Exercise 5.3:

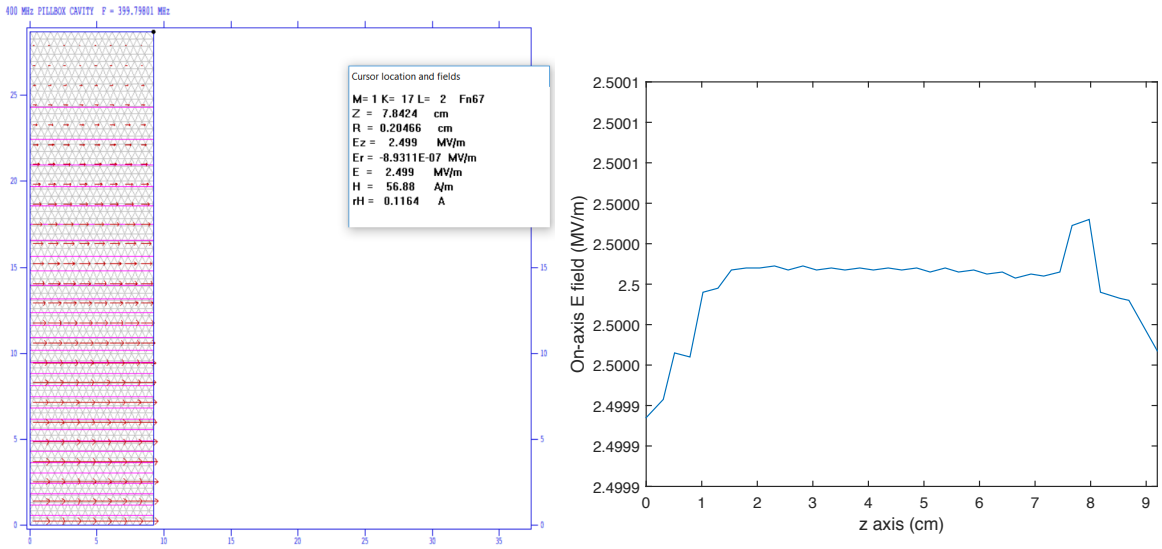
Question a):

Using the same solution of problem sheet #3, exercise 4: assuming the TM_{010} mode in the pillbox and a frequency f=400.8 MHz:

$$f_{010} = \frac{\Lambda_{010}}{2\pi/c} = \left(\frac{P_{01}}{r_0}\right)\frac{c}{2\pi}$$

that gives r_0 = 28.7 cm.

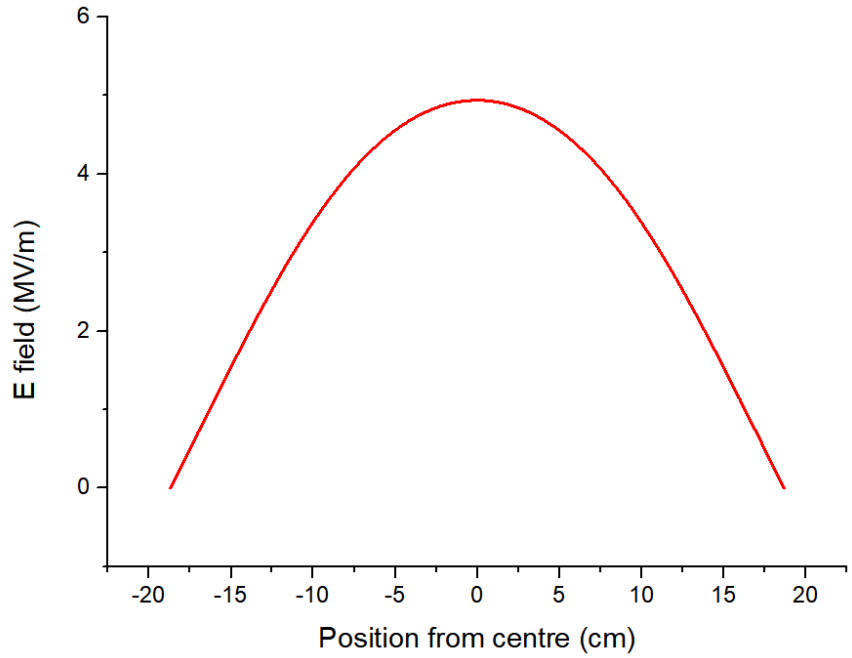
Just scaling down the 40 MHz pillbox cavity of the tutorial of factor 10 and changing the frequency to 400.8 MHZ:



The on axis field is 2.5 MV/m, and the length 9.2 cm. The Cavity length is not incresed in order to keep limited the transit time factor of the cavity. Now to reach the 32 MV required for the HE-LHC, this would require 348 cavities to guarant the necesary accelerating voltage. This is quite impractical. In addition, if a single beam pipe is used to accelerate the two beams spinning in the opposite directions and the maximum field is in the center, the maximum field is in a region not used by the beams.

Question b):

The quality factor of the cavity is Q = 16.2 x 10^9. The average accelerating gradient is 3.2 MV/m and the cavity length is 36 cm, so around 30 cavities are necessary to match the specification.


E field (MV/m)
6
4
2
0
-20
-15
-10
-5
0
5
10
15
20
Position from centre (cm)