Sample Questions on Numerical Differentiation

1. Forward and Backward Difference (First Order, $O(h)$)

Conceptual Questions

Q1. The forward difference formula for the first derivative is:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

What is the order of error in this approximation?

(1) $O(h)$
(2) $O(h^2)$
(3) $O(h^3)$
(4) $O(1/h)$

Q2. The backward difference formula for the first derivative is:

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

Which of the following statements is true?

(1) It uses values at $x+h$ and x.
(2) It has an error of $O(h)$.
(3) It uses values at $x+h$ and $x-h$.
(4) It has an error of $O(h^2)$.

Q3. Forward and backward difference approximations are accurate for:

(1) Large values of h.
(2) Small values of h.
(3) Any value of h.
(4) They are not accurate at all.

Numerical Questions

Q4. Using the forward difference formula, calculate $f'(x)$ for $f(x) = \sin(x)$ at $x = \pi/6$ with $h = 0.1$.

(1) 0.4975

(2) 0.4992

(3) 0.5000

(4) 0.5015

Q5. Using the backward difference formula, approximate the derivative of $f(x) = e^x$ at $x = 0$ with $h = 0.1$.

(1) 1.0513

(2) 1.0488

(3) 1.0500

(4) 1.0495

2. Forward, Backward, and Central Difference (First Order, $O(h^2)$)

Conceptual Questions

Q6. The forward difference formula with $O(h^2)$ error is:

$$f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

What is the key advantage of this formula over the $O(h)$ formula?

(1) It requires fewer evaluations of $f(x)$.

(2) It is more accurate for small values of h.

(3) It works for non-continuous functions.

(4) It has no truncation error.

Q7. The central difference formula for $f'(x)$ with $O(h^2)$ error is:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

This formula is:

(1) Less accurate than forward or backward difference.

(2) As accurate as forward difference for $O(h)$.

(3) More accurate than forward or backward difference for small h.

(4) Only applicable for symmetric functions.

Q8. Which of the following is true for the backward difference formula with $O(h^2)$ error?

(1) It uses three points: $x - 2h, x - h, x$.

(2) It uses two points: $x - h, x$.

(3) It has lower accuracy than forward difference with $O(h)$.

(4) It approximates $f'(x)$ as $(f(x) - f(x - h))/h$.

Numerical Questions

Q9. Use the central difference formula to approximate $f'(x)$ for $f(x) = \ln(x)$ at $x = 2$ with $h = 0.1$:

(1) 0.5002

(2) 0.4999

(3) 0.5010

(4) 0.4985

Q10. Calculate $f'(x)$ using the forward difference formula with $O(h^2)$ error for $f(x) = x^2$ at $x = 1$ with $h = 0.1$:

(1) 1.0000

(2) 1.0020

(3) 0.9990

(4) 1.0015

Q11. Approximate $f'(x)$ using the backward difference formula with $O(h^2)$ error for $f(x) = e^x$ at $x = 1$ with $h = 0.1$:

(1) 2.7185

(2) 2.7189

(3) 2.7180

(4) 2.7191