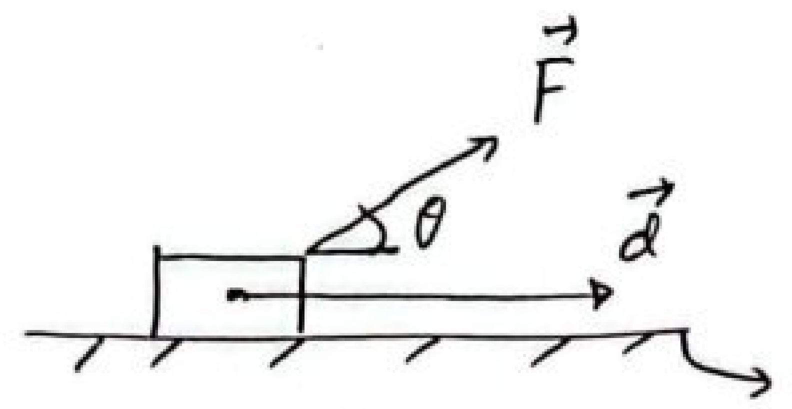
Chapter 7 and 8

"Kinetic Energy and Work"

and

"Potential Energy and Conservation of Energy"

Work:



Work done by Force, $W = \vec{F} \cdot \vec{d}$

$= F d \cos\theta$

$= F (d \cos\theta)$

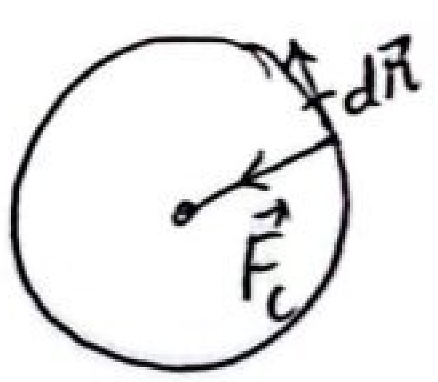
$=$ (Force) $\times$ (component of displacement along the direction of $\vec{F}$)

$= (F \cos\theta) \times d$

$=$ (component of $\vec{F}$ along the direction of $\vec{d}$) $\times$ (displacement)

(i) Work done by Force $\Rightarrow$ zero if $|\vec{d}| = 0$ or $\vec{F} \wedge \vec{d} = 90°$

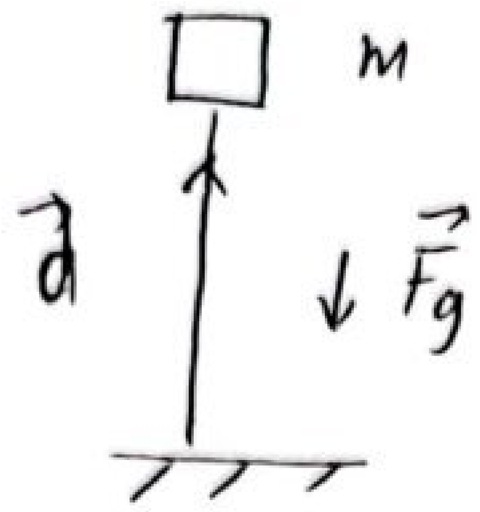
Example: In Uniform circular work done by centripetal Force is zero.



$\vec{F}_c \wedge d\vec{r} = 90°$

$W = \int \vec{F}_c \cdot d\vec{r} = 0$

(ii) Work done by force => negative : if $90° < \vec{F} \wedge \vec{d} \leq 180°$

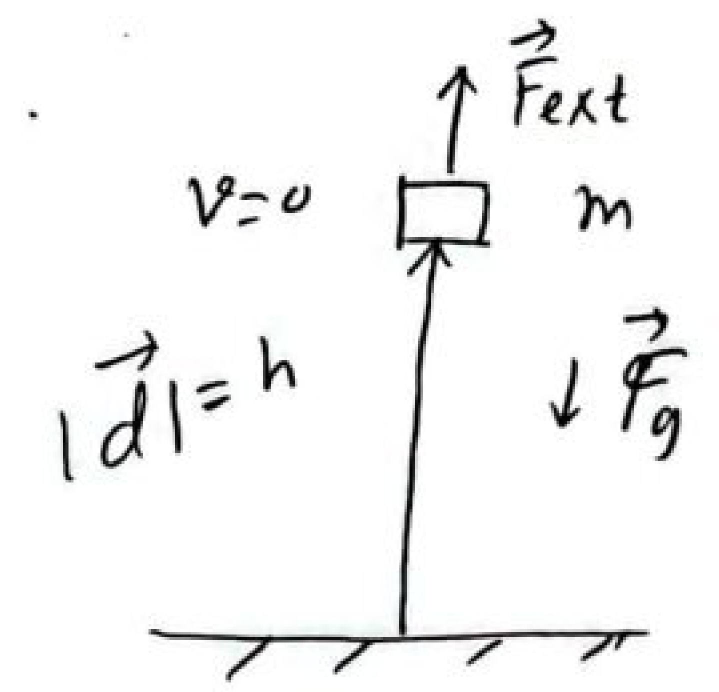


Here work done by $\vec{F_g}$ is negative.

(iii) Work done by Force => positive : if $0 \leq \vec{F} \wedge \vec{d} \leq 90°$

Work done by Constant Force:

Considering here, $\vec{F} \wedge \vec{d}$ = constant. For instance, consider in the figure.



Here, $\vec{F_g} \wedge \vec{d} = 180°$
and $\vec{F}_{ext} \wedge \vec{d} = 0°$

Work done by gravitational Force,

$$W_g = \vec{F_g} \cdot \vec{d}$$
$$= |\vec{F_g}||\vec{d}| \cos 180°$$
$$= -mgh$$

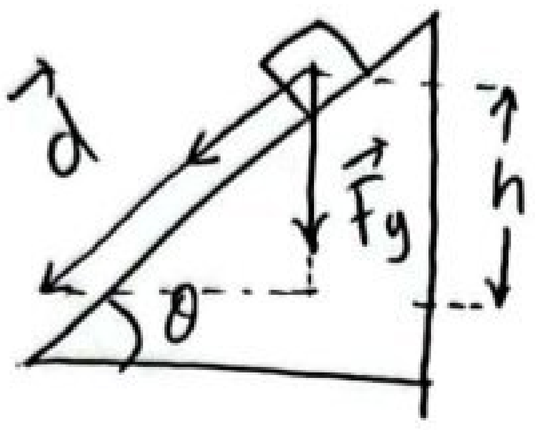
Work done by external Force, $W_{ext} = \vec{F}_{ext} \cdot \vec{d}$

$$= |\vec{F}_{ext}| |\vec{d}| \cos 0°$$
$$= mgh$$
$$= -W_g$$

$\therefore$ Work done by net Force, $W_{net} = W_g + W_{ext} = 0$.

Consider a Frictionless ramp. If the block slide down d distance along the surface of the ramp. Then work done by gravitational Force,

$W_g = \vec{F}_g \cdot \vec{d}$

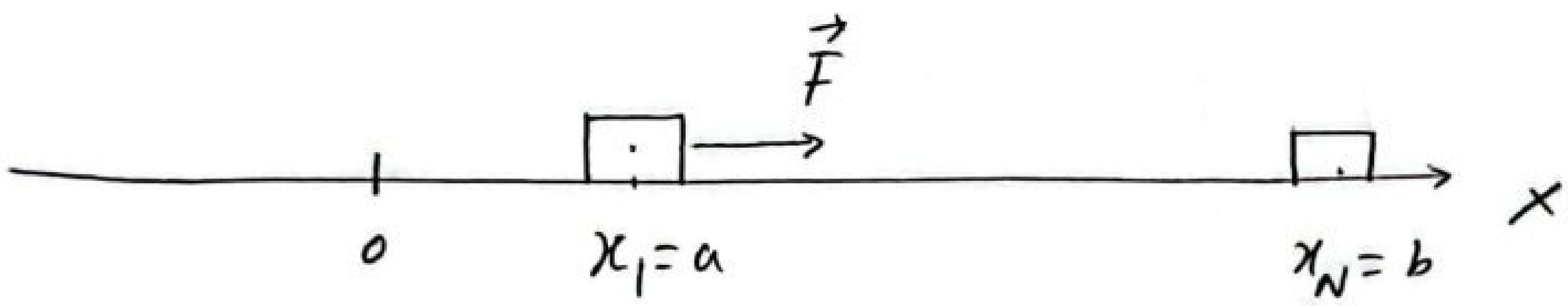


$= |\vec{F}_g||\vec{d}| \cos(90^\circ - \theta)$

$= mg\, d \sin\theta$, but $\sin\theta = \frac{d}{h}$

$= \boxed{mgh}$

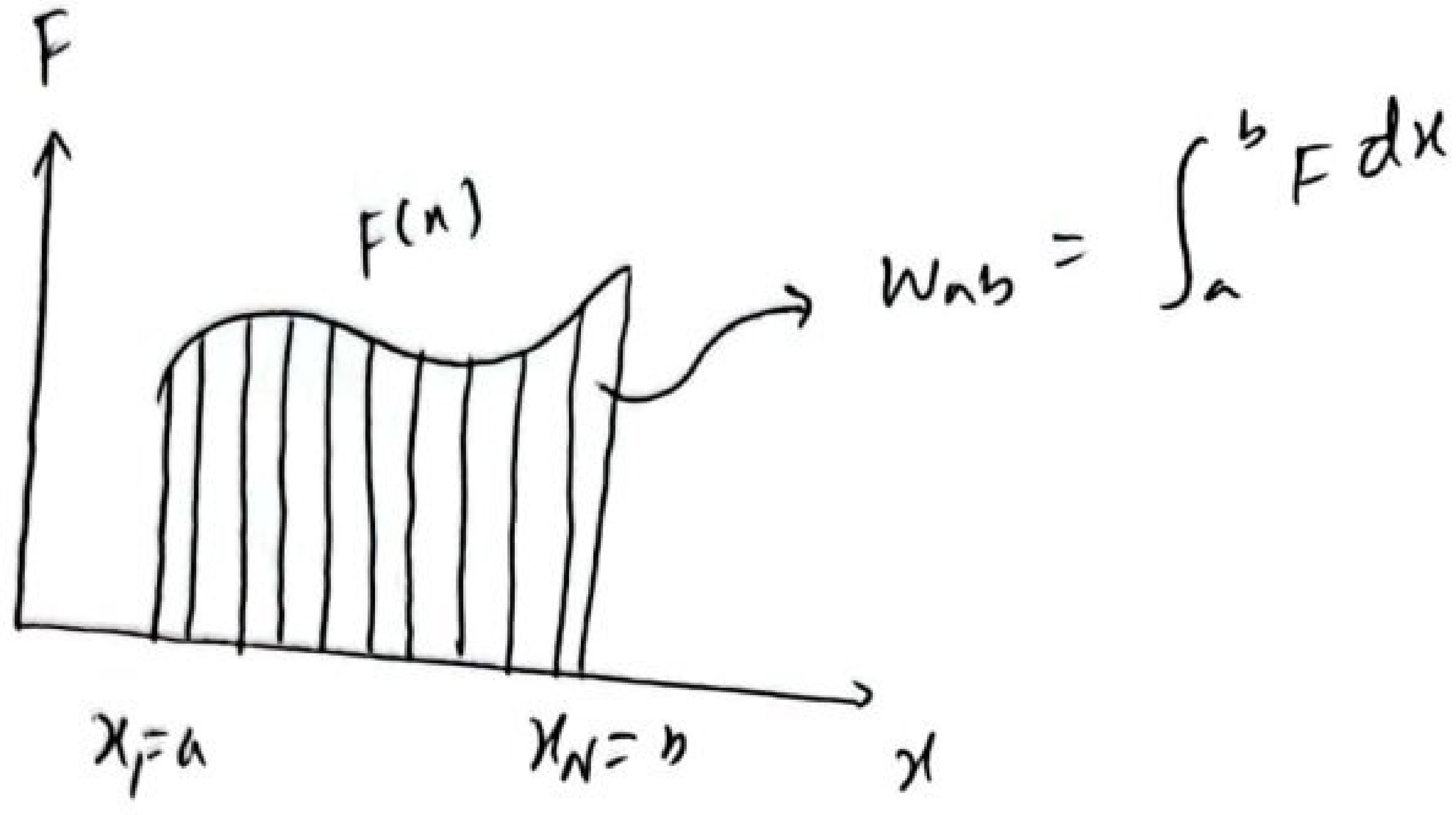
$\boxed{\text{Work done by variable Force}}$ (1D)



Consider a particle is confined along x-axis. This time applied Force may be variable with respect to position x. For simplicity the angle between the force and displacement is always constant (= 0°). If the partice is moved from $x_1 = a$ to $x_N = b$ upon by applied variable force $F(x)$, then the work done by variable Force will be,

$$W_{ab} = F(x_1)\Delta x + F(x_2)\Delta x + \cdots + F(x_{N-1})\Delta x$$

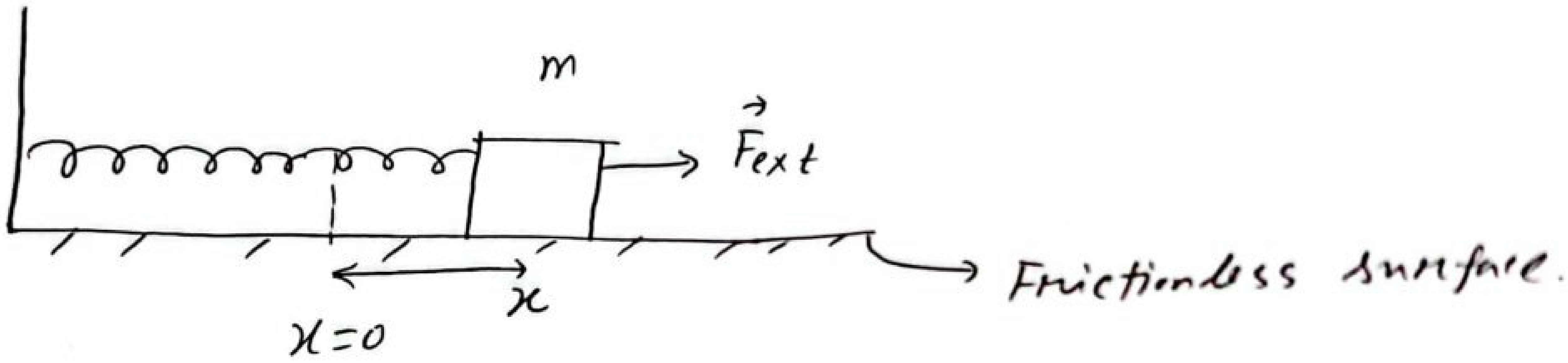
$$= \sum_{i=1}^{N} F(x_i)\Delta x \quad \text{—(1)}$$



If $N \to \infty$, then $\Delta x \to dx$ so the eqn (1) can be written as,

$$W_{ab} = \int_a^b F(x)\,dx$$

Work done by spring Force:



According to Hooke's Law, spring Force,

$$F_s \propto -x$$

$$\Rightarrow F_s = -kx$$

where k is spring constant.

So, Here the force is variable. Work done (From x_i to x_f) done by spring Force can be

$$W^s_{i\to f} = \int_{x_i}^{x_f} F_s\,dx = \int_{x_i}^{x_f} (-kx)\,dx$$

$$W^{s}_{i\to f} = -\left[\frac{1}{2}kx^2\right]_{x_i}^{x_f}$$

$$= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Work done by external Force,

$$W^{ext}_{i\to f} = -W^{s}_{i\to f}$$

$$= \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

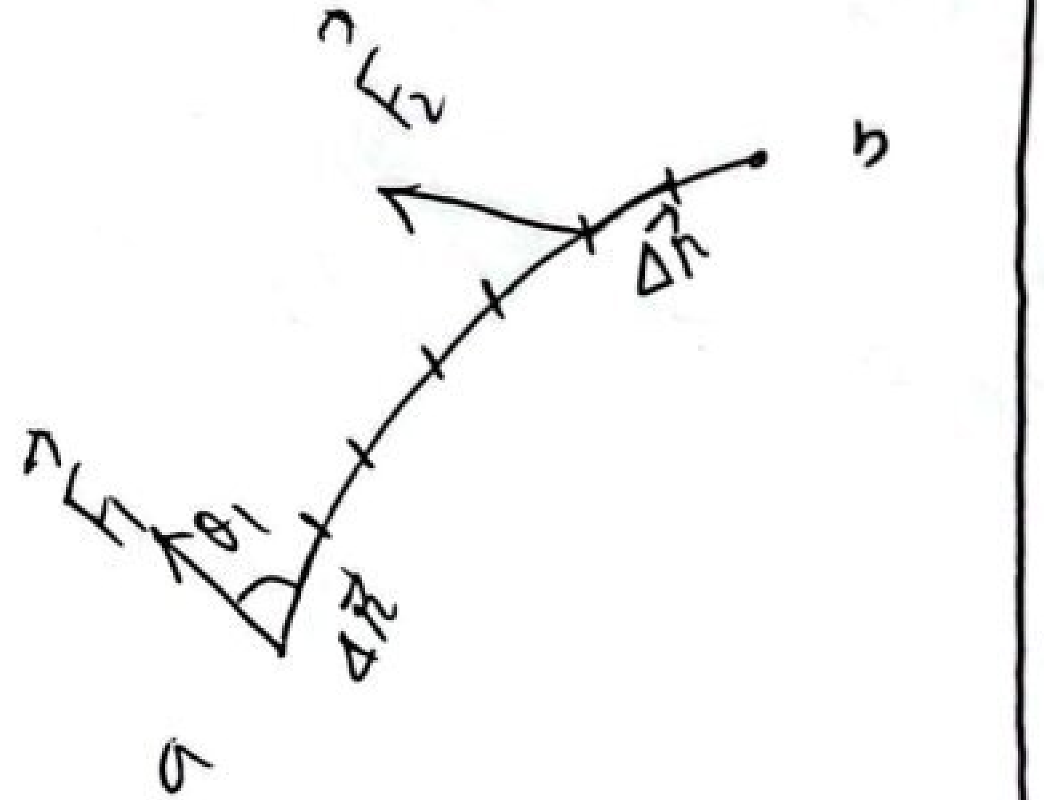
If $x_i = 0$ (equilibrium position) and $x_f = x$

then, Work done by external force here,

$$\boxed{W^{ext} = \frac{1}{2}kx^2}$$

Work done by variable Force: (2D or 3D)

Consider a particle is moving ~~for~~ from a to b under variable force $\vec{F}$.



This time the path is curved(2D) and the angle between $\vec{F}$ and $\Delta\vec{r}$ is not always constant. To find the work done by variable Force $\vec{F}$ upon transition the particle from a to b can be obtained by dividing the path into n equal $\Delta\vec{r}$ intervals.

The work done by $\vec{F}$, can be written as,

$$W_{ab} = W_1 + W_2 + \cdots + W_n$$
$$= \vec{F}_1 \cdot \Delta\vec{r} + \vec{F}_2 \cdot \Delta\vec{r} + \cdots + \vec{F}_N \cdot \Delta\vec{r} = \sum_{i=1}^{N} \vec{F}_i \cdot \Delta\vec{r}$$

If $n \to \infty$, then $\Delta\vec{r} \simeq d\vec{r}$, so we can write down,

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{r} \quad \text{——(1)}$$

But this integral can't be performed only because of the angle betn the force and displacement always changing.

To perform this Integral we have to,

$\vec{F} = F_x\hat{i} + F_y\hat{j}$, and $\vec{r} = x\hat{i} + y\hat{j}$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$$

So the eqn (1) can be written as,

$$W_{ab} = \int_a^b [(F_x\,dx) + [F_y\,dy)]$$

$$W_{ab} = \int_{x_a}^{x_b} F_x\,dx + \int_{y_a}^{y_b} F_y\,dy \quad \text{——(2)}$$

Equation (2) is so-called line integral.

For 3D, $W_{ab} = \int_{x_a}^{x_b} F_x \, dx + \int_{y_a}^{y_b} F_y \, dy + \int_{z_a}^{z_b} F_z \, dz$

[**] Sample problem : 7.08

Applied Force, $\vec{F} = (3x^2 N \hat{i} + 4N \hat{j})$. The particle moves from (2m, 3m) to (3m, 0m).

$W_{ab} = ?$

$$W_{ab} = \int_{x_a}^{x_b} F_x \, dx + \int_{y_a}^{y_b} F_y \, dy$$

$$= \int_2^3 3x^2 \, dx + \int_3^0 4 \, dy$$

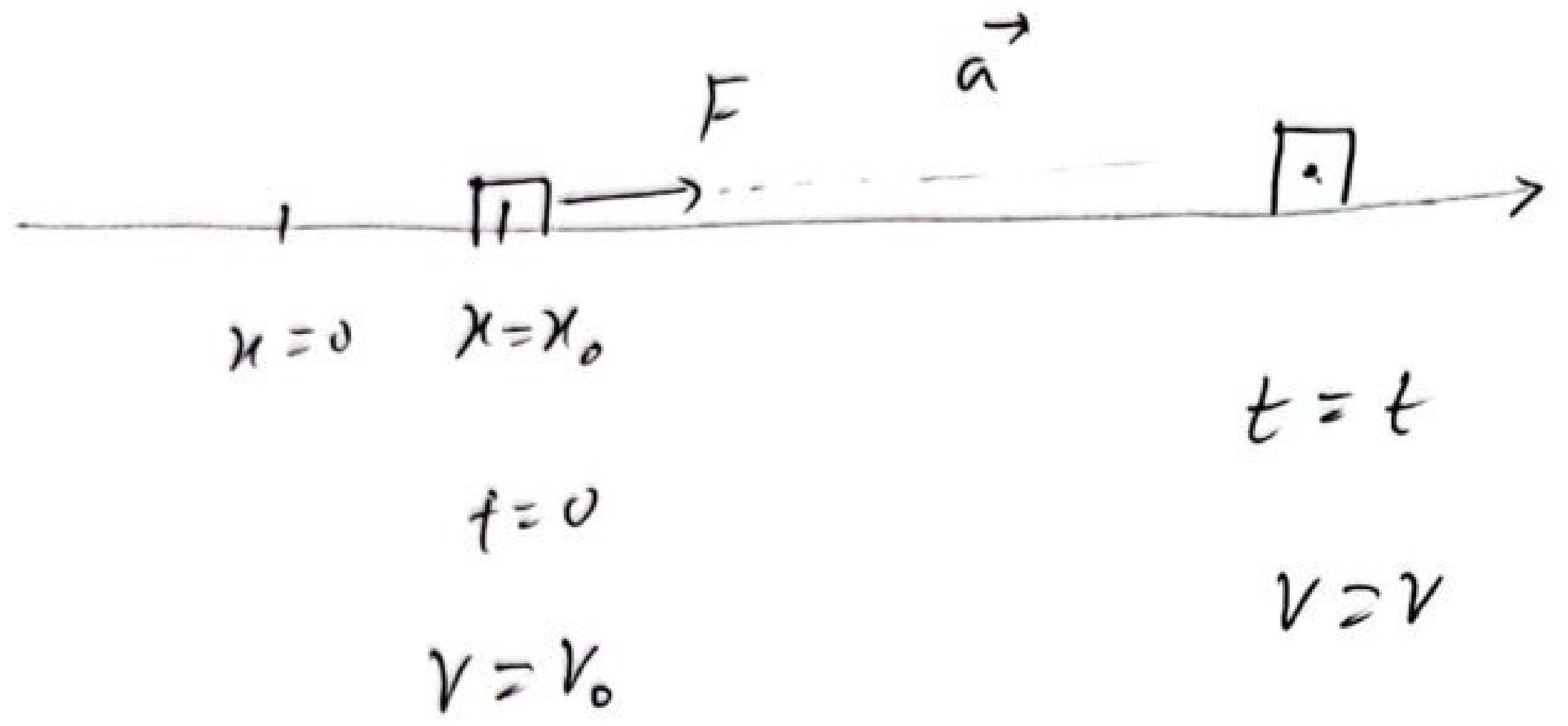
$$= \left([x^3]_2^3 + 4 \cdot [y]_3^0 \right) J$$

$$= [(27 - 8) + 4(-3)] J$$

$$= (19 - 12) J$$

$$= \boxed{7 J}$$

Work – Kinetic Energy Theorem:



Work done by Force,

$$W = \int_{x_0}^{x} F\,dx$$

$$= \int_{x_0}^{x} ma\,dx \qquad \text{[From Newton's 2nd law, } F = ma\text{]}$$

Now, $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} v = v\frac{dv}{dx}$

x	x_0	x
v	v_0	v

So, $$W = m\int_{v_0}^{v} v\frac{dv}{dx}\,dx$$

$$= m\int_{v_0}^{v} v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_{v_0}^{v}$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$= k - k_0 \qquad \text{kinetic energy, } k = \frac{1}{2}mv^2$$

$\boxed{\text{Work-Energy theorem}} \rightarrow \boxed{W = \Delta k}$; $\boxed{\text{unit} \rightarrow J}$

Power : → Work done by force in per unit time.

Average power, $P_{avg} = \frac{W}{\Delta t}$; Δt = time interval

Instantaneous power, $P = \frac{dW}{dt}$

$= \cancel{\frac{d(\vec{F}\cdot d\vec{r})}{}}$

$= \vec{F}\cdot\frac{d\vec{r}}{dt}$

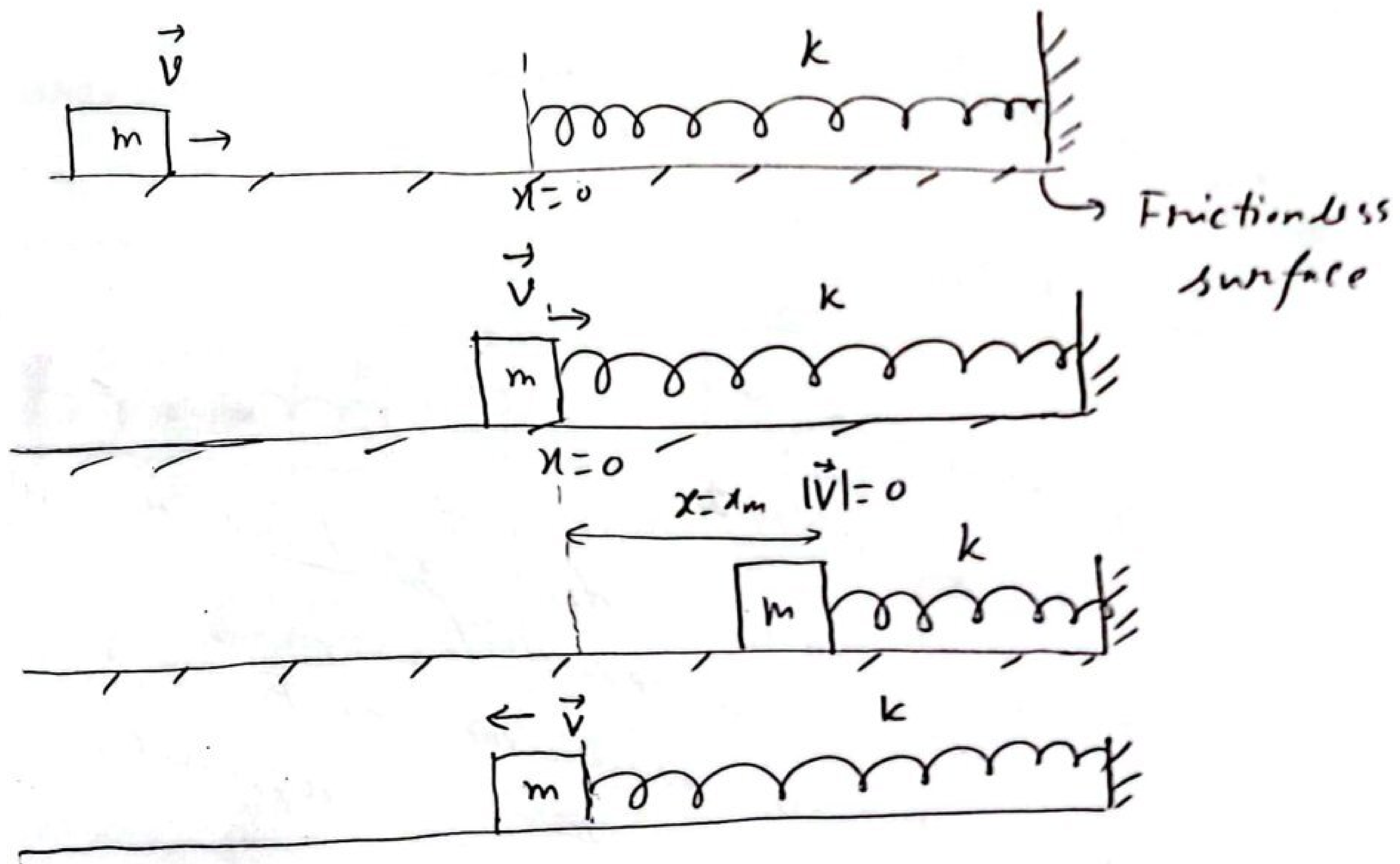
$= \vec{F}\cdot\vec{v}$

$1 \text{ watt} = 1 \text{ J s}^{-1}$

$1 \text{ kW-h} = 10^3 \times 1 \text{ J s}^{-1} \times 3600 \text{ s} = 3.6\times10^6 \text{ J}$

$1 \text{ HP} = 746 \text{ W}$

Conservative Force:



After a round trip, workdone by Force,

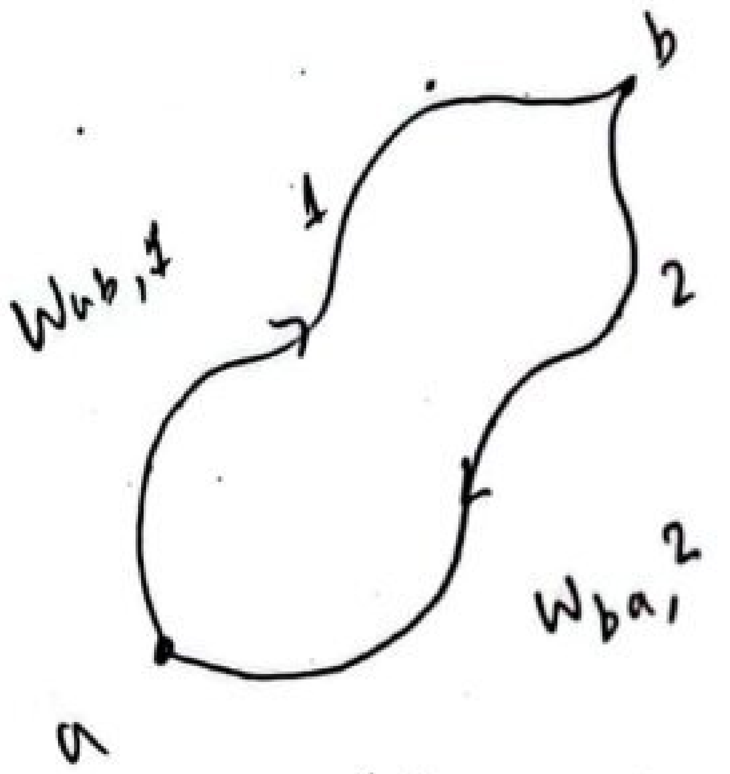
$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv$$
$$= 0$$

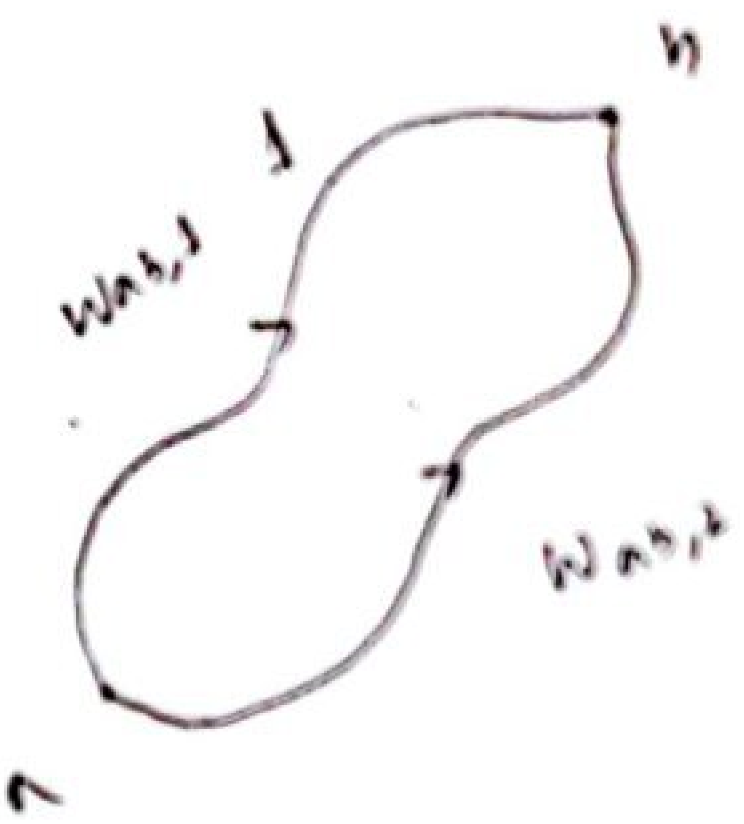
Conservative Force:

→ Work done by Force on a particle after a round trip is zero.

Nonconservative Force:

→ Work don by Force on a particle after a round trip is not zero



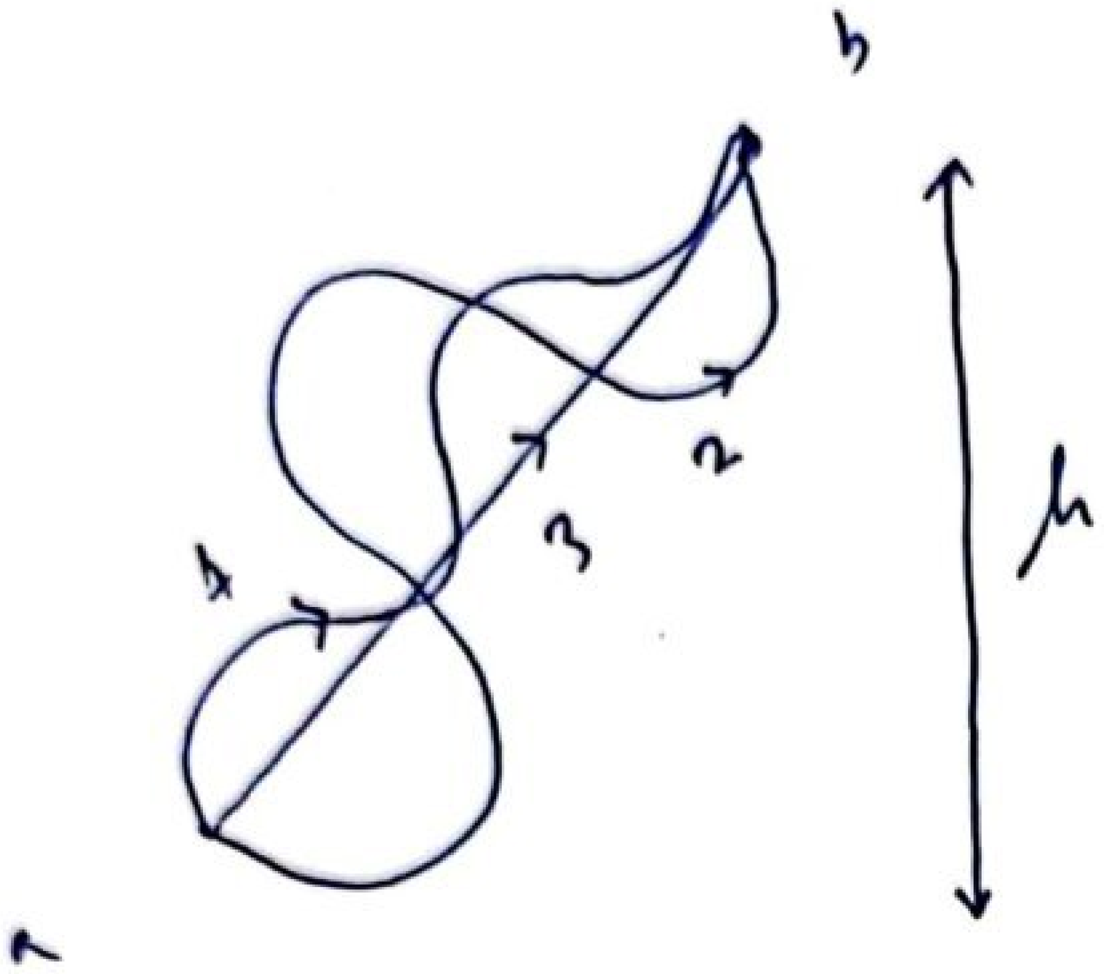


$$W_{ab,1} + W_{ba,2} = 0$$

$$\Rightarrow W_{ab,1} = -W_{ba,2}$$
$$= W_{ab,2}$$

Conservative Force: → Work done by Force is path independent.

Non-Conservative " : → work done by Force is path dependent.



$W_{ab,1} = W_{ab,2} = W_{ab,3} = mgh$

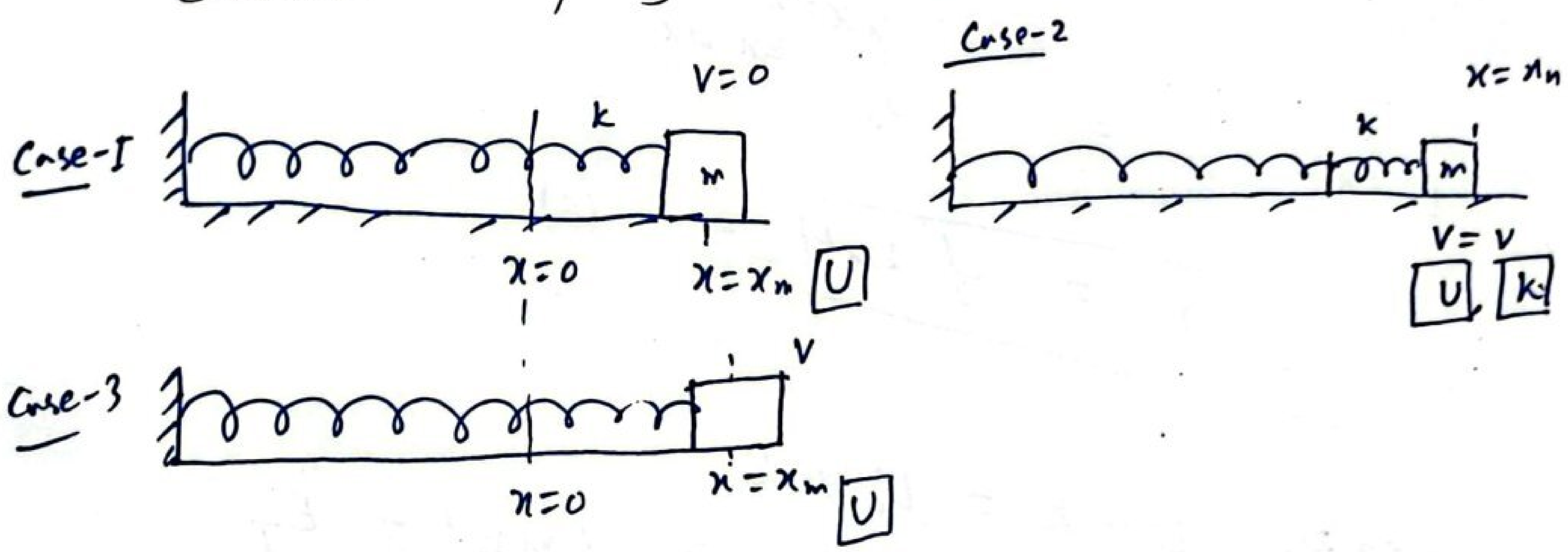
Examples of conservative Force: Spring Force,
Gravitational Force,
Electrostatic Force etc.

Examples of Non-conservative Force: Friction

<u>Potential Energy</u>: * There is no general formula for potential Energy.

Energy of configuration → Potententential Energy.

Consider a spring-block system



$\Delta U + \Delta k = 0$ —(1)

Integrating this eqn: $U + k = \text{Const} = E_{TOT}$ —(2)

Mechanical Energy

Partic

From equation (1),

$$\Delta U = -\Delta k$$

$$= -W$$

$$\Rightarrow \Delta U = -\int_0^x F(x)\,dx \quad \text{--- (3)}$$

Integrating this equation: $\frac{dU}{dx} = -F(x)$ --- (4)

(In 3D) $\vec{F} = -\vec{\nabla} U$ --- (5)

For spring-block system:

$$\Delta U = -\int_{x=0}^{x=x} F(x)\,dx$$

$$\Rightarrow U(x) - U(0) = -\int_{x=0}^{x=x} -kx\,dx$$

$$\Rightarrow U(x) = \int_0^{x=x} kx\,dx$$

$$\therefore \boxed{U(x) = \frac{1}{2}kx^2} \quad \text{--- (6)}$$

From eqn (2), $U + k = U_0 + k_0$

$$\Rightarrow \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = E_T$$

Particle - Earth System:

$y_2 = y \; ; \; U(y_2) = ?$

$y_1 = 0 \; ; \; U(y_1) = 0$

$$\Delta U = -\int_{y_1}^{y_2} F_y \, dy$$

$$= -\int_{y_1}^{y_2} (-mg) \, dy$$

$$= mg \, [y]_{y_1}^{y_2}$$

$$\Delta U = mg\,(y_2 - y_1) \quad \text{—(1)}$$

$$= mg \, \Delta y$$

setting, $y_1 = 0$, $y_2 = y$, $U(y_1) = 0$, $U(y) = ?$

Eqn: (1) $\Rightarrow$

$$U(y) - U(y_1) = mg\,(y - 0)$$

$$\Rightarrow \quad U(y) = mgy \quad \text{... (2)}$$

Problems → [Ch-7 and 8]

$W = \vec{F}.\vec{S}$

$W_{ab} = \int_{x=a}^{x=b} F(x)\, dx$

$W_{ab} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}.d\vec{r}$

$= \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy$

Kinetic Energy, $K = \frac{1}{2} m v^2$

Work - Kinetic Energy Theorem: $W = \Delta K = K_f - K_i$

Power, $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$

Conservative Force, $F(x) = -\frac{dU}{dx}$ (1D)

∴ Potential Energy is function of position (For conservative system)

For conservative system, $\Delta K + \Delta U = 0 \Rightarrow U + K = E_{TOT}$

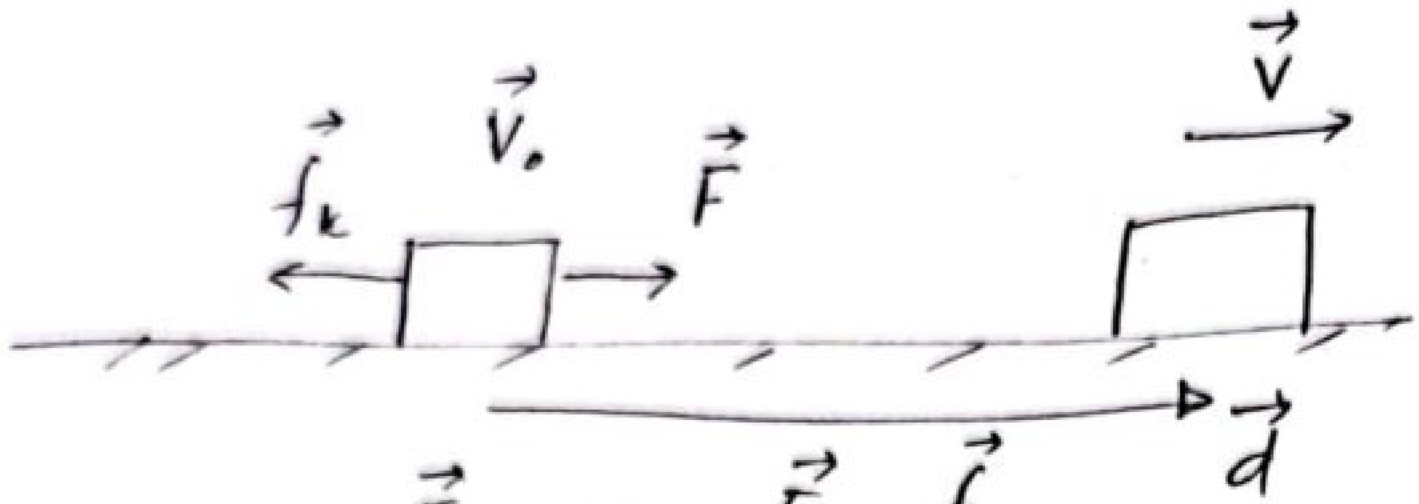
$$\Delta U = - \int_{x_1}^{x_2} F(x)\, dx$$

Conservation of Energy,

$$E_{mec} = K_i + U_i = K_f + U_f$$

Particle - Earth system, Potential Energy, $U(y) = mgy$

Spring - block " , Potential Energy, $U(x) = \frac{1}{2} k x^2$



$$\vec{F}_{net} = \vec{F} - \vec{f_k}$$

For (1D), $F - f_k = ma$

$$v^2 = v_0^2 + 2ad$$

$$\Rightarrow \frac{v^2 - v_0^2}{2} = ad$$

$$\Rightarrow \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = mad$$

$$\Rightarrow \Delta k = (F - f_k) d$$

$$\Rightarrow \Delta k = Fd - f_k d$$

$$\Rightarrow \Delta k + f_k d = Fd$$

$$\Rightarrow Fd = \Delta k + f_k d$$

$$\therefore \boxed{W = \Delta k + \Delta E_{th}}$$

$\Delta E_{th} = f_k d$ = Thermal Energy (produced by friction)

In General : $W = \Delta E_{mec} + \Delta E_{th}$

* Ch-7: SP- [redacted]

CP – 1–3

[redacted]

CP – 1–4

[redacted]

$\boxed{\text{Ch-7}}$

$\boxed{11}$ $|\vec{F}| = 12\,N$

$\vec{d} = (2\hat{i} - 4\hat{j} + 3\hat{k})\ m$

$|\vec{d}| = \sqrt{(2)^2 + (-4)^2 + (3)^2}\ m$

$= 5.39\,m$

(a) when $\Delta k = 30\,J$, $\theta = ?$

(b) when $\Delta k = -30\,J$, $\theta = ?$

(a) According to Work-kinetic Energy theorem,

$$W = \Delta k$$

$$\Rightarrow \vec{F} \cdot \vec{d} = \Delta k$$

$$\Rightarrow |\vec{F}|\,|\vec{d}| \cos\theta = 30$$

$$\Rightarrow 12 \times 5.39 \cos\theta = 30$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{30}{12 \times 5.39}\right) = \boxed{62.37^\circ}$$

(b) $W = \Delta k$

$$\Rightarrow \vec{F} \cdot \vec{d} = \Delta k \Rightarrow |\vec{F}|\,|\vec{d}| \cos\theta = \Delta k$$

$$\Rightarrow \cos\theta = \frac{\Delta k}{|\vec{F}|\,|\vec{d}|}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-30}{12 \times 5.39}\right) = \boxed{117.63^\circ}$$

More problems on ch 7, from 26 no page

Ch-8

6

mass, $m = 0.032$ kg

$h = 5R$, $R = 12$ cm $= 12\times10^{-2}$ m

$= 5\times12\times10^{-2}$ m

(a) $W_g = mg(4R) = 0.032\times9.8\times4\times12\times10^{-2}$ J $= \boxed{0.15\ J}$

(b) $W_g = mg(3R)$

$= 0.032\times9.8\times3\times(12\times10^{-2})$ J

(c) $U = mg(5R)$

(d) $U = mg(R)$

(e) $U = mg(2R)$

(f) same

22 $m = 60$ kg, $H = 20$ m, $\theta = 28°$

(a) $K_i + U_i = K_f + U_f$

$\Rightarrow 0 + mgH = \frac{1}{2}mv^2 + 0$

$\Rightarrow v = \sqrt{2gH}$

$= \sqrt{2\times9.8\times20}$ m/s

$= 19.8$ m/s

$h = \frac{v^2}{2g}\sin^2\theta$

$= \frac{(19.8)^2}{2\times9.8}\times\sin^2(28°)$

$= \boxed{4.4\ m}$

(b) Same.

23

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow mg(h+x) + 0 = \frac{1}{2} k x^2 + 0$$

$$\Rightarrow mgh + mgx - \frac{1}{2} kx^2 = 0$$

$$\Rightarrow 2mgh + 2mgx - kx^2 = 0$$

$$\Rightarrow kx^2 - 2mgx - 2mgh = 0$$

$$\Rightarrow x = \frac{2mg \pm \sqrt{(2mg)^2 - 4k\,2mgh}}{2\cdot k}$$

$$= \frac{2\times 2\times 9.8 \pm \sqrt{(2\times 2\times 9.8)^2 - 4\times 1960\times 2\times 9.8\times 4\times 10^{-2}}}{2\times 1960}$$

$$\therefore x = \boxed{0.1\ m}$$

53

$k = 640$ N/m

$\mu_k = 0.25$

$m = 3.5$ kg, $D = 7.8$ m

(a) $\Delta E_{th} = f_k D = \mu_k F_N D$

$= \mu_k mgD$

$= 0.25\times 3.5\times 9.8\times 7.8$ J

$= \boxed{67\ J}$

(b) $K_{max} = \Delta E_{th} = \boxed{67\ J}$

(c) $\frac{1}{2} kx^2 = K_{max} \Rightarrow x = \sqrt{\frac{2K_{max}}{k}} = \sqrt{\frac{2\times 67}{640}}\ m = \boxed{0.46\ m}$

[57]

$$\frac{1}{2} m v_0^2 = mgh + \Delta E_{th}$$

$$\Delta E_{th} = f_k d$$
$$= \mu_k F_N d$$
$$= \mu_k mgd$$

$$\Rightarrow \frac{1}{2} m v_0^2 = mgh + \mu_k mgd$$

$$\Rightarrow \mu_k mgd = \frac{m v_0^2}{2} - mgh$$

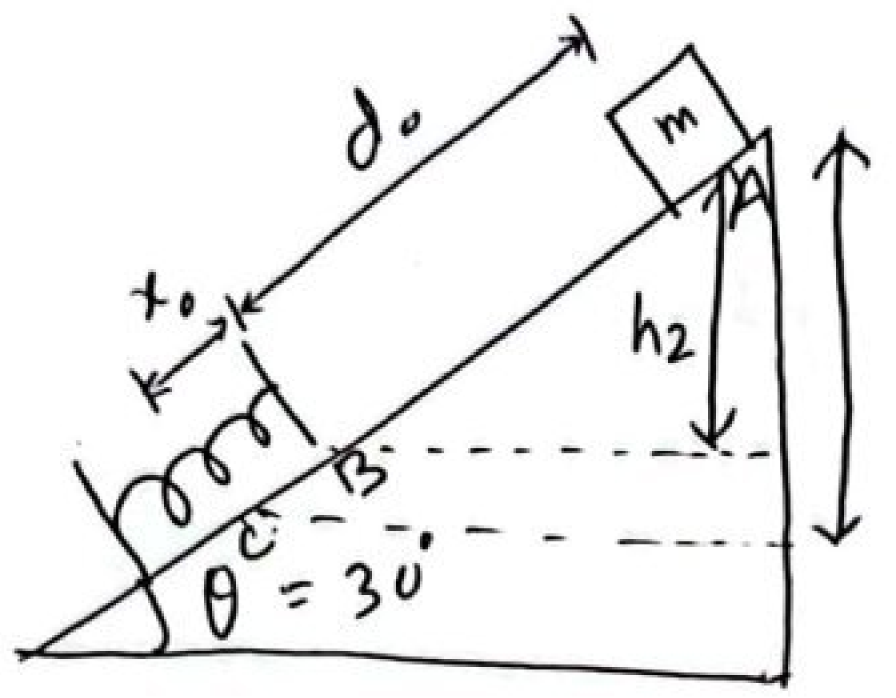
$$\Rightarrow d = \frac{m v_0^2}{2 \mu_k mg} - \frac{mgh}{\mu_k mg}$$

$$= \frac{v_0^2}{2 \mu_k g} - \frac{h}{\mu_k}$$

$$= \left(\frac{(6)^2}{2 \times 0.6 \times 9.8} - \frac{1.1}{0.6} \right) m$$

$$\boxed{\therefore d = \boxed{1.2 m}}$$ Ans

29



$m = 12\ kg$

$K = \frac{F}{x} = \frac{270 N}{0.02 m} = 1.35 \times 10^4 N/m$

$x_0 = 5.5 \times 10^{-2} m$

$d_0 = ?$

(a) According to conservation of Energy,

$$U_A + K_A = U_C + K_B$$

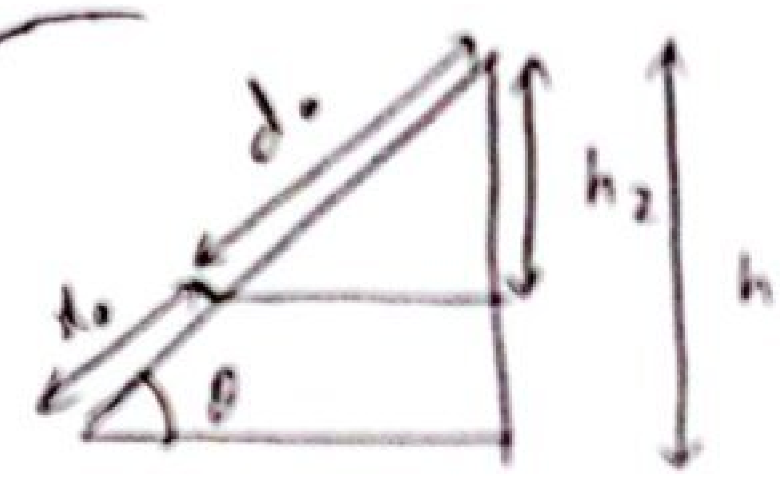
$$mgh_1 + 0 = \frac{1}{2} k x_0^2 + 0$$

$$\Rightarrow mg(x_0 + d_0)\sin\theta = \frac{1}{2} k x_0^2$$

$$\Rightarrow mgx_0 \sin\theta + mg d_0 \sin\theta = \frac{1}{2} k x_0^2$$

$$\Rightarrow mg d_0 \sin\theta = \frac{1}{2} k x_0^2 - mg x_0 \sin\theta$$

$$\Rightarrow d_0 = \frac{\frac{1}{2} k x_0^2 - mg x_0 \sin\theta}{mg \sin\theta}$$



$\sin\theta = \frac{h_1}{x_0 + d_0}$

$\Rightarrow h = (\sin\theta)(x_0 + d_0)$

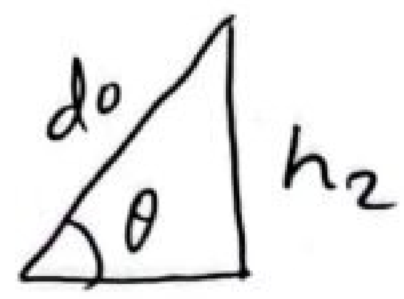
$$= \frac{\frac{1}{2} \times 1.35 \times 10^4 \times (5.5 \times 10^{-2})^2 - 12 \times 9.8 \times 5.5 \times 10^{-2} \sin 30^\circ}{12 \times 9.8 \times \sin 30^\circ} m$$

$$= \boxed{0.292\ m}$$

(b)

$$U_A + K_A = U_B + K_B$$

$$\Rightarrow mgh_2 + 0 = 0 + \frac{1}{2} m v^2$$



$$\Rightarrow g d_0 \sin\theta = \frac{1}{2} v^2$$

$$\sin\theta = \frac{h_2}{d_0}$$

$$\Rightarrow v = \sqrt{2 g d_0 \sin\theta}$$

$$\Rightarrow d_0 \sin 30^\circ = h_2$$

$$= \sqrt{2 \times 9.8 \times 0.292 \times \sin 30^\circ} \text{ m/s}$$

$$\simeq 1.7 \text{ m/s}$$

$\boxed{30}$

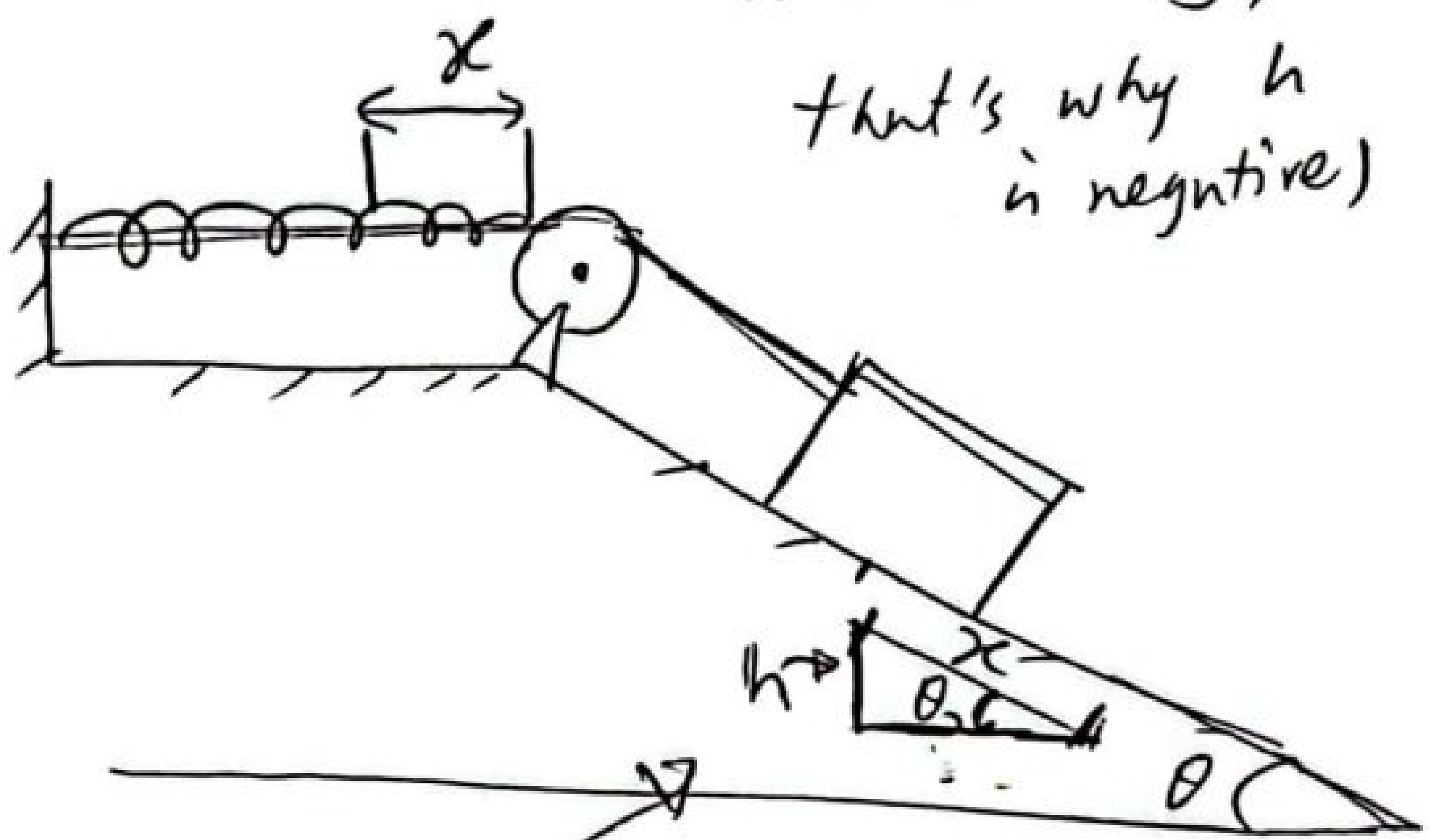
(a) Here the system is conservative, so,

$$K_i + U_i = K_f + U_f \Rightarrow 0 + 0 = K_f + U_f$$

$$\Rightarrow K_f + U_f = 0$$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mg(-h) = 0 \quad \text{——(1)}$$

(gravitational potential energy is decreasing, that's why h is negative)



here, $x = 10\,cm = 10 \times 10^{-2} m$

$m = 2\,kg$

$\theta = 40°$

$k = 120\,N/m$

$$\sin\theta = \frac{h}{x}$$

$$\Rightarrow h = x\sin\theta$$

$$= 10^{-1} \times \sin(40°)\ m$$

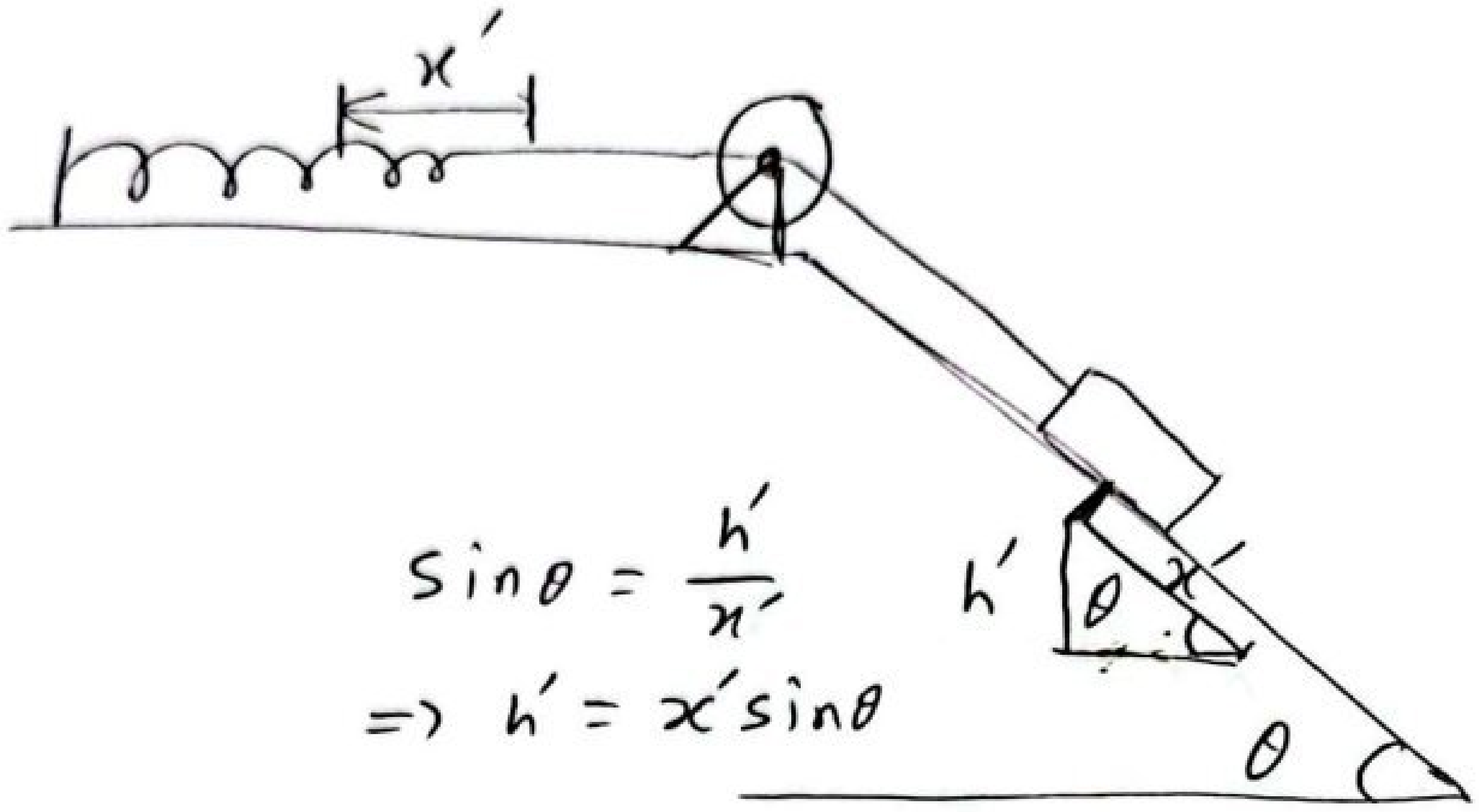
From eqn (1),

$$\frac{1}{2}mv^2 = mgh - \frac{1}{2}kx^2$$

$$\Rightarrow v = \sqrt{\frac{2\left(mgh - \frac{1}{2}kx^2\right)}{m}} = \sqrt{\frac{2 \times \left[2 \times 9.8 \times 10^{-1}\sin(40°) - \frac{1}{2} \times 120 \times (10^{-1})^2\right]}{2}}\ m/s$$

$$= \boxed{0.81\ m/s}$$

(b) $\frac{1}{2} m v'^2 + \frac{1}{2} k x'^2 + mg(-h') = 0$ — (2)



$\sin\theta = \frac{h'}{x'}$

$\Rightarrow h' = x' \sin\theta$

here $v' = 0$,

From eqn (2),

$$\frac{1}{2} k x'^2 - mg x' \sin\theta = 0$$

$$\Rightarrow x' \left(\frac{1}{2} k x' - mg\sin\theta\right) = 0$$

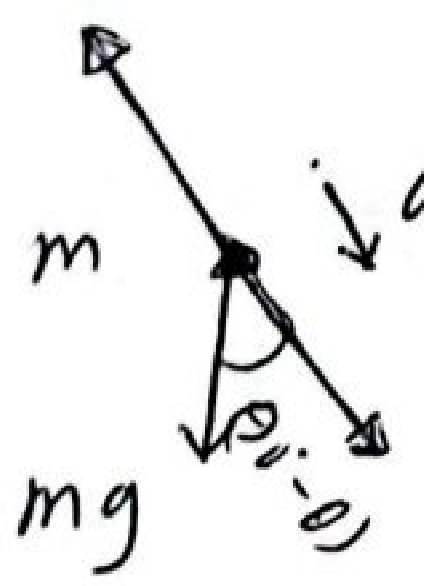
$$\therefore x' = \frac{2 mg\sin\theta}{k}$$

$$= \frac{2 \times 2 \times 9.8 \sin(40^\circ)}{120} \text{ m}$$

$$= 0.21 \text{ m}$$

(c)

$T = kx' = (120 \times .21)\ N = 25.2\ N$



$mg\cos(90^\circ - \theta) = mg\sin\theta$

$(\sin\theta) mg - T = ma$

$$\Rightarrow a = \frac{mg\sin\theta - T}{m}$$

$$= \frac{2 \times 9.8 \times \sin(40^\circ) - 25.2}{2} \ (m/s^2)$$

$$= \frac{-12.6}{2} \ m/s^2 = -6.3 \ m/s^2$$

(d) Direction of the acceleration is upward.

$\boxed{\text{Solve} \rightarrow 31}$ solve it yourself.

$\boxed{36}$

$$\frac{1}{2} k x_{compress}^2 = \frac{1}{2} m v_0^2$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{k}{m}}\, x_{compress}$$

$$v_0 \propto x_{compress}$$

$$\frac{v_{01}}{v_{02}} = \frac{x_{compress,1}}{x_{compress,2}} \quad \text{—— (1)}$$

$$x = v_0 \cos 0^\circ t \qquad \text{and} \quad h = \frac{1}{2} g t^2$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$ so t doesn't depend on v_0.

$\therefore \quad x \propto v_0$

$$\frac{x_1}{x_2} = \frac{v_{01}}{v_{02}} \quad \text{—— (2)}$$

From (1) and (2), $$\frac{x_{compress,1}}{x_{compress,2}} = \frac{x_1}{x_2}$$

$x_1 = (2.20 - 0.27)\,m = 1.93\,m$

$x_2 = 2.20\,m$

$x_{compress,1} = 1.1\,cm$

$x_{compress,2} = ?$

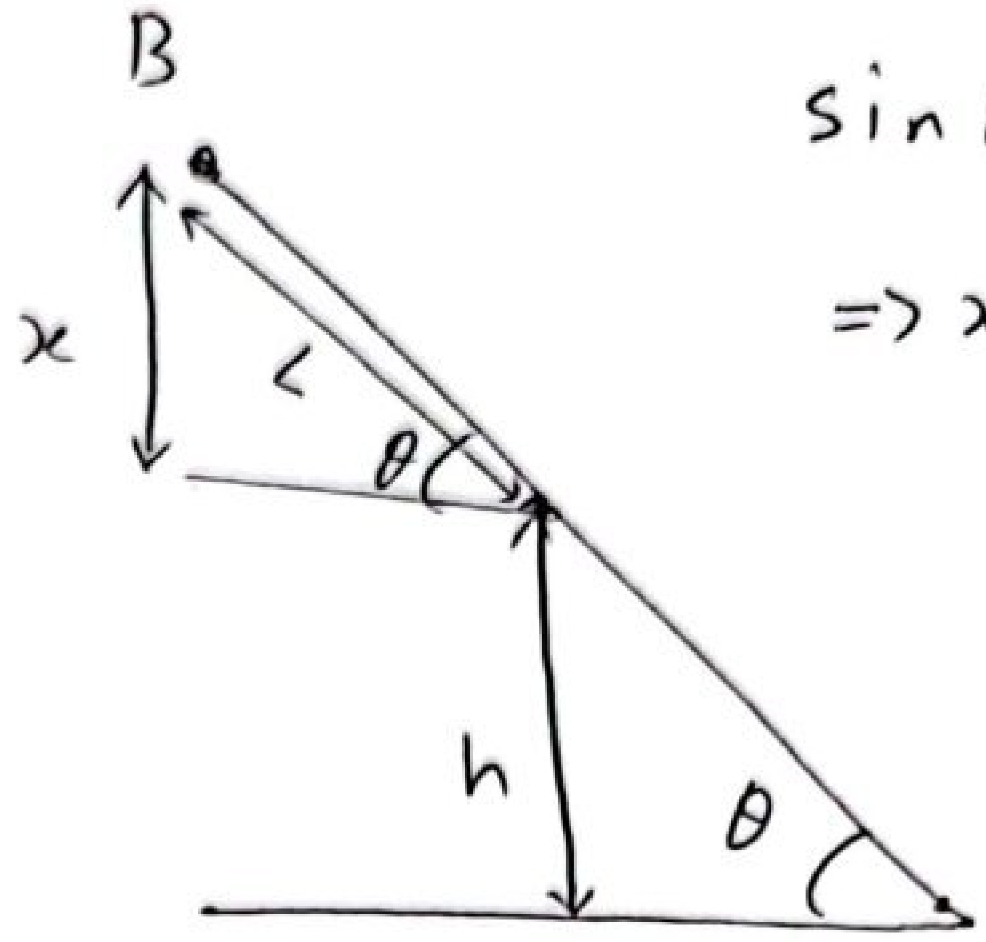
$$\Rightarrow \quad x_{compress,2} = \frac{x_2}{x_1} x_{compress,1}$$

$$= \frac{2.20}{1.93} \times 1.1\,cm$$

$$= \boxed{1.25\,cm}$$

(Ans)

62



$\sin\theta = \frac{x}{L}$

$\Rightarrow x = L\sin\theta$

$= 0.75\sin(30^\circ)$

$= \frac{0.75}{2}$

$$\frac{1}{2}mv^2 = mg(h+x) + \Delta E_{th} + \frac{1}{2}mv'^2$$

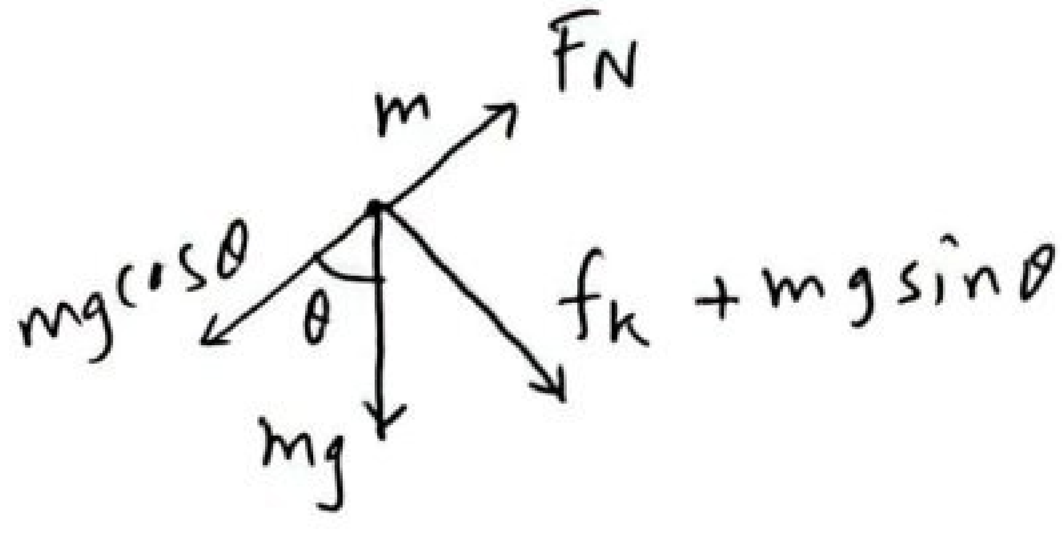
$$\Rightarrow \frac{1}{2}mv'^2 = \frac{1}{2}mv^2 - mg(h+x) - \Delta E_{th}$$

$$\left[\begin{aligned} \Delta E_{th} &= \mu_k F_N L \quad (= f_k L) \\ &= \mu_k mg\cos\theta L \end{aligned}\right]$$

$$\Rightarrow \frac{1}{2}\cancel{m}v'^2 = \frac{1}{2}\cancel{m}v^2 - \cancel{m}g(h+x) - \mu_k \cancel{m}g\cos\theta L \quad \text{——(1)}$$

$$\cancel{\Rightarrow \frac{1}{2}v'^2 = \frac{1}{2}\times 8^2 - g\left(2+\frac{0.75}{2}\right) - 0.4\cos 30^\circ \times (0.75)}$$

$$\cancel{\Rightarrow \frac{1}{2}v'^2 = \frac{64}{2} - 9.8\times\left(2+\frac{0.75}{2}\right) - 0.4\cos(30^\circ)\times 0.75}$$



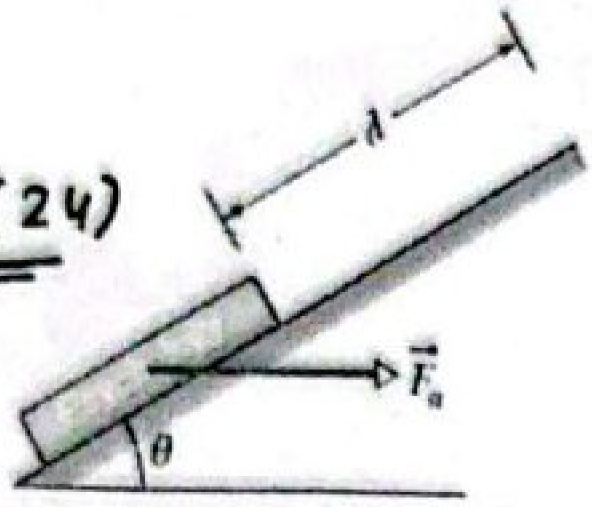
$$\Rightarrow \frac{v'^2}{2} = \frac{8^2}{2} - 9.8\times\left(2+\frac{0.75}{2}\right) - 0.4\times 9.8\cos(30^\circ)\times 0.75$$

$= 6.18$ so, $v'^2 > 0$, so the block can be reached at point B.

The the velocity at the point B, $v' = \sqrt{2\times 6.18}$ m/s

$= \boxed{3.52 \text{ m/s}}$

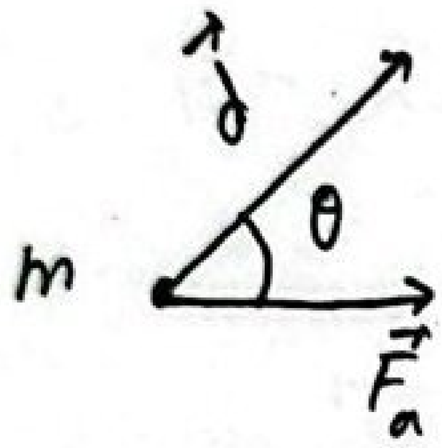
Pr(24)



In Figure, a horizontal force $\vec{F}_a$ of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance $d = 0.500\ m$ up a frictionless ramp at angle $\theta = 30.0°$. (a) During the displacement, what is the net work done on the book by $\vec{F}_a$, the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?

Sol^n:

(a) Work done by $\vec{F}_a$, $\quad W_a = \vec{F}_a \cdot \vec{d}$



$$= |\vec{F}_a||\vec{d}| \cos\theta$$
$$= (20\,N)(0.5m)(\cos(30°))$$
$$= 8.66\ J$$

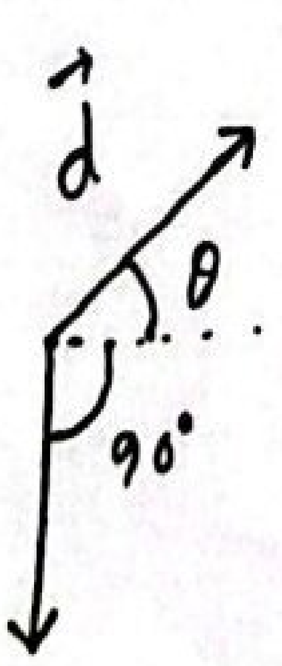
$|\vec{F}_a| = 20N$

$m = 3kg$

$|\vec{d}| = 0.5\ m$

Work done by $\vec{F}_g$, $\quad W_g = \vec{F}_g \cdot \vec{d}$

$$= |\vec{F}_g|\,|\vec{d}| \cos(90° + \theta)$$



$$= -mg\ d \sin\theta$$
$$= -(3\,kg)(9.8\,m/s^2)(0.5m)\sin(30°)$$

$$= -\cancel{14.7} = -7.35\ J$$

Work done by $\vec{F}_N$, $\quad W_N = \vec{F}_N \cdot \vec{d}$

$$= |\vec{F}_N||\vec{d}| \cos(90°) = 0\,J$$

So, Net work done by all forces,

$$W = W_a + W_g + W_N$$

$$= (8.66 - 7.35 + 0)\ J$$

$$= \boxed{1.31\ J}$$

(b) Initial kinetic energy, $K_i = 0$

Final " " , $K_f = \frac{1}{2} m v_f^2$,

where v_f is the final ~~velo~~ speed of the book.

According to work-kinetic energy theorem,

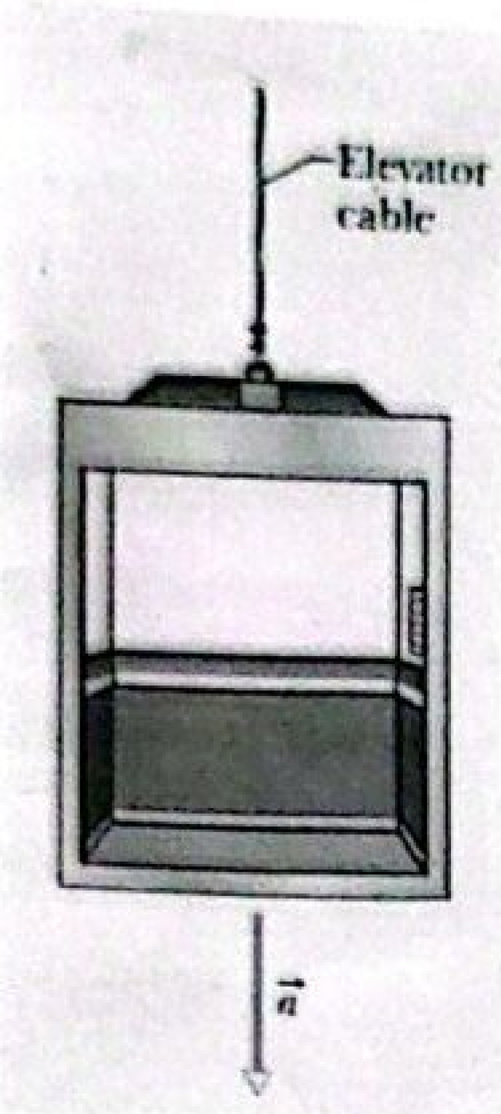
$$W = \Delta K = K_f - K_i$$

$$\Rightarrow 1.31 = \frac{1}{2} m v_f^2 - 0$$

$$\Rightarrow v_f = \sqrt{\frac{2 \times 1.31}{m}}$$

$$= \sqrt{\frac{2 \times 1.31}{3}}\ m/s$$

$$= \boxed{0.935\ m/s}$$



Sample Problem 7.05: An elevator cab of mass $m = 500\,\text{kg}$ is descending with speed $v_i = 4.0\,\text{m/s}$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$.

(a) During the fall through a distance $d = 12\,\text{m}$, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

(c) What is the net work W done on the cab during the fall?

(d) What is the cab's kinetic energy at the end of the 12 m fall?

Solⁿ:

(a) Here, mass of the elevator cab, $m = 500\,kg$

distance, $d = 12\,m$

Work done by gravitational force, $W_g = ?$

$$\therefore \quad W_g = F_g\, d \cos\theta \quad ; \quad \theta = 0^\circ$$

$$= mg\, d \cos 0^\circ$$

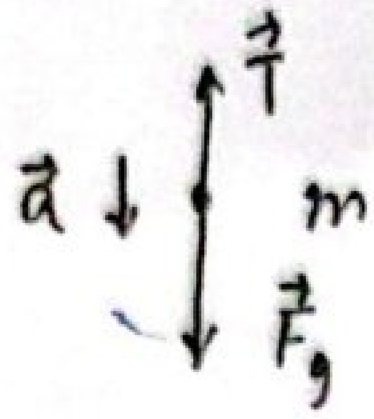
$$= (500 \times 9.8 \times 12 \times 1)\ J$$

$$= 58800\ J$$

$$= \boxed{5.88 \times 10^4\ J}$$

$|\vec{a}| = \frac{|\vec{g}|}{5}$

(b)



$$F_g - T = ma$$

$$\Rightarrow T = F_g - ma$$

$$= mg - ma$$

$$= m(g-a)$$

$$= m\left(g - \frac{g}{5}\right)$$

$$\boxed{\therefore T = \frac{4mg}{5}}$$

Work done by Tension force,

$$W_T = \vec{T}\cdot\vec{d}$$

$$= |\vec{T}|\,|\vec{d}|\cos(180^\circ)$$

$$= \frac{4}{5} mg\, d\,(-1)$$

$$= \frac{4}{5} \times 500 \times 9.8 \times 12 \times (-1)\ \text{J}$$

$$= \boxed{-4.7\times10^4\ \text{J}}$$

(c) The net work done by the acting forces on the cab,

$$W = W_g + W_T$$

$$= (5.88\times10^4 - 4.7\times10^4)\ \text{J}$$

$$= 1.18\times10^4\ \text{J}$$

(d) Initial Kinetic Energy, $K_i = \frac{1}{2} m v_i^2$

$$= \frac{1}{2}\times 500 \times (4)^2\ \text{J}$$

$$= 4\times10^3\ \text{J}$$

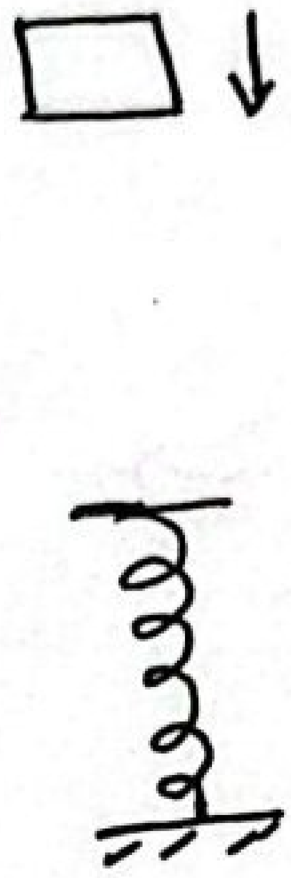
Final kinetic energy $K_f = ?$

According to work-kinetic Energy theorem, $W = \Delta K = K_f - K_i$

$$\Rightarrow K_f = W + K_i = (1.18\times10^4 + 4\times10^3)\ \text{J}$$

$= 1.58\times10^4$ J

62



$m = 250\ g$

$= 250 \times 10^{-3}\ kg$

$K = 2.5\ N/cm$

$= \dfrac{2.5\ N}{10^{-2}\ m}$

$= 2.5 \times 10^{2}\ N m^{-1}$

$x_{compressed} = 12\ cm = 0.12\ m$

During compression,

(a) $W_g = ?$

$$W_g = \vec{F_g} \cdot \vec{d}$$
$$= mgd \cos 0^\circ$$
$$= mg\, x_{compressed}$$
$$= (250 \times 10^{-3} \times 9.8 \times 0.12)\ J$$
$$= \boxed{0.29\ J}$$

(b) $W^{s}_{i \to f} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$

$\begin{cases} x_i = 0\ m \\ x_f = x_{compressed} = 0.12\ m \end{cases}$

$W^{s}_{i \to f} = -\frac{1}{2} \times 2.5 \times 10^{2} \times 0.12^{2}\ J$

$= \boxed{-1.8\ J}$

(c) $v_i = ?$, $v_f = 0\ m/s$

Work-kinetic energy theorem, $W_{net} = K_f - K_i$

$\Rightarrow W_g + W^{s}_{i \to f} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$$\Rightarrow 0.29 - 1.8 = \frac{1}{2} \times 0.25 \times 0^2 - \frac{1}{2} \times 0.25 \times v_i^2$$

$$\Rightarrow -1.51 = -\frac{1}{2} \times 0.25 \times v_i^2$$

$$\Rightarrow v_i = \sqrt{\frac{2 \times 1.51}{0.25}} \text{ m/s}$$

$$= \boxed{3.5 \text{ m/s}}$$

(d) $v_i' = 2v_i = 2 \times 3.5$ m/s

$= 7$ m/s

$v_f' = 0$ m/s , $x_i' = 0$

$x_f' = x'_{compressed} = ?$

$$W_{net} = K_f' - K_i'$$

$$\Rightarrow W_g + W^s_{i \to f} = 0 - \frac{1}{2} m v_i'^2$$

$$\Rightarrow mg x'_{compressed} + \frac{1}{2} k x_i'^2 - \frac{1}{2} k x_f'^2 = -\frac{1}{2} m v_i'^2$$

$$\Rightarrow mg x'_{compressed} + 0 - \frac{1}{2} k x'^2_{compressed} = -\frac{1}{2} m v_i'^2$$

$$\Rightarrow mg x'_{compressed} - \frac{1}{2} k x'^2_{compressed} + \frac{1}{2} m v_i'^2 = 0$$

$$\Rightarrow k x'^2_{compressed} - 2mg x'_{compressed} - m v_i'^2 = 0$$

$$\Rightarrow \quad x'_{compressed} = \frac{2mg \pm \sqrt{4m^2g^2 - 4 \cdot k \cdot (-mv_i'^2)}}{2 \cdot k}$$

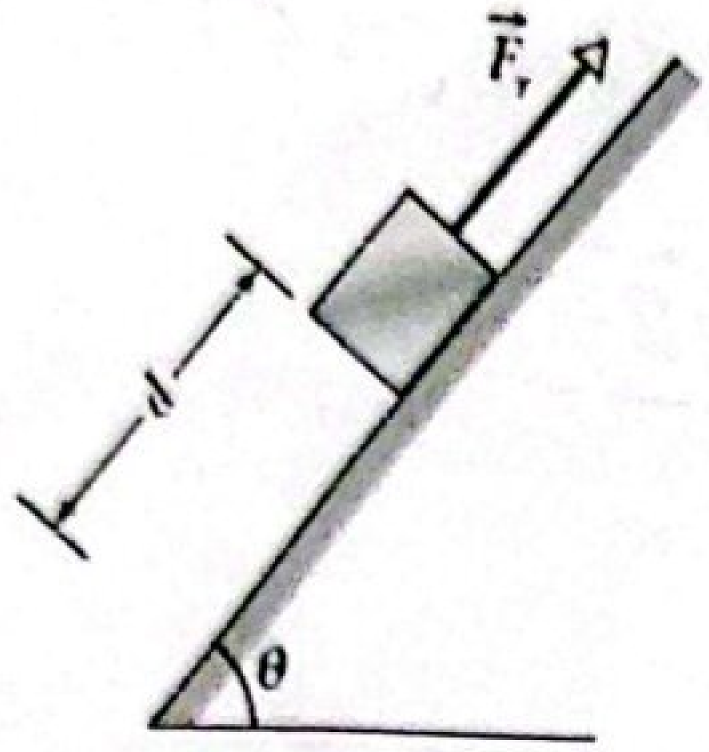
$$= \frac{2mg \pm 2\sqrt{m^2g^2 + mkv_i'^2}}{2k}$$

$$= \frac{mg \pm \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

putting the value of m, g, v_i' and k

$$x'_{compressed} = \boxed{0.23 \text{ m}}$$

Pr-19



Problem 19: In Figure, a block of ice slides down a frictionless ramp at angle $\theta = 50^\circ$ while an ice worker pulls on the block (via a rope) with a force $\vec{F}_r$ that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?

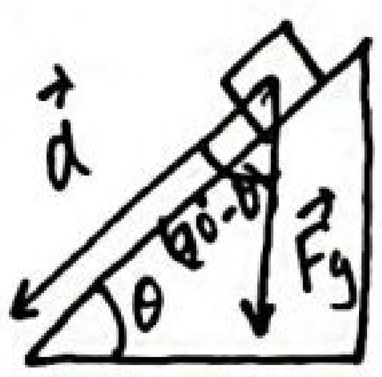
Sol^n:

Net work done on the block by all forces,

$$W = W_g + W_r$$

where, work done by gravitational force,

$$W_g = \vec{F}_g \cdot \vec{d}$$
$$= |\vec{F}_g||\vec{d}| \cos(90^\circ \cancel{+} - \theta)$$
$$= mgd\sin\theta \quad \text{—(1)}$$



Work done by $\vec{F}_r$, $W_r = \vec{F}_r \cdot \vec{d}$

$$= |\vec{F}_r|\,|\vec{d}| \cos(180^\circ)$$
$$= -|\vec{F}_r|\,|\vec{d}| \quad \text{—(2)}$$

According to work-kinetic energy theorem,

$$W = \Delta K$$
$$\Rightarrow W_g + W_r = \Delta K \quad \text{—(3)}$$

In equation (3), W_g is positive and W_r is negative.

If the rope had not been attatched to the block, then $|\vec{F}_r| = 0$, So, $W_r = 0$.

So according to equation (3), the kinetic energy would be ~~increased~~ greater $F_r d = (50)(0.5)$ J amount.

$= \boxed{25\ J}$

Solve:

***

S.P → 7.02, 7.03, 7.04, 7.06

C.P → 1, 2

Pr → 14, 15, 25, 58.