$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \ \ \epsilon_{ij} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right), \ \ i = 1, 2, \ldots, k; j = 1, \ldots, n_i$$

ect to model constraint $\sum_{i=1}^{k} n_i \tau_i = 0$. Total number of observations $N = \sum$

$$SSTR = \sum_{i=1}^{k} n_i \left(\bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet}\right)^2, \ \ SSTO = \sum_{i=1}^{k}\sum_{j=1}^{n_i} \left(Y_{ij} - \bar{Y}_{\bullet\bullet}\right)^2$$

$$SSE = \sum_{i=1}^{k}\sum_{j=1}^{n_i} \left(Y_{ij} - \bar{Y}_{i\bullet}\right)^2 = \sum_{i=1}^{k} (n_i - 1) s_i^2, \ \ s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i} \left(Y_{ij} - \bar{Y}_{i\bullet}\right)^2$$

ANOVA Table:

Source	SS	df	MS	F
Treatment	$SSTR$	$k-1$	$MSTR = \frac{SSTR}{k-1}$	$F = \frac{MSTR}{MSE}$
Error	SSE	$N-k$	$MSE = \frac{SSE}{N-k}$	
Total	$SSTO$	$N-1$		

Expected Mean Square:

$$\mathbf{E}(MSE) = \sigma^2, \ \mathbf{E}(MSTR) = \sigma^2 + \frac{1}{k-1}\sum_{i=1}^{k} n_i \tau_i^2.$$

Primary hypotheses: $H_0 : \tau_1 = \cdots = \tau_k = 0$ vs H_1 : at least one $\tau_i \neq 0$.

$$F = \frac{MSTR}{MSE} \overset{H_0}{\sim} F(k-1, N-k).$$

CI for μ_i:

$$\bar{Y}_{i\bullet} \pm t_{\alpha/2}(N-k) \cdot \sqrt{\frac{MSE}{n_i}}$$

CI for $\mu_i - \mu_j$:

$$(\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}) \pm t_{\alpha/2}(N-k) \cdot \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Fisher's Least Significant Difference for a Balanced Design:

$$(\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}) \pm t_{\alpha/2}(N-k) \cdot \sqrt{\frac{2 \cdot MSE}{n}}$$

Model (Hierarchical $a \times b \times n$ design)

$$Y_{ijk} = \mu_{\bullet\bullet} + \alpha_i + \beta_{j(i)} + \epsilon_{k(i,j)}$$

with constraints and assumptions

$$\sum_i \alpha_i = 0, \ \ \beta_{j(i)} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma_\beta^2\right), \ \ \epsilon_{ijk} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right)$$

and $\beta_{j(i)}$ and $\epsilon_{k(i,j)}$ are independent.

ANOVA Table

Source	SS	df	MS	F
A	SSA	$a-1$	$MSA = \frac{SSA}{a-1}$	$F_A = \frac{MSA}{MSB(A)}$
B(A)	$SSB(A)$	$a(b-1)$	$MSB(A) = \frac{SSB(A)}{a(b-1)}$	$F_{B(A)} = \frac{MSB(A)}{MSE}$
Error	SSE	$ab(n-1)$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$SSTO$	$abn-1$		

Hypothesis Testing

- Test for Factor A Main Effects:

$$H_0 \colon \alpha_i = 0, \forall i \qquad versus \qquad H_1 \colon \text{at least one } \alpha_i \neq 0$$

Test Statistic: $F = \frac{MSA}{MSB(A)} \overset{H_0}{\sim} F(a-1, a(b-1))$

- Testing for Variability of the Effects of Factor B:

$$H_0 \colon \sigma_\beta^2 = 0 \qquad versus \qquad H_1 \colon \sigma_\beta^2 > 0$$

Test Statistic: $F = \frac{MSB(A)}{MSE} \overset{H_0}{\sim} F(a(b-1), ab(n-1))$

Variance Estimate

$$\hat{\sigma}_\beta^2 = \frac{MSB(A) - MSE}{n}$$

Bonferroni Simultaneous CI for $\{(\mu_i - \mu_j), \ \forall (i,j)\}$:

$$(\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}) \pm t_{\alpha*/2}(N-k)\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}, \qquad \alpha^* = \frac{\alpha}{k(k-1)/2}$$

Tukey Simultaneous CI for $\mu_i - \mu_j$:

$$(\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}) \pm q_\alpha(k, N-k)\sqrt{\frac{MSE}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

lom Effect Model (balanced design):

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \ \ \epsilon_{ij} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right), \ \ i = 1, 2, \ldots, k; j = 1, \ldots, n$$

e $\tau_i \overset{iid}{\sim} N(0, \sigma_\tau^2)$, τ_i and ϵ_{ij} are independent. Total number of observations $N =$

$\mathbf{E}(MSTR) = \sigma^2 + n\sigma_\tau^2$, for a balanced design

Primary hypotheses: $H_0 : \sigma_\tau^2 = 0$ vs $H_1 : \sigma_\tau^2 > 0$.

$$F = \frac{MSTR}{MSE} \overset{H_0}{\sim} F(k-1, N-k).$$

Covariance:

$$\mathbf{COV}(Y_{ij}, Y_{i'j'}) = \begin{cases} 0, & \text{if } i \neq i' \\ \sigma_\tau^2, & \text{if } i = i', \ \ j \neq j' \\ \sigma_\tau^2 + \sigma^2, & \text{if } i = i', \ \ j = j' \end{cases}$$

Coefficient of Correlation Between Responses from the Same Factor Level:

$$\rho\left(Y_{ij}, Y_{ij'}\right) = \frac{\sigma_\tau^2}{\sigma_Y^2} = \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma^2}$$

Variance component estimate:

$$\hat{\sigma}^2 = MSE, \ \ \hat{\sigma}_\tau^2 = \frac{MSTR - MSE}{n}.$$

$100(1-\alpha)\%$ CI for μ: $\bar{Y}_{\bullet\bullet} \pm t_{\alpha/2}(k-1)\sqrt{\frac{MSTR}{N}}$

mized Complete Block Design (RCBD):

el with fixed treatment and block factors:

$$Y_{ij} = \mu + \underbrace{\tau_i}_{\substack{\text{trt}\\\text{effect}}} + \underbrace{\beta_j}_{\substack{\text{block}\\\text{effect}}} + \epsilon_{ij}, \ \ i = 1, 2, \ldots, k; \ \ j = 1, 2, \ldots, b$$

ect to $\sum_{i=1}^{k} \tau_i = 0, \sum_{j=1}^{b} \beta_j = 0$.

Model assumption:

$$Y_{ijk} = \mu + \underbrace{\alpha_i}_{\text{Factor } A} + \underbrace{\beta_j}_{\text{Factor } B} + \underbrace{(\alpha\beta)_{ij}}_{\text{Interaction}} + \epsilon_{ijk}$$

where $i = 1, \ldots, a \qquad j = 1, \ldots, b \qquad k = 1, \ldots, n$, and

$$\sum_i \alpha_i = 0, \ \ \beta_j \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma_\beta^2\right), \ \ \sum_i (\alpha\beta)_{ij} = 0, \ \ \epsilon_{ijk} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right)$$

$$(\alpha\beta)_{ij} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \frac{a-1}{a}\sigma_{\alpha\beta}^2\right), \ \ \mathbf{COV}\left((\alpha\beta)_{ij}, (\alpha\beta)_{i'j}\right) = -\frac{1}{a}\sigma_{\alpha\beta}^2, \forall i \neq i'$$

In addition, assume $\{\beta_j\}, \{(\alpha\beta)_{ij}\}, \{\epsilon_{ijk}\}$ are mutually independent.

ANOVA Table

Source	SS	df	MS	F
A	SSA	$a-1$	$MSA = \frac{SSA}{a-1}$	$F_A = \frac{MSA}{MSAB}$
B	SSB	$b-1$	$MSB = \frac{SSB}{b-1}$	$F_B = \frac{MSB}{MSE}$
AB	$SSAB$	$(a-1)(b-1)$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{AB} = \frac{MSAB}{MSE}$
Error	SSE	$ab(n-1)$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$SSTO$	$abn-1$		

Hypotheses and Distributions of F

- Test for Interactions

$$H_0 \colon \sigma_{\alpha\beta}^2 = 0 \qquad versus \qquad H_1 \colon \sigma_{\alpha\beta}^2 > 0$$

$$F_{AB} = \frac{MSAB}{MSE} \sim F\left((a-1)(b-1), \ ab(n-1)\right)$$

- When there is no significant interaction effects, test for main effects
  - Test for Factor A:

$$H_0 \colon \alpha_i = 0, \forall i \qquad versus \qquad H_1 \colon \text{at least one } \alpha_i \neq 0$$

$$F_A = \frac{MSA}{MSAB} \sim F\left((a-1), \ (a-1)(b-1)\right)$$

  - Test for Factor B:

$$H_0 \colon \sigma_\beta^2 = 0 \qquad versus \qquad H_1 \colon \sigma_\beta^2 > 0$$

$$F_B = \frac{MSB}{MSE} \sim F\left((b-1), \ ab(n-1)\right)$$

Source	SS	df	MS	F
Treatment	$SSTR$	$k-1$	$MSTR = \frac{SSTR}{k-1}$	$F_{TR} = \frac{MSTR}{MSE}$
Block	SSB	$b-1$	$MSB = \frac{SSB}{b-1}$	$F_B = \frac{MSB}{MSE}$
Error	SSE	$(k-1)(b-1)$	$MSE = \frac{SSE}{(k-1)(b-1)}$	
Total	$SSTO$	$kb-1$		

Expected Mean Squares

$$\mathbf{E}[MSE] = \sigma^2, \ \mathbf{E}[MSTR] = \sigma^2 + \frac{b}{k-1}\sum_i \tau_i^2, \ \mathbf{E}[MSB] = \sigma^2 + k\frac{k}{b-1}\sum_j \beta_j^2.$$

Primary hypotheses: $H_0 : \tau_1 = \ldots = \tau_k = 0$ vs H_1 : at least one $\tau_i \neq 0$.

$$F_{TR} = \frac{MSTR}{MSE} \overset{H_0}{\sim} F(k-1, (k-1)(b-1)).$$

Test for block effects $H_0 : \beta_1 = \ldots = \beta_b = 0$ vs H_1 : at least one $\beta_j \neq 0$.

$$F_B = \frac{MSB}{MSE} \overset{H_0}{\sim} F(b-1, (k-1)(b-1)).$$

CI for the i^{th} treatment mean: $\bar{Y}_{i\bullet} \pm t_{\alpha/2}\left([k-1][b-1]\right) \cdot \sqrt{\frac{MSE}{b}}$

CI for $\mu_i - \mu_j$ (treatment means): $\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet} \pm t_{\alpha/2}\left([k-1][b-1]\right)\sqrt{\frac{2MSE}{b}}$

Tukey Simultaneous CI for $\mu_i - \mu_j$: $\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet} \pm q_\alpha(k, [k-1][b-1])\sqrt{\frac{MSE}{b}}$

$\times\, b$ Factorial Design (2-way ANOVA with Fixed Effects)

Statistical model

$$Y_{ijk} = \mu + \underbrace{\alpha_i}_{\text{Factor } A} + \underbrace{\beta_j}_{\text{Factor } B} + \underbrace{(\alpha\beta)_{ij}}_{\text{Interaction}} + \epsilon_{ijk}, \epsilon_{ijk} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right)$$

where $i = 1, \ldots, a \qquad j = 1, \ldots, b \qquad k = 1, \ldots, n$, and subject to constraints

$$\sum_{1=1}^{a} \alpha_i = 0, \ \ \sum_{j=1}^{b} \beta_j = 0, \ \ \sum_i (\alpha\beta)_{ij} = 0, \ \ \forall j, \ \ \sum_j (\alpha\beta)_{ij} = 0, \ \ \forall i.$$

Source	SS	df	MS	F
A	SSA	$a-1$	$MSA = \frac{SSA}{a-1}$	$F_A = \frac{MSA}{MSE}$
B	SSB	$b-1$	$MSB = \frac{SSB}{b-1}$	$F_B = \frac{MSB}{MSE}$
AB	$SSAB$	$(a-1)(b-1)$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{AB} = \frac{MSAB}{MSE}$
Error	SSE	$ab(n-1)$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$SSTO$	$abn-1$		

$$\mathbf{E}(MSA) = \sigma^2 + bn\frac{\sum_i \alpha_i^2}{a-1}$$

- $\mathbf{E}(MSAB) = \sigma^2 + n\frac{\sum_i\sum_j (\alpha\beta)_{ij}^2}{(a-1)(b-1)}$

$$\mathbf{E}(MSB) = \sigma^2 + an\frac{\sum_j \beta_j^2}{b-1}$$

- $\mathbf{E}(MSE) = \sigma^2$

Test for interaction $H_0 : (\alpha\beta)_{ij} = 0, \forall (i,j)$ vs H_1 : at least one $(\alpha\beta)_{ij} \neq 0$.

$$F_{AB} = \frac{MSAB}{MSE} \overset{H_0}{\sim} F((a-1)(b-1), ab(n-1))$$

If there is no significant interaction effects, test for main effects

- Test for factor A effect, $H_0 : \alpha_i = 0, i = 1, \ldots, a$ vs H_1 : at least one $\alpha_i \neq 0$.

$$F_A = \frac{MSA}{MSE} \overset{H_0}{\sim} F((a-1, ab(n-1)))$$

- Test for factor B effect, $H_0 : \beta_j = 0, j = 1, \ldots, b$ vs H_1 : at least one $\beta_j \neq 0$.

$$F_B = \frac{MSB}{MSE} \overset{H_0}{\sim} F((b-1, ab(n-1)))$$

- $100(1-\alpha)\%$ CI for $\mu_{i\bullet}$: $\bar{Y}_{i\bullet} \pm t_{\alpha/2}\left[(n-1)ab\right]\sqrt{\frac{MSE}{bn}}$
- Bonferroni Simultaneous CI for $\mu_{i\bullet} - \mu_{i'\bullet}$: $(\bar{Y}_{i\bullet\bullet} - \bar{Y}_{i'\bullet\bullet}) \pm t_{\frac{\alpha}{2m}}\left[(n-1)ab\right]\sqrt{\frac{2MSE}{bn}}$
- Tukey Simultaneous CI for $\mu_{i\bullet} - \mu_{i'\bullet}$: $(\bar{Y}_{i\bullet\bullet} - \bar{Y}_{i'\bullet\bullet}) \pm q_\alpha\left[b, (n-1)ab\right]\sqrt{\frac{MSE}{bn}}$

If interaction effects present, compare within-group treatment means

$$H_0 \colon \mu_{i1} - \mu_{ij} = 0, \forall j = 2, \ldots, b \qquad versus \qquad H_1 \colon \text{ at least one} \mu_{i1} - \mu_{ij} \neq 0$$

Model (Hierarchical $a \times b \times n$ design)

$$Y_{ijk} = \mu_{\bullet\bullet} + \alpha_i + \beta_{j(i)} + \epsilon_{k(i,j)}$$

with constraints and assumptions

$$\sum_i \alpha_i = 0, \ \ \sum_j \beta_{j(i)} = 0, \forall\ i, \ \ \epsilon_{ijk} \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right)$$

ANOVA Table

Source	SS	df	MS	F
A	SSA	$a-1$	$MSA = \frac{SSA}{a-1}$	$F_A = \frac{MSA}{MSE}$
B(A)	$SSB(A)$	$a(b-1)$	$MSB(A) = \frac{SSB(A)}{a(b-1)}$	$F_{B(A)} = \frac{MSB(A)}{MSE}$
Error	SSE	$ab(n-1)$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$SSTO$	$abn-1$		

Hypothesis Testing and Estimation

- Test for Factor A Main Effects:

$$H_0 \colon \alpha_i = 0, \forall i \qquad versus \qquad H_1 \colon \text{at least one } \alpha_i \neq 0$$

Test Statistic: $F = \frac{MSA}{MSB(A)} \overset{H_0}{\sim} F(a-1, ab(n-1))$

- Testing for Variability of the Effects of Factor B:

$$H_0 \colon \beta_{j(i)} = 0, \forall (i,j) \qquad versus \qquad H_1 \colon \text{at least one } \beta_{j(i)} \neq 0$$

Test Statistic: $F = \frac{MSB(A)}{MSE} \overset{H_0}{\sim} F(a(b-1), ab(n-1))$

- $100(1-\alpha)\%$ CI for $\mu_{i\bullet}$ are given by

$$\bar{Y}_{i\bullet\bullet} \pm t_{\alpha/2}(\,ab(n-1))\sqrt{\frac{MSE}{bn}}$$

- $100(1-\alpha)\%$ CI for μ_{ij} are

$$\bar{Y}_{ij\bullet} \pm t_{\alpha/2}\left[ab(n-1)\right]\sqrt{\frac{MSE}{n}}$$

Statistical model setup:

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, i = 1, \ldots, n$$

where $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

1). Least Square Estimation of Coefficients β_0, β_1

- Least Square objective function:

$$Q(\beta_0, \beta_1) = \sum_{i=1}^{n} \left(Y_i - \beta_0 - \beta_1 x_i\right)^2$$

- Least Square Estimates:

$$(\hat{\beta}_0, \hat{\beta}_1) = \arg\min_{\beta_0, \beta_1} Q(\beta_0, \beta_1) \iff \frac{\partial Q}{\partial \beta_0} = 0, \ \frac{\partial Q}{\partial \beta_1} = 0,$$

- Let $s_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}), s_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$, then

$$\hat{\beta}_1 = \frac{s_{xy}}{s_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \sum_i c_i Y_i$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \sum_{i=1}^{n} k_i Y_i$$

Note that

$$c_i = \frac{x_i - \bar{x}}{s_{xx}}, \qquad \sum_i c_i = 0, \qquad \sum_i c_i X_i = 1, \qquad \sum_i c_i^2 = s_{xx}^{-1}, \qquad k_i = \frac{1}{n} - \bar{x}c_i.$$

- Properties of the residuals:

$$\sum_i \hat{\epsilon}_i = 0, \qquad \sum_i \hat{\epsilon}_i x_i = 0, \qquad \sum_i \hat{Y}_i \hat{\epsilon}_i = 0, \qquad \sum_i \hat{\epsilon}_i \left(\hat{Y}_i - \bar{Y}\right) = 0.$$

2). Sampling Distribution of Least Square Estimators

- Sampling Distribution of $\hat{\beta}_1 = \sum_i c_i Y_i$:
  - When σ^2 is known, $\hat{\beta}_1 \sim \mathsf{N}\left(\beta_1, \frac{\sigma^2}{s_{xx}}\right)$
  - When σ^2 is unknown, then $\frac{\hat{\beta}_1 - \beta_1}{\mathbf{SE}\left(\hat{\beta}_1\right)} \sim t\,(n-2)$, $\quad \mathbf{SE}\left(\hat{\beta}_1\right) = \sqrt{\frac{MSE}{s_{xx}}}$

- $100(1-\alpha)\%$ C.I. for β_1: $\qquad \hat{\beta}_1 \pm t_{\alpha/2}(n-2) \cdot \mathbf{SE}\left(\hat{\beta}_1\right)$.

Distribution of $\hat{\beta}_0$: $\hat{\beta}_0 \sim \mathsf{N}\left(\beta_0, \sigma^2\left[\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right]\right)$

Matrix Form $Y = X\beta + \epsilon$, where

$$X = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ x_1 & x_2 & \ldots & x_n \end{pmatrix}^T, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

and the least square estimator $\hat{\beta} \sim \mathsf{N}\left(\beta, \sigma^2\left(X'X\right)^{-1}\right)$.

Analysis of Variance (ANOVA)

Primary Hypotheses $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$.

Decomposition of Sum Squares $SSTO = SSR + SSE$

- $SSTO = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$
- $SSR = \sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right)^2 = \hat{\beta}_1^2 s_{xx}$. Under $H_0 : \beta_1 = 0$, $\frac{SSR}{\sigma^2} \sim \chi^2(1)$.
- $SSE = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2$, $\qquad \frac{SSE}{\sigma^2} \sim \chi^2(n-2)$
- $SSE \perp\!\!\!\perp SSR$

(Basic) ANOVA Table

Source	SS	df	MS	F
Regression	$SSR = \sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right)^2$	1	$MSR = \frac{SSR}{1}$	$F = \frac{MSR}{MSE}$
Error	$SSE = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2$	$n-2$	$MSE = \frac{SSE}{n-2}$	
Total	$SST = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$	$n-1$		

Under $H_0 : \beta_1 = 0$,

$$F = \frac{MSR}{MSE} \sim F(1, n-2)$$

with critical region $\mathcal{C} = \{F > F_\alpha(1, n-2).\}$

$E(MSR) = \sigma^2 + \beta_1^2 s_{xx}, \ \ E(MSE) = \sigma^2$.

tical model with p explanatory variables:

$$Y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + \epsilon_i, \ \ i = 1, 2, \ldots, n \ \text{ where } \ \epsilon_i \overset{\text{iid}}{\sim} \mathsf{N}\left(0, \sigma^2\right)$$

ANOVA Table:

Source	DF	SS	MS	F
Regression	p	$SSR = \sum_i \left(\hat{y}_i - \bar{y}\right)^2$	$MSR = \frac{SSR}{p}$	$F = \frac{MSR}{MSE}$
Error	$n-p-1$	$SSE = \sum_i \left(y_i - \hat{y}_i\right)^2$	$MSE = \frac{SSE}{n-p-1}$	
Total	$n-1$	$SST = \sum_i \left(y_i - \bar{y}\right)^2$		

Note that $\frac{SSE}{\sigma^2} \sim \chi^2(n-1-p)$. Under $H_0 : \beta_1 \ldots = \beta_p = 0$, $\frac{SSR}{\sigma^2} \sim \chi^2(p)$.

Expected Mean Square

$$\mathbf{E}(MSE) = \mathbf{E}\left(\frac{SSE}{n-p-1}\right) = \sigma^2, \ \ \mathbf{E}(MSR) = \sigma^2 + \frac{1}{p}\sum_{j=1}^{p}\left[\beta_j^2 \sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j\right)^2\right]$$

Primary hypotheses: $H_0 : \beta_1 = \cdots = \beta_p = 0$ vs H_1 : at least one $\beta_j \neq 0$.

$$F = \frac{MSR}{MSE} \overset{H_0}{\sim} F(p, n-p-1).$$

with critical region $\mathcal{C} = \{F > F_\alpha(p, n-p-1)\}$

Partial test for individual coefficient $H_0 : \ \beta_j = 0$ vs $H_1 : \beta_j \neq 0$

$$t\left(\hat{\beta}_j\right) = \frac{\hat{\beta}_j}{\mathbf{SE}\left(\hat{\beta}_j\right)} \overset{H_0}{\sim} t(n-p-1)$$

where critical region $\mathcal{C} = \left\{|t| > t_{\alpha/2}(n-p-1)\right\}$

C.I. for the j^{th} partial coefficient β_j:

$$\hat{\beta}_j \pm t_{\alpha/2}(n-p-1) \cdot \mathbf{SE}\left(\hat{\beta}_j\right)$$

Coefficient of Determination:

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO},$$

the percentage of variation in the response explained by the regression model.
Adjusted R^2:

$$R_{adj}^2 = 1 - \frac{SSE/(n-1-p)}{SSTO/(n-1)},$$

the proportionate reduction of variation due to the regression.

Mean μ	σ^2 known, Normal Dist.	$\bar{X} \pm z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}$
	σ^2 unknown, Normal Dist.	$\bar{X} \pm t_{\alpha/2}(n-1)\sqrt{\frac{s^2}{n}}$
	σ^2 unknown, large n	$\bar{X} \pm z_{\alpha/2}\sqrt{\frac{s^2}{n}}$
Mean Difference between Independent Populations $\mu_1 - \mu_2$	σ_1^2 and σ_2^2 known, Normal dist.,	$(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	σ_1^2 and σ_2^2 unknown, large n	$(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
	$\sigma_1^2 = \sigma_2^2$ unknown, Normal Dist.	$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}(n_1 + n_2 - 2)\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$
Paired Difference μ_D	Paired data, Normal Dist.	$\bar{D} \pm t_{\alpha/2}(n-1)\sqrt{\frac{s_D^2}{n}}$
Proportion p	$np > 5$	$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Proportion Difference $p_1 - p_2$	Independent samples $n_1 p_1 > 5, n_2 p_2 > 5,$	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
Variance σ^2	Normal distribution	$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}\right)$
Variance Difference $\frac{\sigma_1^2}{\sigma_2^2}$	Independent Normal Distributions	$\left(\frac{s_1^2/s_2^2}{F_{\alpha/2}(n_1-1, n_2-1)}, \frac{s_1^2/s_2^2}{F_{1-\alpha/2}(n_1-1, n_2-1)}\right)$

$\mu = \mu_0$	σ^2 known, Normal dist. $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$\mu = \mu_0$	σ^2 unknown and n large $z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
$\mu = \mu_0$	σ^2 unknown and n small, Normal dist. $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$; $df = n-1$	$\mu_1 - \mu_2 = 0$	σ_1^2 and σ_2^2 known , Normal dist. $z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
$\mu_1 - \mu_2 = 0$	σ_1^2 and σ_2^2 unknown and n_1, n_2 "large" $z \approx \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	$\mu_D = 0$ paired data	Differences are Normally Distributed $t = \frac{\bar{D}}{s_D/\sqrt{n}}$, $df = n-1$
$\mu_1 - \mu_2 = 0$	$\sigma_1^2 = \sigma_2^2$ unknown, Normal Dist. $t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, $df = n_1 + n_2 - 2$; $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 = 0$	$\sigma_1 \neq \sigma_2$; both unknown, n_1, n_2 "small" Normal Distributions Welch's t-test
$p = p_0$	Normal Approximation $\hat{p} = y/n$ $z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$p_1 = p_2$	Normal Approximation $\hat{p} = \frac{y_1 + y_2}{n_1 + n_2}$ $z \approx \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$
$\sigma_1^2 = \sigma_2^2$	Normal Distributions Independent Samples $F = s_1^2/s_2^2$ $df = n_1 - 1$ and $n_2 - 1$		