2}$$
   (b) Evaluate $f(c)$.
   (c) Determine the subinterval $[a, c]$ or $[c, b]$ where the sign change occurs.
   (d) Update the interval accordingly.
3. Record the values of a, b, c, $f(c)$, and the absolute error for each iteration.

Newton-Raphson Method

1. Derive the first derivative of $f(x)$:

$$f'(x) = e^{\sin(x)} \cdot \cos(x) - 1$$

2. Choose an initial guess x_0 close to the suspected root.

3. Perform the Newton-Raphson Method for 5 iterations using the following formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

4. Record the values of x_n, $f(x_n)$, $f'(x_n)$, x_{n+1}, and the absolute error $|x_{n+1} - x_n|$ for each iteration.

Initial Guess

- Pick any initial guess to your liking based on your graphical analysis.

Comparison

1. After completing 5 iterations for both methods, compare the absolute errors.

2. Discuss which method converges faster based on the errors obtained.

2 Differentiation

Given the dataset below, find the first-order derivative at $x = 3$ using an $\mathcal{O}(h^2)$ approximation. For this problem, $h = 1$.

Dataset

x	0	1	2	3	4	5	6
$f(x)$	3.65	3.98	4.43	4.04	3.88	3.71	3.59

Task

1. Utilize the forward difference, backward difference and central difference formula with $\mathcal{O}(h^2)$ accuracy to estimate the first derivative $f'(3)$.

2. Show all your calculations.

3 Area Under the Curve

Using the trapezoidal rule, estimate the area under the curve for the following dataset.

Dataset

x	0	1	2	3	4	5	6
$f(x)$	3.65	3.98	4.43	4.04	3.88	3.71	3.59

Task

1. Apply the trapezoidal rule to the given dataset to estimate the area under the curve from $x = 0$ to $x = 6$.

2. Show all your calculations and intermediate steps.

Formula

The trapezoidal rule for n intervals with equal spacing h is given by:

$$\text{Area} \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

Submission Requirements

- **Answer Sheets**: Neatly show all calculations and steps for each question.
- **Tables**: Include any necessary tables to support your solutions.
- Use an A4 sheet and scan/upload your working.

Evaluation Criteria

- **Accuracy**: Correctness of calculations and final answers.
- **Clarity**: Clear presentation of steps and logical flow.
- **Completeness**: All parts of each question are addressed.
- **Presentation**: Neatness and organization of the submission.

Good Luck!

If you have any questions or need further assistance, please feel free to reach out during office hours or through the course communication channels.