1 Introduction

k-means is a clustering algorithm that partitions a set of data points into k classes.

Suppose we have a set of data points $\{\mathbf{x}_i\}_{i=1}^n$ where each data point is a vector of generally continuous values $\mathbf{x}_i = [x_i, \ldots, x_p]$. Note that k-means works best with continuous data because it updates clusters with Euclidean distance.

An approximate approach is given by:

(i) Randomly assigning cluster centers $\hat{\mu}_1, \ldots, \hat{\mu}_M$. Generally, we these to be points in the training data.

(ii) Determine which of the clusters $R_1, \ldots, R_M$ each data point $\mathbf{x}_i$ belongs to by computing the closest cluster center $\hat{\mu}$.

(iii) Update the cluster centers $\hat{\mu}_m$ as the average of all $\mathbf{x}_i \in R_m$.

This process is iterated until we reach some max number of iterations or a convergence threshold where the change in cluster centers is deemed negligible.

2 Algorithm

Algorithm 1 k-Means Clustering

Input: Data set $\mathcal{T} = \{\mathbf{x}_i\}_{i=1}^n$, max iterations s, stopping threshold ϵ (optional)
Output: Set of k clusters

function k-MEANS($\mathcal{T}, s, \epsilon,$)
    $\hat{\mu}_i \leftarrow$ random $\mathbf{x}_p \in \mathcal{T}$ for $p = 1, \ldots, k$
    $t \leftarrow 1$
    repeat
        $R_j \leftarrow \emptyset$ for $j = 1, \ldots, k$
        for $\mathbf{x}_i \in \mathcal{T}$ **do**
            $j^* \leftarrow \arg\min_i \mid\mid x_j - \hat{\mu}_i \mid\mid^2$ (assign $\mathbf{x}_j$ to the closest cluster center)
            $R_{j^*} \leftarrow R_j \cup \mathbf{x}_j$
        end for
        for $i = 1, \ldots, k$ **do**
            $\hat{\mu}_i \leftarrow \frac{1}{|R_i|} \sum_{\mathbf{x}_j \in R_i} \mathbf{x}_j$ (update cluster centers to average of points in cluster)
        end for
    until $t = s$ or largest change in cluster center less than ϵ
end function