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Interference Cancelling Codes for Ultra-Reliable Random Access

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Abstract Combinatorial Code Designs (CCDs) are proposed as a means for achieving ultra-reliability in the random access channel. In contrast to traditional access protocols that use random repetition coding, we show that by uniquely allocating repetition patterns to users, successful reception may be guaranteed up to a number of simultaneously active users in small frame sizes. Such codes are particularly robust in the low activity region where mission-critical machine-type communication is expected to operate. We also present deterministic codes designed to work in conjunction with Successive Interference Cancellation (SIC) to further improve reliability. The optimal IC code for frames of 5 access slots is given. Unlike slotted ALOHA, it is shown to limit packet losses to well below the ultra-reliability threshold (10^{-5}). These error performance gains come at the cost of a strict limitation on the supported user population (11 users in the case of 5 slots). We therefore consider larger frames of 24 slots, and analyse heuristic, low-complexity CCDs with fixed repetition factors that support up to 2024 users. While these are sub-optimal IC codes, significant gains are still observed compared to random codes.

Keywords Ultra-reliability · Random Access · Interference Cancelling Codes · Successive Interference Cancellation · Collision Resolution

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1 Introduction

Machine-type communications in both the massive (mMTC) and ultra-reliable (uMTC) varieties will facilitate many new types of use cases and services in fifth generation (5G) mobile networks [1, 2]. Such applications place demands on reliability, availability and end-to-end latencies that contemporary wireless systems cannot meet. Future networks are predicted to be characterised by billions of low power devices, low data rates and sporadic traffic patterns [3]. The ultra-reliable low-latency communications (URLLC) standard for mission-critical, real-time communication will require that transmissions be guaranteed with at least 99.999% certainty within radio latencies of less than 1ms [3]. Such limits on latency preclude the traditional use of re-transmissions in providing robustness to packet loss, even for small packets. Designing the 5G radio access technology (RAT) must necessarily entail many paradigm shifts if it is to support such a diverse range of use cases and meet its connectivity, capacity, reliability and latency targets.

A particular challenge is the provision of ultra-reliable and low-latency multiple-access to large numbers of uncoordinated and infrequently active devices. Uplink access in current wireless networks is based on a random access mechanism, and as such it inevitably forms a performance bottleneck as the number of users the system must serve grows. Networks reserve a portion of their available time-frequency resources as access slots, which together constitute a random access channel (RACH). In cellular systems, where a Base Station (BS) functions as a source of approximate timing in its operational area, RACH coordination has typically been realised by slotted variants of the ALOHA protocol [4, 5]. These protocols employ repetition coding, where packets are transmitted multiple times to improve the probability of successful transmission.

When access is scheduled, the network allocates resources for each communication event at the request of a user. The signalling overhead involved in such dynamic resource allocation is prohibitive for massive machine-type random access. When data packets are relatively small and irregular, conventional multi-stage resource reservation protocols become inefficient, costly and introduce significant potential for error. For these reasons, it is imperative that reliable protocols which provide direct or grant-free access to data transmission without the need for resource allocation or feedback be developed for these systems.

While contention-based, grant-free, random access methods will go some way towards reducing latencies, guaranteeing access to a large population of users without (or at least with limited) re-transmissions and with near certainty becomes a non-trivial problem. The contentious nature of such access inevitably leads to collisions between users, which may cause their respective data to be lost or unrecoverable. Both physical layer and medium access control (MAC) layer approaches to enhance RACH reliability and performance are being studied. Compressive sensing approaches to multi-user detection (MUD) in the physical layer have been shown to be effective [6–10]. In the MAC layer, Successive Interference Cancellation (SIC), a means of iterative collision resolution, may effectively recover colliding packets by decoding all interference-free packets and removing the corresponding users' impact on those remaining [11]. This improves robustness to packet loss, at a cost to decoding complexity and latency. Furthermore, it necessitates that all access packets contain a means of uniquely identifying the responsible user, such that the locations of their packet replicas may be known to the receiver.

Recently this line of research has seen considerable advances. In [12] and [13], the authors use graph theoretical methods to achieve very high throughputs with limited packet loss. However, their approach assumes that communication happens in large, synchronised frames consisting of many sequential time slots. This work was extended in [14] and [15], wherein the authors analyse the performance of the graph based approach for smaller frames. However, as we point out in [16], random placement of packets will inevitably lead to error floors when the frame size is very small. In this paper, we also consider the RACH without feedback from a MAC layer perspective, where synchronous communication is performed in frames of sequential time slots. Active users distribute their access packet and a number of repetitions throughout each frame according to a particular access *pattern*. Given the small frame sizes and low user activity of machine-type traffic, we seek to facilitate URLLC on the RACH by means of Combinatorial Code Designs (CCD), i.e. pre-designed and -allocated repetition patterns, and employing SIC at the receiver. We seek to show that access patterns may be designed in this framework to guarantee decodability with extremely high probability for low channel load. In a preliminary study in [16] we found that, when SIC is available to the receiver, a deterministic CCD provides considerable gains in reliability over classical ALOHA-type random access. Such codes circumnavigate the error floor that random patterns are necessarily subject to, a bound on performance which we will analyse here in detail. Deterministic random access codes are based upon the combinatorial idea that when the number of active users M in a frame is sufficiently limited, we may guarantee that all RACH contenders may be successfully received. This approach leads to two code design criterion, depending on the availability of SIC, which we will elaborate on here. We will also include a complete failure probability analysis for a simple deterministic random access code in short frames under Poisson arrivals, to illustrate the significant gains over the random approach available in the low activity domain. Finally, we will provide some insights into how heuristic CCDs may provide additional performance gains.

The random access code design problem when patterns may be pre-allocated to users has been discussed in [17], wherein the authors also employ combinatorial designs. This work was done under the assumption that collisions between users are always destructive and no means of interference cancellation is available. The corresponding combinatorial problem has been considered in classical coding theory and combinatorics literature [18] under the name *superimposed codes*. Other related works include [19–22]. To the best of our knowledge, the combinatorial approach of designing deterministic patterns that guarantee decoding with interference cancellation for small numbers of active users has been first considered in [23] and independently by us in [16]. In [23], the authors produce a deterministic version of [12] and suggest the use of LDPC code words as access patterns. In order to combat the error floor, they attempt to guarantee that no small stopping sets appear in the corresponding bipartite graph. This approach works well for moderately sized frames, where the user population is not significantly larger than the frame size, and they attain high throughput and a low error floor. In contrast, our approach in [16] provides an explicit condition that access patterns must satisfy in order to guarantee that, when at most M users are active, decoding all users is guaranteed. This approach provides considerable flexibility and is particularly suited to low-activity access scenarios where the user population is large compared to the frame size. We note that the combinatorial structure underlying such patterns has appeared before, as building blocks for collision resolution protocols with feedback under the name *selective families* [24] and, in the context of user tracing, the name $(\leq M, 1, n)$-*locally thin codes* [25].

2 System Model

Here we consider uplink access in a multi-user wireless communication system. The system supports a population of some N machine-type users who randomly and sporadically use the radio resources to transmit access packets to a central base station. We concentrate on the low activity scenario, where the number of active users during a frame is upper-bounded by $M << N$ with very high probability. No feedback is permitted. We assume that the users are synchronised with a base station, such that a rough timing advance may be known and making it possible to have slot and frame synchronisation between users. Frames consist of some n contiguous time slots, and an access packet occupies exactly one time slot. Active users attempt to transmit their packets during the next synchronised frame interval, along with $k < n$ repetitions. We assume that access patterns may be pre-distributed to each of the users prior to any transmission, such that they might operate without coordination with other users with some certainty of limited interference. Access packets follow an (independent) Poisson arrival process with an exponentially distributed inter-arrival time. The expected number of access attempts in a frame interval is denoted by λ. The reliability target is $P_{\text{rel}} = 0.99999$. For simplicity, we limit our study to reliability against and recoverability from collisions in the MAC-layer, and do not consider the impact of the transmission channel.

There are two models of receiver operation considered in this paper. In the first, the receiver simply parses each access frame and if two packets collide in a given slot, then the contained data is considered lost. Conversely, if there is no intra-slot collision, the transmission is considered successful. The lower bound on the supportable user population N for guaranteeing successful reception of up to M active users in n time slots is obviously given by time division. Each of $M \leq n$ users simply transmits during their own preallocated time slot. However, in the case of random access, it would be preferable to guarantee that the transmissions of at most $M << N$ active users are successful, and increase N. This condition immediately precludes us from considering conventional approaches to repetition-coded random access, in which users place their packets and replicas on a frame in a random manner. While this removes any constraints on the supportable user population, the randomness guarantees a non-zero probability of collisions. We therefore consider the scenario where each of N users may be pre-assigned a unique pattern from a deterministic codebook, according to which their k packets will be distributed. In this paper, we endeavour to show that with this approach, when we limit M, we can support considerably more users than time-division while still guaranteeing interference-free reception.

In the second receiver structure, the base station is presumed to be able to perform perfect successive interference cancellation, and thus a degree of collision resolution. As we have already shown in [16], when access packets contain pointers to the time slots occupied by its copies, for a given M and n, SIC algorithms allow us to greatly expand the set of recoverable access patterns. Herein, we will provide optimal and heuristic interference-cancelling codes, and compare their performance in a random access scenario to random patterns. As states, our approach to targeting the ultra-reliability region assumes an upper limit on the number of active users M. However, we will show that, despite this constraint, the resulting access codes remain effective in the realistic scenario that this bound may be exceeded, as long as the likelihood of this occurring (user activity) is sufficiently low.

2.1 Basic Notation

Let us index the n access slots in a frame by the set of indices $S = \{1, 2, 3, \ldots, n\}$. There are at most N users, and when user i becomes active, it transmits an access packet x_i by placing k replicas of it over the next synchronised frame, i.e. selects a subset of S. For example, the subset $\{1, 7\}$ with $k = 2$ corresponds to the user transmitting in the first and seventh slots. Such a subset may also be represented by a binary vector or pattern, where 1s represent slots that contain a replica, e.g. the subset $\{1, 7\}$ corresponds to the pattern $(10000010\ldots0)$. In binary, collision is observed whenever the involved vectors have one or more 1s in the same position, while in set theoretic language there is a collision in the kth slot if at least two of the involved sets include k. We will henceforth move freely between these representations when presenting access patterns.

Consider the following trivial example where we have $n = 7$ and N ordered users, and the second user has been pre-allocated pattern (1110000), the third user pattern (1000011), and the seventh user pattern (0010110). When these users transmit simultaneously, the receiver sees

$$(x_2 + x_3, x_2 + x_7, x_2 + x_7, 0, 0, x_3 + x_7, x_3). \tag{1}$$

If we assume that collisions are destructive, the decoder will fail here: packets of the second and seventh users are unrecoverable. However, with interference cancellation it will succeed. The receiver decodes the interference-free packet from the third user, identifying them and thus the locations of their replicas, and erases them to reveal interference-free packets from the remaining users. This process may continue recursively as necessary, until there are no undecoded packets remaining or the decoder becomes stuck.

3 Random vs. Deterministic Codes with and without IC

In this paper we concentrate on designing deterministic random access codes. The key point here is that the supportable user population N is limited, and deterministic patterns must be pre-distributed to the users. Compared to the completely random approach, this obviously adds some complexity to the system. We aim here to motivate that employing deterministic access patterns is worthwhile from the perspective of ultra-reliability, despite these limitations.

In [16] we analysed a simple random strategy and compared it to a corresponding deterministic code. Specifically, we simulated the performance of a random access code of weight 3 patterns in short frames of 24 timeslots in the cases where active users either randomly select from the code (supporting an infinite user population), or were uniquely pre-allocated patterns (supporting up to 2024 users). We found that gains with deterministic patterns were small when we assumed that collisions were destructive, however considerable gains were observed when the receiver could perform SIC. These performance gains can be attributed to the following. In the random scenario, when a user accesses the channel they distribute three copies of their access packet over the 24 slot frame. If at least two of the M active users happen to have chosen the same pattern, their respective packets will be unrecoverable regardless of whether or not the receiver can perform perfect SIC. The probability of this occurring is surprisingly large, and creates a lower bound for failure probability, or error floor, in the random case.

We now analyse the error floor in the random approach more generally. Let us assume that we have a length n access frame, and that during the frame there is probability $P_a(i)$ that exactly i users are active inside the frame. When users access the channel, they will uniformly and randomly distribute k copies of their access packet into the frame. This is equivalent to randomly selecting one of $\binom{n}{k}$ weight k binary vectors (assuming replacement). A natural comparison can be made to the deterministic approach where we have $N = \binom{n}{k}$ possible users, each assigned a unique weight k binary vector.

Let us now denote with $P_{f,r,n}(i)$ the probability that a failure (f) occurs on reception, when exactly i users are simultaneously active and employing randomly generated (r) patterns, and interference cancellation is unavailable (n). Similarly, we denote by $P_{f,c,n}(i)$ the failure probability under the assumption that we use a deterministic code (c). The probabilities $P_{f,r,n}(i)$ and $P_{f,c,n}(i)$ are connected by the following equations

$$
\begin{aligned}
P_{f,r,n}(i) &= P_b(i) + (1 - P_b(i))P_{f,c,n}(i), &(2)\\
&= P_{f,c,n}(i) + P_b(i)(1 - P_{f,c,n}(i)), &(3)
\end{aligned}
$$

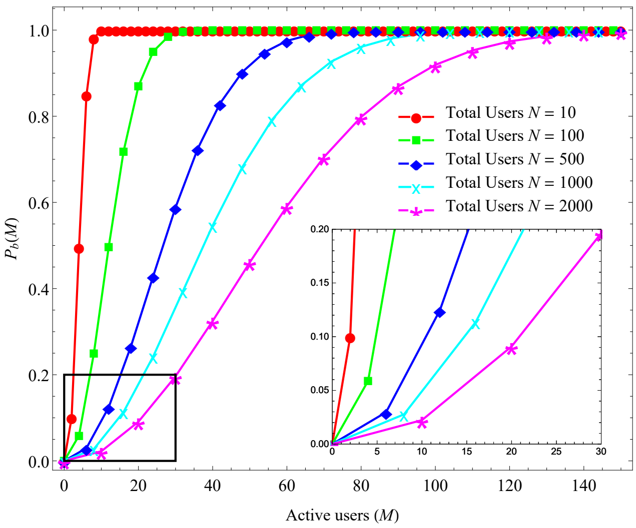

Fig. 1 Birthday paradox providing a lower-bound on the performance of random codes.

where

$$P_b(i) = 1 - \frac{i! \cdot \binom{N}{i}}{N^i}. \tag{4}$$

is the probability that, in the random case, at least two of the i active users have chosen the same pattern. This number grows surprisingly fast with respect to i, a phenomenon referred to as the *birthday paradox*. The impact of the paradox on the system model parameters can be observed in Figure 1.

By using Equation (3), we can write the overall probability of error in the random approach

$$
\begin{aligned}
P_{f,r,n} &= \sum_{i=0}^{N} P_a(i) P_{f,r,n}(i) &(5)\\
&= \sum_{i=0}^{N} P_a(i)(P_{f,c,n}(i) + P_b(i)(1 - P_{f,c,n}(i)))\\
&= P_{f,c,n} + \sum_{i=0}^{N} P_a(i)(P_b(i)(1 - P_{f,c,n}(i))).
\end{aligned}
$$

The last equation pinpoints just how much using random repetitions loses to the underlying deterministic code. We can clearly see that, while taking a deterministic approach is always better, how much gain can be obtained depends heavily on the scenario considered. Initial impressions suggest that gains may be considerable when, for a given i, $P_a(i)$ and $P_b(i)$ are large while $P_{f,c,n}(i)$ is small. However, we note that in the case where the number of possible users of the underlying deterministic code is small and $P_b(i)$ grows strongly as a function of i, gains will be impacted by the fact that $P_{f,c,n}(i)$ will grow as well.

Another interesting result that can be derived from Equations (2) and (3) is

$$P_{f,c,n} + \sum_{i=0}^{N} P_a(i)P_b(i) \geq P_{f,r,n} \geq \sum_{i=0}^{N} P_a(i)P_b(i). \tag{6}$$

This result amounts to a simple lower bound for the error probability of the random approach.

Let us now consider the scenario where the receiver is able to perform successive interference cancellation. We can perform a similar analysis as in the case where collisions are destructive. We simply replace $P_{f,r,n}(i)$ and $P_{f,c,n}(i)$ with $P_{f,r,ic}(i)$ and $P_{f,c,ic}(i)$, respectively. Analogous to Equation (3), we then have

$$P_{f,r,ic}(i) = P_{f,c,ic}(i) + P_b(i)(1 - P_{f,c,ic}(i)). \tag{7}$$

Similarly, Equation (5) becomes

$$P_{f,r,ic} = P_{f,c,ic} + \sum_{i=0}^{N} P_a(i)(P_b(i)(1 - P_{f,c,ic}(i))). \tag{8}$$

As interference cancellation is a strong decoding method, we can expect that $P_{f,c,ic}(i)$ is considerably smaller than $P_{f,c,n}(i)$ in almost every case. Equations (5) and (8) lead us to conclude that using even simple, heuristic deterministic codes instead of random patterns with small frame size will likely give the decoder a far greater advantage, especially when IC is available.

4 Code Design for Ultra-Reliable Random Access

In the previous section, it was concluded that for small frame sizes and low activity it might well be very beneficial to pre-distribute deterministic access patterns to users. This approach opens up the possibility to design the patterns in such a way as to improve the likelihood that decoding all randomly accessing users is successful.

As before, we have length n frames, N possible users and a selection of N individual patterns that have been pre-distributed to the user population. Using the same notation as in the previous section, we have the following expression for the failure probability (irrespective of the decoding method)

$$P_{f,c,n/ic} = \sum_{i=0}^{N} P_a(i)P_{f,c,n/ic}(i). \tag{9}$$

Let us assume further that $P_a(i)$ is always extremely small whenever $i > M$, for some fixed M. In this case, Equation (9) suggests that a good code would be such that $P_{f,c,n/ic}(i) = 0$, whenever $i \leq M$. The drawback becomes that forcing such a condition for fixed n causes the number of possible patterns (and thus the number of possible users N), to diminish. This reveals the following coding problem: given N and n, find the set of patterns for which $P_{f,c,n/ic}(i) = 0$ for all $i \leq M$, where M is as large as possible. The optimal code design will vary depending on whether the decoder can employ IC or not. The following sections elaborate on work in [16].

4.1 Interference-Free Reception

Suppose there are a total of N users, each uniquely allocated a length n pattern and that inter-user collisions are destructive. This means that, given M simultaneously active users, the receiver must obtain at least one copy of each packet from each user free of interference from the other users. Codes providing this guarantee were first considered in [18], where the authors called them *superimposed codes*. A formal definition of these codes can be given as follows. For N users, each in possession of a unique pattern of length n, denote by W the $n \times N$ matrix whose columns consist of the patterns of the N users. We have that

Definition 1 ([18]). *The set of patterns W is a M-superimposed code if any $n \times M$ matrix formed from the columns of W has a submatrix consisting of a $M \times M$ permutation matrix.*

This definition can also be given in the following set theoretic form. Suppose we have the set $S = \{1, 2 \ldots, n\}$. We are interested in collections of $\mathscr{B}$ distinct subsets of S with the following property

Definition 2 ([26]). *A collection of sets $\mathscr{B}$ is M-*covering free *if it satisfies the following condition. Given any subset $X \in \mathscr{B}$, there does not exist sets $Y_1, .., Y_{M-1} \in \mathscr{B}/X$ such that*

$$X \subseteq Y_1 \cup Y_2 \cup \ldots . \cup Y_{M-1},$$

where $Y_i \in \mathscr{B}$.

In the previous section we considered a set of patterns that consisted of all the weight k binary vectors of length n. In set theoretic notation, this corresponds to a collection of all the k element subsets of a set with n elements. It is immediately evident that any such collection is 2-covering free. In order to maximize the number of possible users, we see that subsets with $\lfloor n/2 \rfloor$ give the maximal number of equal weight patterns. In this case, $|\mathscr{B}| = \binom{n}{\lfloor n/2 \rfloor}$. This result is simple, but a theorem by Sperner [27] states that given any collection $\mathscr{A}$ of subsets that are 2-covering free, we have that $|\mathscr{A}| \leq \binom{n}{\lfloor n/2 \rfloor}$. Accordingly, if we are satisfied with codes that guarantee the decoding of any two simultaneously accessing users, we can always support up to $\binom{n}{\lfloor n/2 \rfloor}$ users. Asymptotically, $\binom{2n}{\lfloor n \rfloor}$ grows like $\frac{4^n}{\sqrt{n\pi}}$. This reveals that when $M = 2$, the supportable user population is massive already for small frames. However, it is very likely that maximizing N may lead to dramatic increases in the error probability when there are more than two simultaneously active users.

Guaranteeing the decodability of more than two active users becomes far more complicated and the gains less impressive. Exhaustive search shows that in the case of $M = 3$ we can have only 4 and 5 users for frame sizes $n = 4$ and $n = 5$, respectively. From known results, we find that for three active users and frame length $n = 12$, one may obtain 22 patterns, for $n = 21$ we find 70 patterns, and for $n = 26$ we have 260 patterns.

4.2 Interference Cancelling Codes

Superimposed codes have many desirable properties and provide a possible solution for a very small number of active M and supportable users N. For any given code, it is obvious that if the decoder applies interference cancellation, the error probability is reduced. However, as we will see, with decoder-specific code design, much larger gains are obtained. When the decoder can perform SIC, the system can support a larger number of active and possible users for a given frame size compared to superimposed codes.

Suppose that we have length n frames, N users, and that we would like to be able to support at most M simultaneous users with a guarantee of successful SIC reception. With $S = \{1, \ldots, n\}$, the recursive nature of SIC suggests the following definition

Definition 3 (**[16]**). *We say that a collection B of subsets of S are M-interference cancelling if they satisfy the following. Given any collection of subsets $A_1, \ldots, A_s$, $s \leq M$ from B then at least one A_i contains at least one element that is not contained in any other A_k.*

This set theoretic definition can be translated to the language of binary vectors as follows

Lemma 1 (**[16]**). *Let us suppose we have a random access code C with N length n binary patterns and let us denote with A the $n \times N$-binary matrix formed of these vectors. If then each submatrix Y formed of M or less columns has the property that at least one of the rows has weight one, then C guarantees correct decoding as long as there is at most M active users.*

Such codes may be referred to as *interference cancelling codes* or IC codes. In [25], such codes are called $(\leq M, 1, n)$-locally thin codes. They have also been used as the building blocks of *non-adaptive collision resolution algorithms* in [24] and [28]. These belong to a large body of works beginning from [29–31]. These consider destructive collisions with minor feedback. It is interesting to note that while this channel model differs from ours, the combinatorial constructions share some similarities.

Compared to Definition 1, we see that the assumption of SIC decoding greatly relaxes the code design. Instead of requiring $s \leq M$ rows with a single 1 (and on different columns), it is enough to have single 1 on at least one of the rows. In the set theoretic language, we can see that when $M = 2$, the condition for being able to guarantee decodability is that each user employs a pattern that corresponds to a different set. This leads to $2^n - 1$ different possible users. Compared to superimposed codes, this is a considerably larger set of possible users for a given frame size n. With superimposed codes, we could have only 252 users for $n = 10$. With IC codes, we can have 1023.

Finding optimal IC codes is a non-trivial exercise. It is not yet clear how they might be constructed directly. Algorithm 1 describes a simple, brute force search by which they might be obtained. We note that, given a frame size n, it is not clear what the optimal number of patterns N one might obtain for any given number of active users M is, necessitating multiple searches with various parameters to find the optimum. The search space becomes astronomically large very quickly with n, N, M. Each search consists of a maximum of $\binom{2^n-1}{N}\binom{N}{M}$ comparisons. Searching for the largest codebooks obtainable with $M = 3$ for $n = 4$ requires over 200,000 iterations, and for $n = 5$ almost 14 billion. We performed such an exhaustive search and found the following optimums for these parameters. For $n = 4$ we have the following 7 patterns

$$\begin{matrix} 0&0&0&1&1&1&0 \\ 1&0&0&1&0&0&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&0&0&1&1 \end{matrix} \tag{10}$$

and for $n = 5$ we have the following 11 patterns, as in [16]

$$\begin{matrix} 1&0&0&1&0&1&1&0&0&0&0 \\ 0&1&0&1&0&0&0&1&0&0&1 \\ 0&0&1&1&0&1&0&0&1&0&1 \\ 0&0&0&0&1&1&0&0&0&1&1 \\ 0&0&0&0&0&0&1&1&1&1&1 \end{matrix} \tag{11}$$

In comparison to superimposed codes, these IC codes expand on the number of supported users and increase the resolvable number of collisions for very small frame sizes.

Algorithm 1 Obtaining IC codes by exhaustive search

Input: User population N, max active users M, frame length n
Output: $n \times N$ code matrix

```
Compute all 2^n − 1 choose N possible n × N binary CBs
for i = 1 to 2^n − 1 choose N do
    Replace 1s in CB i with N unique identifiers
    Compute all N choose M possible combinations of active users
    Set found flag
    for j = 1 to N choose M do
        Sum patterns from CB i for user group j into vector v
        while v contains identifiable user u do
            Add u to list of detected users d
            Remove u from v according to pattern
        end while
        if d not user group j then
            clear found
            break
        end if
    end for
    if found then
        return CB i
    end if
end for
```

5 Failure Probability for Small Frame Sizes

In this section, we compare the performance of the length-5 IC code presented in (11) and the random approach by simulation. In both cases, access frames are limited to $n = 5$ slots. Users either transmit according to a random pattern (of weight 1 to 4) or a uniquely pre-assigned pattern from the IC code. The system may therefore support a total of $N = 11$ users. Each user generates access packets and attempts to access the channel according to a Poisson process with rate λ. Thus, m users are active in a given frame with probability $\lambda^m e^{-\lambda}/m!$. The total probability of decoding failure in the RACH is contingent on the load λ and the probability that at least one user's slots are all in collision. Or, in the case of IC, the probability that the recursive algorithm is unable to recover at least one access packet for each user.

Figure 2 illustrates the failure probability for $n = 5$ and $N = 11$. We see that the use of IC codes over the random approach reduces the error probability marginally with a non-IC receiver, but dramatically when IC is available. Clearly, optimising user patterns for IC offers dramatic performance gains in the activity region in question. The probability of decoding failure is well within the ultra-reliability region ($< 10^{-5}$) up to an activity factor of approximately 0.1. The significant gains observed can be attributed to the fact that the IC has been designed to guarantee decodability for up to $M < 4$ simultaneously active users, which occurs with high probability in this region, while the birthday paradox is severely impacting the random case.

Given that the computation of IC codes for larger n and N becomes exponentially more difficult, it may be necessary to consider more heuristic combinatorial code designs. The most obvious of these is the code obtained by taking all $\binom{n}{k}$ size k subsets of $\{1, 2, 3, \ldots, n\}$ and their corresponding binary patterns. In other words, all different weight k and length n binary vectors. Doing so produces a code of equal weight patterns which guarantees decodability for $M < 3$. Figure 3 shows that equal weight codes outperform IC codes when IC is unavailable for frame sizes of $n = 5$. This is as expected, since the IC code includes codewords of low weight which rely heavily on collision resolution, whereas equally weighted codewords are, on average, more robust. However, the ability to guarantee the decodability of an additional active user obviously still grants the IC codes a significant performance advantage over the heuristic code.

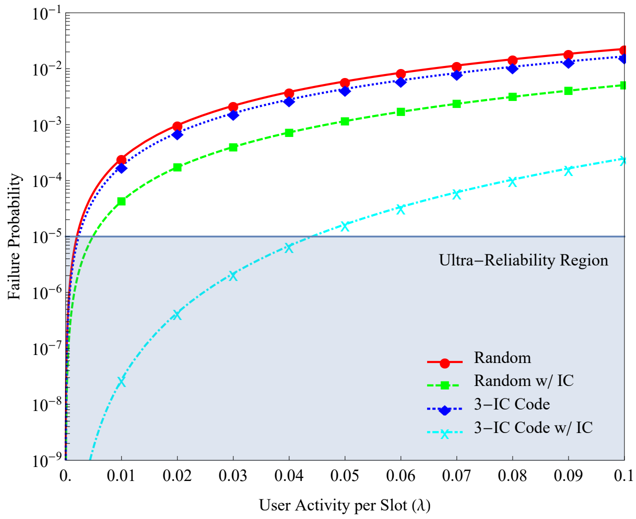

Fig. 2 Simulated performance of random and 3-way user collision resolving IC codes for random access in frames of 5 slots.

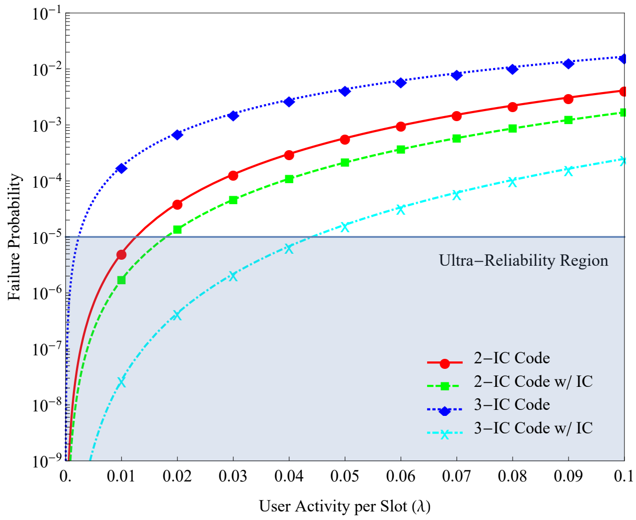

Fig. 3 Simulated performance of 2- and 3-way user collision resolving IC codes for random access in frames of 5 slots.

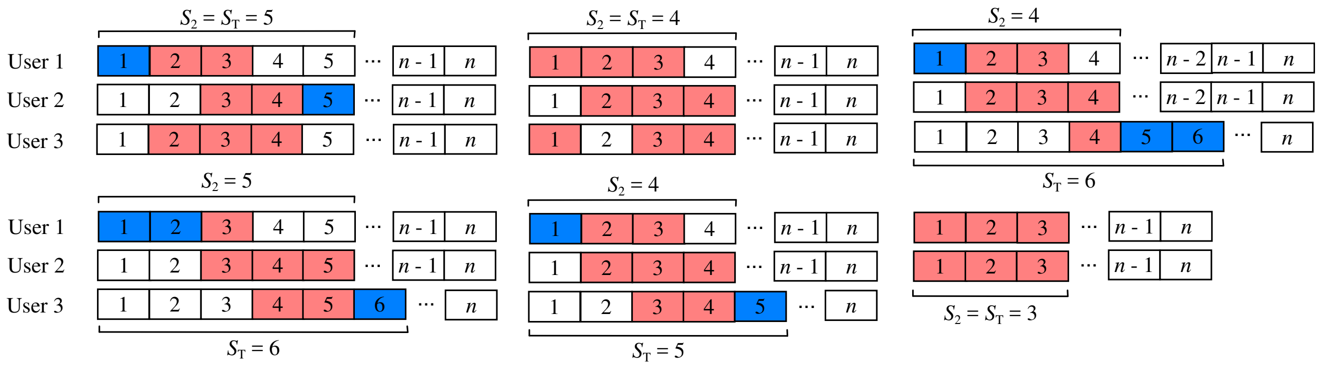

Fig. 4 Examples of 2- and 3-way user collision scenarios for $k = 3$ that result in failures on the RACH.

6 Failure Probability Analysis for $n = 24$

Clearly, frame sizes of 5 slots and supported populations in the 10s of users are too small for practical machine-type random access purposes. We will therefore extend our analysis of combinatorial code designs for ultra-reliable random-access to frame sizes of $n = 24$ access slots. The motivation for this number is that in the narrowest LTE standard, one quarter of a LTE sub-frame and 6 physical resource blocks in the frequency domain constitutes a reasonably sized RACH.

We proceed to establish approximate expressions for the RACH failure probability by assessing the more rudimentary contention cases, and evaluate these by simulation. Examples of such cases are illustrated in Figure 4. To make such an analysis possible, we do not pursue the optimal IC codes, but instead consider a limited cardinality code set. As in the previous section, we obtain a heuristic CCD by taking size 3 subsets of $\{1,2,3,\ldots,24\}$ and their corresponding binary repetition patterns of weight 3. Such a code allows the system to support a finite population of $N = \binom{24}{3} = 2024$ users with distinct patterns.

For this performance analysis we continue to consider a synchronous system. Users that are active in the same frame may have patterns that collide in some of the slots. We note that with the CCD, as before, there are no collisions between two users that would result in decoding failure. We provide an analytical lower bound by considering contention scenarios where there are up to three colliding users, and show by simulation that, for low activity, this is sufficient to approximate the expected failure performance and motivate deterministic codes with SIC decoding for ultra-reliability on the RACH with larger frame sizes.

6.1 Collisions with $M = 3$

Consider the contention scenario where three of N users each transmit k times in a given frame according to their pre-allotted (deterministic) or generated (random) patterns. To determine the failure probability for a given user, i.e. the probability that none of the user's k packet replicas can be successfully decoded, it is sufficient to compute the probability that all k slots in their pattern are occupied by the other two users. In the case where the receiver can perform SIC, failures occur when no undecoded users have a collision-free slot at any stage in the algorithm.

We take the following approach to analysing the failure probability. Note that, without loss of generality and for arbitrary n, in order for two users to collide they must occupy a *span* S_2 of at most five access slots. Repetition coding guarantees decodability of both users. For failure to occur, the third user must then occupy S_2 or expand the span to at most 6 slots. In the latter case, the packet residing outside of span S_2 serves to guarantee decodability of User 3, while their remaining two packets cause failure with some probability. We have thus divided up the probability space into the various collision scenarios that result in failure, as illustrated in Figure 4. The probability of failure for $S_2 = 5$ is given by

$$P_{3\mathrm{u},5\mathrm{s}} = \frac{3\cdot\binom{n-2}{2}}{\binom{n}{3}-\mu/2}\cdot\sum_{i=2}^{3}\frac{\binom{n-5}{3-i}\binom{5}{i}-\mu\,\delta_{K,3}}{\binom{n}{3}-\mu}\cdot P_i, \tag{12}$$

where μ depends on the code: $\mu = 0$ for random patterns, and $\mu = 2$ for CCDs, and where $P_2 = 1/15$ and $P_3 = 1/2$ for $\mu = 2$ and $P_3 = 8/15$ for $\mu = 0$.

In the case where the first two users span only 4 access slots in a frame with a certain probability, their decodability is also guaranteed. A third user must then occupy S_2, or expand the total span to 5 or 6 slots for there to be the possibility of failure occurring. The probability of this when $S_2 = 4$ is given by

$$P_{3\mathrm{u},4\mathrm{s}} = \frac{\binom{3}{2}\binom{n-3}{1}}{\binom{n}{3}-\mu/2}\cdot\sum_{i=1}^{3}\frac{\binom{n-4}{3-i}\binom{4}{i}-\mu\,\delta_{K,3}}{\binom{n}{3}-\mu}\cdot P_i, \tag{13}$$

where $P_1 = 1/6$, $P_2 = 1/3$, and $P_3 = 1$ for $\mu = 2$ and $P_3 = 5/6$ for $\mu = 0$.

It is clear that when $S_2 = 3$, failure may only occur for random patterns, as all users must collide on all slots in the span. Outage probability in this instance is simply

$$P_{2u,3s} = \binom{n}{3}^{-2}. \tag{14}$$

Finally, the total probability of failure in the case of three-way collisions with random patterns is

$$P_{3u,rnd} = P_{3u,5s} + P_{3u,4s} + P_{2u,3s}, \tag{15}$$

and for the heuristic CCD $P_{3u,ccd}$ is the sum of only first two terms in (15). Note that the failure probability in the latter case does not depend on the specific repetition pattern assigned to a user, implying that the user codes are fair.

6.2 Collisions with $M = 3$ and Interference Cancellation

The SIC algorithm is able to progressively remove contending users from an access frame as long as, at each stage, one user occupies an interference-free slot. The receiver identifies this user, decodes their packet and eliminates all replicas from the frame according to their pattern. When the number of repetitions is $k = 3$, and the number of active users is $M = 3$, the only scenario in which IC fails to recover all user data is when all M users occupy a span of four access slots. If the third user was to expand the span of the first two beyond $S_2 = 4$, then there is a guarantee that at least one packet is free of any interference. Consequently, the total probability of failure with IC in the case of three-way collisions with random patterns reduces to the sum of the first term in (13) and (14), while with heuristics IC code it is simply the first term in (13).

6.3 Evaluation by Simulation

Again we consider the random access scenario where each user generates access packets according to a Poisson process with intensity λ. The complete failure probability under Poisson arrivals may be lower bounded as

$$P_{rnd}(\lambda) \geq \frac{e^{-\lambda}\lambda^2}{2} \cdot \binom{n}{3}^{-1} + \frac{e^{-\lambda}\lambda^3}{6} \cdot P_{3u,rnd}, \tag{16}$$

for random patterns. A corresponding expression for CCD may be readily derived, as well as expressions for random and CCD with IC. Figure 5 illustrates the analytically approximated and simulated failure probabilities for Poisson arrivals as a function of the load on the RACH, for random patterns and the heuristic IC code with and without IC, for $k = 3$ repetitions and frames of size $n = 24$. The simulated probabilities account for up to 10-way collisions inside a given frame. We observe that consideration of only up to three-way collisions in the analysis provides an good approximation to the performance, which deviates from the simulation that grows as the probability of $M > 3$ increases.

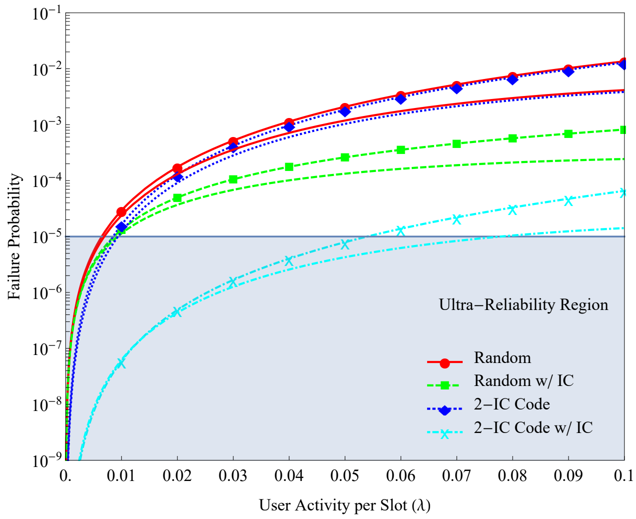

Fig. 5 Analytical lower bound (without markers) and simulated performance of random and 2-way user collision resolving IC codes for random access in frames of 24 slots.

Figure 5 also confirms that deterministic patterns circumnavigate the error floor associated with random selection, and thus reduce the failure probability to a diminishing degree as the load increases. This is as expected, since two-way collisions occur frequently at lower access intensities, for which the deterministic patterns cannot be in error and the birthday paradox negatively impacts random selection. With increasing load, the less probable multi-way collisions start to dominate performance. The stand-out result in Figure 5 is the significant gains obtained by employing even a rudimentary $\binom{n}{k}$ CCD with SIC, in this load region. The fact that no two deterministic patterns may fully overlap reduces the probability that the SIC algorithm becomes stuck during an iteration. For random patterns, the birthday paradox has the consequence that the number of user pairs that select the same pattern grows quadratically with load, thus preventing SIC from resolving the collisions.

These results ultimately show that combinatorial access codes designed in conjunction with interference cancellation have the potential to target the ultra-reliability region in the RACH. Interference cancellation provides moderate gains in the tolerated user activity for slotted repetition ALOHA, whereas a heuristic IC code provides orders of magnitude more gain. From what we observed for $n = 5$, we expect that a designed IC code for $n = 24$ that may fully resolve collisions of $M > 3$ accessing users (at some cost to N), would provide more robustness to higher loads and exhibit an even more dramatic increase in error performance. Given the complexity of obtaining such a code, it may be prudent to explore additional heuristic CCDs.

6.4 Alternative Heuristic Designs

The $\binom{24}{3}$ IC code presented previously is the least complex construction that provides deterministic gain in decoding performance. We observed in Section 6 that IC benefits greatly when the third user has at least one access packet outside the span of the first two colliding users, with high probability. The third user may be decoded and removed from the frame, leaving the remaining users which are guaranteed to have unique patterns. For this reason, additional gains may be obtained by increasing the weight of the patterns. A balance between the number of repetitions k and the size of the codebook N must be struck, since both contribute to the probability of unrecoverable collisions occurring. We may readily increase the probability that at least one user in the case of $M = 3$ will have a contention-free slot over the aforementioned $\binom{24}{3}$ IC code by simply increasing the number of repetitions k to 4, and selecting a random subset of size $N = 2024$. Producing such a pruned CCD is significantly less complex than finding the optimal IC code for $n = 24$. Obviously, when the ratio of frame length to repetitions is small, increasing the weight is counter-productive. The additional ones in the codebook only serve to increase the probability of completely destructive inter-user collisions. However, for larger frame sizes, the increased probability of extra-span slots increases. Figure 6 shows that, for low activity, this slightly increased probability translates to a significant reduction in failure probability on the RACH.

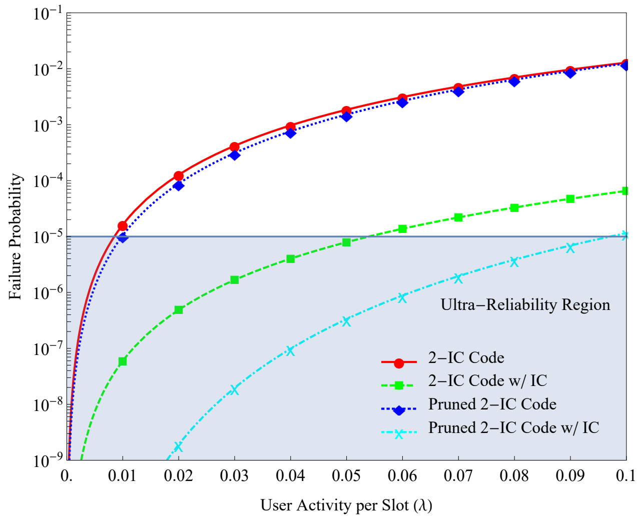

Fig. 6 Simulated performance of 2-way user collision resolving $\binom{24}{3}$ IC codes and $\binom{24}{4}$ IC codes pruned to an equivalent size, for random access in frames of 24 slots.

7 Conclusions and Future Work

In this paper, we considered the challenge of ultra-reliable and low-latency communication in the random access channel. We have shown that, for synchronised frames of small size, pre-distributed and deterministic repetition patterns are necessary to eliminate the error floor to which conventional, random methods are subject. This comes at a cost to the total number of users the system may support. However, we have shown that one may support a larger population of users N than with simple resource division, when the user activity factor is low. In fact, combinatorial code designs exist which guarantee that all users active in a frame can always be received, up to a given threshold $2 < M < N$ on the number of simultaneously active users. The reliability may be further improved when the receiver is able to perform successive interference cancellation. We may then design codes which expand M and/or N and offer dramatic error performance gains. Optimal IC codes for frames of 4 and 5 access slots, supporting 7 and 11 users, respectively, were presented and shown by simulation to offer robustness to packet error that falls well within the five nines ultra-reliability standard for low channel load. Heuristic, low-complexity IC codes were then offered for frames of size 24, supporting 2024 users, which were analysed and shown to also exhibit error performance in the ultra-reliability region.

In this work, we considered very small frames and random access codes optimised by brute force search. For larger frames, the deterministic codes presented here are overly simple and likely suboptimal. A obvious research direction is to find and develop code construction methods that would yield close to optimal codes for larger access frames. The question of how to combine these MAC layer random access codes with physical layer coding, and in particular how to deal with fading, also remains to be addressed.

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References

1. A. Osseiran, J. Monserrat, and P. Marsch, *5G Mobile and Wireless Communications Technology*. Cambridge University Press, 2016.
2. P. Popovski, "Ultra-Reliable Communication in 5G Wireless Systems," *5G for Ubiquitous Connectivity (5GU)*, pp. 146–151, Nov. 2014.
3. J. Andrews, S. Buzzi, W. Choi, S. Hanly, A. Lozano, A. Soong, and J. Zhang, "What will 5G be?," *IEEE J. Sel. Areas Comm.*, vol. 32, pp. 1065–1082, June 2014.
4. L. G. Roberts, "ALOHA Packet System With and Without Slots and Capture," *SIGCOMM Comput. Comm. Rev.*, vol. 5, no. 2, pp. 28–42, 1975.
5. H. Okada, Y. Igarashi, and Y. Nakanishi, "Analysis and Application of Framed ALOHA Channel in Satellite Packet Switching Networks - FADRA Method," *Electronics and Communications in Japan*, vol. 60, pp. 60–72, Aug. 1977.

6. F. Fazel, M. Fazel, and M. Stojanovic, “Random access compressed sensing for energy-efficient underwater sensor networks,” *IEEE J. Select. Areas in Comm.*, vol. 29, no. 8, pp. 1660–1670, 2011.
7. Y. Jin, Y.-H. Kim, and B. D. Rao, “Limits on support recovery of sparse signals via multiple-access communication techniques,” *IEEE Trans. on Inf. Theory*, vol. 57, no. 12, p. 78777892, 2011.
8. J. Scarlett and V. Cevher, “Limits on support recovery with probabilistic models: An information-theoretic framework,” *IEEE Trans. on Inf. Theory*, vol. 63, p. 593620, Jan. 2017.
9. F. Monsees, M. Woltering, C. Bockelmann, and A. Dekorsy, “Compressive Sensing Multi-User Detection for Multicarrier Systems in Sporadic Machine Type Communication,” in *Proc. IEEE Veh. Tech. Conf.*, pp. 1–5, May 2015.
10. Y. Beyene, C. Boyd, K. Ruttik, C. Bockelmann, O. Tirkkonen, and R. Jäntti, “Compressive Sensing for MTC in New LTE Uplink Multi-User Random Access Channel,” in *Proc. IEEE AFRICON*, pp. 1–5, Sept. 2015.
11. E. Casini, R. D. Gaudenzi, and O. del Rio Herrero, “Contention resolution diversity slotted ALOHA (CRDSA): an enhanced random access scheme for satellite access packet networks,” *IEEE Trans. Wireless Comm.*, vol. 6, pp. 1408–1419, Apr. 2007.
12. G. Liva, “Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted ALOHA,” *IEEE Trans. Comm.*, vol. 59, pp. 477–487, Feb. 2011.
13. G. L. E. Paolini, C. Stefanovic and P. Popovski, “Coded Random Access: Applying Codes on Graphs to Design Random Access Protocols,” *IEEE Comm. Mag.*, vol. 53, pp. 144–150, June 2015.
14. E. Paolini, G. Liva, and A. G. i Amat, “Finite length analysis of irregular repetition slotted ALOHA (IRSA) access protocols,” in *Proc. IEEE Int. Conf. on Comm. Workshop*, pp. 2115 – 2120, June 2015.
15. M. Ivanov, F. Brännström, A. G. i Amat, and P. Popovski, “Error floor analysis of coded slotted ALOHA over packet erasure channels,” *IEEE Comm. Lett.*, vol. 19, p. 419422, Mar. 2015.
16. C. Boyd, R. Vehkalahti, and O. Tirkkonen, “Combinatorial Code Designs for Ultra-Reliable IoT Random Access,” in *Proc. IEEE Int. Symp. on Pers., Ind. and Mobile Radio Comm.*, Oct. 2017.
17. G. T. Peeters, R. Bocklandt, and B. V. Houdt, “Multiple Access Algorithms Without Feedback Using Combinatorial Designs,” *IEEE Trans. Comm.*, vol. 57, pp. 2724–2733, Sept. 2009.
18. W. H. Kautz and R. C. Singleton, “Nonrandom Binary Superimposed Codes,” *IEEE Trans. Inf. Theory*, vol. 10, pp. 363–377, Oct. 1964.
19. J. Massey and P. Mathys, “The Collision Channel Without Feedback,” *IEEE Trans. Inf. Theory*, vol. 31, pp. 192–204, Mar. 1985.
20. A. G. Dyachkov and V. V. Rykov, “A coding model for a multiple access adder channel,” *Prob. Perdachi Inf.*, vol. 17, pp. 26–32, 1981.
21. B. S. Tsybakov and N. B. Likhanov, “Packet switching in a channel without feedback,” *Prob. Perdachi Inf.*, vol. 19, pp. 69–84, Apr. 1983.
22. L. A. Bassalygo and M. S. Pinsker, “Restricted asynchronous multiple access,” *Prob. Perdachi Inf.*, vol. 19, pp. 92–96, 1983.
23. E. Paolini, G. Liva, and A. G. i Amat, “A structured irregular repetition slotted ALOHA scheme with low error floors,” in *Proc. IEEE Int. Conf. on Comm.*, pp. 1–6, May 2017.
24. J. Komlos and A. Greenberg, “An asymptotically fast nonadaptive algorithm for conflict resolution in multiple-access channels,” *IEEE Trans. on Inf. Theory*, vol. 31, pp. 302–306, Mar. 1985.
25. M. Csűrös and M. Ruszinkó, “Single-User Tracing and Disjointly Superimposed Codes,” *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1606–1611, 2005.
26. P. Erdös, P. Frankl, and Z. Furedi, “Families of Finite Sets in which No Set Is Covered by the Union of r Others,” *Israel Journal of Math*, vol. 51, no. 1-2, pp. 75–89, 1985.
27. E. Sperner, “Ein Satz über Untermengen einer endlichen Menge,” *Mathematische Zeitschrift*, vol. 27, pp. 544–548, 1928.
28. S. Basagni, D. Bruschi, and I. Chlamtac, “A Mobility-Transparent Deterministic Broadcast Mechanism for Ad Hoc Networks,” *IEEE Trans. Networking*, vol. 7, no. 6, pp. 799–807, 1999.
29. J. F. Hayes, “An adaptive technique for local distribution,” *IEEE Trans. Comm.*, vol. COM-26, pp. 1178–1186, Aug. 1978.
30. B. S. Tsybakov and V. A. Mikhailov, “Free synchronous packet access in a broadcast channel with feedback,” *Prob. Perdachi Inf.*, vol. 16, pp. 32–59, Oct. 1978.
31. J. Capetanakis, “Generalized TDMA: The multi-accessing tree protocol,” *IEEE Trans. Comm.*, vol. COM-27, pp. 1479–1484, Oct. 1979.

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