Methods HW 7

2. Completed entirely on MATLAB

3. $h(x) = 1 - \frac{x^2}{4}$

$X_0 = 2$

$X_{n+1} = 1 - \frac{X_n^2}{4}$

$X_1 = 1 - \frac{2^2}{4} = 1 - 1 = 0$

$X_2 = 1 - \frac{0^2}{4} = 1 - 0 = 1$

$X_3 = 1 - \frac{1^2}{4} = 1 - .25 = .75$

$X_4 = 1 - \frac{.75^2}{4} = 1 - .140625 \approx 0.8594$

$X_5 = 1 - \frac{X_4^2}{4} = 1 - .184631 \approx 0.81537$

$X_6 = 1 - \frac{X_5^2}{4} = 1 - .166207 \approx 0.83379$

$X_7 = 1 - \frac{X_6^2}{4} = 1 - .173803 \approx \boxed{0.826197}$

$X_8 = 1 - \frac{X_7^2}{4} = 1 - .17065 \approx \boxed{0.829396}$

$X_9 = 1 - \frac{X_8^2}{4} = 1 - .17196 \approx \boxed{0.828045}$

$X_{10} = 1 - \frac{X_9^2}{4} = 1 - .171414 \approx \boxed{0.8285}$

By fixed point iteration X_n lies between 0.82 to 0.83

4. $X_{n+1} = g(X_n) = 1 + e^{-X_n} \rightarrow g'(x) = -e^{-x} \rightarrow |g'(x)| = |-e^{-x}| < 1$

$X_{n+1} = g(X_n)$ converges $\forall X_0 \in [1,2]$

$X_0 = 1$

$X_1 = 1.36788$

$X_2 = 1.25465$

$X_3 = 1.28518$

$X_4 = 1.2766$

$X_5 = 1.27898$

$X_6 = 1.27832$

$X_7 = 1.27785$

$X_8 = 1.27845$

$\left.\begin{array}{l} X_9 = 1.27847 \\ X_{10} = 1.27846 \end{array}\right]$ 10^{-5} diff

$\boxed{\text{10 iterations}}$

Methods HW 7

5. $x_{n+1} = g(x_n)$

$f(\alpha) = 0$

$\alpha = g(\alpha)$

$\to x_{n+1} - \alpha = g(x_n) - g(\alpha)$ (Taylor)

$R_n(x) = \frac{(x-x_0)^{n+1}}{n+1} f^{(n+1)} \xi_x$ where ξ_x is between x, x_0

derive from Taylor

$g(x_n) = g(\alpha) + g'(\alpha)(x_n - \alpha) + \frac{g''(\varepsilon)}{2}(x_n - \alpha)^2$ where ε is between α, x_n

$\frac{f(x)f''(x)}{f'(x)^2}$

$g'(\alpha) = \frac{f(\alpha)f''(\alpha)}{[f'(\alpha)]^2} = 0$

$g(x_n) = g(x) + 0 + \frac{g''(\varepsilon)}{2}(x_n - \alpha)^2$

$= g(x) + \frac{g''(\varepsilon)}{2}(x_n - \alpha)^2$

$x_n - \alpha = \varepsilon_n$

$\varepsilon_{n+1} = x_{n+1} - \alpha = g(x_n) - g(\alpha) = \frac{g''(\varepsilon)}{2}\varepsilon_n^2$

For Taylor

$r = 1$

- $C = 1$
  - convergence is sub-linear
- $C < 1$
  - covergence is linear
- $C > 1$
  - convergence is superlinear

$r = 2$

- convergence is quadratic

because $r=2$, the equation would be quadratically convergent

1. $|x_{n+1} - x_n| = |g(x_n) - g(x_{n-1})| \leq \gamma |x_n - x_{n-1}|$

$|x_{n+1} - x_n| \leq \gamma^n |x_1 - x_0|$, $r = x_{n+\varepsilon}$ for some ε

$\sum_{n=0}^{\varepsilon-1} \gamma^n \leq \sum_{n=0}^{\infty} \gamma^n = \frac{1}{1-\gamma}$, therefor $0 < \gamma < 1$

s.t. $|x_n - r| \leq \frac{\gamma^n}{1-\gamma} |x_1 - x_0|$

confused about where to go / what to prove?