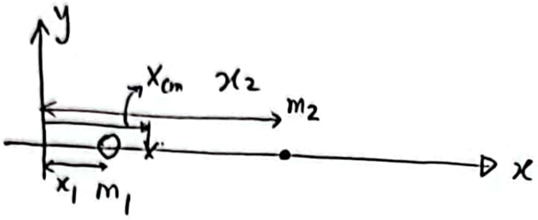
"Center of Mass and Linear Momentum"

Center of Mass (mass - weighted mean)



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

For ~~n~~- number of particles, $x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n}$

$$= \frac{1}{M} \sum_i m_i x_i$$

In three-dimension,

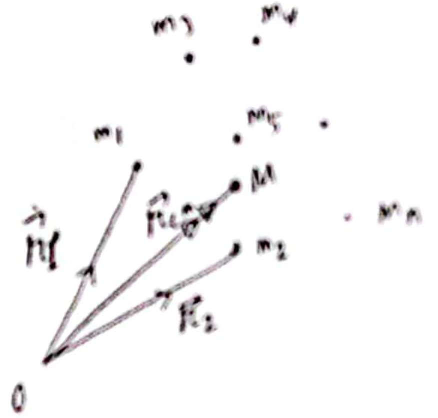
$$x_{cm} = \frac{1}{M} \sum_i m_i x_i , \quad y_{cm} = \frac{1}{M} \sum_i m_i y_i , \quad z_{cm} = \frac{1}{M} \sum_i m_i z_i$$

$$\vec{r}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k}$$

$$= \frac{1}{M} \sum_i m_i x_i \hat{i} + \frac{1}{M} \sum_i m_i y_i \hat{j} + \frac{1}{M} \sum_i m_i z_i \hat{k}$$

$$= \frac{1}{M} \sum_i m_i (x_i \hat{i} + y_i \hat{j} + z_i \hat{k})$$

$$\boxed{\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i}$$



For, Solid bodies,

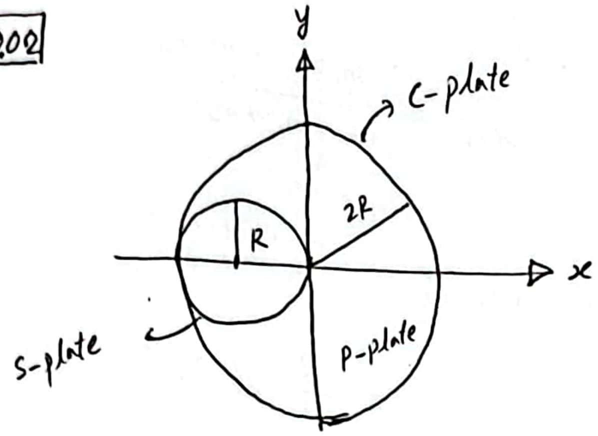
$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} \, dm$$

$$x_{cm} = \frac{1}{M} \int x \, dm , \quad y_{cm} = \frac{1}{M} \int y \, dm , \quad z_{cm} = \frac{1}{M} \int z \, dm$$

density, $\rho = \frac{dm}{dV} = \frac{M}{V} \Rightarrow dm = \frac{M}{V} dV$

$x_{cm} = \frac{1}{V} \int x \, dV$, $y_{cm} = \frac{1}{V} \int y \, dV$, $z_{cm} = \frac{1}{V} \int z \, dV$

$\boxed{SP : 9.02}$



$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P} \qquad \text{——(1)}$$

$x_{S+P} = 0$

$x_S = -R$

$x_P = ?$

From eqn. (1),

$$0 = \frac{m_S x_S + m_P x_P}{m_S + m_P}$$

$$\Rightarrow m_S x_S + m_P x_P = 0$$

$$\Rightarrow x_P = -\frac{m_S}{m_P} x_S$$

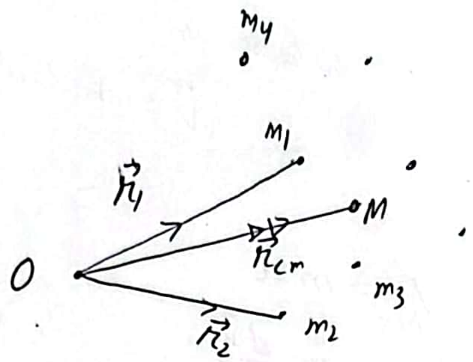
$$= -\frac{m_S}{m_P}(-R)$$

$$= \frac{m_S}{m_P} R$$

$$\frac{m_S}{m_P} = \frac{\rho_S V_S}{\rho_P V_P} = \frac{\cancel{\rho}\,(\text{Area})_S \text{ thickness}}{\cancel{\rho}\,(\text{Area})_P \text{ thickness}} = \frac{\pi R^2}{\pi (2R)^2 - \pi R^2} = \frac{\pi R^2}{3\pi R^2} = \frac{1}{3}$$

$$\therefore \boxed{x_P = \frac{1}{3} R}$$

Newton's 2nd Law for a System of Particles:



$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\Rightarrow M \vec{r}_{cm} = \sum_i m_i \vec{r}_i = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots\cdots + m_n \vec{r}_n$$

Taking $\frac{d}{dt}$

$$\Rightarrow M \frac{d\vec{r}_{cm}}{dt} = m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + \cdots\cdots + m_n \frac{d\vec{r}_n}{dt}$$

$$\Rightarrow M \vec{V}_{cm} = m_1 \vec{V}_1 + m_2 \vec{V}_2 + \cdots\cdots + m_n \vec{V}_n$$

Taking $\frac{d}{dt}$

$$\Rightarrow M \frac{d\vec{V}_{cm}}{dt} = m_1 \frac{d\vec{V}_1}{dt} + m_2 \frac{d\vec{V}_2}{dt} + \cdots\cdots + m_n \frac{d\vec{V}_n}{dt}$$

$$\Rightarrow M \vec{a}_{cm} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \cdots\cdots + m_n \vec{a}_n$$

$$\therefore \boxed{M \vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \cdots\cdots + \vec{F}_n}$$

Linear momentum:

Linear momentum of a particle,

$$\vec{P} = m\vec{V}$$

where m = mass of the particle
$\vec{V}$ = velocity of the particle

Newton's 2nd Law, $\vec{F}_{net} = m\vec{a}$

$$= m\frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}(m\vec{v})$$

$$\therefore \vec{F}_{net} = \frac{d}{dt}(\vec{P})$$

If $\vec{F}_{net} = 0$, $\vec{P}$ = constant.

So, $\vec{P}_i = \vec{P}_f$ (conservation of Linear momentum)

Linear momentum for a system of Particles,

$$\vec{P}_{cm} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n$$

Newton's 2nd Law,

$$\vec{F}_{net} = \frac{d}{dt}(\vec{P}_{cm})$$

If $\vec{F}_{net} = 0$; $\vec{P}_{cm}$ = Constant

$$\Rightarrow \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n = \text{Constant}$$

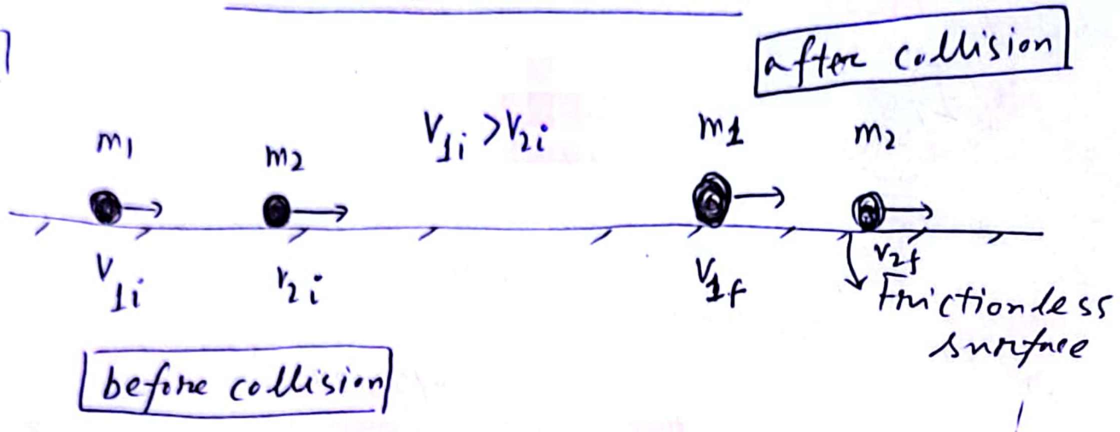
$$\vec{P}_{cm_i} = \vec{P}_{cm_f}$$

$$\vec{P}_{1i} + \vec{P}_{2i} + \dots + \vec{P}_{ni} = \vec{P}_{1f} + \vec{P}_{2f} + \dots + \vec{P}_{nf}$$

Conservation of Linear Momentum for a system of particles

Collisions (1D and 2D)

1D



According to conservation of Linear momentum.

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow \vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

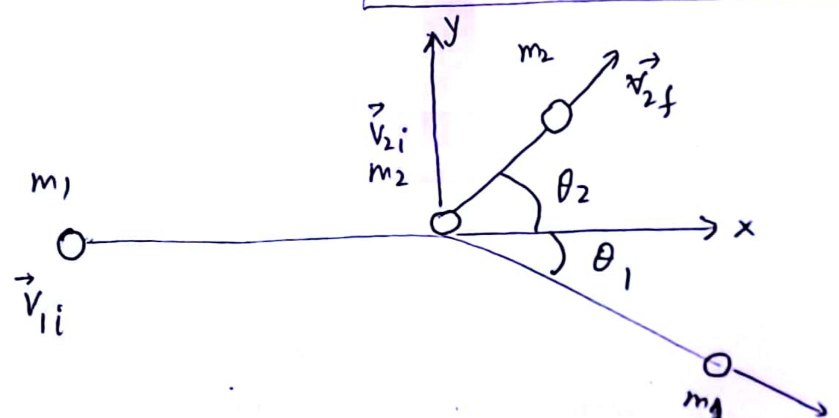
$$\Rightarrow P_{1i} + P_{2i} = P_{1f} + P_{2f}$$

$$\therefore \boxed{m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}}$$

For elastic Collision, $K_{1i} + K_{2i} = K_{1f} + K_{2f}$

$$\Rightarrow \boxed{\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2}$$

2D



$V_{1i} > V_{2i}$

Before collision velocity of m_1 and m_2 are $\vec{V}_{1i}$ and $\vec{V}_{2i}$ respectively

After collision velocity of m_1 and m_2 are $\vec{V}_{1f}$ and $\vec{V}_{2f}$ respectively

According to Conservation of Linear Momentum,

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow \vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f} \quad \cdots (1)$$

For eqn (1), components along x-axis,

$$P_{1i} + P_{2i} = P_{1f} \cos\theta_1 + P_{2f} \cos\theta_2$$

$$\Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \quad \cdots (2)$$

For eqn (2) components along y-axis,

$$\Rightarrow 0 = - P_{1f} \sin\theta_1 + P_{2f} \sin\theta_2$$

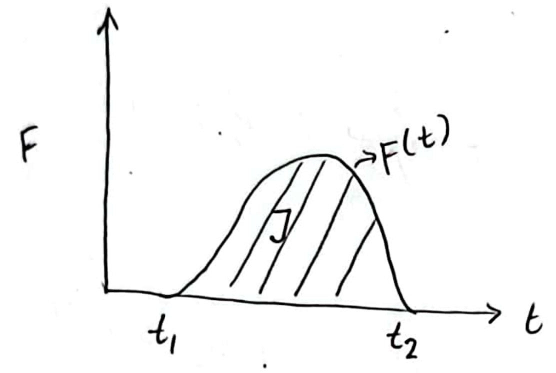
$$\Rightarrow 0 = - m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2 \quad \cdots (3)$$

For elastic Collision,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \cdots (4)$$

Impulse: (Change of the momentum)

Impulsive Force
↓
Large amount of Force in infinitesimal time



Impulse, $\boxed{\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt}$

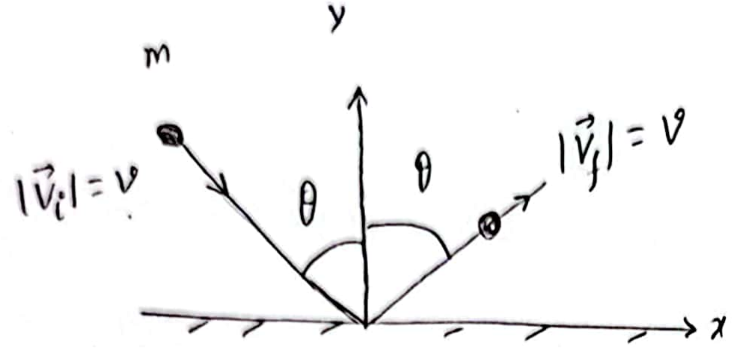
According to Newton's 2nd Law, $\vec{F} = \frac{d\vec{p}}{dt}$

$\Rightarrow d\vec{p} = \vec{F}\, dt$

$\Rightarrow \int_{\vec{P}_i}^{\vec{P}_f} d\vec{P} = \int_{t_1}^{t_2} \vec{F} dt$

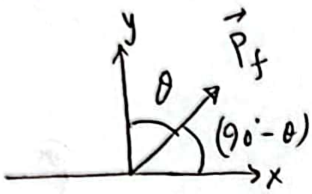
$\Rightarrow \boxed{\vec{P}_f - \vec{P}_i = \vec{J}}$

Impulse, $\boxed{\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt = \vec{P}_f - \vec{P}_i = \Delta \vec{P}}$

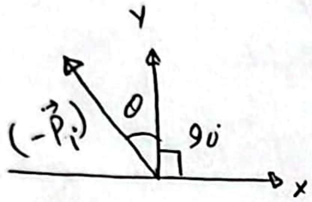


Impulse, $\vec{J} = ?$

* Impulse $\vec{J} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = \vec{P}_f + (-\vec{P}_i)$



$$\vec{P}_f = |\vec{P}_f| \cos(90^\circ-\theta)\,\hat{i} + |\vec{P}_f| \sin(90^\circ-\theta)\,\hat{j}$$
$$= mv \sin\theta\,\hat{i} + mv \cos\theta\,\hat{j}$$



$$(-\vec{P}_i) = |-\vec{P}_i| \cos(90^\circ+\theta)\,\hat{i} + |-\vec{P}_i| \sin(90^\circ+\theta)\,\hat{j}$$
$$= -mv \sin\theta\,\hat{i} + mv \cos\theta\,\hat{j}$$

Impulse, $\vec{J} = \vec{P}_f + (-\vec{P}_i)$

$$= (mv \sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) + (-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j})$$

$$\boxed{\vec{J} = 2mv\cos\theta\,\hat{j}}$$

{** Ch-9: Pr 2, 4, 25, 37, 38, 51, 55, 60, 68, 70
SP: 9.03, 9.04}

Chapter-9

[2]

$m_1 = 3\ kg$

$m_2 = 4\ kg$

$m_3 = 8\ kg$

$x_1 = 0\ m, \quad y_1 = 0\ m$

$x_2 = 2\ m, \quad y_2 = 1\ m$

$x_3 = 1\ m, \quad y_3 = 2\ m$

(a) Center of mass, $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$= \left(\dfrac{3 \times 0 + 4 \times 2 + 8 \times 1}{3 + 4 + 8}\right) m$

$= \boxed{1.1\ m}$ Ans a

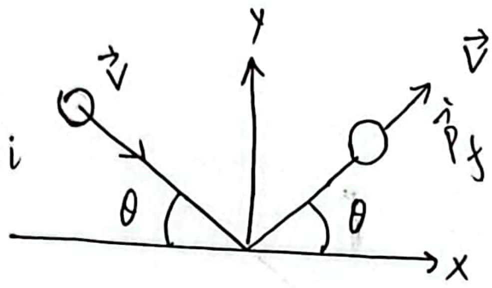
(b) and, $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$

$= \left(\dfrac{3 \times 0 + 4 \times 1 + 8 \times 2}{3 + 4 + 8}\right) m$

$= \boxed{1.3\ m}$ Ans. b

(c) If m_3 is increased then the center of mass will shift toward to the m_3.

38



$m = 300g = \cancel{3\times10^{-3}kg} = 300\times10^{-3}kg = 0.3\ kg = 3\times10^{-1}kg$

$|\vec{v}| = 6\ m/s$

$\theta = 30^\circ$

$\Delta t = 10\ ms = 10\times10^{-3}s$

(a) $\vec{J} = ?$

Impulse, $\vec{J} = \vec{P}_f - \vec{P}_i$

$\vec{P}_f = |\vec{P}_f|\cos\theta\,\hat{i} + |\vec{P}_f|\sin\theta\,\hat{j}$

$= mv\cos\theta\,\hat{i} + mv\sin\theta\,\hat{j}$

$= \{[3\times10^{-1}\times 6\times\cos(30^\circ)]\,\hat{i} + [3\times10^{-1}\times 6\times\sin(30^\circ)]\,\hat{j}\}\ kgms^{-1}$

$= 3\times10^{-1}\,[5.2\,\hat{i} + 3.0\,\hat{j}]\ kgms^{-1}$

$\vec{P}_i = |\vec{P}_i|\cos(-\theta)\,\hat{i} + |\vec{P}_i|\sin(-\theta)\,\hat{j}$

$= mv\cos\theta\,\hat{i} + mv(-\sin\theta)\,\hat{j}$

$= [(3\times10^{-1}\times 6\times\cos30)\,\hat{i} - (3\times10^{-1}\times6\times\sin30)\,\hat{j}]\ kgms^{-1}$

$= 3\times10^{-1}\,[5.2\,\hat{i} - 3.0\,\hat{j}]\ kgms^{-1}$

$\vec{P}_i = 3\times10^{-1} [5.2\,\hat{i} - 3.0\,\hat{j}]$ kgms-1 (3)

Impulse, $\vec{J} = \vec{P_f} - \vec{P_i}$

$= 3\times10^{-1} [5.2\,\hat{i} + 3.0\,\hat{j} - 5.2\,\hat{i} + 3.0\,\hat{j}]$ kgms-1

$= (3\times10^{-1} \times 6)\,\hat{j}$ kgms-1

$= (1.8\ Ns)\,\hat{j}$

(b) Average Force on the ball, $\vec{F} = \frac{\vec{J}}{\Delta t}$

$= \frac{(1.8\ Ns)\,\hat{j}}{10\times10^{-3}\,s}$

$= 180\ N\,\hat{j}$

Average Force on the wall $(-180\ N\hat{j})$

58

$m_1 = 2 \text{ kg}$

$m_2 = 1 \text{ kg}$

$v_1 = 4 \text{ m/s}$

$v_2 = 0 \text{ m/s}$

According to conservation of Linear momentum,

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$\Rightarrow (2 \times 4 + 1 \times 0) = (2+1) v$$

$$\Rightarrow v = \frac{2 \times 4}{2+1} \text{ m/s}$$

$$= 2.7 \text{ m/s}$$

According to conservation of Energy,

$$\frac{1}{2}(m_1 + m_2) v^2 = \frac{1}{2} k x^2$$

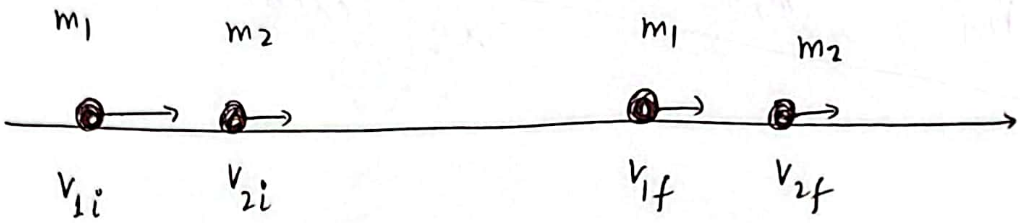
$$\Rightarrow x = \sqrt{\frac{m_1 + m_2}{k}}\, v$$

$$= \sqrt{\frac{3}{200}}\,(2.7) \text{ m}$$

$$= \boxed{0.33 \text{ m}}$$

68

Note: For problem 68 and 70



before collision

after collision

According to conservation of linear momentum,

$$\vec{P}_I = \vec{P}_f$$

$$\Rightarrow \vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$\Rightarrow P_{1i} + P_{2i} = P_{1f} + P_{2f}$$

$$\Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{---} (1)$$

For completely inelastic collision, (particle will stick together). ~~Let~~ Then,

Condition for completely inelastic collision,

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) V}$$

For, Elastic collision,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{---} (2)$$

From eqn (1) and (2), we can find,

$$\boxed{\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2 m_2}{m_1 + m_2} v_{2i} \\ v_{2f} &= \frac{2 m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{aligned}}$$

58

$$m_1 g h = \frac{1}{2} m_1 v_{1i}^2$$

$$\Rightarrow \quad v_{1i} = \sqrt{2gh}$$

v_{1i} = initial velocity of particle 1, (before collision)

$$= \sqrt{2 \times 9.8 \times 2.5} \text{ m/s}$$

(a)

For elastic collision,

$$v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

$m_2 = 2 m_1$

$v_{2i} = 0$ m/s

$$v_{2f} = \frac{2 m_1}{m_1 + 2m_1} v_{1i} + \frac{2m_1 - m_1}{m_1 + 2m_1} \times 0$$

$$= \frac{2}{3} \times \sqrt{2 \times 9.8 \times 2.5} \text{ m/s}$$

$$\frac{1}{2} m_2 v_{2f}^2 = \Delta E_{th}$$

$$\Rightarrow \frac{1}{2} \cancel{m_2} v_{2f}^2 = \mu_k \cancel{m_2} g d$$

$$\Rightarrow d = \frac{v_{2f}^2}{2 \times \mu_k g} = \frac{\left(\frac{2}{3} \times \sqrt{2 \times 9.8 \times 2.5}\right)^2}{2 \times 0.5 \times 9.8} m$$

$$= \boxed{2.2 m} \text{ Ans.}$$

(b) For completely inelastic collision,

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$$

$$\Rightarrow v = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$v_{2i} = 0$ m/s

$v_{1i} = \sqrt{2 \times 9.8 \times 2.5}$ m/s

$$= \frac{m_1}{3 m_1} \times \sqrt{2 \times 9.8 \times 2.5} \text{ m/s}$$

$$= \frac{\sqrt{2 \times 9.8 \times 2.5}}{3} \text{ m/s}$$

$$\frac{1}{2} (m_1 + m_2) v^2 = \Delta E_{th}$$

$$\Rightarrow \frac{1}{2} (m_1 + m_2) v^2 = \mu_k (m_1 + m_2) g d$$

$$\Rightarrow d = \frac{v^2}{2 \times \mu_k g} = \frac{\left(\frac{\sqrt{2 \times 9.8 \times 2.5}}{3}\right)^2}{2 \times 0.5 \times 9.8} m$$

$$= \boxed{0.556 m}$$

$\boxed{8}$

$\boxed{70}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2 m_2}{m_1 + m_2} v_{2i} \quad \text{---(1)}$$

$$v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \quad \text{---(2)}$$

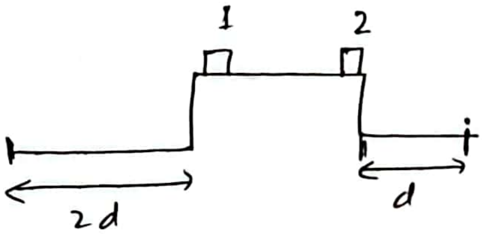
$$v_{2i} = 0 \, m/s$$

$$m_1 = 0.2 \, kg$$

Here,

$$\therefore \; v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} v_{1i} \right)$$

$$v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i}$$



$$(-2d) = v_{1f} \, \Delta t \quad \text{---(3)}$$

$$(d) = v_{2f} \, \Delta t \quad \text{---(4)}$$

$$(3) \div (4) \Rightarrow \frac{-2\cancel{d}}{\cancel{d}} = \frac{v_{1f} \cancel{\Delta t}}{v_{2f} \cancel{\Delta t}}$$

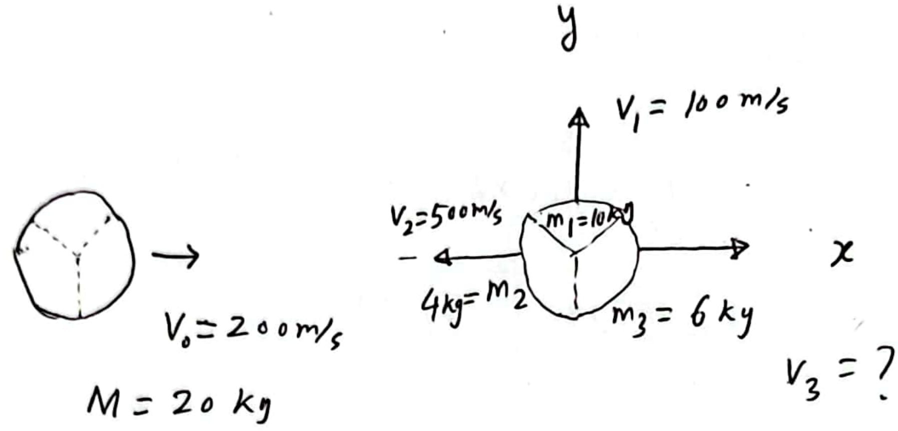
$$\Rightarrow 2 = \frac{-\left(\frac{m_1 - m_2}{m_1 + m_2} \right) \cancel{v_{1i}}}{\left(\frac{2 m_1}{m_1 + m_2} \right) \cancel{v_{1i}}}$$

$$\Rightarrow 2 = \left(\frac{m_2 - m_1}{\cancel{m_1 + m_2}} \right) \times \left(\frac{\cancel{m_1 + m_2}}{2 m_1} \right) = \frac{m_2 - m_1}{2 m_1}$$

$\Rightarrow \quad 4m_1 = m_2 - m_1 \quad \Rightarrow \quad 5m_1 = m_2$

$\Rightarrow \quad m_2 = 5 \times 0.2\,kg$

$= \boxed{1\ kg}$



(a) According to Conservation of Linear momentum,

$$\vec{P_I} = \vec{P_f} \Rightarrow \vec{P} = \vec{P_{1f}} + \vec{P_{2f}} + \vec{P_{3f}}$$

$$\Rightarrow M\vec{V_0} = m_1\vec{V_1} + m_2\vec{V_2} + m_3\vec{V_3}$$

$$\Rightarrow M(V_0\hat{i}) = m_1(V_1\hat{j}) + m_2(V_2(-\hat{i})) + m_3\vec{V_3}$$

$$\Rightarrow 20 \times 200\hat{i} = 10 \times 100\hat{j} - 4 \times 500\hat{i} + 6\vec{V_3}$$

$$\Rightarrow 4000\hat{i} - 1000\hat{j} + 2000\hat{i} = 6\vec{V_3}$$

$$\Rightarrow 6\vec{V_3} = 6000\hat{i} - 1000\hat{j}$$

$$\Rightarrow \vec{V_3} = \left[\frac{6000}{6}\hat{i} - \frac{1000}{6}\hat{j}\right] m/s$$

$$\boxed{\vec{V_3} = (10^3\hat{i} - 0.167 \times 10^3\hat{j})\ m/s}$$ Ans (a)

Magnitude, $|\vec{V_3}| = \sqrt{(10^3)^2 + (-0.167 \times 10^3)^2} = \boxed{1.01 \times 10^3\ m/s}$

Angle $\theta = \tan^{-1}\left(\frac{-0.167 \times 10^3}{10^3}\right) = \boxed{-9.48°}$

(b)

Energy released,

$$\Delta k = K_f - k_i = \frac{1}{2}MV_0^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_3v_3^2$$

$$= \left[\frac{1}{2}\times 20\times 200^2 - \frac{1}{2}\times 10\times 100^2 - \frac{1}{2}\times 4\times 500^2 - \frac{1}{2}6\times(1.01\times 10^3)^2\right]J$$

$$= \boxed{3.23\times 10^6\ J}$$ Am (b)

60

$m_A = 1.6$ kg

$m_B = 2.4$ kg

$V_{Ai} = 5.5$ m/s

$V_{Bi} = 2.5$ m/s

$V_{Bf} = 4.9$ m/s

(a) $V_{Af} = ?$

According to Conservation of Linear momentum,

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow \vec{P}_{Ai} + \vec{P}_{Bi} = \vec{P}_{Af} + \vec{P}_{Bf}$$

For (1D), $P_{Ai} + P_{Bi} = P_{Af} + P_{Bf}$

$$\Rightarrow m_A V_{Ai} + m_B V_{Bi} = m_A V_{Af} + m_B V_{Bf}$$

$$\Rightarrow V_{Af} = \frac{m_A V_{Ai} + m_B V_{Bi} - m_B V_{Bf}}{m_A}$$

$$= \frac{1.6\times 5.5 + 2.4\times 2.5 - 2.4\times 4.9}{1.6}\ m/s$$

$$= \boxed{1.9\ m/s}$$ (Am a)

(b) Direction of the velocity, V_{Af} is from left to right.

(c)

Before Collision, Kinetic Energy,

$$K_i = \frac{1}{2} m_A V_{Ai}^2 + \frac{1}{2} m_B V_{Bi}^2$$

$$= \left[\frac{1}{2} \times 1.6 \times 5.5^2 + \frac{1}{2} \times 2.4 \times (2.5)^2\right] J$$

$$= \boxed{31.7 J}$$

After collision, kinetic Energy,

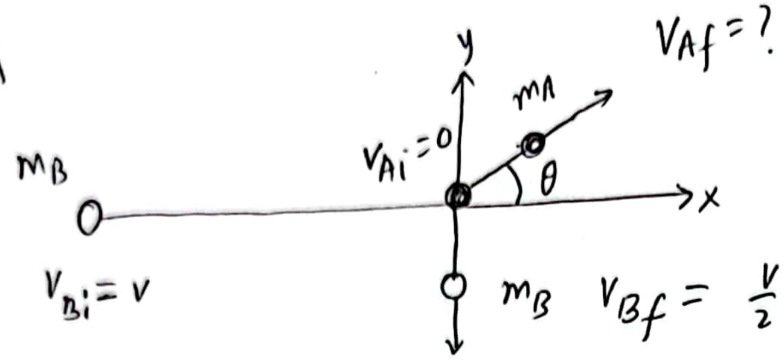
$$K_f = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2$$

$$= \left[\frac{1}{2} \times 1.6 \times (1.9)^2 + \frac{1}{2} \times 2.4 \times (4.9)^2\right] J$$

$$= \boxed{31.7 J}$$

$\therefore K_i = K_f$ so, the collision is elastic.

[72]



Applying Conservation of Linear momentum along x-axis,

$$m_B v_{Bi} = m_A v_{Af} \cos\theta + m_B v_{Bf} \cos(-90^\circ)$$

$$\Rightarrow m_B v = m_A v_{Af} \cos\theta \quad \text{---} (1)$$

Applying Conservation of Linear momentum along y-axis,

$$\Rightarrow 0 = m_A v_{Af} \sin\theta + m_B v_{Bf} \sin(-90^\circ)$$

$$\Rightarrow m_B \frac{v}{2} = m_A v_{Af} \sin\theta \quad \text{---} (2)$$

$(2) \div (1)$ $\quad \tan\theta = \frac{1}{2}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \boxed{27^\circ}$$ (Ans)

~~Putting~~ Here, From equation (1) and (2), m_A, m_B and v_{Af} are unknown. So, We can't solve these two equations. That's why we can't find v_{Af}. Ans

14

$m_A = 2\,kg$

$\vec{V}_A = (15\hat{i} + 30\hat{j})$ m/s

$m_B = 2\,kg$

$\vec{V}_B = (-10\hat{i} + 5\hat{j})$ m/s

$V_A' = (-5\hat{i} + 20\hat{j})$ m/s ; $V_B' = ?$

(a)

According to conservation of linear momentum,

$$m_A \vec{V}_A + m_B \vec{V}_B = m_A \vec{V}_A' + m_B \vec{V}_B'$$

$$\Rightarrow \vec{V}_A + \vec{V}_B = \vec{V}_A' + \vec{V}_B' \quad [m_A = m_B]$$

$$\Rightarrow \vec{V}_B' = \vec{V}_A + \vec{V}_B - \vec{V}_A'$$

$$= [(15\hat{i} + 30\hat{j}) + (-10\hat{i} + 5\hat{j}) - (-5\hat{i} + 20\hat{j})] \text{ m/s}$$

$$= [(15 - 10 + 5)\hat{i} + (30 + 5 - 20)\hat{j}] \text{ m/s}$$

$$= [10\hat{i} + 15\hat{j}] \text{ m/s}$$

(b) Initial kinetic Energy,

$$k_i = \frac{1}{2} m_A \vec{V}_A^2 + \frac{1}{2} m_B \vec{V}_B^2$$

$$= [\frac{1}{2} \times 2 \times (15^2 + 30^2) + \frac{1}{2} \times 2 \times ((-10)^2 + 5^2)] \text{ J}$$

$$= 1.3 \times 10^3 \text{ J}$$

Final kinetic Energy, $k_f = \frac{1}{2} m_A \vec{V}_A'^2 + \frac{1}{2} m_B \vec{V}_B'^2$

$$= [\frac{1}{2} \times 2 \times [(-5)^2 + (20)^2] + \frac{1}{2} \times 2 \times [10^2 + 15^2]] \text{ J}$$

$$= 8 \times 10^2 \text{ J}$$

Change of the kinetic energy,

$$\Delta K = K_f - K_i$$

$$= (8 \times 10^2 - 1.3 \times 10^3)\ J$$

$$= \boxed{-5 \times 10^2\ J}$$