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In this thesis, we have presented a investigate the resolution of stiff ordinary differential equation using PINNs. We’ve shown that stiffness is a reel challenge for PINNs to learn and encode the solution because of behaviour charactristique of the stiffness such as the transient phase. We then show that by training a PINNs in a low stiff regime using our multihead architecture we can then used this knowledge to compute the solution in a stiffer regime. This approach can compete with numerical methods in term of compational time and been even faster when infering on initiale condition and force function inside a stiff domain.

In this thesis, we have explored the resolution of stiff ordinary differential equations (ODEs) using Physics-Informed Neural Networks (PINNs). Our investigation revealed that the inherent characteristics of stiffness, particularly during the transient phase, pose a significant challenge for PINNs in learning and encoding the solution accurately. Subsequently, we demonstrated that training PINNs in a low-stiffness regime through our multihead architecture enables the model to acquire knowledge that can be effectively applied to compute solutions in stiffer regimes. This innovative approach competes with numerical methods in terms of computational time because it doesn’t require to retrain when inferring on the stiffness parameter. It also exhibits enhanced efficiency, especially when changing initial conditions and force functions within a stiff domain and revealed faster than numerical methods for this task.

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\noindent Future research endeavors could explore the adaptation of these methods to Partial Differential Equations (PDEs), as suggested by Wang et al. (2020) \cite{wang2020understanding}. This extension might encompass linear PDEs or non-linear PDEs possessing a perturbation solution.

Furthermore, there is potential for advancing the methodology by enhancing its capacity to train in stiffer domains. This could be achieved through the allocation of additional computational resources during the training process or by employing larger models. By pushing the boundaries of stiffness that the model can handle during training, may subsequent in transferring to even more stiff regime. Exploring these avenues would provide valuable insights into the scalability of the proposed approach, in the realm of solving stiff problems.

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In this thesis, we have explored the resolution of stiff ordinary differential equations using Physics-Informed Neural Networks. The result confirmed that the inherent characteristics of stiffness, particularly during the transient phase, pose a significant challenge for PINNs in learning and encoding the solution accurately. The results then showed, using a multi-head architecture, that training PINNs in a low-stiffness regime enables the model to encode a general representation of the equation, as well as stiffness-related behavior. These elements can then be used to calculate solutions in stiffer regimes, without the need to re-train the model. This innovative approach rivals numerical methods in terms of computation time, although it still falls short of numerical methods like the Radau for calculating solutions in regimes of different stiffness. Nevertheless, it also proves more efficient than numerical methods when modifying initial conditions and force functions in a rigid domain.

It is worth noting that this a relatively new area of research that does not have the well established theory and vast number of contributions which numerical methods provide such as error bound or robustness. Namely, it is known that neural networks do not provide a guarantee in terms of training stability or convergence. Hence, for most real-world, critical applications where accuracy of solutions is paramount, our network-based solution should be used in conjunction with numerical approaches.

Based on these conclusions, future research should consider the to Partial Differential Equations (PDEs), following the transfer learning approach of Wang et al. (2020) [12]. This extension might encompass linear PDEs or non-linear PDEs like the one dimensional advection-reaction system \cite{SANTILLANA2016372} in Global tropospheric chemistry transport.

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