



# Neutron dark matter decays and correlation coefficients of neutron $\beta^-$ -decays

A.N. Ivanov<sup>a,\*</sup>, R. Höllwieser<sup>a,b</sup>, N.I. Troitskaya<sup>a</sup>, M. Wellenzohn<sup>a,c</sup>,  
Ya.A. Berdnikov<sup>d</sup>

<sup>a</sup> *Atominstytut, Technische Universität Wien, Stadionallee 2, A-1020 Wien, Austria*

<sup>b</sup> *Department of Physics, Bergische Universität Wuppertal, Gausstr. 20, D-42119 Wuppertal, Germany*

<sup>c</sup> *FH Campus Wien, University of Applied Sciences, Favoritenstraße 226, 1100 Wien, Austria*

<sup>d</sup> *Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya 29, 195251, Russian Federation*

Received 29 August 2018; received in revised form 5 November 2018; accepted 10 November 2018

Available online 15 November 2018

Editor: Hong-Jian He

## Abstract

As we have pointed out in (arXiv:1806.10107 [hep-ph]), the existence of neutron dark matter decay modes  $n \rightarrow \chi + \text{anything}$ , where  $\chi$  is a dark matter fermion, for the solution of the neutron lifetime problem changes priorities and demands to describe the neutron lifetime  $\tau_n = 888.0(2.0)$  s, measured in beam experiments and defined by the decay modes  $n \rightarrow p + \text{anything}$ , in the Standard Model (SM). The latter requires the axial coupling constant  $\lambda$  to be equal to  $\lambda = -1.2690$  (arXiv:1806.10107 [hep-ph]). Since such an axial coupling constant is excluded by experimental data reported by the PERKEO II and UCNA Collaborations, the neutron lifetime  $\tau_n = 888.0(2.0)$  s can be explained only by virtue of interactions beyond the SM, namely, by the Fierz interference term of order  $b \sim -10^{-2}$  dependent on scalar and tensor coupling constants. We give a complete analysis of all correlation coefficients of the neutron  $\beta^-$ -decays with polarized neutron, taking into account the contributions of scalar and tensor interactions beyond the SM with the Fierz interference term  $b \sim -10^{-2}$ . We show that the obtained results agree well with contemporary experimental data that does not prevent the neutron with the rate of the decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, to have dark matter decay modes  $n \rightarrow \chi + \text{anything}$ . © 2018 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

\* Corresponding author.

E-mail addresses: [ivanov@kph.tuwien.ac.at](mailto:ivanov@kph.tuwien.ac.at) (A.N. Ivanov), [roman.hoellwieser@gmail.com](mailto:roman.hoellwieser@gmail.com) (R. Höllwieser), [naitroitskaya@yandex.ru](mailto:naitroitskaya@yandex.ru) (N.I. Troitskaya), [max.wellenzohn@gmail.com](mailto:max.wellenzohn@gmail.com) (M. Wellenzohn), [berdnikov@spbstu.ru](mailto:berdnikov@spbstu.ru) (Ya.A. Berdnikov).

## 1. Introduction

Recently Fornal and Grinstein [1,2] have proposed to explain the neutron lifetime anomaly, related to a discrepancy between experimental values of the neutron lifetime measured in bottle and beam experiments, through contributions of the neutron dark matter decay modes  $n \rightarrow \chi + \gamma$ ,  $n \rightarrow \chi + \phi$  and  $n \rightarrow \chi + \gamma^* \rightarrow \chi + e^- + e^+$ , where  $\chi$  and  $\phi$  are dark matter fermion and scalar boson, respectively,  $\gamma$  and  $\gamma^*$  are real and virtual photons, and  $(e^-e^+)$  is the electron–positron pair. However, according to recent experimental data [3,4], the decay modes  $n \rightarrow \chi + \gamma$  and  $n \rightarrow \chi + e^- + e^+$  are suppressed. In turn, the neutron decay mode  $n \rightarrow \chi + \phi$  has not been investigated experimentally. In [5] the experimental data on the decay mode  $n \rightarrow \chi + e^- + e^+$  in [4] have been interpreted as follows. An unobservability of the decay mode  $n \rightarrow \chi + e^- + e^+$ , which is not mediated by a virtual photon, may also mean that the production of the electron–positron pair in such a decay is below the reaction threshold, i.e. a mass  $m_\chi$  of dark matter fermions  $\chi$  obeys the constraint  $m_\chi > m_n - 2m_e$ , where  $m_n$  and  $m_e$  are masses of the neutron and electron (positron), respectively. Then, we have proposed that the neutron lifetime anomaly can be explained by the decay mode  $n \rightarrow \chi + \nu_e + \bar{\nu}_e$ , where  $(\nu_e\bar{\nu}_e)$  is a neutrino–antineutrino pair [5]. Since neutrino  $\nu_e$  and electron  $e^-$  belong to the same doublet in the Standard Electroweak Model (SEM) [6], neutrino–antineutrino  $(\nu_e\bar{\nu}_e)$  pairs couple to the neutron–dark matter current with the same strength as electron–positron  $(e^-e^+)$  pairs [5]. For the UV completion of the effective interaction  $(n\chi\ell\bar{\ell})$ , where  $\ell(\bar{\ell})$  is electron (positron) or neutrino (antineutrino), we have proposed a gauge invariant quantum field theory model with  $SU_L(2) \times U_Y(1) \times U'_R(1) \times U''_L(1)$  gauge symmetry, where  $Y$  is the weak hypercharge [6]. Such a quantum field theory model contains the sector of the SEM (or the Standard Model (SM) sector) [6] with  $SU_L(2) \times U_Y(1)$  gauge symmetry and the dark matter sector with  $U'_R(1) \times U''_L(1)$  gauge symmetry. In the physical phase the dark matter sectors with  $U'_R(1)$  and  $U''_L(1)$  symmetries are responsible for the UV completion of the effective interaction  $(n\chi\ell\bar{\ell})$  [5] and interference of the dark matter into dynamics of neutron stars [7–10], respectively. The dark matter sector with  $U''_L(1)$  gauge symmetry we have constructed in analogue with scenario proposed by Cline and Cornell [10]. This means that dark matter fermions with mass  $m_\chi < m_n$  couple to a very light dark matter spin-1 boson  $Z''$  providing a necessary repulsion between dark matter fermions in order to give a possibility for neutron stars to reach masses of about  $2M_\odot$  [11], where  $M_\odot$  is the mass of the Sun [6]. We have shown that in the physical phase the predictions of the dark matter sector with  $U'_R(1)$  gauge symmetry do not contradict constraints on i) dark matter production in ATLAS experiments at the LHC, ii) the cross section for the low-energy dark matter fermion–electron scattering ( $\chi + e^- \rightarrow \chi + e^-$ ) [12], and iii) the branching ratio of the Higgs-boson decay  $H^0 \rightarrow Z + Z'$  [13], where  $H^0$ ,  $Z$  and  $Z'$  are the Higgs-boson with mass  $M_{H^0} = 125 \text{ GeV}$  [6], the electroweak  $Z$ -boson [6] and the dark matter spin-1 boson  $Z'$  of the dark matter sector with  $U'_R(1)$  gauge symmetry [5]. We have also proposed that the reactions  $n \rightarrow \chi + \nu_e + \bar{\nu}_e$ ,  $n + n \rightarrow \chi + \chi$ ,  $n + n \rightarrow \chi + \chi + \nu_e + \bar{\nu}_e$  and  $\chi + \chi \rightarrow n + n$ , allowed in our model, can serve as URCA processes for the neutron star cooling [14–16]. In addition we have discussed a possible quark level formulation of our model with  $SU_L(2) \times U_Y(1) \times U'_R(1) \times U''_L(1)$  gauge symmetry.

Having assumed that the results of the experimental data [3,4] can be also interpreted as a production of electron–positron pairs below reaction threshold of the decay mode  $n \rightarrow \chi + e^- + e^+$ , we have proposed to search for traces of dark matter fermions induced by the  $n\chi e^-e^+$  interaction in the low-energy electron–neutron inelastic scattering  $e^- + n \rightarrow \chi + e^-$ . Such a reaction can be compared experimentally with low-energy electron–neutron elastic scattering  $e^- + n \rightarrow n + e^-$  [17–21]. The differential cross section for the reaction  $e^- + n \rightarrow \chi + e^-$  possesses the following

properties: i) it is inversely proportional to a velocity of incoming electrons, ii) it is isotropic relative to outgoing electrons and iii) momenta of outgoing electrons are much larger than momenta of incoming electrons. Because of these properties the differential cross section for the reaction  $e^- + n \rightarrow \chi + e^-$  can be in principle distinguished above the background of the elastic electron–neutron scattering  $e^- + n \rightarrow n + e^-$ . In order to have more processes with particles of the SM in the initial and final states allowing to search dark matter in terrestrial laboratories we have proposed to search dark matter fermions by means of the electrodisintegration of the deuteron into dark matter fermions and protons  $e^- + d \rightarrow \chi + p + e^-$  close to threshold [22], induced by the electron–neutron inelastic scattering  $e^- + n \rightarrow \chi + e^-$  with energies of incoming electrons larger than the deuteron binding energy, which is of about  $|\varepsilon_d| \sim 2 \text{ MeV}$ . We have calculated the triple-differential cross section for the reaction  $e^- + d \rightarrow \chi + p + e^-$  close to threshold, and proposed to detect dark matter fermions from the electrodisintegration of the deuteron  $e^- + d \rightarrow \chi + p + e^-$  above the background  $e^- + d \rightarrow n + p + e^-$  by detecting outgoing electrons, protons and neutrons in coincidence. A missing of neutron signals at simultaneously detected signals of protons and outgoing electrons should testify an observation of dark matter fermions in the final state of the electrodisintegration of the deuteron close to threshold.

As has been pointed out in [5], the acceptance of existence of the neutron dark matter decay modes  $n \rightarrow \chi + \text{anything}$  is not innocent and demands to pay the following price. Indeed, the neutron lifetime time  $\tau_n = 879.6(1.1) \text{ s}$ , calculated in the SM [23] for the axial coupling constant  $\lambda = -1.2750(9)$  [24] by taking into account the complete set of corrections of order  $10^{-3}$ , caused by the weak magnetism and proton recoil, taken to next-to-leading order in the large nucleon mass  $M$  expansion of order  $O(E_e/M)$ , where  $E_e$  is the electron energy, and radiative corrections of order  $O(\alpha/\pi)$ , where  $\alpha$  is the fine-structure constant [6], agrees well with the world averaged lifetime of the neutron  $\tau_n = 880.1(1.0) \text{ s}$  [6], and the neutron lifetime  $\tau_n = 879.6(6)$ , averaged over the experimental values measured in bottle experiments [25–30] included in the Particle Data Group (PDG) [6]. It agrees also with the value  $\tau_n = 879.4(6) \text{ s}$  and the axial coupling constant  $\lambda = -1.2755(11)$ , obtained by Czarnecki et al. [31] by means of a global analysis of the experimental data on the neutron lifetime and axial coupling constant. At first glimpse such an agreement rules out fully any dark matter decay mode  $n \rightarrow \chi + \text{anything}$  of the neutron. For a possibility to the neutron to have any dark matter decay mode  $n \rightarrow \chi + \text{anything}$  the SM should explain the neutron lifetime  $\tau_n = 888.0(2.0) \text{ s}$ , measured in beam experiments, instead of to explain the neutron lifetime  $\tau_n = 879.6(6) \text{ s}$ , measured in bottle ones. As has been shown in [5], using the analytical expression for the neutron lifetime (see Eq. (41) and (42) of Ref. [23]) the value  $\tau_n = 888.0(2.0) \text{ s}$  can be fitted by the axial coupling constant equal to  $\lambda = -1.2690$ . Since such a value of the axial coupling constant is ruled out by recent experiments [32–35] and a global analysis by Czarnecki et al. [31], so the hypothesis of the existence of the neutron dark matter decay modes should state that the SM, including a complete set of corrections of order  $10^{-3}$  caused by the weak magnetism, proton recoil and radiative corrections [36] (see also [23]), is not able to describe correctly the rate and correlation coefficients of the neutron decay modes  $n \rightarrow p + \text{anything}$ . Hence, the theoretical description of the neutron lifetime, measured in beam experiments, should go beyond the SM. Indeed, keeping the value of the axial coupling constant equal to  $\lambda = -1.2750$  or so [32–35] and having accepted the existence of the dark matter decay modes  $n \rightarrow \chi + \text{anything}$  we have also to accept a sufficiently large contribution of the Fierz interference term  $b$  [37], dependent on the scalar and tensor coupling constants of interactions beyond the SM [37–52] (see also [36] and [23]). Using the results obtained in [23], the neutron lifetime  $\tau_n = 888.0 \text{ s}$  can be fitted by the axial coupling constant  $\lambda = -1.2750$ , the Cabibbo–Kobayashi–Maskawa (CKM) matrix element  $V_{ud} = 0.97420$  [6] and the Fierz interference term

$b = -1.44 \times 10^{-2}$  calculated at the neglect of the quadratic contributions of scalar and tensor coupling constants of interactions beyond the SM [5]. Thus, in order to confirm a possibility for the neutron to have any dark matter decay modes  $n \rightarrow \chi + \text{anything}$  we have to show that a tangible influence of the Fierz interference term  $b = -1.44 \times 10^{-2}$  is restricted only by the rate  $1/\tau_n = 1/888.0 \text{ s}^{-1} = 1.126 \times 10^{-3} \text{ s}^{-1}$  of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and such a term does not affect the correlation coefficients of the electron-energy and angular distributions of the neutron  $\beta^-$ -decay.

This paper is addressed to the analysis of the contributions of scalar and tensor interactions beyond the SM to the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and correlation coefficients of the neutron  $\beta^-$ -decay with polarized neutron, polarized electron and unpolarized proton. We take into account the contributions of the SM, including a complete set of corrections of order  $10^{-3}$ , caused by the weak magnetism and proton recoil, calculated to next-to-leading order in the large nucleon mass  $M$  expansion [53,54] (see also [36] and [23,55]), and radiative corrections of order  $O(\alpha/\pi)$ , calculated to leading order in the large nucleon mass expansion [56–59] (see also [36] and [23,55]). We search anyone solution for the values of scalar and tensor coupling constants of interactions beyond the SM allowing to fit the rate  $1/\tau_n = 1.126 \times 10^{-3} \text{ s}^{-1}$  of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and the experimental data on the correlation coefficients of the neutron  $\beta^-$ -decay under consideration. The existence of such a solution for the values of scalar and tensor coupling constants of interactions beyond the SM should imply an allowance for the neutron to have the dark matter decay modes  $n \rightarrow \chi + \text{anything}$ .

The paper is organized as follows. In section 2 we give the electron-energy and angular distribution of the neutron  $\beta^-$ -decay with polarized neutron, polarized electron and unpolarized proton. We write down the expressions for the correlation coefficients including the contributions of the SM corrections of order  $10^{-3}$ , caused by the weak magnetism and proton recoil to next-to-leading order in the large nucleon mass expansion of order  $O(E_e/M)$  and radiative corrections of order  $O(\alpha/\pi)$ , and the contributions of scalar and tensor interactions beyond the SM, calculated to leading order in the large nucleon mass  $M$  expansion. In section 3 we analyse the rate  $1/\tau_n = 1.126 \times 10^{-3} \text{ s}^{-1}$  of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and fit it by the contribution of the Fierz interference term by taking into account the quadratic contributions of scalar and tensor interactions beyond the SM. In section 4 we analyse possible solutions for the values of scalar and tensor coupling constants. On this way for a search of one of possible solutions we follow [46,52] and assume that the scalar and tensor coupling constants are real, and set  $\tilde{C}_S = -C_S$  and  $C_T = -\tilde{C}_T$ . Such a solution implies also that in scalar and tensor interactions beyond the SM the neutron and proton couple to right-handed electron and antineutrino only. We take into account the constraints on the scalar coupling constant  $C_S$ , i.e.  $|C_S| = 0.0014(13)$  and  $|C_S| = 0.0014(12)$  obtained by Hardy and Towner [60] and González-Alonso et al. [61], respectively, from the superallowed  $0^+ \rightarrow 0^+$  transitions. Since in the superallowed  $0^+ \rightarrow 0^+$  transitions the scalar coupling constant  $C_S$  is commensurable with zero we propose the solution  $\tilde{C}_S = -C_S = 0$ . For real scalar and tensor coupling constants and for  $\tilde{C}_S = -C_S = 0$  and  $C_T = -\tilde{C}_T$  we solve Eq. (6) and obtain the solution Eq. (10). In the linear approximation we get  $C_T = -\tilde{C}_T = 1.11 \times 10^{-2}$  and the Fierz interference term equal to  $b = -1.44 \times 10^{-2}$ . We define the correlation coefficients in terms of the coupling constant  $C_T = 1.11 \times 10^{-2}$  and the Fierz interference term  $b = -1.44 \times 10^{-2}$ . We show that the contributions of quadratic terms  $C_T^2$  are of the standard order  $10^{-4}$ . In turn, the contributions of linear terms are of order  $10^{-2} - 10^{-3}$ . In section 5 we analyse i) the contributions of the Fierz interference term to the electron and antineutrino asymmetries, defined by the neutron

spin and electron and antineutrino 3-momentum correlations, respectively, and to the asymmetry, caused by the correlations of the electron and antineutrino 3-momenta, and ii) the averaged values of the correlation coefficients  $N(E_e)$  and  $R(E_e)$ . The correlation coefficient  $N(E_e)$  defines the neutron–electron spin–spin  $\vec{\xi}_n \cdot \vec{\xi}_e$  correlations, whereas the correlation coefficient  $R(E_e)$  is caused by P-odd (parity odd) and T-odd (time reversal odd) correlations defined by  $\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)$ , where  $\vec{\xi}_n$  and  $\vec{\xi}_e$  are unit vectors of the neutron and electron polarizations, and  $\vec{k}_e$  is the electron 3-momentum. We show that the contribution of the Fierz interference term  $b = -1.44 \times 10^{-2}$  does not contradict the experimental data on the measurements of the correlation coefficients  $A_0$ ,  $B_0$  and also  $a_0$ , defined to leading order in the large nucleon mass  $M$  expansion [24,62]. We would like to emphasize that the Fierz interference term  $b = -1.44 \times 10^{-2}$  does not contradict the result  $b = -0.0028(26)$  extracted by Hardy and Towner [60] from the superallowed  $0^+ \rightarrow 0^+$  transitions. Indeed, in the superallowed  $0^+ \rightarrow 0^+$  transitions the Fierz interference term is defined only by the scalar coupling constants. At  $C_S = -\bar{C}_S$  it is equal to  $b \simeq 2C_S$ . In turn, in the neutron  $\beta^-$ -decay at  $C_S = -\bar{C}_S = 0$  the Fierz interference term is induced by the tensor coupling constants only. For  $C_T = -\bar{C}_T$  we get  $b \simeq 6\lambda C_T / (1 + 3\lambda^2)$  and  $b = -1.44 \times 10^{-2}$  for  $C_T = 1.11 \times 10^{-2}$  and  $\lambda = -1.2750$  [24] (see also [23,52,55]). We show that the averaged value of the correlation coefficient  $\langle N(E_e) \rangle$ , obtained in this paper, agrees with the experimental value within one standard deviations. In turn, the averaged value of the correlation coefficient  $\langle R(E_e) \rangle$  acquires the relative contributions of order  $10^{-4}$ , caused by interactions beyond the SM. In section 6 we discuss the obtained results. We argue that the obtained agreement between theoretical values of the correlation coefficients, defined by the contributions of the SM to order  $10^{-3}$ , the Fierz interference term  $b = -1.44 \times 10^{-2}$  and other linear and quadratic coupling contributions of tensor interactions beyond the SM, implies an allowance for the neutron to have dark matter decay modes  $n \rightarrow \chi + \text{anything}$ . We would like to emphasize that the results, obtained in this paper, are model-independent in the sense that they should be valid and actual for any model of the solution of the neutron lifetime problem in terms of neutron dark matter decay modes (see for example [63]).

## 2. Electron-energy and angular distribution of neutron $\beta^-$ -decay with polarized neutron, polarized electron and unpolarized proton

The electron-energy and angular distribution of the neutron  $\beta^-$ -decay with polarized neutron and electron and unpolarized proton takes the form [23,52,55]

$$\begin{aligned}
 & \frac{d^5 \lambda_n(E_e, \vec{k}_e, \vec{k}_v, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_v} \\
 &= (1 + 3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z=1) \zeta(E_e) \\
 & \times \left\{ 1 + b \frac{m_e}{E_e} + a(E_e) \frac{\vec{k}_e \cdot \vec{k}_v}{E_e E_v} + A(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + B(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_v}{E_v} \right. \\
 & + K_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_v)}{E_e^2 E} + Q_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_v)(\vec{k}_e \cdot \vec{k}_v)}{E_e E_v^2} + D(E_e) \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_v)}{E_e E_v} \\
 & \left. - 3 \frac{E_e}{M} \frac{1 - \lambda^2}{1 + 3\lambda^2} \left( \frac{(\vec{k}_e \cdot \vec{k}_v)^2}{E_e^2 E_v^2} - \frac{1}{3} \frac{k_e^2}{E_e^2} \right) + G(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} + N(E_e) \vec{\xi}_n \cdot \vec{\xi}_e \right\}
 \end{aligned}$$

$$+ Q_e(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{\xi}_e)}{E_e(E_e + m_e)} + R(E_e) \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)}{E_e} + \dots \}, \quad (1)$$

where  $G_F = 1.1664 \times 10^{-11} \text{ MeV}^{-2}$  and  $V_{ud} = 0.97420(21)$  are the Fermi weak coupling constant and the Cabibbo–Kobayashi–Maskawa (CKM) matrix element [6], respectively,  $\lambda = -1.2750(9)$  is a real axial coupling constant [24],  $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2927 \text{ MeV}$  is the end-point energy of the electron spectrum, calculated for  $m_n = 939.5654 \text{ MeV}$ ,  $m_p = 938.2720 \text{ MeV}$  and  $m_e = 0.5110 \text{ MeV}$  [6],  $\vec{\xi}_n$  and  $\vec{\xi}_e$  are unit polarization vectors of the neutron and electron, respectively,  $F(E_e, Z = 1)$  is the relativistic Fermi function [42,64–66]

$$F(E_e, Z = 1) = \left(1 + \frac{1}{2}\gamma\right) \frac{4(2r_p m_e \beta)^{2\gamma}}{\Gamma^2(3 + 2\gamma)} \frac{e^{\pi\alpha/\beta}}{(1 - \beta^2)^\gamma} \left| \Gamma\left(1 + \gamma + i \frac{\alpha}{\beta}\right) \right|^2, \quad (2)$$

where  $\beta = k_e/E_e = \sqrt{E_e^2 - m_e^2}/E_e$  is the electron velocity,  $\gamma = \sqrt{1 - \alpha^2} - 1$ ,  $r_p$  is the electric radius of the proton. In the numerical calculations we use  $r_p = 0.841 \text{ fm}$  [67]. The Fermi function  $F(E_e, Z = 1)$  describes final-state Coulomb proton–electron interaction. Then,  $b$  is the Fierz interference term [37–52] (see also [36] and [23]). The infinitesimal solid angles  $d\Omega_e = \sin \vartheta_e d\vartheta_e d\varphi_e$  and  $d\Omega_\nu = \sin \vartheta_\nu d\vartheta_\nu d\varphi_\nu$  are defined relative to the 3-momenta  $\vec{k}_e$  and  $\vec{k}_\nu$  of the decay electron and antineutrino, respectively.

The correlation coefficients  $\zeta(E_e)$ ,  $a(E_e)$  and so on, taking into account the contributions of the SM and scalar and tensor interactions beyond the SM [23,52], and the Fierz interference term  $b$  are given by

$$\begin{aligned} \zeta(E_e) &= \zeta^{(\text{SM})}(E_e) \left(1 + \zeta^{(\text{BSM})}(E_e)\right), \quad b = \frac{b_F}{1 + \zeta^{(\text{BSM})}(E_e)}, \\ X(E_e) &= \frac{X^{(\text{SM})}(E_e) + X^{(\text{BSM})}(E_e)}{1 + \zeta^{(\text{BSM})}(E_e)}, \end{aligned} \quad (3)$$

where  $X = a, A, B, D, G, N, Q_e$  and  $R$ , respectively. The correlation coefficients  $\zeta^{(\text{SM})}(E_e)$  and  $X^{(\text{SM})}(E_e)$  for  $X = a, A$  and so on are calculated within the SM including the complete set of corrections caused by the weak magnetism and proton recoil of order  $O(E_e/M)$  and radiative corrections of order  $O(\alpha/\pi)$  [23,55]. In turn, the correlation coefficients  $b_F$ ,  $\zeta^{(\text{BSM})}(E_e)$  and  $X^{(\text{BSM})} = a, A$  and so on are defined by [23,52]

$$\begin{aligned} b_F &= \frac{1}{1 + 3\lambda^2} \text{Re}((C_S - \bar{C}_S) + 3\lambda(C_T - \bar{C}_T)), \\ \zeta^{(\text{BSM})}(E_e) &= \frac{1}{2} \frac{1}{1 + 3\lambda^2} (|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2), \\ a^{(\text{BSM})}(E_e) &= -\frac{1}{2} \frac{1}{1 + 3\lambda^2} (|C_S|^2 + |\bar{C}_S|^2 - |C_T|^2 + |\bar{C}_T|^2), \\ A^{(\text{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_T^* + \bar{C}_S C_T^* + 2C_T \bar{C}_T^*), \\ B^{(\text{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2} \text{Re}(2C_T \bar{C}_T^* - C_S \bar{C}_T^* - \bar{C}_S C_T^*) - b_N \frac{m_e}{E_e}, \\ D^{(\text{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2} \text{Im}(C_S C_T^* + \bar{C}_S \bar{C}_T^*), \\ G^{(\text{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_S^* + 3C_T \bar{C}_T^*), \end{aligned}$$

$$\begin{aligned}
N^{(\text{BSM})}(E_e) &= \frac{m_e}{E_e} \frac{1}{1+3\lambda^2} \text{Re}(C_S C_T^* + \bar{C}_S \bar{C}_T^* + |C_T|^2 + |\bar{C}_T|^2) + b_E, \\
Q_e^{(\text{BSM})}(E_e) &= \frac{1}{1+3\lambda^2} \text{Re}(C_S C_T^* + \bar{C}_S \bar{C}_T^* + |C_T|^2 + |\bar{C}_T|^2) - b_E, \\
R^{(\text{BSM})}(E_e) &= \frac{1}{1+3\lambda^2} \text{Im}(\lambda(C_S - \bar{C}_S) + (1+2\lambda)(C_T - \bar{C}_T)), \\
b_E &= \frac{1}{1+3\lambda^2} \text{Re}(\lambda(C_S - \bar{C}_S) + (1+2\lambda)(C_T - \bar{C}_T)), \\
b_N &= \frac{1}{1+3\lambda^2} \text{Re}(\lambda(C_S - \bar{C}_S) + (1-2\lambda)(C_T - \bar{C}_T)). \tag{4}
\end{aligned}$$

Now we may proceed to the analysis of contributions of scalar and tensor interactions beyond the SM to the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and correlation coefficients under consideration.

### 3. Neutron lifetime measured in beam experiments

Using the results, obtained in [23,52], the neutron lifetime is given by the following expression

$$\begin{aligned}
\frac{1}{\tau_n} &= (1+3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{2\pi^3} f_n \left( 1 + \frac{1}{2} \frac{1}{1+3\lambda^2} \left( (\text{Re}C_S)^2 + (\text{Im}C_S)^2 + (\text{Re}\bar{C}_S)^2 + (\text{Im}\bar{C}_S)^2 \right. \right. \\
&\quad \left. \left. + 3(\text{Re}C_T)^2 + 3(\text{Im}C_T)^2 + 3(\text{Re}\bar{C}_T)^2 + 3(\text{Im}\bar{C}_T)^2 \right) + b_F \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} \right), \tag{5}
\end{aligned}$$

where  $\langle m_e/E_e \rangle_{\text{SM}} = 0.6556$  [23,52], and  $f_n = 0.0616 \text{ MeV}^5$  is the Fermi integral [23,52]. Since  $(1+3\lambda^2)G_F^2|V_{ud}|^2 f_n/2\pi^3 = 1/879.6 \text{ s}^{-1}$  [23], we get

$$\begin{aligned}
\frac{1}{2} \frac{1}{1+3\lambda^2} \left( (\text{Re}C_S)^2 + (\text{Im}C_S)^2 + (\text{Re}\bar{C}_S)^2 + (\text{Im}\bar{C}_S)^2 + 3(\text{Re}C_T)^2 + 3(\text{Im}C_T)^2 \right. \\
\left. + 3(\text{Re}\bar{C}_T)^2 + 3(\text{Im}\bar{C}_T)^2 \right) = -b_F \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} - \frac{\Delta\tau_n}{\tau_n}, \tag{6}
\end{aligned}$$

where  $\Delta\tau_n = 8.4 \text{ s}$ ,  $\tau_n = 888.0 \text{ s}$  and  $\Delta\tau_n/\tau_n = 9.46 \times 10^{-3}$ . It is seen that the correlation coefficient  $b_F$ , defining the sign of the Fierz interference term  $b$  (see Eq. (3)) should be negative.

### 4. Correlation coefficients and numerical analysis of scalar and tensor coupling constants

The aim of this paper is to find anyone plausible solution for the scalar and tensor coupling constants of interactions beyond the SM, which should be compatible with present time accuracy of the definition of the correlation coefficients of the neutron  $\beta^-$ -decay. As a first step of the analysis of the contributions of scalar and tensor interaction beyond the SM we follow [46] and [52] and set i) zero the imaginary parts of the scalar and tensor coupling constants and ii)  $\bar{C}_j = -C_j$  for  $j = S, T$ . This gives

$$\frac{1}{1+3\lambda^2} (C_S^2 + 3C_T^2) = -b_F \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} - \frac{\Delta\tau_n}{\tau_n} \tag{7}$$

and



$$\begin{aligned}
b_F &= \frac{2}{1+3\lambda^2} (C_S + 3\lambda C_T), \quad \zeta^{(\text{BSM})}(E_e) = \frac{1}{1+3\lambda^2} (C_S^2 + 3C_T^2) \\
a^{(\text{BSM})}(E_e) &= -\frac{1}{1+3\lambda^2} (C_S^2 - C_T^2), \quad A^{(\text{BSM})}(E_e) = \frac{2}{1+3\lambda^2} (C_S C_T + C_T^2), \\
B^{(\text{BSM})}(E_e) &= \frac{2}{1+3\lambda^2} (C_T^2 - C_S C_T) - b_N \frac{m_e}{E_e}, \quad D^{(\text{BSM})}(E_e) = 0, \\
G^{(\text{BSM})}(E_e) &= \frac{1}{1+3\lambda^2} (C_S^2 + 3C_T^2), \quad N^{(\text{BSM})}(E_e) = \frac{m_e}{E_e} \frac{2}{1+3\lambda^2} \\
&\quad \times (C_S C_T + C_T^2) + b_E, \\
Q_e^{(\text{BSM})}(E_e) &= \frac{2}{1+3\lambda^2} (C_S C_T + C_T^2) - b_E, \quad R^{(\text{BSM})}(E_e) = 0, \\
b_E &= \frac{2}{1+3\lambda^2} (\lambda C_S + (1+2\lambda) C_T), \quad b_N = \frac{2}{1+3\lambda^2} (\lambda C_S + (1-2\lambda) C_T).
\end{aligned} \tag{8}$$

The simplest solution, which sticks a mile and is comparable with the constraints  $|C_S| = 0.0014(12)$  and  $|C_S| = 0.0014(13)$  obtained by Hardy and Towner [60] and González-Alonso et al. [61] from the superallowed  $0^+ \rightarrow 0^+$  transitions, is  $C_S = -\bar{C}_S = 0$ . Setting  $C_S = 0$  we transcribe the correlation coefficients into the form

$$\begin{aligned}
a(E_e) &= \frac{a^{(\text{SM})}(E_e) + \frac{1}{1+3\lambda^2} C_T^2}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \quad A(E_e) = \frac{A^{(\text{SM})}(E_e) + \frac{2}{1+3\lambda^2} C_T^2}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \\
B(E_e) &= \frac{B^{(\text{SM})}(E_e) + \frac{2}{1+3\lambda^2} C_T^2 - b_N \frac{m_e}{E_e}}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \quad D(E_e) = \frac{D^{(\text{SM})}(E_e)}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \\
G(E_e) &= \frac{G^{(\text{SM})}(E_e) + \frac{3}{1+3\lambda^2} C_T^2}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \quad N(E_e) = \frac{N^{(\text{SM})}(E_e) + \frac{2}{1+3\lambda^2} C_T^2 \frac{m_e}{E_e} + b_E}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \\
Q_e(E_e) &= \frac{Q_e^{(\text{SM})}(E_e) + \frac{2}{1+3\lambda^2} C_T^2 - b_E}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \quad R(E_e) = \frac{R^{(\text{SM})}(E_e)}{1 + \frac{3}{1+3\lambda^2} C_T^2}, \\
b_F &= \frac{6\lambda}{1+3\lambda^2} C_T, \quad b_N = 2 \frac{1-2\lambda}{1+3\lambda^2} C_T, \quad b_E = 2 \frac{1+2\lambda}{1+3\lambda^2} C_T.
\end{aligned} \tag{9}$$

At  $C_S = 0$  Eq. (7) reduces to the quadratic algebraical equation with the solution

$$C_T = -\lambda \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}} \left( 1 - \sqrt{1 - \frac{1+3\lambda^2}{3\lambda^2} \frac{\Delta\tau_n}{\tau_n} \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}}^{-2}} \right), \tag{10}$$

where we have chosen only the solution obeying the constraint  $|C_T| \ll 1$ . In the linear approximation we get



$$\begin{aligned}
C_T &= -\frac{1+3\lambda^2}{6\lambda} \frac{\Delta\tau_n}{\tau_n} \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}}^{-1} = 1.11 \times 10^{-2}, \quad \frac{3}{1+3\lambda^2} C_T^2 = 6.27 \times 10^{-5}, \\
b_F &= -\frac{\Delta\tau_n}{\tau_n} \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}}^{-1} = -1.44 \times 10^{-2}, \quad b_N = -\frac{1-2\lambda}{3\lambda} \frac{\Delta\tau_n}{\tau_n} \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}}^{-1} = 1.34 \times 10^{-2}, \\
b_E &= -\frac{1+2\lambda}{3\lambda} \left\langle \frac{m_e}{E_e} \right\rangle_{\text{SM}}^{-1} = -5.85 \times 10^{-3}.
\end{aligned} \tag{11}$$

Plugging Eq. (11) into Eq. (9) we obtain the following correlation coefficients, corrected by the contributions of tensor interactions beyond the SM:

$$\begin{aligned}
a(E_e) &= a^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}) + 2.09 \times 10^{-5}, \\
A(E_e) &= A^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}) + 4.18 \times 10^{-5}, \\
B(E_e) &= B^{(\text{SM})}(E_e) \left( 1 - 1.34 \times 10^{-2} \frac{m_e}{E_e} \right) - 2.09 \times 10^{-5}, \\
D(E_e) &= D^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}), \\
G(E_e) &= G^{(\text{SM})}(E_e) (1 + 1.25 \times 10^{-4}), \\
N(E_e) &= N^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}) + 4.18 \times 10^{-5} \frac{m_e}{E_e} - 5.85 \times 10^{-3}, \\
Q_e(E_e) &= Q_e^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}) + 5.89 \times 10^{-3}, \\
R(E_e) &= R^{(\text{SM})}(E_e) (1 - 6.27 \times 10^{-5}).
\end{aligned} \tag{12}$$

Thus, we have shown that there exists at least one solution  $C_S = -\bar{C}_S = 0$  and  $C_T = -\bar{C}_T = 1.11 \times 10^{-2}$  for real scalar and tensor coupling constants with the Fierz interference term  $b = -1.44 \times 10^{-2}$ , which does not contradict the constraints  $|C_S| = 0.0014(13)$  [60] and  $|C_S| = 0.0014(12)$  [61] obtained from the superallowed  $0^+ \rightarrow 0^+$  transitions and determines reasonable contributions of interactions beyond the SM to the correlation coefficients  $a(E_e)$  and  $A(E_e)$  of order  $10^{-4}$  [68] and to the correlation coefficients  $G(E_e)$ ,  $N(E_e)$  and  $Q_e(E_e)$  of order  $10^{-3}$ . In turn, the correlation coefficient  $B(E_e) \sim 1$  acquires the correction of order  $10^{-2}$ .

Now we have to compare the obtained results with the experimental data. For this aim we have to analyse the asymmetries of the neutron  $\beta^-$ -decay and the averaged values of correlation coefficients.

## 5. Asymmetries and averaged values of correlation coefficients

### 5.1. Electron asymmetry of neutron $\beta^-$ -decay

The most sensitive asymmetry of the neutron  $\beta^-$ -decay is the electron asymmetry, caused by correlations of the neutron spin  $\vec{\xi}_n$  and the electron 3-momentum  $\vec{k}_e$  and described by the scalar product  $\vec{\xi}_n \cdot \vec{k}_e$ . The experimental electron asymmetry  $A_{\text{exp}}(E_e)$  of electrons emitted forward and backward with respect to the neutron spin  $\vec{\xi}_e$  into the solid angle  $\Delta\Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$  with  $0 \leq \varphi \leq 2\pi$  and  $\theta_1 \leq \theta_e \leq \theta_2$  is equal to [23,52]

$$A_{\text{exp}}(E_e) = \frac{1}{2} \beta \bar{A}_W(E_e) P_n (\cos\theta_1 + \cos\theta_2), \tag{13}$$

where  $P_n = |\vec{\xi}_n| \leq 1$  is the neutron spin polarization, and the correlation coefficient  $\bar{A}_W(E_e)$  is [23,52]

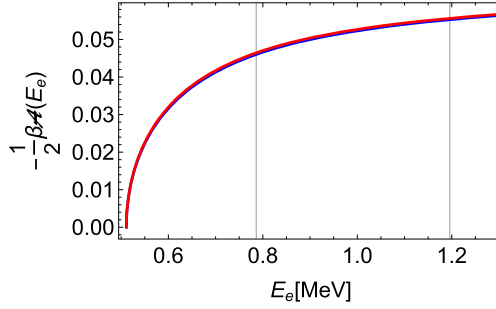


Fig. 1. The theoretical electron asymmetry, calculated in the SM (blue curve) at  $b = C_T = 0$  and with interactions beyond the SM (red curve) for  $C_T = 1.11 \times 10^{-2}$  and  $b = -1.44 \times 10^{-2}$ , respectively. (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

$$\bar{A}_W(E_e) = \frac{A_W(E_e)}{1 + b \frac{m_e}{E_e}}. \quad (14)$$

The correlation coefficient  $A_W(E_e)$  can be obtained from the correlation coefficient  $A(E_e)$  given by Eq. (9) and Eq. (12), respectively, with  $A^{(\text{SM})}(E_e)$  replaced by  $A_W^{(\text{SM})}(E_e)$  taken in the form

$$A_W^{(\text{SM})}(E_e) = \left(1 + \frac{\alpha}{\pi} f_n(E_e)\right) A_W(E_e), \quad (15)$$

where the function  $f_n(E_e)$  defines the radiative corrections, calculated by Shann [57], and the function  $A_W(E_e)$  has been calculated by Bilen'kii et al. [53] and by Wilkinson [54] by taking into account the contributions of order  $O(E_e/M)$  caused by the weak magnetism and proton recoil (see also [23]). Following [32] we plot in Fig. 1 the function  $-\frac{1}{2}\beta A(E_e)$  in the electron energy region  $m_e \leq E_e \leq E_0$ , where the electron asymmetry calculated in the SM at  $b = C_T = 0$  is given by the blue curve, whereas the electron asymmetry calculated with the account for the contributions of interactions beyond the SM at  $C_T = 1.11 \times 10^{-2}$  and  $b = 6\lambda C_T/(1 + 3\lambda^2) = -1.44 \times 10^{-2}$  is presented by the red curve. One may see that these two theoretical curves cannot be practically distinguished in experiments. The results presented by the theoretical curves in Fig. 1 can be also confirmed by the following estimates.

We propose to estimate the correlation coefficient  $\bar{A}_W(E_e)$ , calculated to leading order in the large nucleon mass  $M$  expansion and at the neglect of the radiative corrections. The result is equal to

$$\lim_{M \rightarrow \infty} \bar{A}_W(E_e) = \frac{A_0 \left(1 - \frac{3}{1 + 3\lambda^2} C_T^2\right) + \frac{2}{1 + 3\lambda^2} C_T^2}{1 + b \frac{m_e}{E_e}}, \quad (16)$$

$$A_0 = -2 \frac{\lambda(1 + \lambda)}{1 + 3\lambda^2} = -0.11933,$$

where  $A_0 = -0.11933$  is calculated at  $\lambda = -1.2750$  [24]. For  $C_T = 1.11 \times 10^{-2}$  and  $b = 6\lambda C_T/(1 + 3\lambda^2) = -1.44 \times 10^{-2}$  we get  $-0.12102 \leq \lim_{M \rightarrow \infty} \bar{A}_W(E_e) \leq -0.11997$  for the electron-energy region  $m_e \leq E_e \leq E_0$ . These values agree well with recent experimental values  $A_0 = -0.12015(34)_{\text{stat.}}(63)_{\text{syst.}}$  and  $(A_0)_{\text{exp}} = -0.12054(44)_{\text{stat.}}(68)_{\text{syst.}}$  [35].

The analogous estimate we may make for the correlation coefficient  $\bar{a}(E_e) = a(E_e)/(1 + b m_e/E_e)$ , describing the asymmetry of electron–antineutrino 3-momentum correlations, defined by the scalar product  $\vec{k}_e \cdot \vec{k}_\nu$ . To leading order in the large nucleon mass  $M$  expansion we get

$$\lim_{M \rightarrow \infty} \bar{a}(E_e) = \frac{a_0 \left(1 - \frac{3}{1 + 3\lambda^2} C_T^2\right) + \frac{1}{1 + 3\lambda^2} C_T^2}{1 + b \frac{m_e}{E_e}}, \quad a_0 = \frac{1 - \lambda^2}{1 + 3\lambda^2} = -0.1065, \quad (17)$$

where  $a_0 = -0.10645$  is calculated at  $\lambda = -1.2750$ . At  $C_T = 1.11 \times 10^{-2}$  and  $b = 6\lambda C_T/(1 + 3\lambda^2) = -1.44 \times 10^{-2}$  the correlation coefficient  $\lim_{M \rightarrow \infty} \bar{a}(E_e)$  is constrained by  $-0.1080 \leq \lim_{M \rightarrow \infty} a(E_e) \leq -0.1071$  in the electron-energy region  $m_e \leq E_e \leq E_0$ . The values  $-0.1080 \leq \lim_{M \rightarrow \infty} \bar{a}(E_e) \leq -0.1071$  agree well with recent experimental value  $(a_0)_{\text{exp}} = -0.1090 \pm 0.0030_{\text{stat.}} \pm 0.0028_{\text{sys.}}$  from the aCORN experiment reported in [71].

### 5.2. Antineutrino asymmetry of neutron $\beta^-$ -decay

The antineutrino asymmetry of the neutron  $\beta^-$ -decay is caused by correlations of the neutron spin  $\vec{\xi}_n$  and antineutrino 3-momentum  $\vec{k}_\nu$ , defined by the scalar product  $\vec{\xi}_n \cdot \vec{k}_\nu$ . Following [69] we define the antineutrino asymmetry  $B_{\text{exp}}(E_e)$  as follows [23]

$$B_{\text{exp}}(E_e) = \frac{N^{--}(E_e) - N^{++}(E_e)}{N^{--}(E_e) + N^{++}(E_e)}. \quad (18)$$

This expression determines the asymmetry of the emission of the antineutrinos into the forward and backward hemisphere with respect to the neutron spin, where  $N^{\mp\mp}(E_e)$  is the number of events of the emission of the electron–proton pairs as functions of the electron energy  $E_e$ . The signs  $(++)$  and  $(--)$  show that the electron–proton pairs were emitted parallel  $(++)$  and antiparallel  $(--)$  to a direction of the neutron spin. This means that antineutrinos were emitted antiparallel  $(++)$  and parallel  $(--)$  to a direction of the neutron spin. The number of events  $N^{--}(E_e)$  and  $N^{++}(E_e)$  are defined by the electron-energy and angular distribution of the neutron  $\beta^-$ -decay, integrated over the forward and backward hemisphere relative to the neutron spin, respectively [23]. The analytical expression for  $B_{\text{exp}}(E_e)$  is given by [23]

$$B_{\text{exp}}^{(r \leq 1)}(E_e) = \frac{2P}{3} \frac{(3 - r^2)\bar{B}(E_e) - (3 - 2r)\beta\bar{A}(E_e) + \left(1 - \frac{3}{5}r^2\right)\beta^2\bar{K}_n(E_e) - \left(1 - \frac{2}{5}r^3\right)\beta\bar{Q}_n(E_e)}{(4 - 2r) - \left(1 - \frac{1}{2}r^2\right)\beta\bar{a}(E_e) - \frac{1}{2}\bar{a}_0(E_e)\beta^2 r(1 - r^2) \frac{E_e}{M}} \quad (19)$$

and

$$B_{\text{exp}}^{(r \geq 1)}(E_e) = \frac{2P}{3} \frac{\bar{B}(E_e) - \frac{1}{2}\left(\bar{A}(E_e) + \frac{3}{5}\bar{Q}_n(E_e)\right)\frac{\beta}{r} + \frac{1}{5}\bar{K}_n(E_e)\frac{\beta^2}{r^2}}{1 - \bar{a}(E_e)\frac{\beta}{4r} + \frac{1}{4}\bar{a}_0(E_e)\beta^2\left(1 - \frac{1}{r^2}\right)\frac{E_e}{M}}, \quad (20)$$

where  $r = k_e/E_\nu = k_e/(E_0 - E_e)$  [69] (see also [23]). For  $r \leq 1$  and  $r \geq 1$  the electron kinetic energies are restricted by  $0 \leq T_e \leq (E_0 - m_e)^2/2E_0 = 236 \text{ keV}$  and  $(E_0 - m_e)^2/2E_0 = 236 \text{ keV} \leq T_e \leq E_0 - m_e$ , respectively. At  $r = 1$  or at the electron kinetic energy  $T_e = (E_0 - m_e)^2/2E_0 =$

236 keV the antineutrino asymmetry given by Eq. (19) and Eq. (20) is continuous. The correlation coefficients  $\tilde{X}(E_e)$  for  $X = a, A, B, Q_n$  and  $K_n$  take the form  $\tilde{X}(E_e) = X(E_e)/(1 + b m_e/E_e)$ , where  $X(E_e)$  are defined by Eq. (9) and Eq. (12), respectively, with  $X^{(\text{SM})}(E_e)$  calculated in [23]. Then,  $\tilde{a}_0(E_e)$  is given by  $\tilde{a}_0(E_e) = (a_0(1 - 6.27 \times 10^{-5}) + 2.09 \times 10^{-5})/(1 + b m_e/E_e)$  (see Eq. (12)), where the numerator is calculated to leading order in the large nucleon mass  $M$  expansion. The main contribution to the antineutrino asymmetry  $B_{\text{exp}}(E_e)$  comes from the term  $B^{(\text{SM})}(E_e) \sim 1$ , which does not depend on the tensor coupling constant  $C_T$ . Since the correlation coefficients  $a^{(\text{SM})}(E_e)$  and  $A^{(\text{SM})}(E_e)$  are of order  $a^{(\text{SM})}(E_e) \sim A^{(\text{SM})}(E_e) \sim -0.1$  and the values of correlations coefficients  $K_n^{(\text{SM})}(E_e)$  and  $Q_n^{(\text{SM})}(E_e)$  are of order  $10^{-3}$ , the contribution of the Fierz interference term  $b = -1.44 \times 10^{-2}$  to the antineutrino asymmetry is of about 0.1 % or even smaller. The experimental value of the correlation coefficients  $(B_0)_{\text{exp}} = 0.9802(50)$  [69] is defined with an accuracy of about 0.5 %. This means that at the present level of experimental accuracy the antineutrino asymmetry of the neutron  $\beta^-$ -decay is not sensitive to the contribution of the tensor interactions beyond the SM with  $C_T = 1.11 \times 10^{-2}$  and the Fierz interference term  $b = 6\lambda C_T/(1 + 3\lambda^2) = -1.44 \times 10^{-2}$ .

### 5.3. Averaged values of correlation coefficients

In the neutron  $\beta^-$ -decay with polarized neutron and electron and unpolarized proton the averaged values have been measured only for the correlation coefficients  $N(E_e)$  and  $R(E_e)$ :  $N_{\text{exp}} = \langle N(E_e) \rangle = 0.067 \pm 0.011 \pm 0.004$  and  $R_{\text{exp}} = \langle R(E_e) \rangle = 0.004 \pm 0.012 \pm 0.005$  [70]. Using Eq. (12) and the results, obtained in [52,55], we get

$$\langle N(E_e) \rangle = 0.07185 \quad , \quad \langle R(E_e) \rangle = 0.00089. \quad (21)$$

The averaged value of the correlation coefficient  $\langle N(E_e) \rangle = 0.07185$  agrees with the experimental one within one standard deviation. In turn, as it follows from Eq. (12) the averaged value of the correlation coefficient  $\langle R(E_e) \rangle$  can acquire only the relative correction of order  $10^{-4}$ , caused by tensor interactions beyond the SM. This does not contradict the experimental data by [70].

## 6. Discussion

The main aim of this paper is to show that the fit of the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, by the Fierz interference term  $b = -1.44 \times 10^{-2}$  does not contradict contemporary experimental data on the values of the correlation coefficients of the neutron  $\beta^-$ -decay with polarized neutron, polarized electron and unpolarized proton. We have found that there exist as minimum one solution of our interest for real scalar and tensor coupling constants  $C_S = -\bar{C}_S = 0$  and  $C_T = -\bar{C}_T = 1.11 \times 10^{-2}$ . This solution agrees well with the constraint  $|C_S| = 0.0014(13)$  and  $|C_S| = 0.0014(12)$  extracted from the superallowed  $0^+ \rightarrow 0^+$  transitions by Hardy and Towner [60] and González-Alonso et al. [61], respectively, and defines the Fierz interference term  $b = -1.44 \times 10^{-2}$ . The contributions of this solution of order  $10^{-4} - 10^{-2}$  to the correlation coefficients of the electron-energy and angular distributions of the neutron  $\beta^-$ -decays do not contradict contemporary experimental data on the correlation coefficients and asymmetries of the neutron  $\beta^-$ -decays with polarized neutron, polarized electron and unpolarized proton. We would like to emphasize that the problem of such a fit is not related only to our model [5] for the solution of the neutron lifetime anomaly, but it appears always as soon as the neutron dark matter decay modes  $n \rightarrow \chi + \text{anything}$  would have been observed. For example, the neutron dark matter decay mode  $n \rightarrow \chi + e^- + e^+$ , proposed by Fornal and

Grinstein [1], can be in principle observed in the region  $100 \text{ keV} < T_{-+}$  of kinetic energies by the UCNA and PERKEO II Collaborations [4,5]. This concerns also the decay mode  $n \rightarrow \chi + \phi$  [1,2], which is not still investigated experimentally. Hence, one may argue that in any model of neutron dark matter decays (see, for example, [63]) our analysis of the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and the correlation coefficients should be meaningful and actual.

The results of our analysis on the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and correlation coefficients of electron-energy and angular distributions of the neutron  $\beta^-$ -decays can be interpreted as an allowance for the neutron to have the dark matter decay modes  $n \rightarrow \chi + \text{anything}$ , which have been analysed in [5] in the physical phase of a quantum field theory model with  $SU_L(2) \times U_Y(1) \times U'_R(1) \times U''_L(1)$  gauge symmetry or in any other model of neutron dark matter decays (see, for example, [63]). In our model the traces of the dark matter fermions  $\chi$  with mass  $m_\chi < m_n$  can be searched in terrestrial laboratories through the measurements of the differential cross section for the low-energy inelastic electron–neutron scattering  $e^- + n \rightarrow \chi + e^-$  [5] and the triple-differential cross section for the electrodisintegration of the deuteron into dark matter fermions and protons  $e^- + d \rightarrow \chi + p + e^-$  close to threshold [22].

Our analysis of the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments, and the correlation coefficients of the neutron  $\beta^-$ -decay, taking into account the complete set of corrections of order  $10^{-3}$ , caused by the weak magnetism and proton recoil of order  $O(E_e/M)$  and radiative corrections of order  $O(\alpha/\pi)$ , and the tensor coupling constants  $C_T = -\bar{C}_T = 1.11 \times 10^{-2}$  and the Fierz interference term  $b = -1.44 \times 10^{-2}$ , does diminish an important role of the SM corrections of order  $10^{-5}$ , which has been pointed out in [52,55,72,73]. As has been shown in [55] the SM corrections of order  $10^{-5}$  concern i) Wilkinson's corrections [54], i.e. the higher order corrections caused by 1) the proton recoil in the Coulomb electron–proton final-state interaction, 2) the finite proton radius, 3) the proton–lepton convolution and 4) the higher-order *outer* radiative corrections, and then ii) the higher order corrections defined by 1) the radiative corrections of order  $O(\alpha^2/\pi^2)$ , calculated to leading order in the large nucleon mass expansion, 2) the radiative corrections of order  $O(\alpha E_e/M)$ , calculated to next-to-leading order in the large nucleon mass expansion, which depend strongly on contributions of hadronic structure of the nucleon [72,73], and 3) the corrections of the weak magnetism and proton recoil of order  $O(E_e^2/M^2)$ , calculated to next-to-next-to-leading order in the large nucleon mass expansion [52,72]. These theoretical corrections should provide for the analysis of experimental data of “discovery” experiments the required  $5\sigma$  level of experimental uncertainties of a few parts in  $10^{-5}$  [55].

An important role of strong low-energy interactions and contributions of hadronic structure of the nucleon for a correct gauge invariant calculation of radiative corrections of order  $O(\alpha E_e/M)$  and  $O(\alpha^2/\pi^2)$  as functions of the electron energy  $E_e$  has been pointed out in [52,72]. This agrees well with Weinberg's assertion about important role of strong low-energy interactions in decay processes [74]. A procedure for the calculation of these radiative corrections to the neutron  $\beta^-$ -decays with a consistent account for contributions of strong low-energy interactions, leading to gauge invariant observable expressions dependent on the electron energy  $E_e$  determined at the confidence level of Sirlin's radiative corrections [56], has been proposed in [72,73].

The calculation of the SM corrections of order  $10^{-5}$  should also give a theoretical background for the experimental analysis of the corrections caused by the second class currents [75], which has been analysed in the neutron  $\beta^-$ -decay with polarized neutron and unpolarized proton and

electron by Gardner and Zang [50] and Garner and Plaster [51], and in the neutron  $\beta^-$ -decay with polarized neutron and electron and unpolarized proton by Ivanov et al. [52].

Finalizing our discussion we would like to emphasize that perspectives of development of investigations of the neutron  $\beta^-$ -decays with the contribution of the Fierz interference term  $b = -1.44 \times 10^{-2}$  become real only in case of discovery of the neutron dark matter decay modes  $n \rightarrow \chi + \text{anything}$  in terrestrial laboratories by measuring the differential cross sections for the inelastic low-energy electron–neutron scattering  $e^- + n \rightarrow \chi + e^-$  and for the electrodisintegration of the deuteron into dark matter fermions and protons  $e^- + d \rightarrow \chi + p + e^-$  close to threshold. These processes are induced by the same interaction  $n\chi e^- e^+$  having the strength of the interaction  $n\chi \nu_e \bar{\nu}_e$  responsible for the dark matter decay mode  $n \rightarrow \chi + \nu_e + \bar{\nu}_e$ , allowing to explain the neutron lifetime anomaly. Of course, indirect confirmations of existence of the neutron dark matter decay mode  $n \rightarrow \chi + \nu_e + \bar{\nu}_e$  through the evolution of neutron stars and neutron star cooling should also testify a revision of the experimental data on the neutron  $\beta^-$ -decays with substantially improved accuracies of the measurements of correlation coefficients and asymmetries allowing to feel the contribution of the tensor interactions with the tensor coupling constant  $C_T = -\bar{C}_T = 1.11 \times 10^{-2}$ , giving the contributions of order  $10^{-4}$  to the correlation coefficients, and the Fierz interference term  $b = -1.44 \times 10^{-2}$ . Such a value of the Fierz interference term is required by the necessity to fit the rate of neutron decay modes  $n \rightarrow p + \text{anything}$ , measured in beam experiments. It is obvious that in this connection a relative accuracy of measurements of the rate of the neutron decay modes  $n \rightarrow p + \text{anything}$  (or the neutron lifetime  $\tau_n = 888.0(2.0)$  s) in beam experiments should be improved up to a few parts of  $10^{-4}$  or even better.

## Acknowledgements

We are grateful to Hartmut Abele for fruitful discussions stimulating the work under this paper and to Martin González-Alonso for his comments on the value of the scalar coupling constant. The work of A. N. Ivanov was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under contracts P31702-N27, P26781-N20 and P26636-N20 and “Deutsche Forschungsgemeinschaft” (DFG) AB 128/5-2. The work of R. Höllwieser was supported by the Deutsche Forschungsgemeinschaft in the SFB/TR 55. The work of M. Wellenzohn was supported by the MA 23 (FH-Call 16) under the project “Photonik – Stiftungsprofessur für Lehre”.

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