EART97051 Environmental Data (EDSML) - Earth Observation & Remote Sensing

3 Algebraic operations & spectral indices

Dr P. J. Mason

Department of Earth Science & Engineering
Imperial College London, SW7 2AZ, UK
E-mail: p.j.mason@ic.ac.uk

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Lecture Plan

1. Reflectance spectroscopy & spectral processing
2. General formulae for multi-band point operations:
   - Image addition, subtraction, multiplication & division
   - Formulation & uses plus examples
3. Development of spectral indices
   - Formulation & uses plus examples
4. Summary
5. Exercise 3

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2. Spectroscopy & image spectral profiles

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Causes of spectral behaviour

Physical

- Scattering effects
  - Diffuse and/or specular reflection
  - Volume and/or surface scattering
  - Single and/or multiple scattering
  - Wavelength, particle & surface – dependent

Chemical

- Refractive index (ratio of the speed of light through one material to that through another)
  - Reflectance, transmission
  - Wavelength-dependent
- Absorption coefficient
  - Chemical composition & crystallography
  - Electronic and/or vibrational processes
  - Wavelength-dependent

Hunt (1977)

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Sub-atomic processes causing spectral absorption

i. Electronic transitions occurring in VIS & NIR

- **a) Crystal Field effects (sub-atomic)**
- **b) Charge Transfer effects (sub-atomic)**
- c) Conduction bands
- d) Colour centres

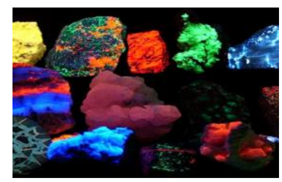
ii. Vibrational transitions occurring in SWIR & TIR

- **Bond stretch, bending & rotation**

iii. Fluorescence (UV)

- High energy required – all UV absorbed by atmosphere so only in laboratory conditions!

All are wavelength dependent



In all cases, absorbed energy is **later emitted at longer wavelength** than it is absorbed

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Reflectance spectroscopy how to exploit spectral profiles

1. Three forms of interaction when energy is incident (I) upon a material:
   - Selective **Reflection (R), Absorption (A) and Transmittance (T)**
   - Proportions of R, A & T depend on the **wavelength of the energy and the chemistry of the material (and condition of its surface).**
2. Later photons may also be **Emitted (E)** from a surface
   - Also wavelength and chemistry dependent
3. By **measuring the energy reflected (and/or emitted)** by targets on the Earth's surface **over a variety of different wavelengths**, we can build up a spectral response or ***signature*** for a particular target substance

See Hunt (1977), Farmer (1974), Hunt (1982); Clark and Roush (1984), Salisbury (1993)

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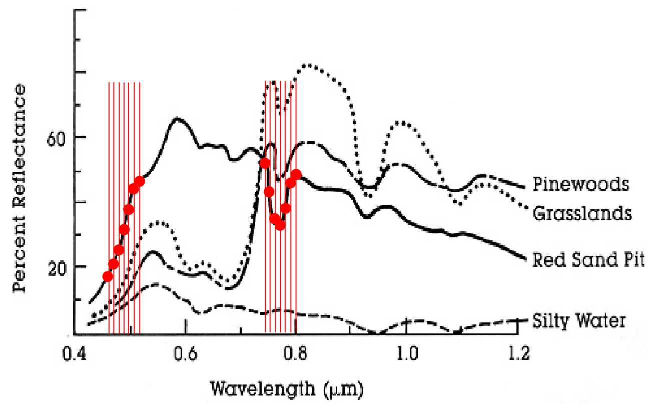
4. By comparing the signatures of different target materials we can distinguish between them, where we might not be able to, if we compared them at a single wavelength (band) only

Spectral signatures of the common ground objects



5. Knowing where to 'look' spectrally and understanding the factors which influence the spectral response are critical to correctly interpreting the contributions of significant different materials to that signature
   - Fundamental to understanding the appearance of features in multispectral images

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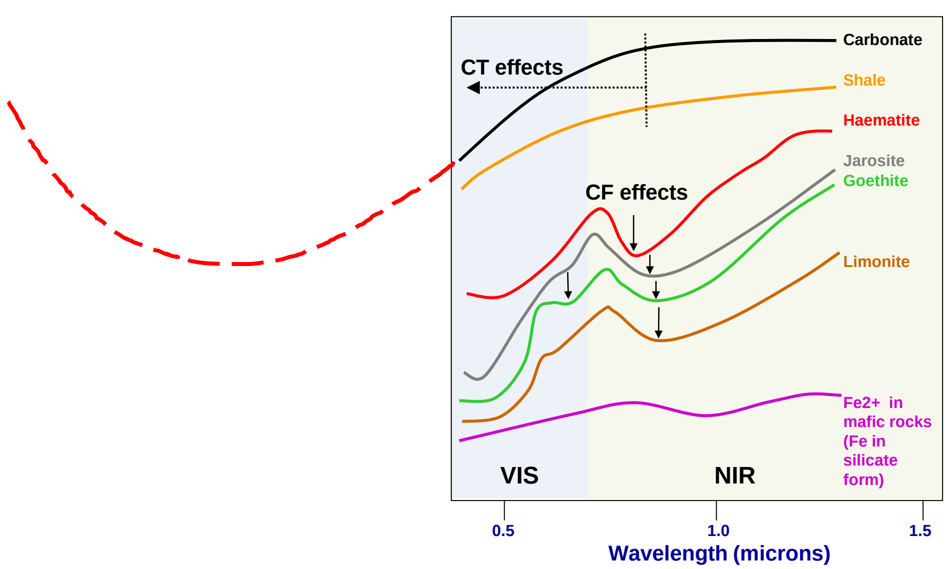
Example of spectra from rock-forming minerals and rocks



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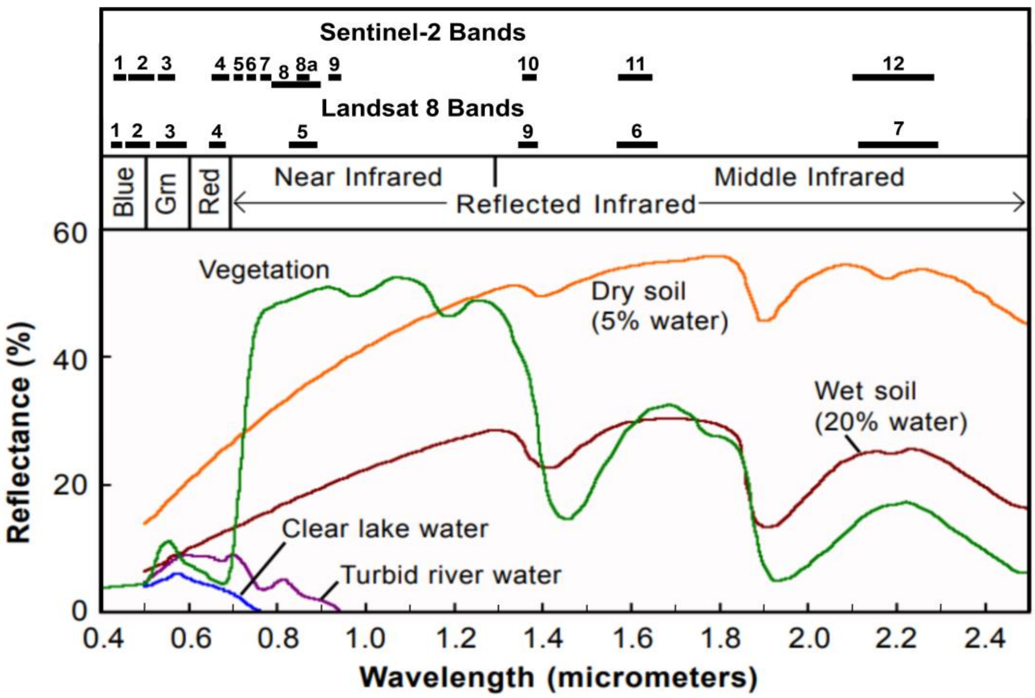

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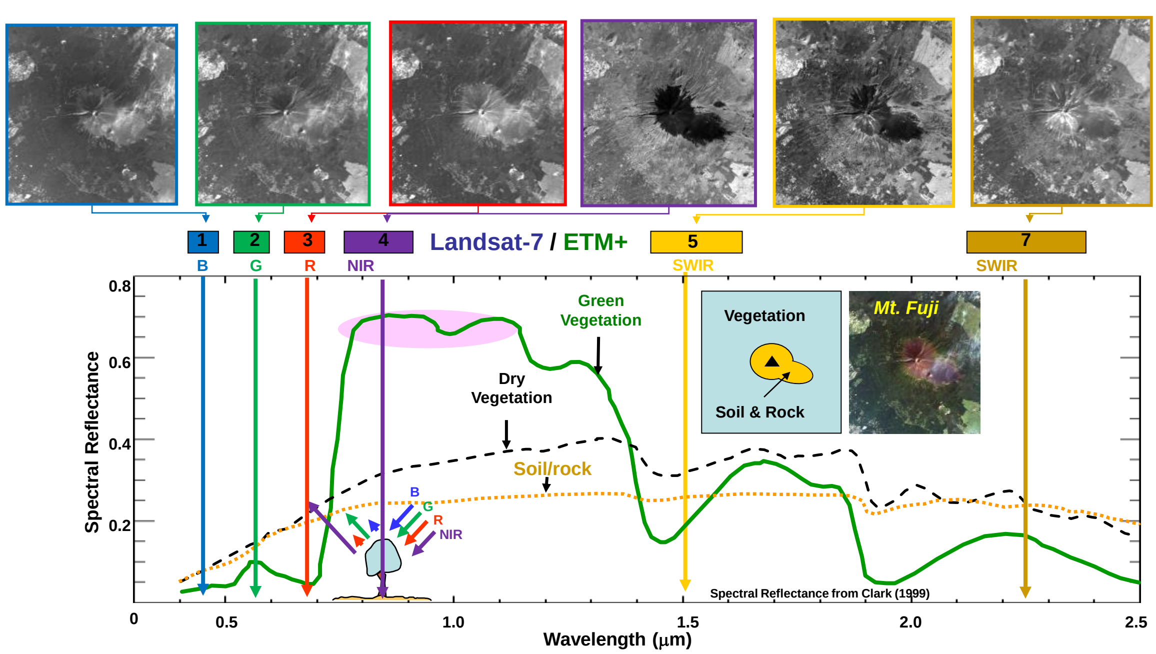
Understanding single-band (monochromatic) images of multispectral datasets



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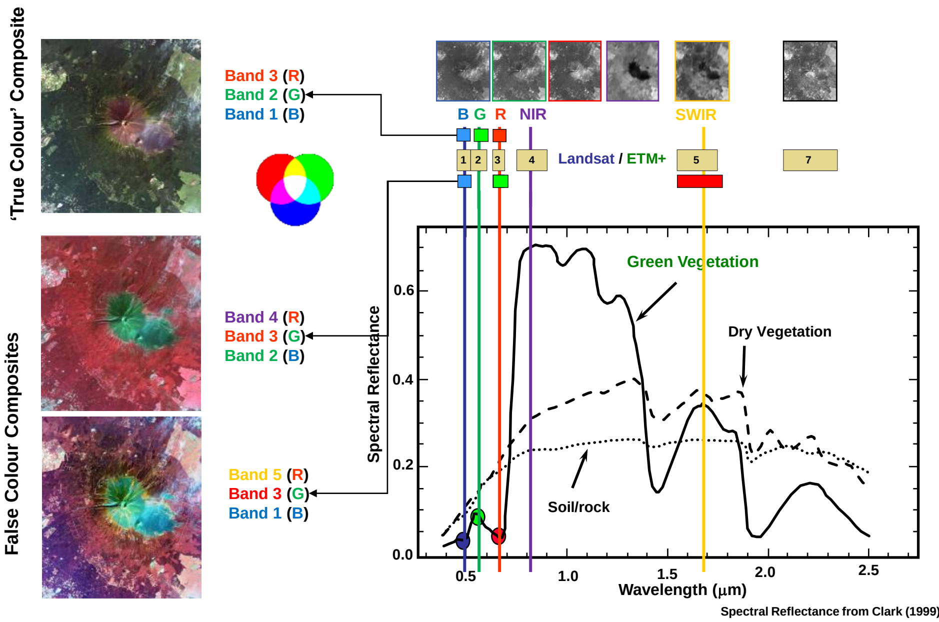
Understanding colour composite images from multispectral datasets



- Multispectral (or 'Broad-band') sensors have a few bands with broad bandwidths defined in microns
- Spectral bands are positioned in atmospheric windows
- Spatial resolutions vary from 10s metres to < 1 m
- Examples: Landsat, Sentinel-2 and most VHR sensors*

*but only WV-3 provides bands beyond VIS-NIR

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2. Algebraic Operations (Multi-band point operations)

- For multi-spectral bands (multi-layer images) algebraic operations, i.e. using the 4 basic arithmetic operations (+, -, x, /), logarithmic, exponential, sin, tan, *etc.*, **can be applied to the DNs of different bands, at each pixel to produce a new image**. Such processing is called an **image algebraic operation**.
- Algebraic operations are performed on each pixel, among DNs of spectral bands (or layers), without involving neighbourhood pixels. They can therefore be considered as **multi-band point operations** defined as:

$$y = f(x_1, x_2, \cdots x_n)$$ where n is the number of bands or layers.

- Obviously, all images involved in algebraic operations should be **precisely co-registered**. As the image algebraic operation is entirely pixel-to-pixel based, we can generalise the description in the following sections: let $X_i, i = 1, 2, \cdots, n$ represent both the i^{th} band image, and any pixel in the i^{th} band image belongs to an n band image dataset $\mathbf{X}$, where $X_i \in \mathbf{X}$, and Y is the output image as well as any pixel in the output image.
- NB Unlike contrast enhancement, **algebraic operations are position relevant**; they must be performed on a per-pixel basis.

Chapter 3, Algebraic operations, Liu & Mason, Image Processing & GIS for remote sensing, Wiley, 2016

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2.1 Image addition

This operation produces a weighted summation of two or more images:

$$Y = \frac{1}{k}\sum_{i=1}^{n} w_i X_i$$

where w_i is the weight of image X_i and k is a scaling factor.

- Important applications of image addition are **noise reduction and increasing the signal-to-noise ratio (SNR).**
- For example, if each image band of an n band multi-spectral image is contaminated by an additive but random noise source, the noise pixels are unlikely to occur at the same positions in each bands and thus a noise pixel DN in band i will be averaged with the non-noise DNs in the other n-1 bands.
- As a result, random noise will be largely suppressed. For n duplications of an image each contaminated by a same level of random noise, then the SNR of the summation image of these n duplications is:

$$SNR_y = \sqrt{n} \cdot SNR_i$$

- This implies that for n-band multi-spectral imagery, the summation of all the bands can increase SNR by about $\sqrt{n}$ times.

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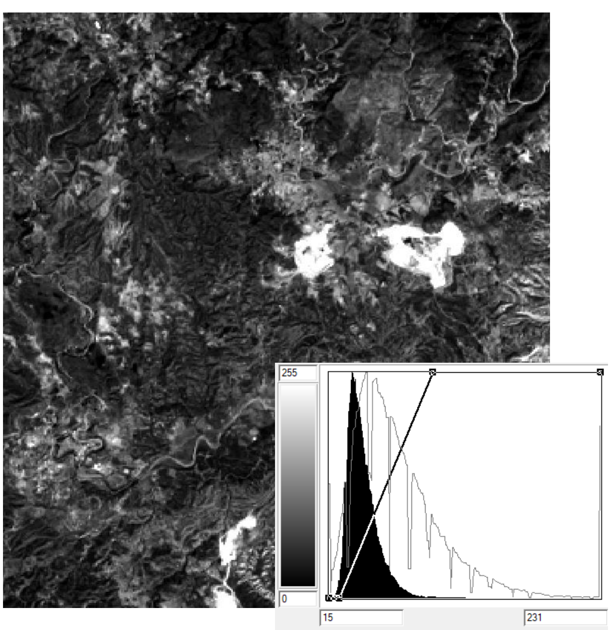
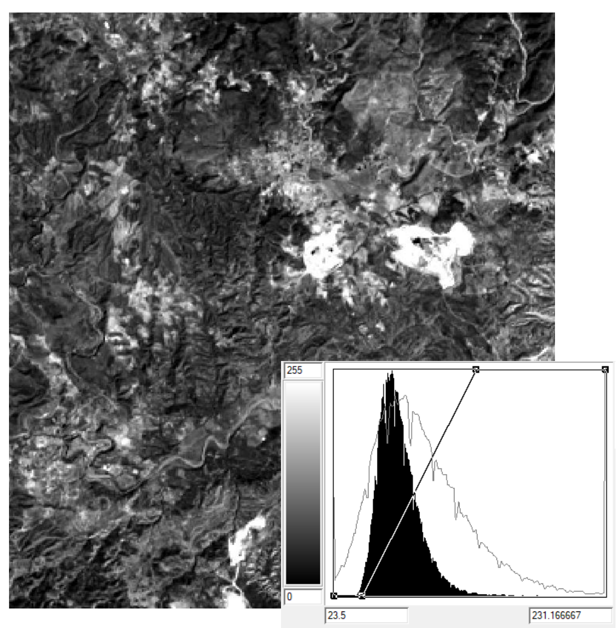
Band 1



Weighed summation of bands 1-5 +7



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2.2 Image subtraction (Differencing)

Image subtraction produces a difference image from two input images:

$$Y = \frac{1}{k}(w_i X_i - w_j X_j)$$

- The weights w_i and w_j are important to assure a balanced differencing is performed.
  - If the brightness of X_i is significantly higher than that of X_j, for instance, the difference image will be dominated by X_i and the true difference between the two images, per pixel, will not be effectively revealed!
- Subtraction is one of the simplest and most effective techniques for **selective spectral enhancement.** It is also useful for **change detection** and **removal of background illumination bias.**
- In general though, a **subtraction operation reduces the image information and decrease image SNR**.
  - Because image subtraction removes the common features while retaining the random noise in both images.

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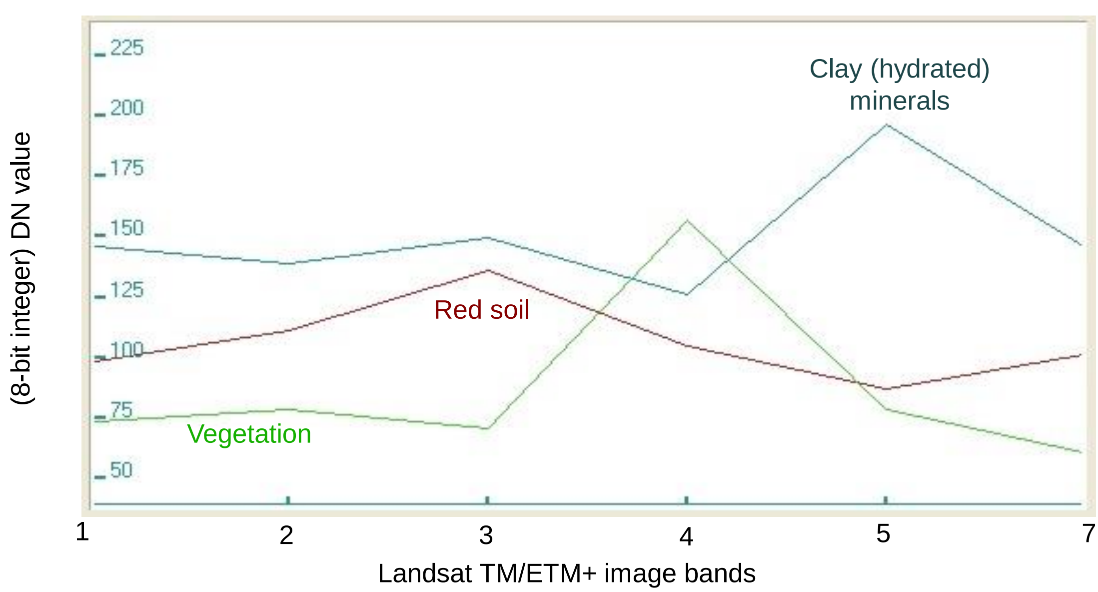
Image spectral profiles (from a Landsat image) representing vegetation, red soil and hydrated minerals



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Difference images and colour composite:

(a) TM3 - TM1 highlights red features often associated with iron oxides.

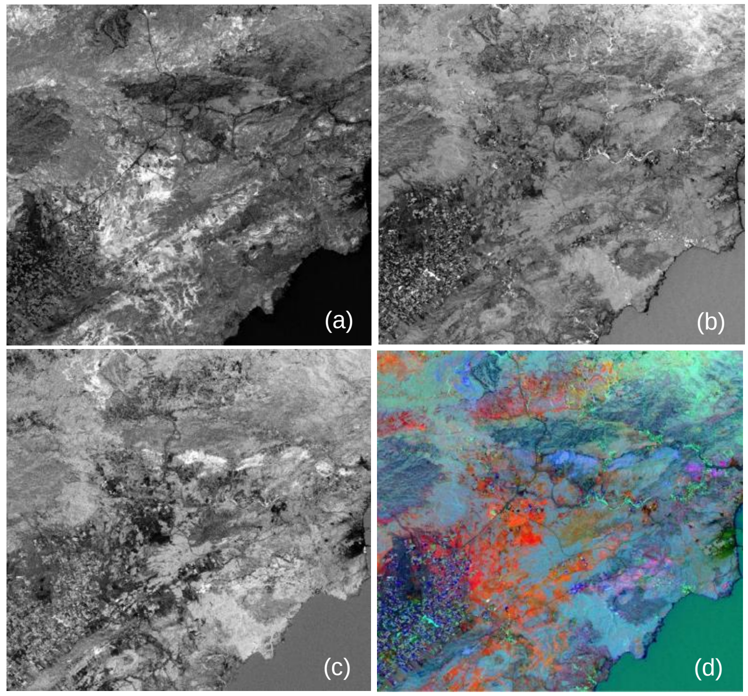
(b) TM4 - TM3 detects the diagnostic ‘red edge’ features of vegetation (chlorophyll).

(c) TM5 - TM7 enhances hydrated mineral absorption features in SWIR spectral range.

(d) Colour composite of:

- (a) TM3 - TM1 (R),
- (b) TM4 - TM3 (G)
- (c) TM5 - TM7 (B)

Highlights iron oxide, vegetation and hydrated minerals, in R, G and B colours respectively.



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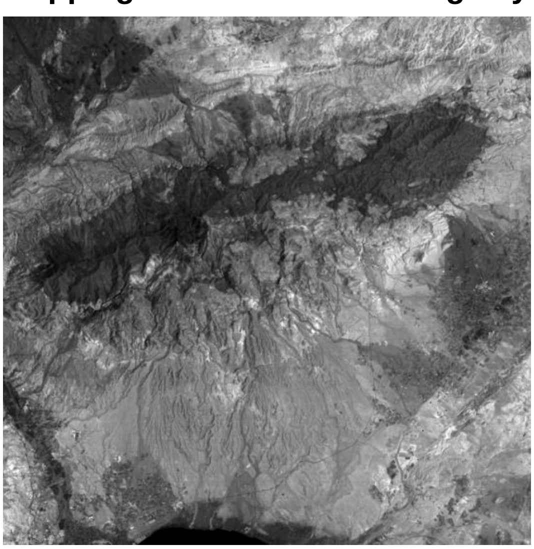
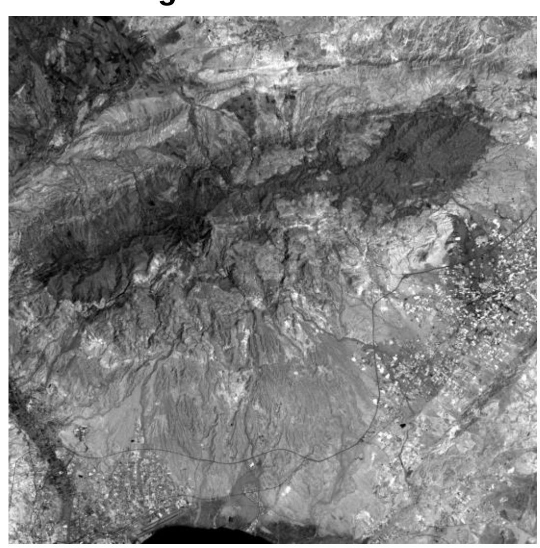
Mapping environmental change by differencing



1984 Landsat-4 TM image band 5



1999 Landsat-7 ETM+ image band 5

Images have been precisely co-registered on pixel-wise basis – this forces the cancellation of any topographic parallax shift in feature position

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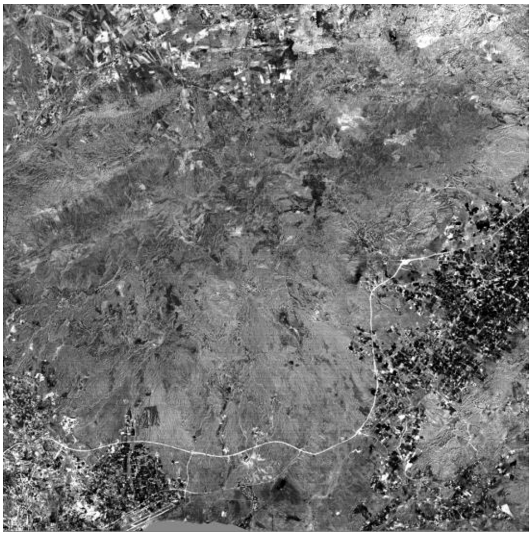
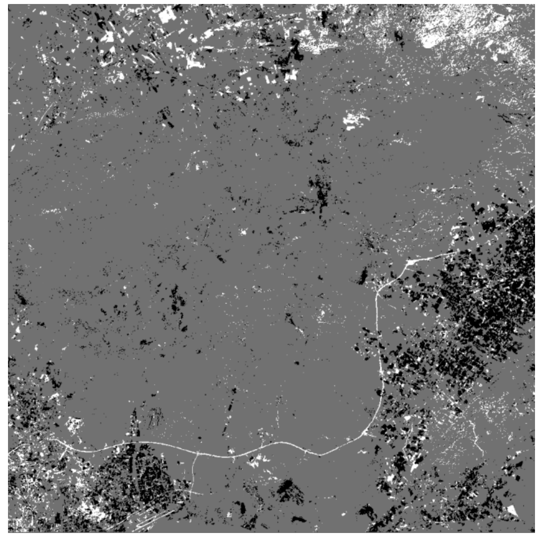
Mapping environmental change by differencing



Difference image: 1984 - 1999



Thresholded difference image with 3 levels

Black = features with significant increase in DN value from 1984 to 1999; White = features with significant decrease in DN value between the two dates and Grey background = no significant change.

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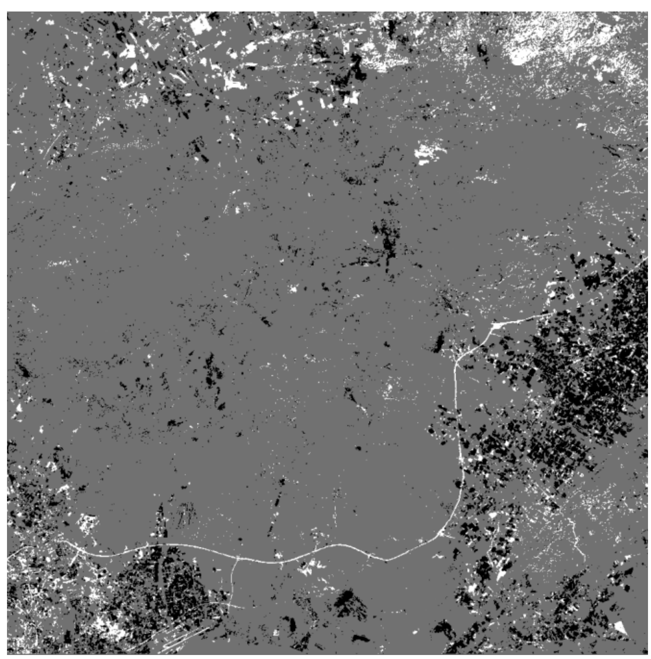
Quantitative environmental change mapping

Temporal changes between1984 Landsat-4 TM and 1999 Landsat-7 ETM+ images



- Black = brighter from ‘84 to ‘99;
- White = darker from ‘84 to ‘99
- Grey = no change

>> 15 yrs of change: major motorway, greenhouses & forest fire burn scars

On GEE, this kind of analysis can be automated and multitemporal image statistics derived from long time-series achieve them very quickly

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2.3 Image multiplication

$$Y = X_i \cdot X_j$$

- Here image multiplication is performed on each image pixel - band i DN is multiplied by band j DN.
  - NB This is fundamentally different from matrix multiplication - remember a digital image is a 2D array, it is **not** a matrix.

Key applications of image multiplication:

a) ***Masking*** – e.g. using an image to remove part of another image. For instance, if X_i is a mask image composed of DN values 0 and 1, the pixels in image X_j which corresponding to 0 in X_i will become 0 (masked or clipped out) and the other DN values remain unchanged in the output image Y.

- This operation can also be done (more efficiently) using a logical operation of a given condition, i.e. *if ... then … else*

b) ***Modulation*** – e.g. using one image to *modulate* another. For instance, topographic shading can be added to a coloured (flat) map or classified image by using a panchromatic image (intensity component) to modulate the three colour components (red, green and blue) of the scanned map or to a pseudocolour classification image as below:

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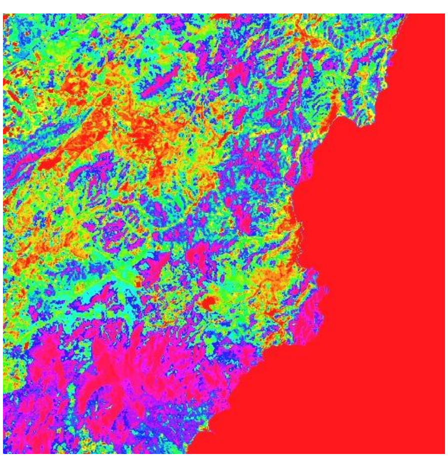
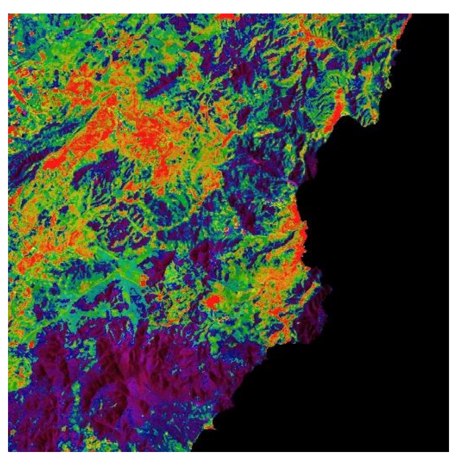
Add-back texture to a classified image

1. Produce red (R), green (G) and blue (B) component images from the colour coded classification image.
2. Use a relevant panchromatic image (I) to modulate R, G and B components: R × I, G × I and B × I.
3. Colour composition using R × I, G × I and B × I.



A pseudo-colour coded classification image - a rather ‘flat’ colour map



Intensity modulated colour coded classification image – including some topographic shading.

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2.4 Image division (Ratio)

Image division is a very popular technique known as ***ratio***. The operation is defined as:

$$Y = \frac{X_i}{X_j}$$

- For processing involving image division, certain protection is usually needed to avoid overflow (where a number is divided by zero).
  - A common trick is to shift up the value range of denominator image by adding 1 to avoid zero.
- The output ratio image Y is an image of real numbers instead of integers.
- NB If both X_i and X_j are 8-bit images, the possible output value ranges for Y are 0, [1/255, 1], or [1, 255]. The value range [1/255, 1] may contain just as much information as that in the much wider value range [1, 255] !
- NB If you then linear stretch the result, you could achieve **up to 50% information loss** because the information recorded in value range [1/255, 1] could be compressed into a very few DN levels. So beware and stretch carefully!

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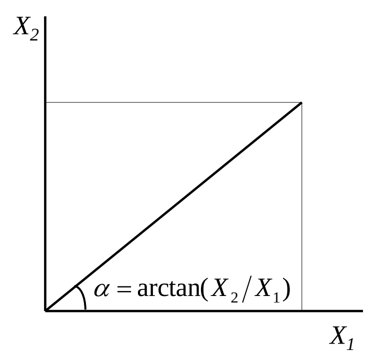
Another way of thinking about image ratio is consider the *ratio* as a coordinate transformation from a Cartesian to a polar coordinate system, then

$$Y = \frac{X_i}{X_j} = \tan(\alpha)$$

$$\alpha = \arctan(\frac{X_i}{X_j})$$



- Ratio image Y is actually a tangent image of the angle α. The information in a ratio image is evenly represented by angle α in the value range [0, π/2] instead of $Y = \tan(\alpha)$ in value range [0,255].
- Therefore, to achieve a 'fair' linear stretch when displaying a ratio image, it is desirable to convert Y to α. An automatic linear scale can then be performed as

$$\beta = 255\frac{\alpha - \mathrm{Min}(\alpha)}{\mathrm{Max}(\alpha) - \mathrm{Min}(\alpha)}$$

In the end, this transform may not be always necessary but is interesting to consider.

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2.4 Image division (Ratio) cont'd

- Ratios are often designed to high**light particular target features in high value DNs**.
  - Direct stretch of ratio image Y may enhance the target features better, at the cost of losing the information represented by low ratio DNs.
  - Thus it is important to notice that, although ratios TM1/TM3 and TM3/TM1 are reciprocal of each other and contain the same information, they will be different after linear scale contrast enhancement!
- **NB when you design a ratio, make sure the target information has high DN values in the output ratio image (more intuitive)**.
- Ratio is very useful for selective enhancement of spectral features. Ratio images derived from different band pairs are often displayed in RGB system to generate ratio colour composites.
  - e.g. a colour composite of TM5/TM7 (blue), TM4/TM3 (green) and TM3/TM1 (red) may highlight clay minerals in blue, vegetation in green and iron oxide in red.
- Many indices, such as Normalised Difference Vegetation Index (NDVI), have been developed based on both differencing and ratio operations.

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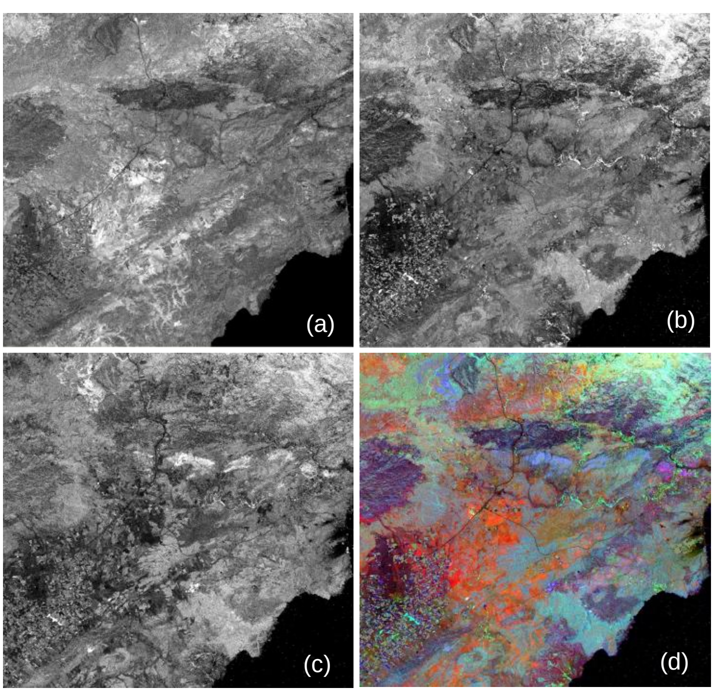

Ratio images and ratio colour composite:

(a) TM3 / TM1

(b) TM4 / TM3

(c) TM5 / TM7

(d) Ratio colour composite of:
- TM3 / TM1 (R),
- TM4 / TM3 (G)
- TM5 / TM7 (B)

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Ratio for suppression of topography

- For a given incident angle of solar radiation, the radiation energy received by an area of land surface **depends on the angle between the land surface and the incident radiation**.
- Therefore, solar illumination on land surface varies with terrain slope and aspect, which causes topographic shadows. The DNs in different spectral bands of a multi-spectral image **are proportional to the solar radiation received by land surface and its spectral reflectance.**
- Let $DN(\lambda)$ represent the digital number of a pixel in an image of spectral band λ, then:

$$DN(\lambda) = \rho(\lambda)E(\lambda)$$

- Where $\rho(\lambda)$ and $E(\lambda)$ are the spectral reflectance and irradiance of spectral band λ. Irradiance is the total solar radiation received at the land surface corresponding to the pixel.

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- Suppose a pixel representing a land surface facing the sun (*DN2*) receives *n* times radiation energy received by another pixel on the land surface facing away from the sun (*DN1*), then the DNs of the two pixels in spectral bands *i* and *j* are as below.

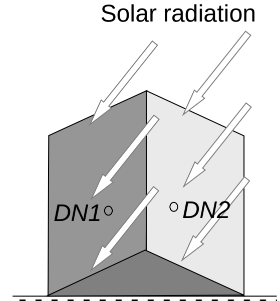
Shaded pixel: $DN1(i) = \rho(i)E(i) \quad DN1(j) = \rho(j)E(j)$

Illuminated pixel: $DN2(i) = n\rho(i)E(i) \quad DN2(j) = n\rho(j)E(j)$

Shaded ratio: $R1_{i,j} = \dfrac{DN1(i)}{DN1(j)} = \dfrac{\rho(i)E(i)}{\rho(j)E(j)}$

Illuminated ratio: $R2_{i,j} = \dfrac{DN2(i)}{DN2(j)} = \dfrac{n\rho(i)E(i)}{n\rho(j)E(j)} = \dfrac{\rho(i)E(i)}{\rho(j)E(j)}$

Therefore: $R1_{i,j} = R2_{i,j}$



- Result = topographic shading is suppressed by ratio
- Since topography often accounts for more than 90% information of a multi-spectral image, ratio images therefore **reduce SNRs** significantly.

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2.5 Spectral index derivation & supervised enhancement

- **Unlimited combinations** of algebraic operations can be derived using basic arithmetic operations and algebraic functions.
- For a meaningful and effective operation, knowledge of the **spectral properties of the target** is essential.
  - Formulae composed from a spectral study or a physical model, for the enhancement of particular targets are known as ***indices or spectral indices***, e.g. the Normalised Difference Vegetation Index (NDVI).
  - A spectral index is guided by physical principles and can therefore be considered a kind of '***supervised enhancement'***.
- Here we briefly introduce a few commonly used spectral indices based on Landsat TM/ETM+ image data.
- You can design your own indices for any given image processing objective, based on the spectral properties of the target.

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a. Vegetation Indices

- Healthy vegetation has a high reflection peak in near infrared (NIR) and an absorption trough in red (R) because of the spectral properties of chlorophyll.
- If we could see NIR, vegetation would be NIR rather than green (and very bright). This significant difference in brightness between R and NIR bands is called the '***red edge***' that is a unique spectral property making photosynthesising vegetation distinctively different from all other ground objects.
- This diagnostic spectral feature of vegetation can be very effectively enhanced by both differencing and ratio operations.

	Conceptual formula	Landsat TM formula
Vegetation Ratio Index (VRI):	$Y = \frac{NIR - \mathrm{Min}(NIR)}{Red - \mathrm{Min}(Red) + 1}$	$Y = \frac{TM4 - \mathrm{Min}(TM4)}{TM3 - \mathrm{Min}(TM3) + 1}$
Normalised Difference Vegetation Index (NDVI):	$NDVI = \frac{NIR - Red}{NIR + Red}$	$Y = \frac{TM4 - TM3}{TM4 + TM3}$

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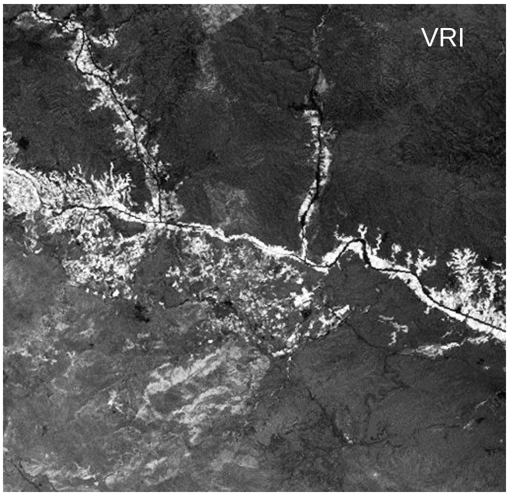
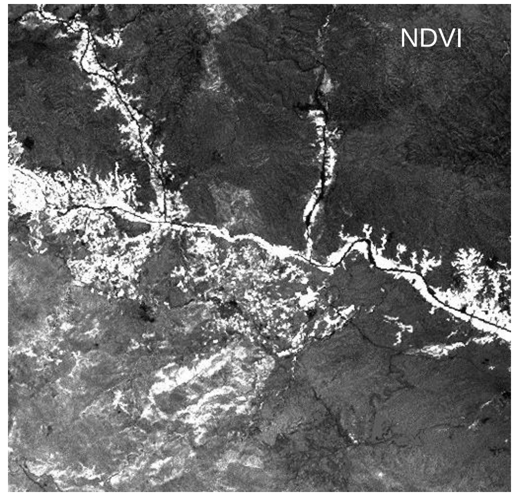
Vegetation Indices





There are several other variations on the NDVI theme, e.g. **SAVI** (Soil Adjusted Vegetation Index), **NDWI** (Normalised Difference Water Index), **EVI** (Enhanced Vegetation Index, and **NDSI** (Normalised Difference Snow Index)

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Other spectral indices

SAVI
(Soil Adjusted Vegetation Index)

$$\mathbf{SAVI} = \frac{\mathbf{(1+L)(NIR-Red)}}{\mathbf{(NIR+Red+L)}}$$

where L is a canopy background adjustment factor. An L value of 0.5 has been found to minimize soil brightness variations and eliminate the need for additional calibration.

NDWI
(Normalised Difference Water Index)

$$\mathbf{NDWI} = \frac{(Xnir - Xswir)}{(Xnir + Xswir)}$$

Water content in leaves (Gao, 1996). Also serves as a Normalized Burn Ratio (BNR)

$$\mathbf{NDWI} = \frac{(Xgreen - Xnir)}{(Xgreen + Xnir)}$$

Water in water bodies (liquid water)

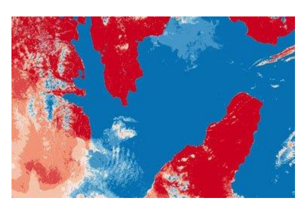
BI
Brightness Index
(of soils)

$$BI = \sqrt{\left(\frac{(X_{red} * X_{red})}{(X_{green} * X_{gren})}\right)^2}$$

At 1.6 μm, snow absorbs sunlight, and so it appears darker than clouds, so you can distinguish between the two brightest objects affecting image brightness, and remove the ice & snow

NDSI
(Normalised Difference Snow Index)

$$NDSI = \frac{(X_{green} - X_{swir})}{(X_{green} + X_{swir})}$$



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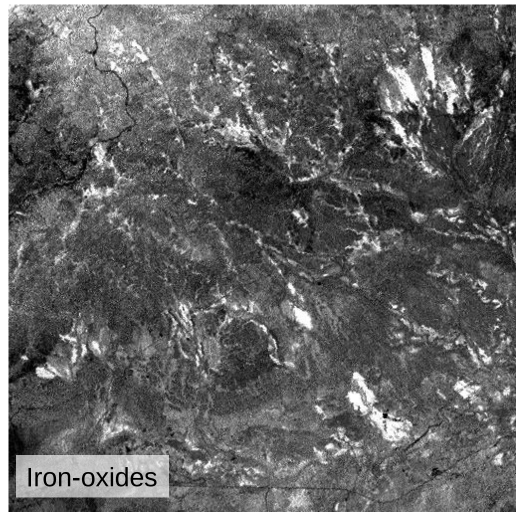
b. Iron-oxide spectral index

- Iron-oxides are one of the most common and widely spread mineral groups in the natural environment.
- They appear in red or reddish brown as a result of high reflectance in Red and absorption in Blue parts of the spectrum.
- Red soils are closely associated with natural iron-oxides & hydroxides (weathering) and with hydrothermal alteration
- We can enhance iron-oxides using the ratio between red and blue spectral band images.

$$Y = \frac{Red - \mathrm{Min}(Red)}{Blue - \mathrm{Min}(Blue) + 1}$$

$$Y = \frac{TM3 - \mathrm{Min}(TM3)}{TM1 - \mathrm{Min}(TM1) + 1}$$



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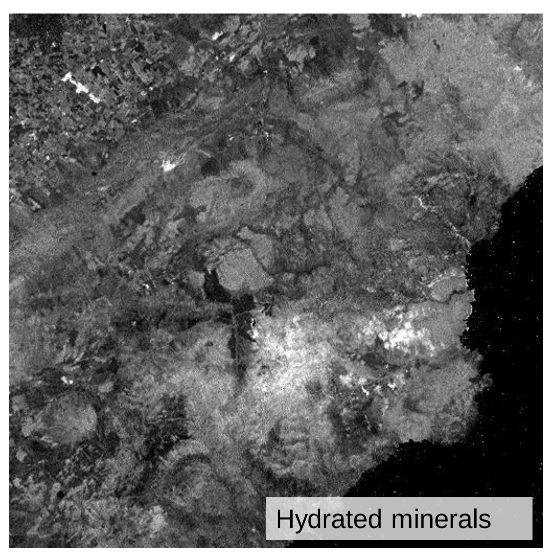
c. Hydrated (clay) mineral ratio index

$$Y = \frac{SWIR1 - \mathrm{Min}(SWIR1)}{SWIR2 - \mathrm{Min}(SWIR2) + 1}$$

$$Y = \frac{TM5 - \mathrm{Min}(TM5)}{TM7 - \mathrm{Min}(TM7) + 1}$$

- Hydrated minerals are effective indicators of rock & mineral alteration and are therefore very useful for mineral exploration using remote sensing.
- The typical spectral signature making hydrated minerals different from unaltered rocks is that they all have strong reflectance in SWIR ~ 1.66 *μm* (corresponding to TM band 5) and strong absorption ~ 2.2 *μm* (corresponding to TM band 7) – more on this later
- Thus all hydrated minerals can be enhanced by the ratio between these two SWIR bands (TM5 & TM7).



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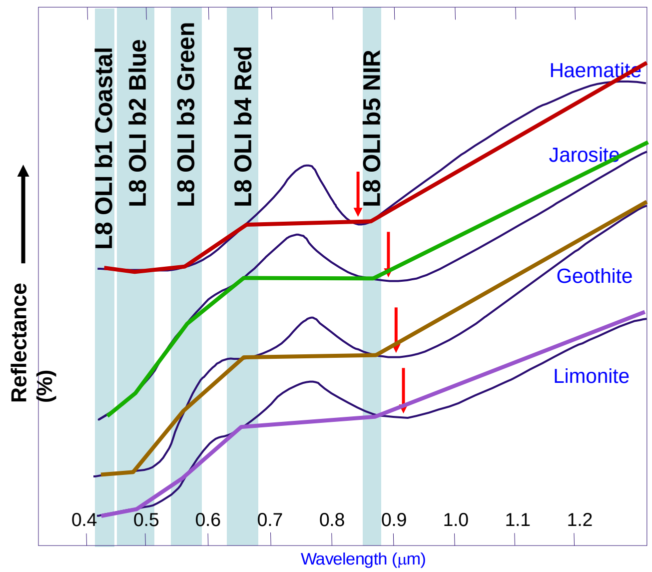
Iron oxides/hydroxides spectral indices & Landsat 8 & 9 OLI

Landsat 8 OLI :

- Band widths are much narrower
- Extra Coastal (new b1) added
- Allows better discrimination (potentially)
- -((b1+b2+b3)/(i4+i5)) haematite
- b3/b2 or b4/b2 goethite & jarosite
- B4/b1 jarosite (possibly)



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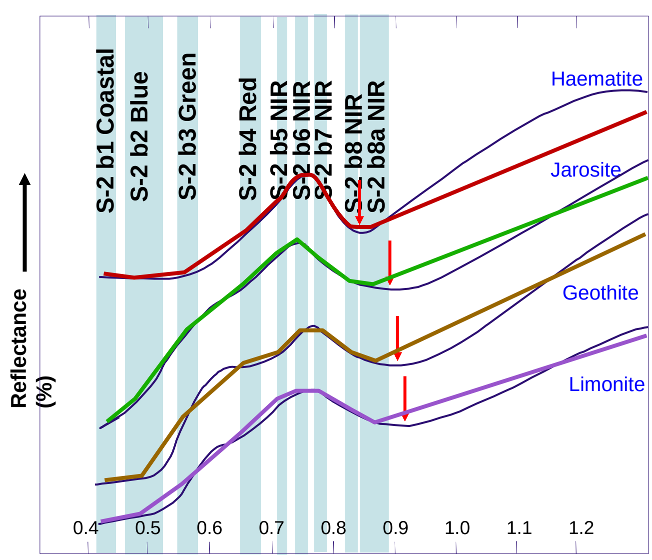
Iron oxides/hydroxides spectral indices & Sentinel-2

Sentinel-2:

- Similar to Landsat 8
- More VNIR bands and narrower bandwidths
- Allows potentially much better discrimination (and possibly identification)
- Treating STL-2 as a hyperspectral sensor may allow better discrimination/identification



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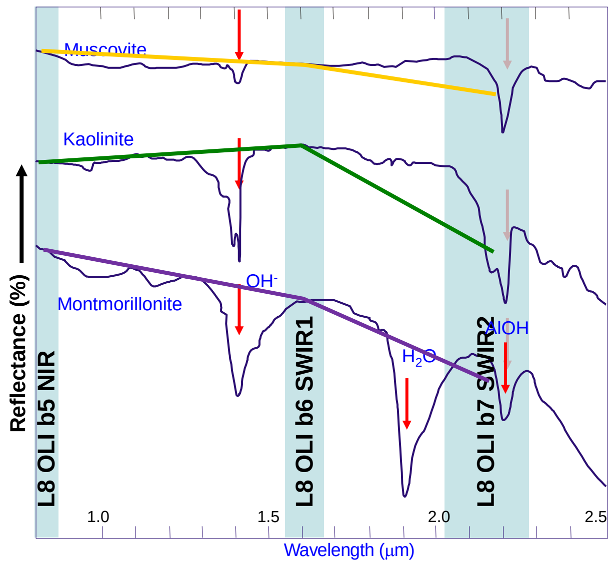
Hydrated minerals spectral indices & Landsat 8 & 9 OLI

Landsat 8 OLI :

- There are still only 2 SWIR bands….
- Band widths narrower than L5 & 7
- Still only allows discrimination of general clays
- Confusion with carbonates around 2.2 mm region
- Made more tricky if vegetation present



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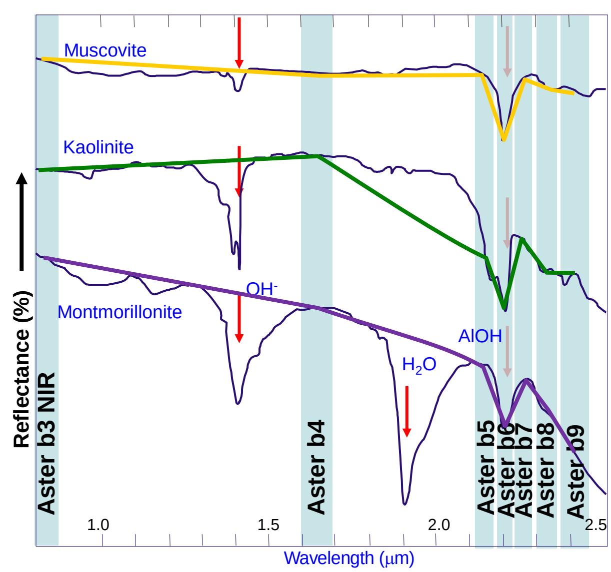
Hydrated minerals spectral indices & ASTER

ASTER with 6 SWIR bands

- Allows discrimination of mineral groups and associations which are important in exploration
- Separable groups include:
  - Kaolinite/alunite & pyrophilite
  - Muscovite/sericite/illite
  - Biotite/phengite
  - Montmorillonite/smectite
  - Chlorite
- Argillic, advanced argillic, silicic and propylitic alteration styles can be separated



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2.6 Summary

- In this chapter, we learnt simple arithmetic operations between images and discussed their major applications for image spectral enhancement.
- The key point is that all the image algebraic operations are point based and performed among the corresponding pixels in different images without involvement of neighbourhood pixels. We can therefore regard algebraic operations as ***multi-image point operations***.
- A major application of image algebraic operations is for selective enhancement of spectral signatures of particular target materials in a multi-spectral image.
- For this purpose, investigating the spectral properties of these targets is essential to compose effective algebraic operations rather than just 'having a go'.
- This process, from spectral analysis to composing an algebraic formula, is generally called ***supervised enhancement***. If such a formula is not image scene dependent and can be widely used, it is called an ***index*** image, for instance, NDVI is a well know vegetation index image.

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2.7 Revision Questions

1. Why is an image algebraic operations also called a multi-image point operation? Write down the mathematical definition of the multi-image point operation.
2. Why does image addition improves image SNR? Using a stationary camera to take 9 pictures of the same scene under identical illumination conditions and then summing them to then generate an average image, by how many times is the SNR is improved in comparison with any an individual picture?
3. Describe image difference (subtraction) and ratio (division) operations and compare the two techniques in terms of change detection, selective enhancement and processing efficiency.
4. What is the importance of the weights in image subtraction? Suggest the most desirable pre-processing step for image differencing…
5. Why does image differencing decrease the SNR?
6. Describe image multiplication and its major application.
7. Explain the characteristics of the value range of a ratio image. Do you think that two reciprocal ratio images contain the same information when displayed after a linear stretch, and explain why?
8. Using a diagram to describe ratio image as a coordinate transformation from a Cartesian coordinates system to a polar coordinates system.
9. Explain the principle of topographic suppression using image ratio technique.
10. What is the NDVI and how it is designed? Explain the different functionalities of differencing and ratio operations in NDVI.
11. Describe the design and functionality of TM or ETM+ iron oxide and hydrated mineral (incl. clay & gypsum) indices.
12. Try the normalised differencing approach, similar to NDVI, to enhance iron oxide and clay minerals. Compare the results with the corresponding ratio indices and explain why the ratio based indices are more effective for these two minerals?

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