Stat 481
Hw 8

Problem 1

1 It is Completly Randomized Design. This is a one-way analysis of variance model, which is also ANOVA model.

Assumptions
* the observations in each level of Cotton are independent.
* The response variable is normally distributed.
* The Variance of the response variable is equal in all levels of Cotton.

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

y_{ij}: pounds per inch^2 for the jth observation in the ith level of cotton

μ: mean

τ: effect of the i_{th} level of Cotton

ϵ_{ij}: random error term.

(2)

(Trt) Source	SS	df	MS	F
Cotton	114.53	2	57.27	7.07
Error	97.20	12	8.10	
Total	211.73	14		

$df_{trt} = k - 1 = 3 - 1 = 2$
$df_{error} = N - k = 15 - 3 = 12$
$\Big\}$ $df_{total} = 12 + 2 = 14$

$MSE = 97.20/12 = 8.10$

$MST = 114.53/2 = 57.27$

$F = 57.27/8.10 = 7.07$

(3) $H_0: \tau_1 = \tau_2 = \tau_3$

H_1: at least one $\tau_i \neq 0$

$F = 7.07$

Critical Region:

Reject H_0 if $F > F_{0.05}(2, 12) = 3.88$

Reject H_0 since $7.07 > 3.88$

p-value $= P(F > 7.07) = 0.00936$ (small)

$p < 0.05$.

There is significant difference in levels.

(4) Based on the ANOVA table:
Variance component for Cotton =
$= MST = 57.27$
Variance component for Error =
$= MSE = 8.10$

Problem 2

1 In this problem we will use a two-way analysis of variance model. It's Randomized Complete Block Design.

Assumptions & constraints:

* Treatments and blocks should be normally distributed.
* Observations should be independent.
* The variance of the response variable is equal in all treatments and blocks.
* treatments and blocks should be random.

$$y_{ij} = \mu + T_i + \beta_i + \epsilon_{ij}$$

y_{ij}: Plant available sulfate content in the ith treatment and jth block.

μ: mean

T_i: effect of ith treatment.

β_i: effect of jth block

ϵ: random error term

R Code:

```
data <- read.csv("/Users/liza/Desktop/stat481/soil.csv", header = TRUE)
data
      Soil   Solution Sulfur
1    Troop       CaCl   5.07
2    Troop     NH4OAc   4.43
3    Troop Ca(H2PO4)2   7.09
4    Troop      Water   4.48
5 Lakeland       CaCl   3.31
6 Lakeland     NH4OAc   2.74
7 Lakeland Ca(H2PO4)2   2.32
8 Lakeland      Water   2.35
9     Leon       CaCl   2.54
10    Leon     NH4OAc   2.09
11    Leon Ca(H2PO4)2   1.09
12    Leon      Water   2.70
13 Chipley       CaCl   2.34
14 Chipley     NH4OAc   2.07
15 Chipley Ca(H2PO4)2   4.38
16 Chipley      Water   3.85
17 Norfolk       CaCl   4.71
18 Norfolk     NH4OAc   5.29
19 Norfolk Ca(H2PO4)2   5.70
20 Norfolk      Water   4.98

anova_result <- aov(Sulfur ~ Solution * Soil, data = data)
anova_table <- summary(anova_result)
anova_table
              Df Sum Sq Mean Sq
Solution       3   1.62   0.540
Soil           4  33.96   8.491
Solution:Soil 12   9.64   0.803
```

Test for Treatment effects

$H_0: \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$

$H_1:$ at least one $\tau_i \neq 0$

Based on the ANOVA table
we have from the code
we can find F_{TR}
for Treatment we have
$k = 4$;
for block we have $b = 5$

$F_{TR} = MSTrt / MSE = 0.540 / 0.803 = 0.67$

p-value $= P(F_{0.05}(3,12) > \bar{F}_{TR}) =$
$= 0.58 < 0.67$

Based on p-value reject H_0,
there is difference among
treatment means

Test for Blocks Effects.

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
$H_1:$ at least one $\beta_j \neq 0$

$F_B = MSB / MSE = 8.491 / 0.803 =$
$= 10.57$

p-value $= P(F_{0.05}(4,12) > F_B) =$
$= 0.000662$ which is less than
F_B (10.57)
Reject H_0

Blocking was effective